Discrete Mathematics Notes

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1 Propositions

Definition 1.1. A **proposition** is a declarative statement that is either true or false, but not both nor none.

True (T) and False (F) denote **truth values** such that propositions either have a true value (T) or a false value (F).

Example 1.2. Here are some examples of some simple numerical propositions:

(i) $1 + 2 = 3$ has a truth value T because the result, or **conclusion** holds.

(ii) $2 + 3 = 4$ has a truth value F because the conclusion does not hold for the given assumption, or **condition**.

(iii) $71951 = 2 * 17 * 59 * 239$ has a truth value F because the conclusion does not hold for the given condition (the 2 on the right-hand side makes the product even, whereas 71951 is odd).

(iv) $4^{101} - 1$ is divisible by 3.
We will show the truth value of this statement by performing the following operations: $4^3 - 1 = a^3 - b^3 = (a-b)(a^2+ab+b^2)$. If $(a-b) \implies (4-1) = 3$, then $4^3 - 1$ is divisible by 3. Therefore $4^{101} - 1$ is divisible by 3 as well.

Example 1.3. Here are examples where we introduce **compound propositions**, or new propositions formed by logical operators. More information on why these compound propositions are true will be explained in the next section. Assume $a, b \in \mathbb{R}$:

(i) $2 * 3 = 3 * 2$ and $2 + 3 = 3 + 2$ holds true (T) because if both propositions are true, then the compound proposition must be true.

(ii) If $a > 3$, then $2a > 6$ is true (T) because if a is always greater than 3, then the conclusion must always hold true.

(iii) $a \leq 3$ or $2a > 6$ is true (T) since the truth value depends on whether a is 3 or less than 3, thus we assume it is true.

(iv) If $a + b = 0$ then $a = 0$ or $b = 0$ is false (F) because a and b both have to be 0 in order for the if-statement to be true.

(v) $a * b = 0$ if and only if $a = 0$ or $b = 0$ is true (T) because only a or b has to equal 0 in order for the proposition to be true.

2 Logical Operators

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