Bargaining

- Couples bargain over about matters of housing, child rearing, and adjustments each other must make for the other's career. Buyers and sellers bargain over price, workers and bosses over wages. In short, we engage in bargaining almost constantly.

- Bargaining situations have two things in common
  - the total payoff that the parties are able to agree to must be greater than the sum of the payoffs they could achieve separately
  - it is not a zero-sum game

- Game theory approaches bargaining along two distinct lines
  - In *cooperative* game theory parties jointly implement a solution
  - In *noncooperative* game theory parties choose strategies separately and search for an equilibrium

- We will exploit sequential gameplay where parties make offers and counteroffers that will lead to an equilibrium

Nash's Cooperative Solution - An Example

Consider two entrepreneurs, Andy and Bill. Andy produces microchips that sell to manufacturers for $900, and Bill has a software package that he can retail for $100.

They realize they can extract an additional $2000 surplus by packaging their products together. They expect to sell millions of these. The only sticking point is how to divide the surplus.

Perhaps "splitting the difference" seems natural, but of what difference? Of the surplus? In relation to their initial contribution?

Nash's Cooperative Solution

Suppose they hire an arbitrator, and the arbitrator decides that the division of profit should be 4:1 in favor of Andy. What is the division of revenue under this scheme?

Suppose Andy gets a total of x and Bill a total of y. Andy's profit is $(x - 900)$ and Bill's is $(y - 100)$. The arbitrator implies that Andy's profit should be $4\times$Bill's profit, so $x - 900 = 4(y - 100) \Rightarrow x = 4y + 500$.

The total revenue available to both is $3,000$, so it must also be true $x + y = 3000$ or $x = 3000 - y$. Then $x = 4y + 500 = 3000 - y$ or $y = 500$, and thus $x = 2,500$. This makes Andy's profit $2,500 - 900 = 1,600$ and Bill's profit $500 - 100 - 400$, the $4 : 1$ split the arbitrator wants.

General Theory

Suppose two bargainers, A and B, seek to split a total value of ν which they can achieve iff they agree on a specific division.

- No agreement reached $\rightarrow$ A will get a and B will get b; these are *backstop payoffs*, or **BATNAs** (best alternative to a negotiated agreement)
- a and b are often zero, but more formally $a + b < \nu$, so that there is a positive **surplus** from agreement

Each player is to be given their BATNA plus a share of the surplus, fraction h for A and fraction k for B $\text{s.t. } h + k = 1$

General Theory

Writing x as the amount A finally ends up with and y for B we have

$$\begin{aligned} x &= a + h(\nu - a - b) = a(1-h) + h(\nu - b) \\ x - a &= h(\nu - a - b) \\ &\text{and} \\ y &= b + k(\nu - a - b) \ = b(1-k) + k(\nu - a) \\ y - b &= k(\nu - a - b) \end{aligned}$$

These expressions are the **Nash formulas**.

