Economic Growth: Capital Accumulation and Population Growth

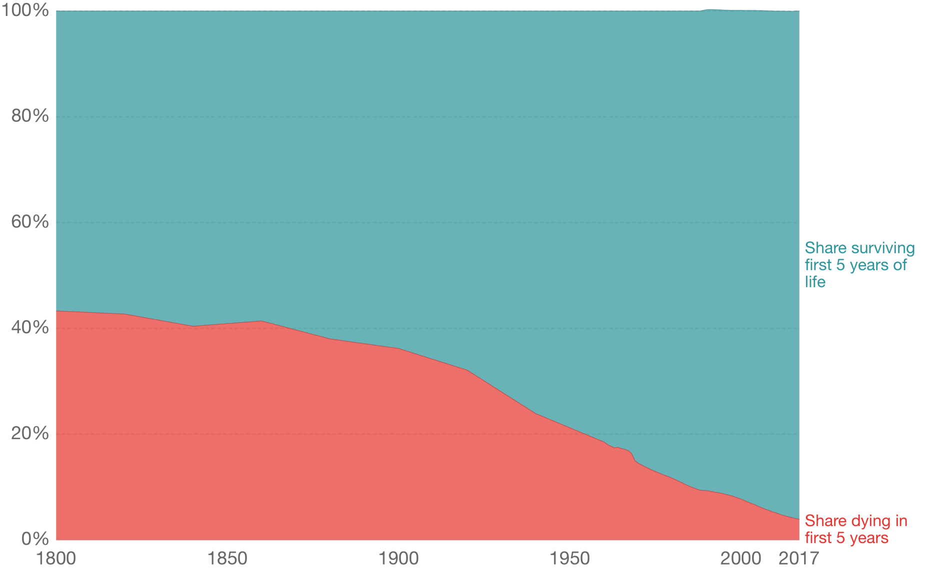
July 26, 2019

The question of growth is nothing new but a new disguise for an age-old issue, one which has always intrigued and pre-occupied economics: the present versus the future.

—James Tobin

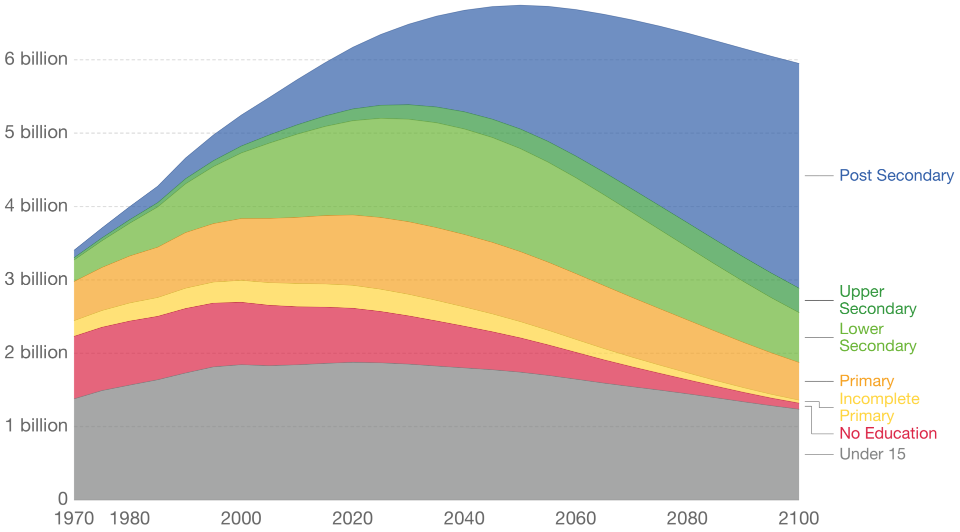
Global child mortality

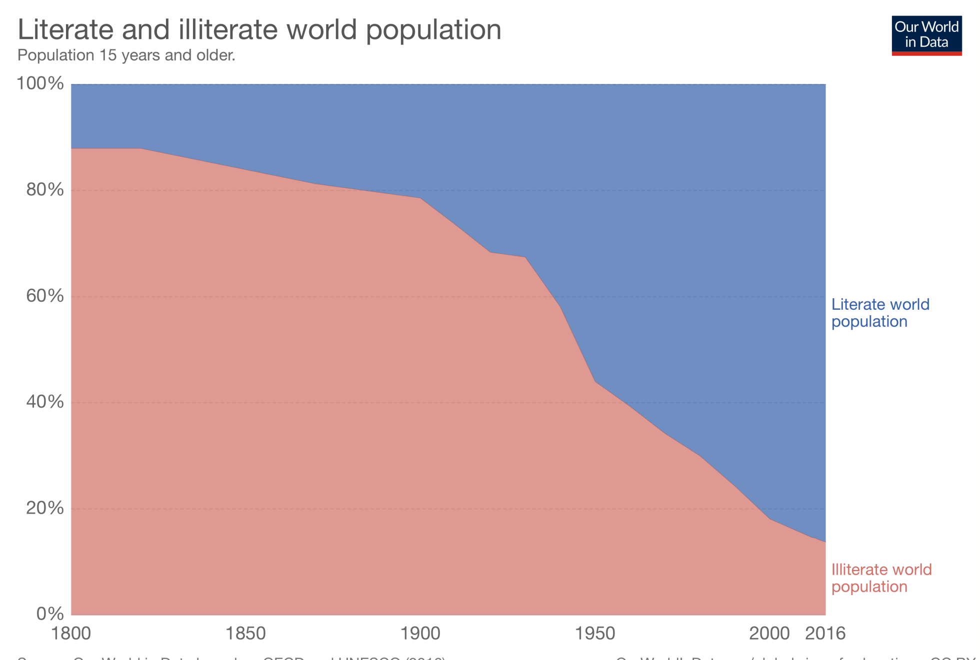
Share of the world population dying and surviving the first 5 years of life.



