Strategy & Voting

- Last time we learned about simultaneous move games with continuous strategies
- Today we will discuss voting
- The number of voters involved, the number of candidates, tallying procedures all affect the system of voting
- Voting procedures vary widely because certain procedures have attributes that make them better for specific voting situations
- Given that different voting procedures produce different voting outcomes it is important to consider strategic decisions made therein

An award with a moderately decent voting method

Voting Rules & Procedures

Binary Methods

Binary methods require voters to choose between only two alternatives at a time.

- Votes can be aggregated using **majority rule** when there are exactly two candidates
- We use **pairwise voting**, a repetition of binary votes, when dealing with more than two candidates
- Pairwise voting is **multistage**; they entail voting on pairs of alternatives in a series of majority votes

Binary Methods

- The **Condorcet method** is a pairwise procedure in which each alternative is put up against each of the others in a round-robin of majority votes

- The **Copeland index** measures an alternative's win-loss record in a round-robin of contests

- The **amendment procedure** can be used to consider any three alternatives by pairing two in a first-round election & then putting up the third against the winner in a second round vote

The first round of the World Cup uses a Copeland index to determine who advances.

Plurative Methods

Plurative Methods allow voters to consider three or more alternatives simultaneously.

- **Postional methods** apply information on the position of alternatives on a voter's ballot to assign points
- **Plurality rule** is a positional method in which each voter casts a single vote for their most-preferred alternative
- The **antiplurality method** asks voters to vote *against* one of the available alternatives
- **Borda counts** require voters to rank-order all of the possible alternatives in an election. Points are assigned to each alternative on the basis of position

Plurative Methods

- Other positional methods might be minor deviations from the ones described
- **Approval voting** is similar to positional voting, but instead of assigning points to each alternative, voters are asked whether or not they *approve* of each alternative

Mixed Methods

Multistage voting procedures combine plurative and binary voting in **mixed methods**.

The 1991 La. gubernatorial election saw the choices between a man who would later be sent to prison when he was 75 and a former klansman. The crook won.

- The **Majority-runoff** procedure is a two-stage method where in the first stage, voters indicate their most-preferred alternative, and voting moves two a second stage if no candidate obtains an absolute majority

Mixed Methods

- Another procedure consists of voting in successive **rounds**
- One can eliminate the need for rounds through the use of a **single transferable vote**
- This method is alternatively known as **instant-runoff voting** or **rank-choice voting**
- The single transferable vote can be combined with **proportional representation**

Strategic misrepresentation of preferences can also alter election outcomes under any set of rules

Voting Paradoxes

The Condorcet Paradox

The **Condorcet paradox** is one of the most famous and important of the voting paradoxes

- **3** city councillors (Left, Center, Right) vote on **3** alternatives (A, D, G) using the Condorcet method
- They are then asked to establish a council ranking, or **social ranking**
- The "curly" greather than symbol $\succ$ can be read as "is preferred to" i.e. "G $\succ$ A" is "Generous welfare is preferred to Average welfare policy."

LEFT	CENTER	RIGHT
G > A > D	A > D > G	D > G > A

Councillor preferences over welfare policies.

- $G \succ A \succ D \succ G$, the group's preferences are cyclical

G > A A > D G > D
A > D D > G
A > G D > A D > G
G > A

The Condorcet Paradox

- The cycle of preferences is an example of an intransitive ordering
- The concept of **rationality** is usually taken to mean the individual preference orderings are **transitive**
- i.e. A preference ordering is transitive if $A \succ B$ and $B \succ C$ then $A \succ C$
- All the councillors have transitive preferences, but the council does not
- Condorcet paradox: even if all individual preference orderings are transitive, there is no guarantee that the social-preference ordering induced by Condorcet's voting procedure also will be transitive

The Agenda Paradox

In a parliamentary setting with a committee chair who determines the order of voting for a three-alternative election, substantial power over the final outcome lies with the chair.

