

共振器の光エネルギーの立ち上がり，利得媒質の反転分布の立ち上がり，に時間的ずれがあるため，過渡的な振動減少が生じる。これが緩和発信。

$$\begin{aligned}
N'_1 &= R - \frac{1 + Bt_c N_1}{Bt_c \tau} - \frac{RBt_c \tau - 1}{Bt_c \tau} - \frac{\rho_1}{t_c} - \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= R - \frac{Bt_c N_1 + RBt_c \tau}{Bt_c \tau} - \frac{\rho_1}{t_c} - \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= R - \frac{N_1 + R\tau}{\tau} - \frac{\rho_1}{t_c} - \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= -\frac{N_1}{\tau} - \frac{\rho_1}{t_c} - \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= -\frac{RBt_c \tau}{\tau} N_1 - \frac{\rho_1}{t_c} \\
&= -RBt_c N_1 - \frac{\rho_1}{t_c} \\
\rho'_1 &= -\frac{\frac{RBt_c \tau - 1}{B\tau} + \rho_1}{t_c} + \frac{RBt_c \tau - 1}{Bt_c \tau} + \frac{\rho_1}{t_c} + \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= -\frac{RBt_c \tau - 1 + B\tau \rho_1}{Bt_c \tau} + \frac{RBt_c \tau - 1}{Bt_c \tau} + \frac{\rho_1}{t_c} + \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= -\frac{B\tau \rho_1}{Bt_c \tau} + \frac{\rho_1}{t_c} + \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= -\frac{\rho_1}{t_c} + \frac{\rho_1}{t_c} + \frac{RBt_c \tau - 1}{\tau} N_1 \\
&= \frac{RBt_c \tau - 1}{\tau} N_1 \\
N_1 &= \frac{\tau}{RBt_c \tau - 1} \rho'_1
\end{aligned}$$

$$\begin{aligned}
\rho''_1 &= \frac{RBt_c \tau - 1}{\tau} N'_1 \iff N'_1 = \frac{\tau}{RBt_c \tau - 1} \rho''_1 \\
\frac{\tau}{RBt_c \tau - 1} \rho''_1 &= -RBt_c N_1 - \frac{\rho_1}{t_c} \\
\frac{\tau}{RBt_c \tau - 1} \rho''_1 &= -RBt_c \frac{\tau}{RBt_c \tau - 1} \rho'_1 - \frac{\rho_1}{t_c} \\
\frac{\tau}{r-1} \rho''_1 &= -\frac{r}{r-1} \rho'_1 - \frac{\rho_1}{t_c} \\
0 &= \frac{\tau}{r-1} \rho''_1 + \frac{r}{r-1} \rho'_1 + \frac{\rho_1}{t_c}
\end{aligned}$$

$$\rho = Ae^{pt} \text{ とすると}$$

$$0 = \frac{\tau}{r-1} Ap^2 e^{pt} + \frac{r}{r-1} Ape^{pt} + \frac{1}{t_c} Ae^{pt}$$

$$0 = \frac{\tau}{r-1} p^2 + \frac{r}{r-1} p + \frac{1}{t_c}$$

$$0 = p^2 + \frac{r}{\tau} p + \frac{r-1}{t_c \tau}$$

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{r}{\tau} \pm \sqrt{\frac{r^2}{\tau^2} - 4\frac{r-1}{t_c \tau}}}{2}$$

$$= -\frac{r}{2\tau} \pm \sqrt{\left(\frac{r}{2\tau}\right)^2 - \frac{r-1}{t_c \tau}}$$

$$\frac{r-1}{t_c \tau} \gg \left(\frac{r}{2\tau}\right)^2$$

$$p = -\frac{r}{2\tau} \pm i\sqrt{\frac{r-1}{t_c \tau}}$$