Projected world population by level of education

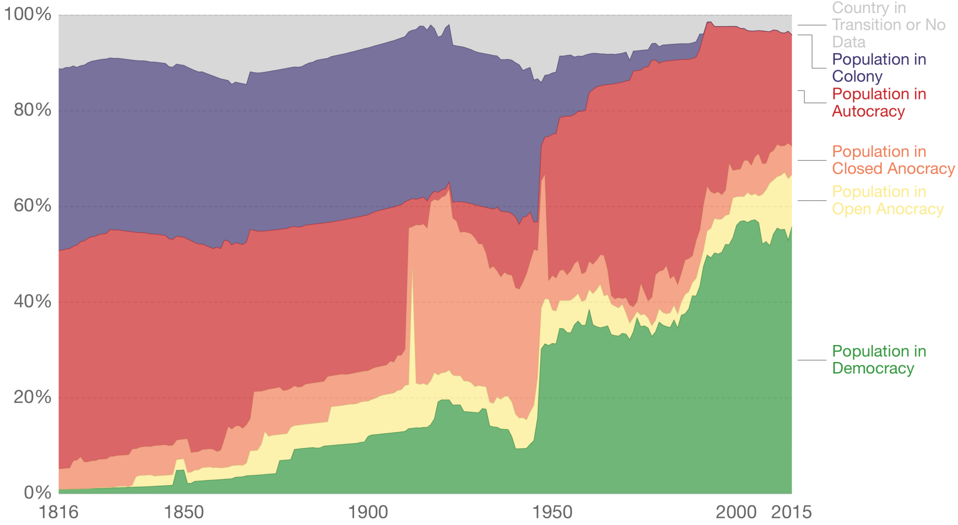
This visualization shows the Medium projection by the International Institute for Applied Systems Analysis (IIASA). The researchers who created this projection describe it as their "middle of the road scenario that can also be seen as the most likely path".




Literate and illiterate world population
Population 15 years and older.
Our World in Data
100%
80%
60%
40%
20%
0%
1800
1850
1900
1950
2000
2016
Literate world population
Illiterate world population

Number of world citizens living under different political regimes

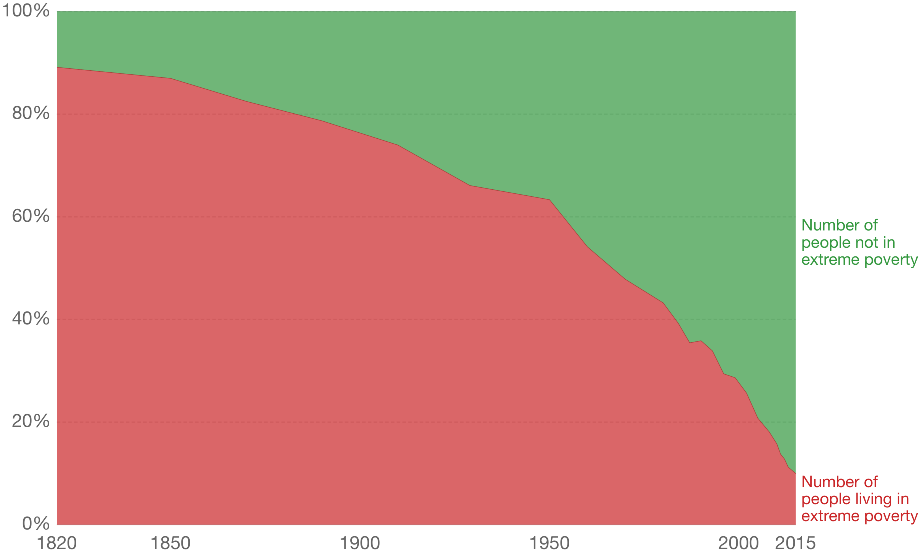
The scale goes from -10 (full autocracy) to 10 (full democracy). Anocracies are those scoring between -5 and 5."Colony" (coded as -20) includes not only colonies, but also countries that were not yet sovereign states (e.g. the CzechRepublic and Slovakia in 1945–92).