General Theory

Another way to write this is the surplus gets divided in proportion $h : k$, or

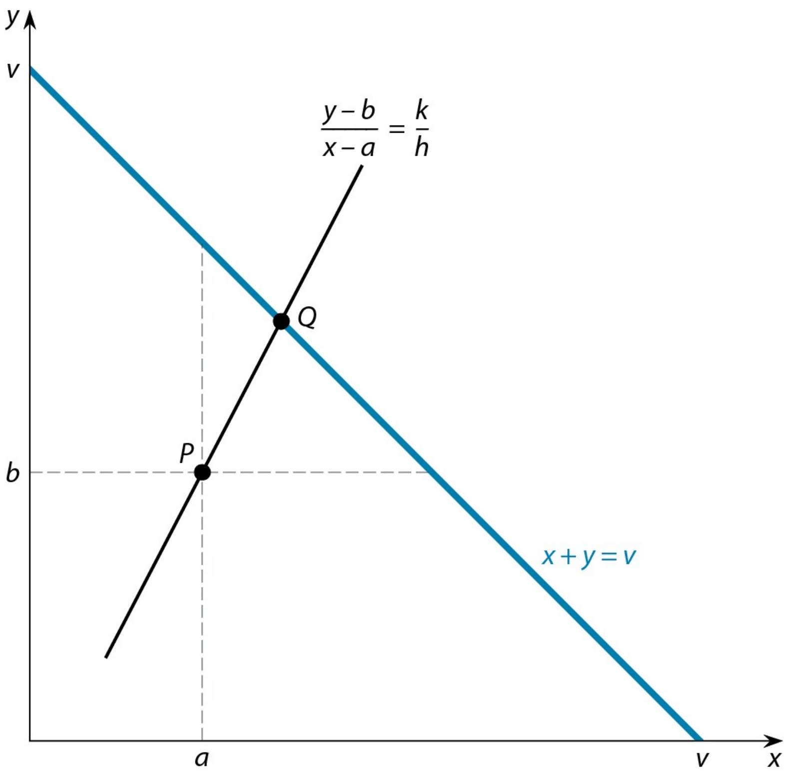
$$\frac{y-b}{x-a} = \frac{k}{h}$$

or, in slope-intercept form

$$y = b + \frac{k}{h}(x-a) = \left(b - \frac{ak}{h}\right) + \frac{k}{h}x$$

To use up the entire surplus x and y must also satisy $x + y = \nu$. The Nash formulas are actually the solution to these last two simultaneous equations.

General Theory



The Nash bargaining solution in the simplest case

- Nash's cooperative solution is shown at point Q, the payoffs after agreement
- The BATNA is shown at P, (a, b)
- The Nash formula says nothing about how or why such a solution might come about
- We may consider this a shorthand description of the outcome given two parties' bargaining strengths h and k

General Theory

We've departed from our usual consideration to examine bargaining from the cooperative angle. Cooperative bargaining generally follows three principles:

1. The outcome should be invariant if the scale in which the payoffs are measured changes linearly
2. The outcome should be **efficient**; no mutual gain should go unexploited
3. The outcome should be unchanged if irrelevant alternatives are not considered

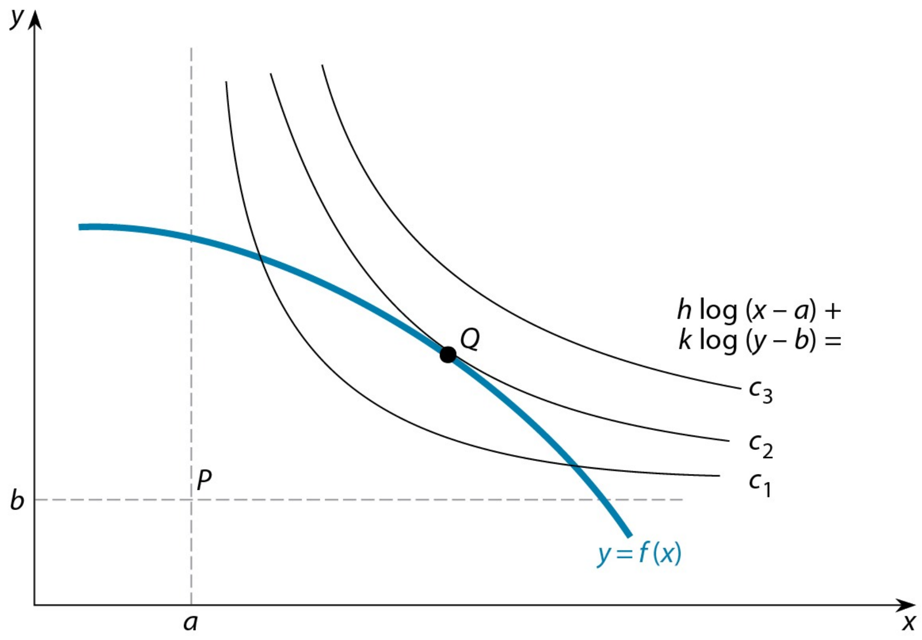
General Theory

- $y = f(x)$ is the **efficient frontier** of the bargaining problem

- Nash proved that the cooperative outcome that satisfied all three properties is characterized by

$$\max (x-a)^h (y-b)^k$$
$$\text{s.t. } y = f(x)$$

- x & y are the outcomes, a & b are the backstops, and h & k two positive numbers summing to 1



