Model Output: New Infections Ratio

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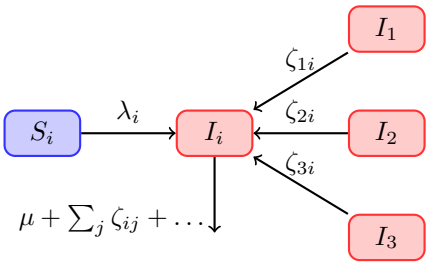
1 Objective

In a heterogeneous system which includes turnover between sub-populations (groups), the prevalence in any group is a function of both "conduction" (incidence) and "convection" (turnover). Therefore we may be interested in the ratio of new infections among a given group which arise from the force of infection versus from other groups via turnover.

2 System & Proposed Output

Consider the following open system, comprising sub-groups of $\mathcal{X}_i$ indexed by i, internal flow rates ζ, and force of infection λ:



We define the new output as: the ratio of new infections in group i which are due to turnover from group j among all new infections in group i:

$$\mathcal{R}_{ji} = \frac{\zeta_{ji} I_j}{\lambda_i S_i + \sum_j \zeta_{ji} I_j} \tag{1}$$

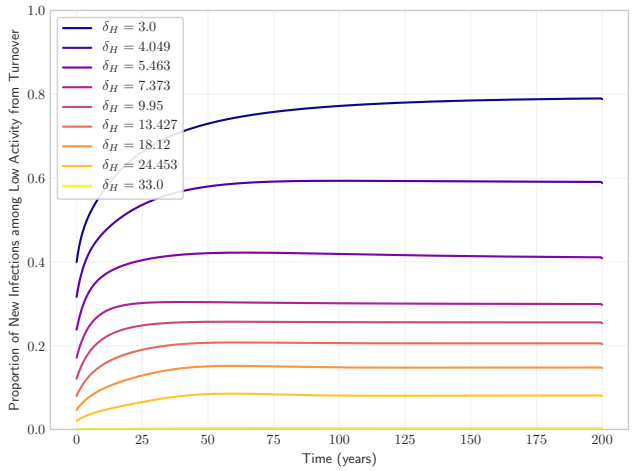
where I indicates people living with HIV and S indicates susceptible people.

3 Experiment

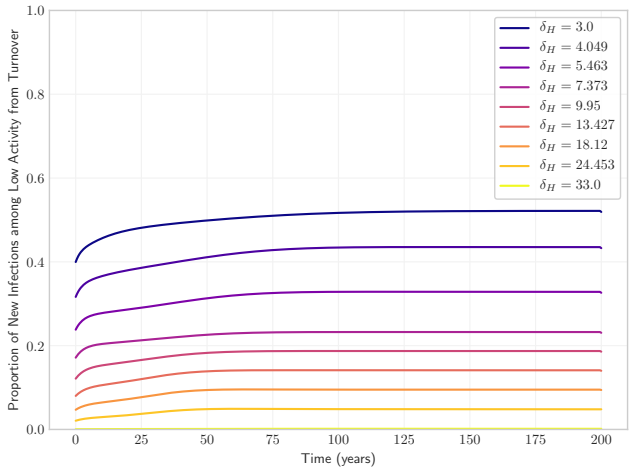
We previously observed that turnover increases equilibrium prevalence among the low risk group. In some cases, this also coincided with decreases in incidence among the same group – an apparent contradiction for models without turnover. However, here we can show that the increase in prevalence is attributable to movement of infected people from high to low risk groups, using the new output described above.

Using the project model, we computed the proportion of new infections among the low-activity group which are attributable to turnover from *any other group* (medium- and high-activity) – i.e. $\sum_j \mathcal{R}_{ij}$. We varied the rate duration of infectiousness $\delta_I \in [5, 10, 20, 50]$.

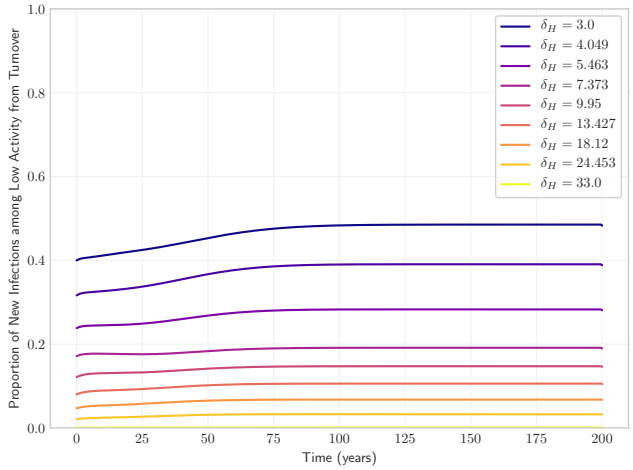
Figure 1 illustrates this result, where we can see that up to 80% individuals entering the "infected & low activity" state acquired their infections at a higher activity level. For typical parameters $\delta_H \approx 10$ years and $\delta_I \approx 10$ years, we see that about 20% of infections in the low activity group were acquired elsewhere. We also note that this proportion is proportional to the rate of treatment (inverse of duration of infectiousness).



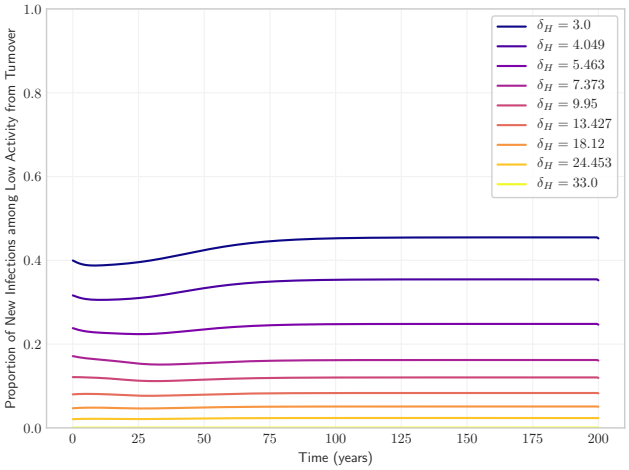
(a) Duration of infectiousness $\delta_I = 05$ years



(b) Duration of infectiousness $\delta_I = 10$ years



(c) Duration of infectiousness $\delta_I = 20$ years



(d) Duration of infectiousness $\delta_I = 50$ years

Figure 1: Proportion of new infections among the low-activity group from turnover

As such, it is reasonable to conclude that increases in prevalence among the low-activity group with turnover (being on the order of 0 – 50%; `docs/debug/surface/surface.pdf`) can be mainly attributed to movement of infected individuals from high-risk to low-risk groups.