Source: World Population by Political Regime they live in (OWID (2016))

World population living in extreme poverty, 1820-2015

Extreme poverty is defined as living on less than 1.90 international-$ per day. International-$ are adjusted for price differences between countries and for price changes over time (inflation).



Country	Income per Person (2012)	Country	Income per Person (2012)
U.S.	$51,749	Phillipines	6,110
Japan	35,618	Nigeria	5,535
Russia	23,589	India	5,138
Mexico	16,426	Vietnam	4,998
Brazil	14,551	Pakistan	4,437
China	10,960	Bangladesh	2,405
Indonesia	9,011	Ethiopia	1,240

Questions We Want To Answer

What causes differences in income over time and across countries?

How Do We Answer the Question?

We develop a model of economic growth, the **Solow growth model** to describe how an economy produces and uses its outputs over time.

The Accumulation of Capital

The Solow growth model shows how growth in the capital stock, growth in the labor force, and advances in technology interact in an economy as well as how they affect output.

1. Examine how supply and demand for goods determine capital accumulation
2. Introduce changes in the labor force
3. Introduce changes in technology later

The Supply of Goods and the Production Function

The production function states that output depends on the capital stock and the labor force:

$$Y = F(K, L)$$

We'll further assume that the production function exhibits CRS, that is

$$zY = F(zK, zL)$$

CRS production functions allow us to analyze all quantities in the economy relative to the size of the labor force. Set z = 1/L

$$Y/L = F(K/L, 1)$$

We can interpret this as the amount of output per worker is a function of the amount of capital per worker. *CRS implies that the size of the economy does not affect the relationship per worker and capital per worker.*

We designate per-worker quantities in lowercase, so $y = Y/L$, and $K/L = k$. Then, the production function is

$$y = f(k)$$

The slope of the production function shows how much extra output a worker produces when given an extra unit of capital. This amount is the marginal product of capital.

$$MPK = f(k+1) - f(k)$$

The Demand for Goods and the Consumption Function

The demand for goods in the Solow model comes from consumption and investment. i.e. output per worker is divided amongst consumption and investment.

$$y = c + i$$

This is just the per-worker version of the NIAI. The Solow model assumes that people save a fraction of their income each year, s, and consume a fraction, $(1 - s)$.

$$c = (1 - s)y$$
$$0 \leq s \leq 1$$

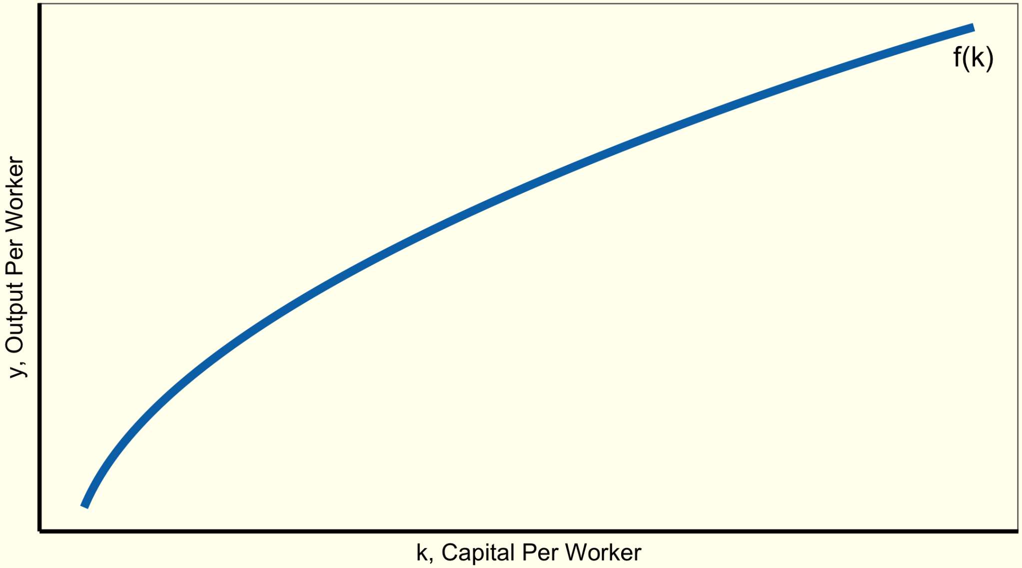
For now we'll take the saving rate as given, but later we will want to answer what is a desirable saving rate.

To see what this consumption function implies, we can sub it into the national income accounts identity

$$y = (1 - s)y + i$$
$$i = sy$$

This is the familiar saving equals investment equation.

The Production Function



Recap

For any given capital stock, k, the production function, $y = f(k)$ determines how much output the economy produces.

The saving rate s determines the allocation of that output between consumption and investment.