The general form of the Nash bargaining solution

Variable-Threat Bargaining

Let's embed the Nash cooperative solution into the second stage of a sequential-play game.

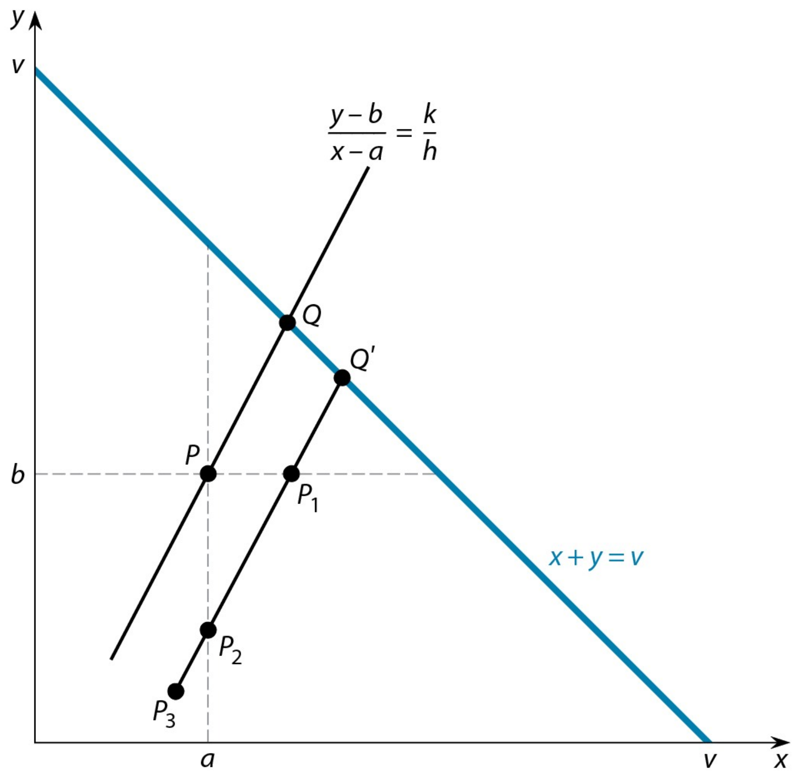
Previously, we had assumed the backstops were fixed, but let's now suppose the players can manipulate them to some extent in the first stage.

After the BATNAs have been manipulated in the first round, the cooperative solution is implemented in the second round. This type of game is called **variable-threat bargaining**.

The question we want to answer is what kind of manipulation is in the player's best interest?

Variable-Threat Bargaining

- The original backstops are at P & the Nash solution at Q
- If player A is able to increase his BATNA, thus, moving to P_1 then the Nash solution leads to Q'
- So, improving your own BATNA improves your own outcome
- Suppose instead, player A makes a strategic move to lower player B's BATNA, the point P_2
- What about lowering both to P_3?



Bargaining game of manipulating BATNAs

Alternating-Offers Model I: Total Value Decays

Let's return to the noncooperative-game theory side. Bargaining is portrayed as a game of **alternating offers**.

- One player A makes an offer
- B can then either accept the offer or make a counteroffer

This is a sequential game, so we can use rollback and look for an equilibrium.

But where is the end point? Why would the game end at all?

Alternating-Offers Model I: Total Value Decays

We'll first consider that the total surplus **decays** with each offer. Suppose two players A and B engage in sequential bargaining.