- Consider the previous figure, but assume one of the councilors sits as chair
- If Left were chair, she can put A vs D in the first round and have the winner confront G in the second round. Other councilors as chair can create different outcomes
- The result that any final ordering can be obtained by choosing an appropriate procedure is known as the agenda paradox
- This outcome only obtains if the councilors engage in sincere voting

The Reversal Paradox

The Borda count can yield the **reversal paradox** when the slate of candidates open to voters change

- Suppose there are 4 candidates for a special commemorative Cy Young to be given to a retired major-league baseball pitcher

- Seaver - $20pts$; Koufax - $19pts$; Carlton - $18pts$; Roberts - $13pts$;

ORDERING 1 (2 voters)	ORDERING 2 (3 voters)	ORDERING 3 (2 voters)
Koufax > Seaver > Roberts > Carlton	Carlton > Koufax > Seaver > Roberts	Seaver > Roberts > Carlton > Koufax

- Ignoring Roberts, Carlton - $15pts$; Koufax - $14pts$; Seaver - $13pts$;

- Preferences haven't changed but the order is reversed

Change The Voting Method, Change The Outcome

GROUP 1 (40 voters)	GROUP 2 (25 voters)	GROUP 3 (35 voters)
A > B > C	B > C > A	C > B > A

- Consider 100 voters who can be broken down into three groups

- **Plurality Rule**- A wins even though 60% of voters rank A lowest
- **Borda count** - B wins with $225pts$. C is second. A is last.
- **Majority or Instant Runoff** - C wins

Evaluating Voting Systems

Given the previous discussion, is there one voting system that satisfies certain regularity conditions and is the most "fair"? Kenneth Arrow's **impossibility theorem** tells us that the answer is no.

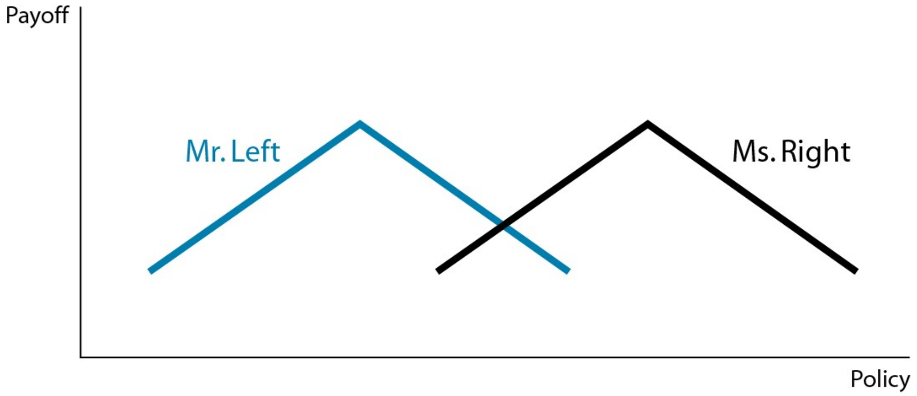
Arrow argued that no preference-aggregation method could satisfy all 6 of the critical principles he identified:

1. The social ranking must be complete
2. It must be transitive
3. It should satisy the Pareto property
4. The rankings cannot be indpendent of preferences
5. There is no dictator
6. It should be independent of irrelevant alternatives

Black's Condition

If we impose **single-peaked preferences** we can satisfy the transitivity property using pairwise voting.

- For a preference ordering to be single-peaked, it must be the case that the alternatives can be considered along a single dimension
- Payoffs uniformly diminish as we move further from the "most-preferred" alternative



Intensity

Mathematician Donald Saari focuses on the difficulty satisfying IIA. He suggests incorporating additional information about voter preferences. Here, we can measure *intensity* as the number of alternatives a voter places between two policies, X and Y.

6′. Society's relative ranking of any two alternatives should be determined only by (1) each voter's relative ranking of the pair and (2) the intensity of this ranking

The Borda count satisfies Arrow's theorem with this amendment, the only positional voting method to do so.