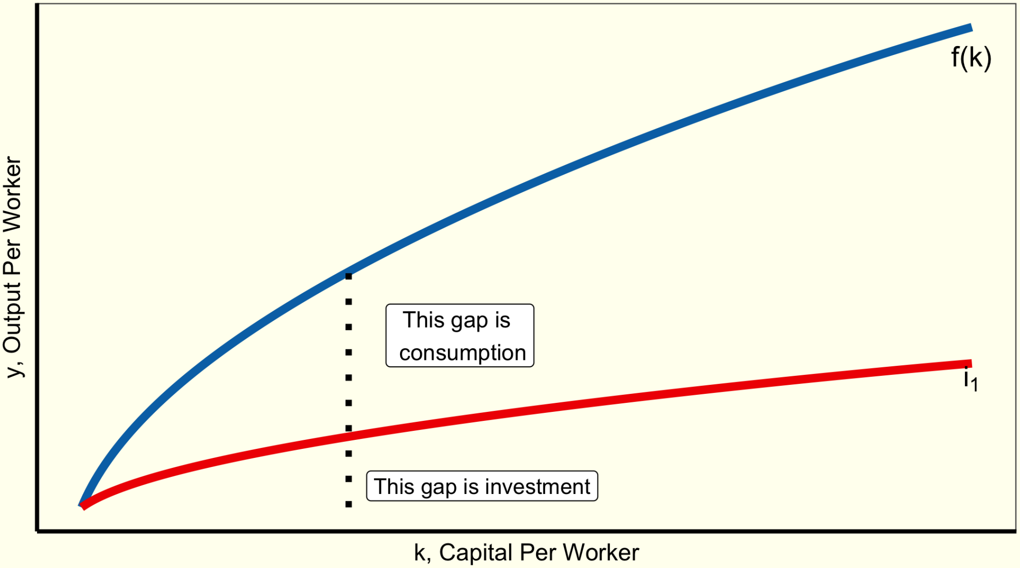
Growth in the Capital Stock and the Steady State

The capital stock is a key determinant in output. Two factors, in particular, influence changes in the capital stock.

- Investment
  - *Investment* is expenditure on new plant and equipment, and it causes capital stock to rise.

- Depreciation
  - *Depreciation* is the wearing out of old capital due to aging and use, and it cause capital stock to fall.

Output, Consumption, and Investment



Investment

We have already noted $i = sy$. We can substitute in the production function to denote investment as a function of the capital stock per worker:

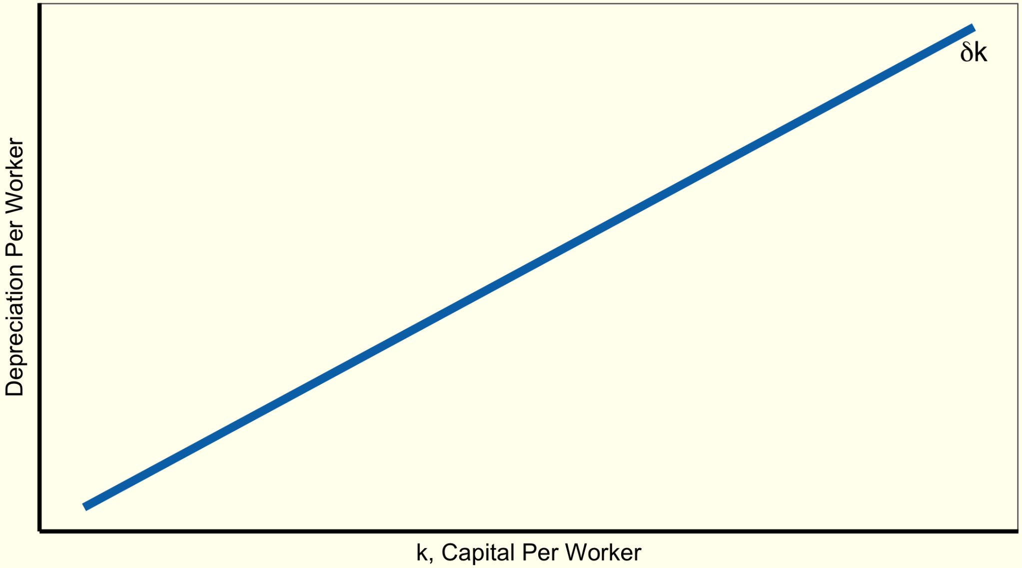
$$i = sf(k)$$

This function relates existing capital to the accumulation of new capital.

Depreciation

We assume that a certain fraction δ of the capital stock wears out each year. δ is called the *depreciation rate*. For example if capital last an average of 25 years, then $\delta = 0.04$ (it loses 1/25 of its productive value each year). The amount of capital that depreciates each year is denoted as δk.

Depreciation



We can express the impact of investment and depreciation on the capital stock with this equation:

$$\text{Change in the Capital Stock} = \text{Investment} - \text{Depreciation}$$
$$\Delta k = i - \delta k$$

We can replace investment with the function of it in terms of capital per-worker,

$$\Delta k = sf(k) - \delta k$$

Steady-State Level of Capital

There is a single level of capital, k^*, at which the amount of investment equals the amount of depreciation.

