Disease Convection:
Modelling Population Turnover in Simulated Epidemics

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1 Background

1.1 Notation

We denote the variable representing the size of group $i \in [1 \dots \mathrm{G}]$ as x_i and the vector of all x_i as $\boldsymbol{x}$. The total population size is denoted N, and the proportions represented by each group by $\hat{x}_i = x_i \mathrm{N}^{-1}$. The rate of population entry into group i is denoted by ν_i, and the rate of exit by μ_i (which may include group-specific disease-attributable death). The proportion of the entering population who are in group i, which may not be equal to the proportion of the current population in group i, is denoted $\hat{z}_i$. Since the rate of entry ν_i is typically expressed as a function of the total population size N, we model each group in the entering population $\boldsymbol{z}$ as having size $z_i = \hat{z}_i \mathrm{N}$.

Turnover transitions can occur between any two groups, in either direction; therefore we denote the turnover rates as a $\mathrm{G} \times \mathrm{G}$ matrix ζ, where ζ_{ij} corresponds to the transition $x_i \rightarrow x_j$. An explicit definition is given in Eq. (1.1), where the diagonal elements are denoted $*$ since they represent transitions from a group to itself, which is inconsequential.

$$\zeta = \begin{bmatrix} * & x_1 \rightarrow x_2 & \cdots & x_1 \rightarrow x_{\mathrm{G}} \\ x_2 \rightarrow x_1 & * & \cdots & x_2 \rightarrow x_{\mathrm{G}} \\ \vdots & \vdots & \ddots & \vdots \\ x_{\mathrm{G}} \rightarrow x_1 & x_{\mathrm{G}} \rightarrow x_2 & \cdots & * \end{bmatrix} \tag{1.1}$$

These transition flows and the associated rates are summarized for $\mathrm{G} = 3$ in Figure 1.

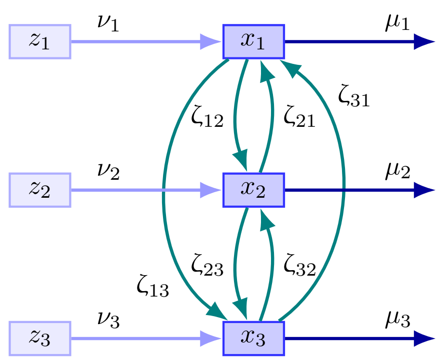

Figure 1: System of compartments and flows among them for $\mathrm{G} = 3$

1.2 Data Sources

No doubt, the system in Figure 1 is pretty, but how can this model be parametrized – which data can be used to define the values of N, $\boldsymbol{x}$, $\boldsymbol{z}$, $\boldsymbol{\nu}$, $\boldsymbol{\mu}$, and ζ?

1.2.1 Population Size N

1.2.2 Population Proportions x

1.2.3 Entering Proportions z

1.2.4 Entry ν and Exit μ

1.2.5 Turnover ζ

1.3 Approaches to Turnover

In this section, we will examine previous approaches to modelling turnover and highlight their assumptions.

1.3.1 Assumptions

The major assumptions which might be made when modelling turnover are summarized as follows:

1. There is no turnover: $\zeta_{ij} = 0$
2. Parameter values are constant:
   - 2.1. The population size N does not change with time
   - 2.2. The proportions of each group $\hat{x}_i$ do not change with time
   - 2.3. The proportions of the entering population $\hat{z}_i$ to not change with time
   - 2.4. The rates of entry ν_i and exit μ_i do not change with time
   - 2.5. The rates of turnover ζ_{ij} do not change with time
3. Parameter values are known:
   - 3.1. The population size N is known
   - 3.2. The proportions of one or more groups $\hat{x}_i$ are known
   - 3.3. The proportions of the entering population $\hat{z}_i$ are known
   - 3.4. The rates of entry ν_i and exit μ_i are known
   - 3.5. The proportion of group i who transfer to group j in a given year is known: ζ_{ij}
   - 3.6. The average duration of an individual in group i is known: D_i
4. Some parameters are equal:
   - 4.1. The group proportions in the entering population are equal to those in the general population: $\hat{z}_i = \hat{x}_i$
   - 4.2. The rate of entry into the population is the same for all groups: $\nu_i = \nu, \ \forall i$
   - 4.3. The rate of exit from the population is the same for all groups: $\mu_i = \mu, \ \forall i$

2 Methods

2.1 Equations

In this work, we propose a framework for implementing turnover, namely methods for estimating the parameters outlined in Figure 1, given a that some are already known from data. This framework is rooted in a set of equations, which we can summarize as follows:

1. **Mass Balance:** the rate of change of group i is equal to the net sum of the flows in and out of the group:

$$\frac{d}{dt}x_i = \nu_i\, z_i + \sum_j \zeta_{ji}\, x_j - x_i \left(\mu_i + \sum_j \zeta_{ij}\right) \tag{2.1}$$

2. **Duration:** the average duration in group i, D_i, is the inverse of all efferent flow rates:

$$D_i = \left(\mu_i + \sum_j \zeta_{ij}\right)^{-1} \tag{2.2}$$

2.2 Solving the System

2.3 Experiment

In this section, we introduce a simple epidemic model. With this model, we aim to answer the following questions:

1. What are the differences in the simulated epidemic using different turnover implementations?

3 Results

4 Discussion

5 Conclusions