Intensity

The Borda count is also the only procedure that observes ties within collections of ballots. Ties can occur two ways: **Condorcet terms** and through **reversal terms**

- Condorcet terms are the preference orderings, $A \succ B \succ C$, $B \succ C \succ A$, and $C \succ A \succ B$ $\ni$ each ballot should offset each other or constitute a tie

- Reversal terms are preference orderings that contain the reversal in the *location* of a pair of alternatives. In the same election, two elections with preference orderings, $A \succ B \succ C$ and $B \succ A \succ C$ should logically lead to a tie in the pairwise contest of A and B

Strategic Manipulation Of Votes

Plurality Rule

Perhaps the greatest presidential spoiler. Apologies to Ross Perot.

A **spoiler** is a late entry by a 3rd candidate who is considered to have no chance of winning but can alter the final outcome of the election.

Duverger's law states that we tend to see two major parties in plurality systems and several parties in countries with proportional systems. This often arises in plurality systems because of strategic third party voting.

Pairwise Voting

You can use your prediction of the outcome in the second round of voting to determine your strategy in the first round.

(a) A versus G election

Right votes:

A

		CENTER	
		A	G
LEFT	A	A	A
	G	A	G

G

		CENTER	
		A	G
LEFT	A	A	G
	G	G	G

(b) D versus G election

Right votes:

D

		CENTER	
		D	G
LEFT	D	D	D
	G	D	G

G

		CENTER	
		D	G
LEFT	D	D	G
	G	G	G

Pairwise Voting

Councilor Center has a strong incentive to misrepresent their preferences and vote for D

Remember Councilor Left set the agenda policy in order for G to win, but that has backfired.

You should verify, however, that Councilor Left can still enact G by setting D against G in the first round of voting.

Right votes:

A

		CENTER	
		A	D
LEFT	A	G	G
	D	G	D

D

		CENTER	
		A	D
LEFT	A	G	D
	D	D	D

Election outcomes based on the first round of votes.

Scope For Manipulability

The economist William Vickrey did some of the earliest work considering strategic behavior of voters. He noted that procedures satisfying IIA were the most immune to manipulation.

Weakening the scope of IIA, while useful for satsifying Arrow's impossibility theorem, gave greater scope to manipulation. e.g. Saari's intensity ranking.

The **Gibbard-Satterthwaite theorem**, a manipulability result, shows that if there are more than three alternatives to consider, the only procedure that prevents strategic voting is dictatorship

The Median Voter Theorem

- All of the preceding sections have focused on the behavior of voters in multiple alternatives of elections
- We can also apply this to *candidate* behavior in elections
- Given a particular distribution of voters and voter preferences, candidates will determine an optimal policy platform in order to win
- Given some regularity conditions and two candidates, the **median voter theorem** tells us that both candidates will position themselves on the political spectrum in the same place as the **median voter**, the voter at the 50th percentile of the distribution