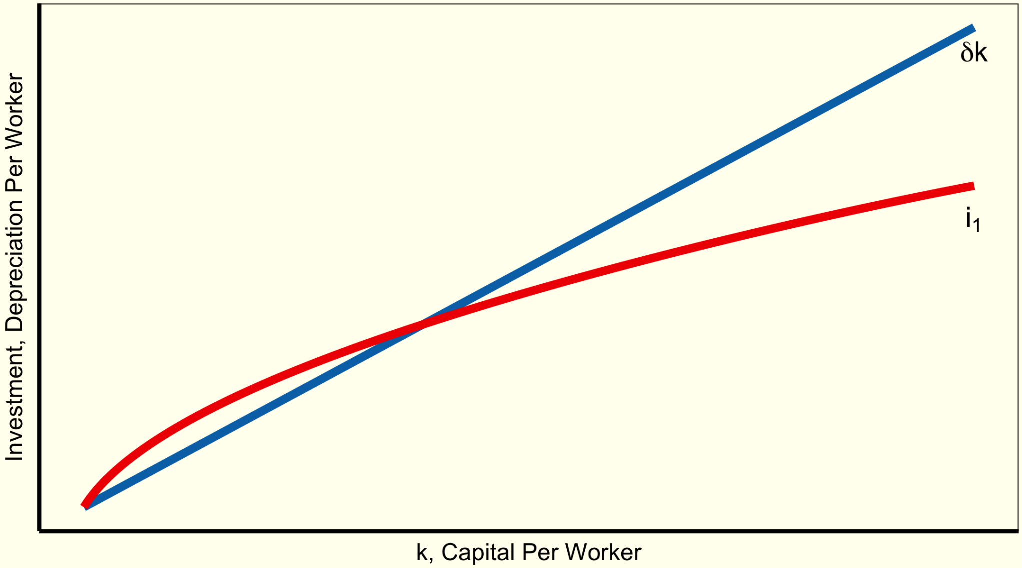
At this level of capital, $\Delta k = 0$, so the capital stock and output per-worker are steady over time.

k^* is called the steady-state level of capital.

Steady-State Level of Capital

- An economy at the steady state will remain there.
- An economy *not* at the stead state will go there.
- *The steady state represents the long-run equilibrium of the economy.*

Investment, Depreciation, and the Steady State



A Numerical Example

Assume the production function is

$$Y = K^{1/2} L^{1/2}$$

To derive the per-worker production function divide both sides by L

$$\frac{Y}{L} = \frac{K^{1/2} L^{1/2}}{L}$$
$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{1/2}$$
$$y = k^{1/2} = \sqrt{k}$$

Output per worker equals the square root of capital per worker.

A Numerical Example

Further assume that 30 percent of output is saved (s = 0.3), and that 10 percent of the capital stock depreciates every year ($\delta = 0.1$), and the economy starts off with 4 units of capital per worker (k=4).

$$y = \sqrt{k} = 2$$
$$i = sy = 0.6$$
$$c = (1 - s)y = 1.4$$
$$\delta k = 0.4$$
$$\Delta k = i - \delta k = 0.2$$

Thus the economy begins with 4.2 units of capital in the following year. If we were to continue with this example, we would observe capital reach its steady state of 9 units.

We could calculate the progress over the economy over many years, but there is a quicker way to determine the steady-state level of capital. Recall,

$$\Delta k = sf(k) - \delta k$$

k^* solves the steady-state equation

$$0 = sf(k^*) - \delta k^*$$

Alternatively,

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}$$

From our example,

$$\frac{k^*}{f(k^*)} = \frac{0.3}{0.1}$$
$$k^* = 9$$

How Saving Affects Growth

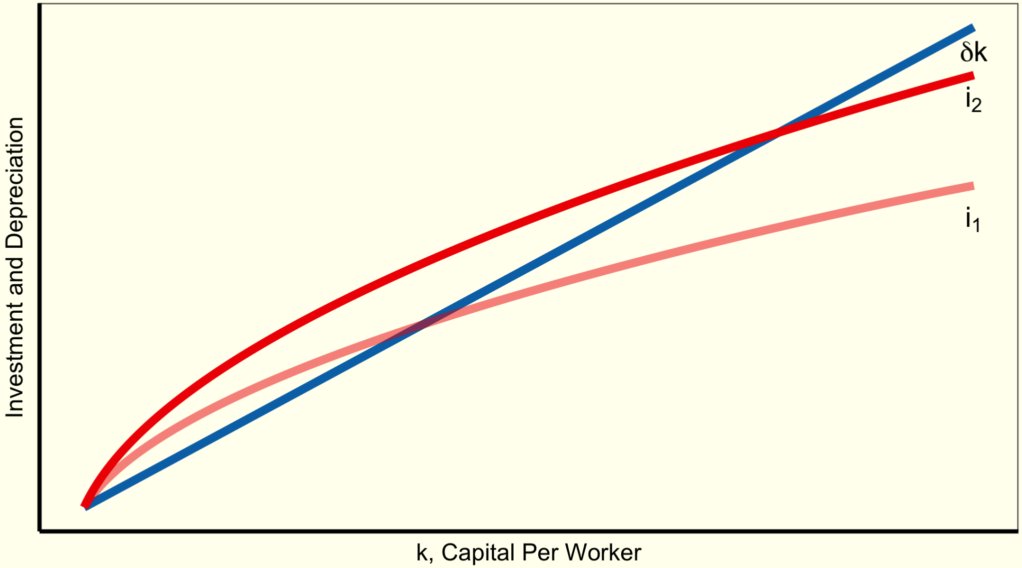
Consider what happens to an economy when its saving rate increases.

