Simultaneous-Move Games: Discrete Strategies

Econ 2160 - Chapter 4

- In the previous chapter we discussed pure sequential-move games where players take turns moving
- In this chapter we will discuss simultaneous-move games where the players move at the same time
- Games where moves are chosen in isolation are also simultaneous-move games, more formally games of imperfect information
- Production decisions, elections, & penalty kicks are all examples of simultaneous move games
- We'll develop a simple concept of equilibrium that has considerable explanatory and predictive power

Depicting Simultaneous-Move Games With Discrete Strategies

- In a purely simultaneous game, strategies are equivalent to actions
- Strategies, however, can be a probabilistic choice from the basic actions specified
- Probabilistic choices are called **mixed strategies**, but we'll focus on **pure strategies**
- Simultaneous-move games are often depicted using a **payoff table**
- Payoff tables are called the **normal** or **strategic form** of the game

			Column	
		Left	Middle	Right
	Top	3, 1	2, 3	10, 2
Row	High	4, 5	3, 0	6, 4
	Low	2, 2	5, 4	12, 3
	Bottom	5, 6	4, 5	9, 7

Example Of A Normal-Form Game - What's The Call?

			Defense	
		Run	Pass	Blitz
	Run	2, -2	5, -5	13, -13
Offense	Short Pass	6, -6	5.6, -5.6	10.5, -10.5
	Medium Pass	6, -6	4.5, -4.5	1, -1
	Long Pass	10, -10	3, -3	-2, 2

NE : offense → SP
defense → Pass defense

Nash Equilibrium

- Each player wants to choose an action that yields them the highest payoff
- To do this we consider the action of their opponent
- The optimal choice to your opponent's action is called the **best response**
- A game is in equilibrium when an outcome cell is a best response for both players
- This is a **Nash equilibrium** --- a list of strategies, one for each player, such that no player can get a better payoff by switching (deviating) to another strategy while other players adhere to the strategies specified

Some Further Explanation of the Concept of Nash Equilibrium

		Column		
		Left	Middle	Right
	Top	3, 1	2, 3	10, 2
Row	High	4, 5	3, 0	6, 4
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	Bottom	5, 6	5, 5	9, 7

The same game as before but with one payoff altered

- The concept of Nash equilibrium (NE) does not require choices to be strictly better than other available choices
- A NE does not have to be jointly best for the players either
- "Bottom, Right" perhaps only possible with cooperation/cheating

R; L
C; M $\Rightarrow$ 5, 4

R; B
C; R $\Rightarrow$ 9, 7

Nash Equilibrium As A System Of Beliefs And Choices

- I've been saying players choose their "best response" to another's actions but the game is simultaneous. How does one respond to something that hasn't happened?

- *Experience* and *Observation*; *Logic*

- The notion of acquiring insight into what other players may choose has been commonly called **belief**

- A NE can then be defined as a set of strategies, one for each player, such that (1) each player has correct beliefs about the strategies of the others and (2) the strategy of each is the best for themself given their beliefs about the strategies of the others

Dominance

- Some games have a property where one strategy is uniformly better or worse than another
- The **prisoners' dilemma** on the right illustrates this
- *Confess* is a **dominant strategy**
- Dominance, when it exists, provides a compelling basis for the theory of solutions to simultaneous move games

		Wife	
		Confess(Defect)	Deny(Cooperate)
Husband	Confess(Defect)	10 yr 10 yr	1yr, 25 yr
	Deny(cooperate)	25 yr, 1 yr	3yr, 3 yr

Prisoners' Dilemma

Both Players Have Dominant Strategies

- In the preceding game, *confess* dominates *deny* for both players
- The optimal strategy is independent of whether beliefs about the other are correct
- Any game with the same general pattern as this one is called a "prisoners' dilemma" which has three components:

  1. Each player has two strategies: cooperate or defect
  2. Each player has a dominant strategy
  3. The dominance solution is worse for both players than playing a dominated strategy

One Player Has A Dominant Strategy

- When only one player has a dominant strategy, the other player can be assured they will play it
- Congress is under pressure to lower taxes & increase spending
- Here the payoffs are rank-ordered, least (1) to most (4) preferred
- Why can't we reach *Budget balance; Low interest rates*

		Federal Reserve	
		Low Interest Rates	High Interest Rates
Congress	~~Budget Balance~~	~~3, 4~~	~~1, 3~~
	Budget Deficit	4, 1	2, 2

Game of fiscal and monetary policy

iterated deletion (elimination of strictly dominated strategies

Successive Elimination Of Dominated Strategies

- Removing dominated strategies reduces the size of the game
- The "new" game may have other dominated strategies
- **Iterated elimination of dominant strategies (IESDS)** is the process by which this is done
- If IESDS yields a unique outcome, the game is said to be **dominance solvable**

			Column	
		~~Left~~	Middle	~~Right~~
	~~Top~~	3, 1	2, 3	10, 2
Row	~~High~~	4, 5	3, 6	6, 4
	Low	2, 2	5, 4	12, 3
	~~Bottom~~	5, 6	4, 5	9, 7