Discrete Political Spectrum

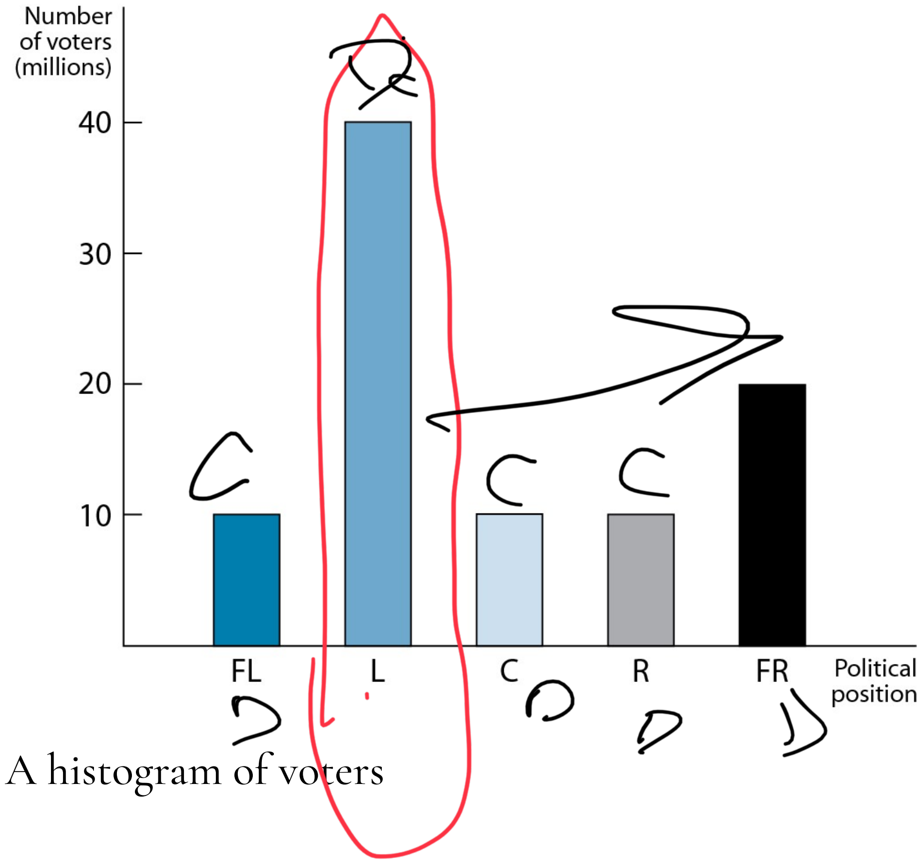
Consider a population of 90 million voters, each of whom has a preferred position on a 5-point political spectrum: Far Left (FL), Left (L), Center (C), Right (R), and Far Right (R).

- These voters are symmetrically spread around the center of the political spectrum displayed using a **histogram**
- Voters will vote for the candidate who publicly identifies theirself as being closer to their own position on the spectrum in an election
- If both candidates are politically equidistant from a group of like-minded voters, each voter flips a coin to decide which candidate to choose

Discrete Political Spectrum

Suppose there is an upcoming election between two former first ladies, Florence and Lucretia.

- We can construct a payoff table for the two candidates showing the number of votes each can expect to receive under all of the different combinations of platform choices



A histogram of voters

Discrete Political Spectrum

		DOLORES				
		FL	L	C	R	FR
CLAUDIA	FL	45, 45	10, 80	30, 60	50, 40	55, 35
	L	80, 10	45, 45	50, 40	55, 35	60, 30
	C	60, 30	40, 50	45, 45	60, 30	65, 25
	R	40, 50	35, 55	30, 60	45, 45	70, 20
	FR	35, 55	30, 60	25, 65	20, 70	45, 45

Discrete Political Spectrum

We found that candidates located at (L,L) in the game table. We can additionally note three important characteristics of the equilibrium in the candidate-location game.