- The economy starts at initial saving rate s_1 and capital stock k_1^*.
- When the saving rate increases from s_1 to s_2, the investment curve shifts upward.
- At the initial saving rate the capital stock, k_1^* just offsets the amount of depreciation.
- Immediately after the change in saving, investment rises but there is no change to the capital stock or depreciation.
- The capital stock gradually rises until the economy reaches the new steady state, k_2^*.

If the saving rate is high, the economy wil have a large capital stock and a high level of output in the steady state. If the saving rate is low, the economy will have a small capital stock and a low level of output in the steady state.

Policies that alter the steady-state growth rate of income per person are said to have a *growth effect*. By contrast, a higher saving rate is said to have a *level effect*.

An Increase in the Saving Rate



Why are Some Countries Richer Than Others —A Partial Answer

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Differences in saving

The Golden Rule Level of Capital

Comparing Steady States

Let's assume that a policymaker (central planner) can set the economy's rate at any level. What steady state should the policy maker choose?

- The policymaker's goal is to maximize well-being
- Individuals have no preferences over capital levels. Their interest is in consumption.
- This implies the policymaker chooses the steady state with the highest level of consumption.
- The steady state level of capital that maximizes consumption is called the **Golden Rule of Capital**.

How Can We Tell Whether an Economy is at the Golden Rule Level?

We first need to find steady-state consumption per worker,

$$y = c + i$$
$$c = y - i$$

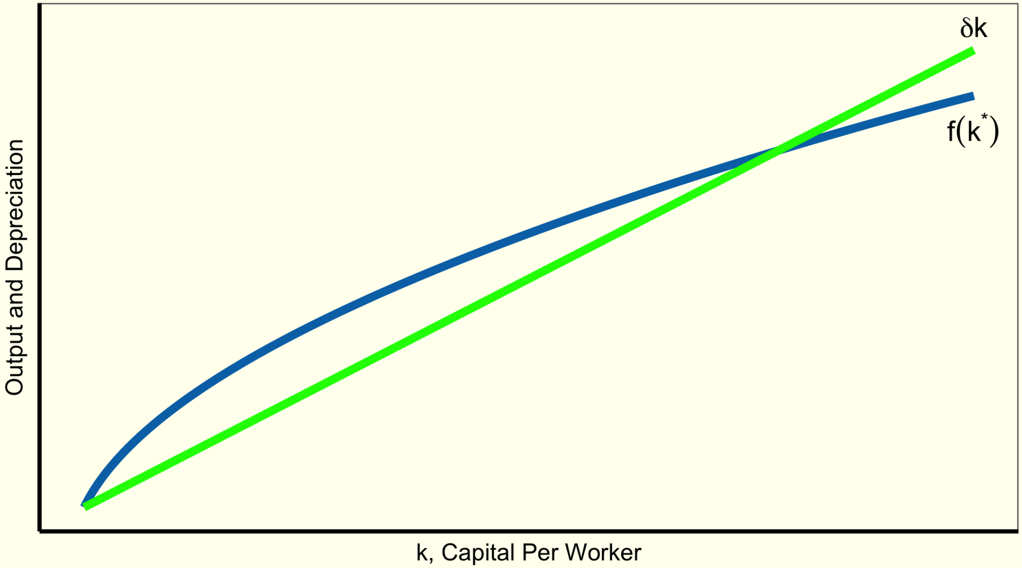
Consumption is output minus investment. We can write the steady-state consumption per worker as

$$c^* = f(k^*) - \delta k^*$$

This equation indicates two things:

- More capital means more output
- With more capital, more output is being used to replace old capital

Steady-State Consumption



Takeaway

The Golden Rule level of capital, k^*_{gold} is the level of capital that maximizes the difference between output and depreciation.