IESDS yields (Low, Middle)

- Other games may not be dominance solvable or yield a unique outcome
- **Warning:** IESDS may get rid of NE
- If using IESDS, its good to check that you're not eliminating any other NE using other concepts

		Colin	
		Left	Right
Rowena	Up	0, 0	1, 1
	Down	1, 1	1, 1

Elimination of weakly dominated strategies

Best-Response Analysis

- Many games don't have dominant or dominated strategies
- We need to develop a method for finding NE in situations where IESDS fails
- "For each choice that the other player(s) make, what is the best choice for this player?"
- **Best response analysis** is a comprehensive way of locating all possible NE
- If the game has no equilibrium in best-response analysis, it has no equilibria in pure strategies

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Three Players

- All of the strategies used so far can be extended to solve simultaneous move games with multiple players
- e.g Let's return to the street-garden game in Chapter 3, but this time make it simultaneous move
- Let's also say the size and appeal of the garden depends on each contribution
- This implies there are six possible outcomes: "contribute" or "not" when
    - Neither contributes
    - Both contribute
    - Exactly one contributes

Talia Chooses

Contribute			
		Nina	
		Contribute	Don't
Emily	Contribute	5, 5, 5	3, 6, 3
	Don't	6, 3, 3	4, 4, 1

Don't			
		Nina	
		Contribute	Don't
Emily	Contribute	3, 3, 6	1, 4, 4
	Don't	4, 1, 4	2, 2, 2

Remember, we're holding other players' strategies fixed

Multiple Equilibria In Pure Strategies

- All the games we've previously analyzed has had one pure strategy equilibrium
- Not all games exhibit unique NE properties
- **Coordination games** are a class of games that do not have unique pure strategy equilibria
- Actors in coordination games have common (though, not necessarily perfectly aligned) interests
- Because they act independently, they don't always achieve their jointly preferred outcome

Jerry always gets hummingbird duty.

They didn't know their boo.

Will Harry Meet Sally? Pure Coordination

Suppose two students, Harry & Sally, decide to meet each other at a coffee shop, but first they have to go to class. While in class they both realize they forgot to mention which coffee shop.

They would both love to meet at the same coffee shop. Of which to meet is unimportant, so this is a game of **pure coordination**.

		Sally	
		Starbucks	Local Latte
Harry	Starbucks	1, 1	0, 0
	Local Latte	0, 0	1, 1

A game of pure coordination

Will Harry Meet Sally? And Where? Assurance

Behavioral differences between classes may alter the payoffs. e.g. Let's suppose Harry & Sally are seniors & prefer Local Latte over Starbucks independently.

Confirm that we still have two NE. There still is no guarantee that they will meet at Local Latte unless there is a **convergence of expectations**.

		Sally	
		Starbucks	Local Latte
Harry	Starbucks	1, 1	0, 0
	Local Latte	0, 0	2, 2

Players get the preferred outcome only if there are **assurances** of the other's actions.

Will Harry Meet Sally? And Where? Battle Of The Sexes

		Sally	
		Starbucks	Local Latte
Harry	Starbucks	2, 1	0, 0
	Local Latte	0, 0	1, 2

Players get the preferred outcome only if there are **assurances** of the other's actions.

In another scenario, Harry and Sally have differing preferences over coffee shops. This particular payoff matrix is known as the **battle of the sexes**.

Confirm that we still have two NE. To achieve either outcome, the players will have to form a focal point as in the previous games.

The risk of coordination failure is greater here. How can players break the asymmetry?

Will James Meet Dean? Chicken

		Dean	
		Swerve (Chicken)	Straight (Tough)
James	Swerve (Chicken)	0, 0	-1, 1
	Straight (Tough)	1, -1	-2, -2

Players get the preferred outcome only if there are **assurances** of the other's actions.

In this final example, players would like to avoid agreeing actions. In addition, the consequences of coordination failures are asymmetric between cells.

The payoffs for **chicken** depend on how negatively one rates the "bad" outcome against being labeled a chicken.

- Each player has a "tough" and "weak" strategy
- There are two pure strategy NE
- Each player strictly prefers the outcome where his opponent chooses "weak"
- The NE "tough" is ... not good

No Equilibrium In Pure Strategies

- All the games so far have had at least one pure strategy NE
- Not all games have pure strategy NE
- On the right is an example from tennis. Evert is about to attempt a passing shot. She can
  - Hit it down the line (hard)
  - Send it across the court (soft)
  - Navratilova simultaneously chooses to defend against one of these shots
- Confirm using IESDS that the game is not dominance-solvable. Then try best response analysis.

		Navratilova	
		Down-the-Line	Crosscourt
Evert	Down-the-Line	50, 50	80, 20
	Crosscourt	90, 10	20, 80

What should Evert do?

Summary

- Players make their strategy choices without knowledge of the choices being made by other players in simultaneous-move games
- Simultaneous-move games are illustrated using game tables, also called the normal form
- Nash equilibrium is the solution concept used to solve simultaneous-move games
- Nash equilibrium can be found using IESDS or best-response analysis