- A makes the first offer to split the surplus v
- If B refuses the offer, the total falls by x_1 to $(v - x_1)$
- If A refuses B's, the total falls by x_2 to $(v - x_1 - x_2)$
- … After 10 rounds the surplus dwindles to nothing $(v - x_1 - x_2 - \ldots - x_{10}) = 0$

If the game has reached round 10, B can make an offer to keep most of the remaining surplus and A gets very little. A has the choice to accept or keep nothing. A should accept. For simplicity we'll say the split is x_{10} to B and 0 to A.

Alternating-Offers Model I: Total Value Decays

- Knowing what happens in round 10, A will need to offer at least x_{10} of the remaining surplus $(x_{10} + x_9)$, so the offer is x_9 to A and x_{10} to B
- Moving to round 8, B will offer x_9 to A and retain $(x_8 + x_{10})$
- We can do rollback and we find that A will offer a split in the first round of $(x_1 + x_3 + x_5 + x_7 + x_9)$ for herself and $(x_2 + x_4 + x_6 + x_8 + x_{10})$ to B. This offer will be accepted
- We can remember the formulas by *hypothesizing* a sequence in which all offers are refused. Then add up the amounts that would be destroyed by the refusal of one player. This total is what the *other* player gets in equilibrium

Alternating-Offers Model I: Total Value Decays

- e.g. when B refused the first offer, the surplus dropped x_1, so x_1 is part of what goes to A in the equilibrium of the game

- We need to slightly amend this procedure when the BATNAs are non-zero. In the final round, B must offer A at least a. If $x_{10} > a$ then B is left with $x_{10} - a$; if not, the game ends before this round is reached

- A must offer B the larger of b or $(x_{10} - a)$ in round $9 \ldots$

- *Graudal decay* results in a fairer split than *sudden decay*

Experimental Evidence

The simplest bargaining experiment is the **ultimatum game** in which there is only one round. A makes an offer, and B can accept the split or reject and both parties receive nothing.

Rollback predicts that A will offer B next to nothing, and that B should accept; however, the most common offer is $50 : 50$. Proposers rarely offer splits worse than $75 : 25$, and when those offers are made, they are summarily rejected.

The simplest explanation is that there are more to the payoffs the money received. An additional consideration is that anonymity isn't guaranteed in these small experimental contexts.

We typically say that it is due to "fear of rejection" or an "ingrained sense of fairness."

Experimental Evidence

We can remove "fear of rejection" as a reason using the "dictator game". Here A determines a split and B receives what A decides.

The splits move away from equity but still rarely reach $70 : 30$, so this suggests a role for an "ingrained sense of fairness" within some limits.

When the dictator game was played with proposers who believed they "earned" the role, the splits were often all and nothing, but there were still $50 : 50$ proposals about 5% of the time.

Moderation and fairness often pay off in the experimental setting.

Alternating-Offers Model II: Impatience

Now, let's suppose players have a "time value of money," and therefore prefer early agreement to later agreement. For concreteness suppose that $0.95 today is as good as having $1 tomorrow (implies $\delta = 0.95$).

Suppose we consider two players with zero BATNAs. A starts by making an offer to split $1. If B rejects, then she will have the opportunity to propose a split of the $1. We label x the amount that goes to the person currently making the offer.

We can use rollback analysis to solve for x.

Alternating-Offers Model II: Impatience

A starts the alternating offer process. He knows that B can get x in the next round when it's B's turn $\therefore$ A will offer B $0.95x$ now, and be left with $1 - 0.95x$, but $1 - 0.95$ actually equals x now, so

$$x = 1 - 0.95x \Rightarrow x = 0.512.$$

- Note that this process can go on for some time, but is immediately determined in the first offer

- The player who makes the first offer gets more than half the split

Alternating-Offers Model II: Impatience

Now suppose the two players are not equally patient.