A Condition to Find the Golden Rule Level of Capital

Recall that the slope of the production function is the MPK. The slope of δk^* is δ. These two slopes are equal at k^*_{gold},

$$MPK = \delta$$

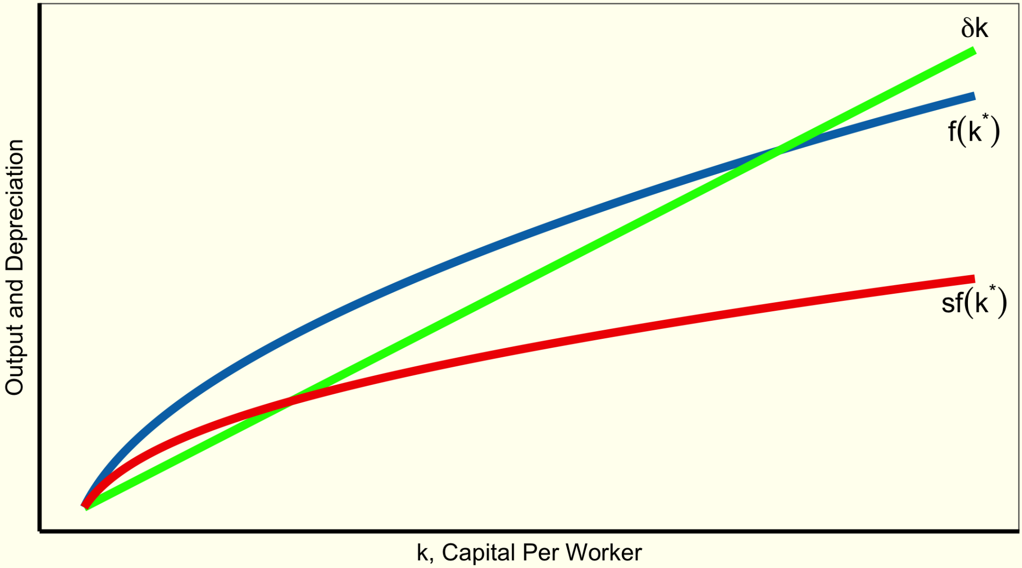
At the Golden Rule level of capital, the marginal product of capital equals the depreciation rate.

Suppose the economy starts at some steady-state capital stock, k^*, and the policymaker is considering increasing the capital stock to $k^* + 1$.

- The amount of additional output is given by $f(k^* + 1) - f(k^*) = MPK$
- If $MPK - \delta > 0$, then an increase in the capital stock increases consumption
- If $MPK - \delta < 0$, then an increase in the capital stock decreases consumption

$$\textbf{MPK} = \delta$$

The Saving Rate and the Golden Rule



A Numerical Example

We begin with the following production function and rate of depreciation.

$$y = \sqrt{k} \tag{1}$$

$$\delta = 0.1 \tag{2}$$

Recall the following equation holds in the steady state

$$\frac{k^*}{f(k^*)} = \frac{s}{\delta}$$

$$\frac{k^*}{f(k^*)} = \frac{s}{0.1}$$

$$k^* = 100s^2$$

These are the combinations of capital stock possible for a given saving rate.

Alternatively, we can use the condition we just derived. The marginal product of capital for the production function is

$$MPK = \frac{1}{2\sqrt{k^*}}$$

We set this equal to the stated level of depreciation

$$\frac{1}{2\sqrt{k^*}} = 0.1$$
$$k^*_{gold} = 25$$
$$\Rightarrow s = 0.5$$

Transition to the Golden Rule Steady State

Starting With Too Much Capital

- In this case, the planner pursues policies aimed at decreasing saving
- Suppose at some point t_0, the policymaker succeeds. What happens to consumption, investment, and output?
-
    - The reduction in saving causes an immediate increase in consumption
    - The reduction causes an immediate decrease in investment
    - Gradually, consumption, output, and investment fall to the new steady-state level

Starting With Too Little Capital

- The planner pursues policies aimed at increasing saving

- Suppose at some point t_0, the policymaker succeeds. What happens to consumption, investment, and output?
  - The increase in saving causes an immediate increase in investment
  - The increase cause and immediate decrease in consumption
  - Gradually, consumption, output, and investment rise to the new steady-state level

Main Takeaway

When the economy begins above the Golden Rule, reaching the Golden Rule produces higher consumption at all points in time. When the economy begins below the Golden Rule, reaching the Golden Rule requires initially reducing consumption to increase consumption in the future.

Population Growth

The Steady State With Population Growth

In our previous model, investment and capital depreciation affected the capital stock. Now, we incorporate population growth into the model

$$\Delta k = i - (\delta + n)k$$

We can think of $(\delta - n)k$ as *break-even investment* —the amount of investment needed to keep the capital stock per worker constant.