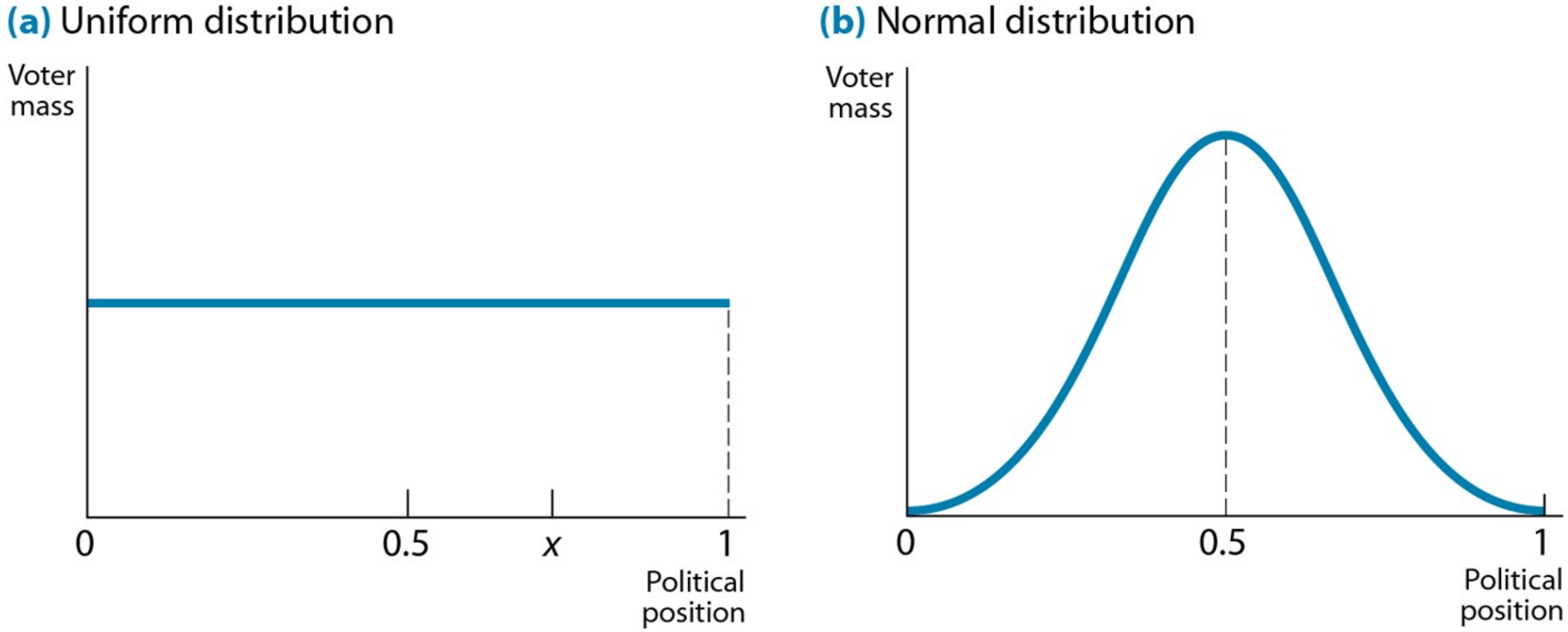
1. Both candidates locate at the same position in equilibrium.This is the **principle of minimum differentiation**, a general result of two-player location games

2. Both candidates locate at the position of the median voter in the population

3. Observe the location of the median voter need not coincide with the geometric center of the spectrum

Continuous Political Spectrum

A **continuous distribution** assumes there are effectively an infinite number of political positions located along the real number line between 0 and 1.



Continuous Political Spectrum

It is not feasible to construct a payoff table for our two candidates in the continuous-spectrum case. We use the techniques developed in Chapter 5 to solve this game.

Suppose Florence chooses platform x on the spectrum. Lucretia can calculate her best response from all the possible positions she may choose. It turns out that this boils down to a few cases.

- Left of x
- Right of x
- Same as x

Continuous Political Spectrum

- Left of x - Lucretia gets all the votes to the left of her position plus one-half of the votes between her position and x
- Right of x - Lucretia gets all the votes to the right of her position plus one-half of the votes between her position and x
- Same as x - 1/2 of all votes

So which position is best? Depends on x relative to the median voter.

Florence reasons similar to Lucretia and both arrive on the location of the median voter.

Summary

- Voting procedures are classified as binary (majority rule, Condorcet method, amendment procedure), plurative (plurality rule, Borda, approval), or mixed methods (majority runoffs, instant runoffs, proportional representation)
- Voting paradoxes show how counterintuitive results can arise owing to difficulties aggregating preferences
- Arrow's impossibility theorem shows that no one system satisfies certain criteria of voting
- Voters may strategically misrepresent their preferences to achieve their most-preferred or to avoid their least preferred-outcome
- Candidates also behave strategically. The median voter theorem shows that in elections with only two candidates, both locate at the position of the median voter