- Player B still regards \$1 today as \$0.95 tomorrow
- Player A now regards \$1 today as \$0.90 tomorrow

Write x as the amount A gets in equilibrium and y as what B gets.

- When A makes the offer he knows that B must receive at least $0.95y$, so $x = 1 - 0.95y$
- When B makes the offer he knows that A must receive at least $0.9x$, so $y = 1 - 0.9x$

We can take these equations and simultaneously solve for x and y

Alternating-Offers Model II: Impatience

For player A,

$$x = 1 - 0.95(1 - 0.9x)$$

$$[1 - 0.95(0.9)]x = 1 - 0.95$$

$$0.145x = 0.05$$

$$x = 0.345$$

And for player B,

$$y = 1 - 0.9(1 - 0.95y)$$

$$[1 - 0.9(0.95)]y = 1 - 0.9$$

$$0.145y = 0.10$$

$$y = 0.690$$

Note, that the payoffs don't add to one because these are the values when each player makes their *first* offer, so B actually gets 0.655 when A makes the first offer.

The more impatient player gets a lot less than the patient player even when he makes the first offer.

A General Model Of Impatience

Let $a = \frac{1}{1+r_a}$ and $b = \frac{1}{1+r_b}$ each players subjective discount rate. If player required rate of return r_a is high, then player A is impatient. Similarly, player B is impatient if r_b is high.

- When A makes the first offer he knows that he must offer B at least by, so $x = 1 - by$
- When B makes the first offer she knows that she must offer A at least ax, so $y = 1 - ax$

For x, we have $x = 1 - b(1 - ax)$ or $(1 - ab)x = 1 - b$. Expressed in terms of the required rates of return we have

$$x = \frac{1-b}{1-ab} = \frac{r_b + r_a r_b}{r_a + r_b + r_a r_b}.$$

A General Model Of Impatience

Similarly for y, we have $y = 1 - a(1 - by)$, or $(1 - ab)y = 1 - a$. Thus, we have

$$y = \frac{1-a}{1-ab} = \frac{r_a + r_a r_b}{r_a + r_b + r_a r_b}.$$

Once again take note that x and y sum to more than 1:

$$x + y = \frac{r_a + r_b + 2r_a r_b}{r_a + r_s + r_a r_b} > 1$$

A General Model Of Impatience

In general r_a and r_b are small and $r_a \times r_b$ is often close to zero, so we ignore it. Then we have an approximate solution for the split that does **not** depend on which player makes the first offer:

$$x = \frac{r_b}{r_a + r_b} \quad \text{and} \quad y = \frac{r_a}{r_a + r_b}$$

Now, $x + y$ is approximately equal to 1, and most importantly the approximate solution are the shares of the surplus that go to the two players

$$\frac{y}{x} = \frac{r_a}{r_b},$$

That is, their shares are inversely proportional to their rates of impatience.

Manipulating Information In Bargaining

We considered games where each players BATNAs and impatience were common knowledge; however, this may not be true. These amount to games of asymmetic information where each party uses screening and signaling to determine each player's BATNA or their level of impatience.

Multi-Issue Bargaining

When two or more issues are on the bargaining table and the two parties are willing to trade more of one against less of the other at different rates, then a mutually beneficial deal exists.

Multiparty Bargaining

Having many parties engaged in bargaining may be helpful in finding deals because parties need not engage in pairwise agreements; though, enforcement may be difficult.

Summary

- Bargaining attempts to divide the surplus that is available if agreement can be reached
- Bargaining can be analyzed as a cooperative game or an uncooperative game
- In noncooperative settings of alternating offer and counteroffer, we use rollback to find an equilibrium
    - If the surplus value decays, the amount that is lost owing to refusal will be owed to the other player in equilibrium
    - If delay in agreement is costly because of impatience, the equilibrium shares are in inverse proportion to the players rates of impatience