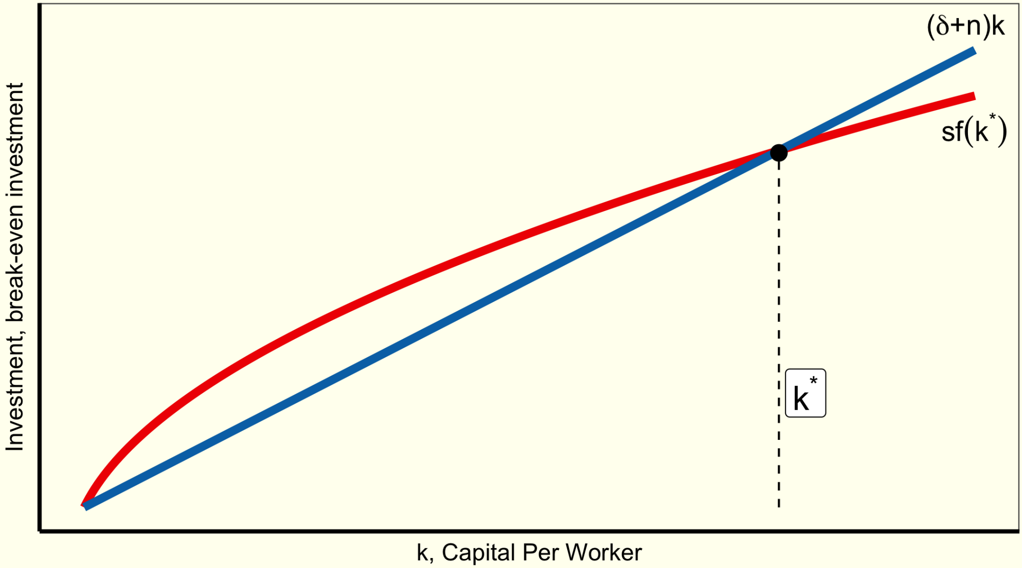
First, let's substitute i for $sf(k)$

$$\Delta k = sf(k) - (\delta + n)k$$

At k^*, $\Delta k = 0$, and $i^* = \delta k^* + nk^*$.

$$0 = sf(k^*) - \delta k^* - nk^*$$

Population Growth in the Solow Model



The Effects of Popultion Growth

Population growth alters the Solow model in three ways:

1. It helps explain a component of sustained growth
   - This doesn't help us explain the increase in standard of living

2. An additional answer as to why some countries are richer than others

3. It affects the criterion of the Golden Rule
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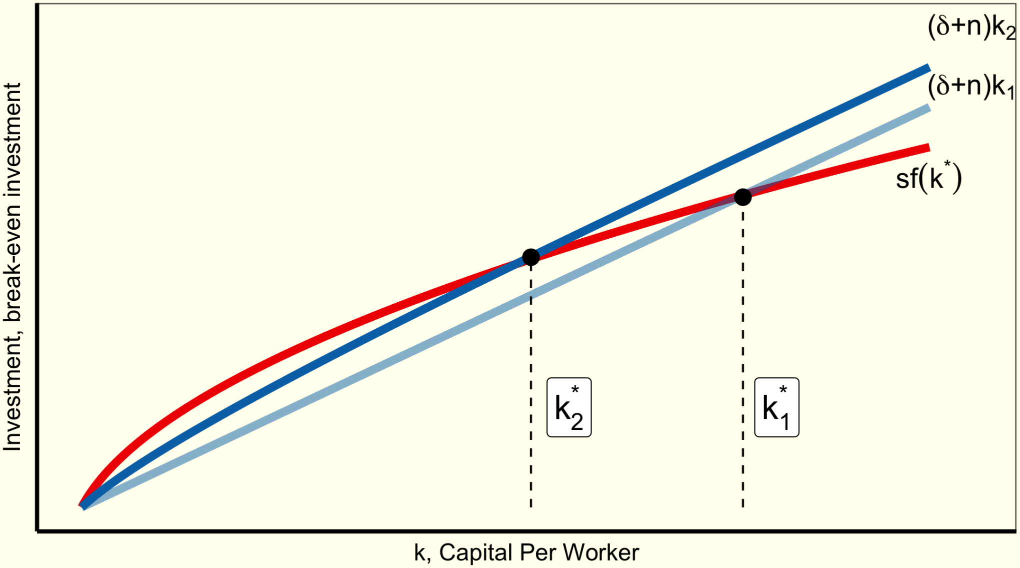
$$c = y - i$$
$$c^* = f(k^*) - (\delta + n)k^*$$
$$MPK = \delta + n$$

Impact of a Change in Population Growth



Alternative Perspectives of Population Growth

Malthusian Model

- Thomas Malthus (1766-1834) argued that an ever-increasing population would continually strain society's ability to provide for itself
- He argued the only check on population growth were "misery and vice." Thus povery alleviation was counter-productive policy
- In general this has proven to be wrong. The population has increased sevenfold since Malthus's time, yet standards of living are much higher. Also, birth control.

Kremerian Model

- Michael Kremer suggested population growth is a key driver of advancing economic prosperity.
- More people implies more potential scientists, inventors, and engineers contributing toward innnovation
- As evidence, he pointed out that nations with the largest populations grew the most after the end of the ice age

Conclusion

- We've started to build the Solow growth model
- We've seen why some countries are richer than others : higher saving, slower population growth
- We have yet to determine how increases in living standards persist. For that we'll turn to technology in the next chapter.