Maximizing Paper Helicopter Flight Time: An Experimental Analysis

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Group H

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Contents

Summary

This study "Maximizing Paper Helicopter Flight Time" examined how blade shape, blade length, and added weight affect the flight time of paper helicopters. Researchers used ANOVA, Tukey HSD tests, correlation analyses, linear regression, and interaction plots to analyze the data. The results showed that rectangular blades significantly increased flight time compared to curved and tapered designs. Blade length showed a positive trend, particularly for rectangular blades, but its effect was not statistically significant at the 0.05 level. Added weight had a strong negative correlation with flight time, consistently reducing flight duration. Interaction plots suggested possible interactions between blade shape, blade length, and added weight, though the interaction between blade length and weight was not significant in the regression model. These findings highlight the importance of blade shape and minimizing weight for optimizing flight performance. Future research should eliminate the effects of limitations on interactions and explore additional factors influencing flight time.

1 Introduction

It is well known that, paper helicopters are widely used in experimental research to explore aerodynamic principles. George E.P. Box's famous paper helicopter experiment laid the foundation for this study, and provided valuable insight into optimizing flight performance and refining experimental designs.

This report aims to investigate how the blade shape(rectangular, tapered, curved) and other quantitative variables, such as blade length and body weight, affect flight time. The goal is to statistically determine the optimal combination of variables that maximizes the helicopter's flight time. The experiment employs a controlled variables approach, measuring flight time under standardized conditions and using ANOVA and regression modeling to assess the significance of each variable and potential interactions.

This study hopes to contribute to the improvement of helicopter flight design. However, the key limitations are the uncertainty introduced by the manual construction of paper helicopters, and the impossibility of conducting experiments with more variable groups due to time constraints, which may affect the results.

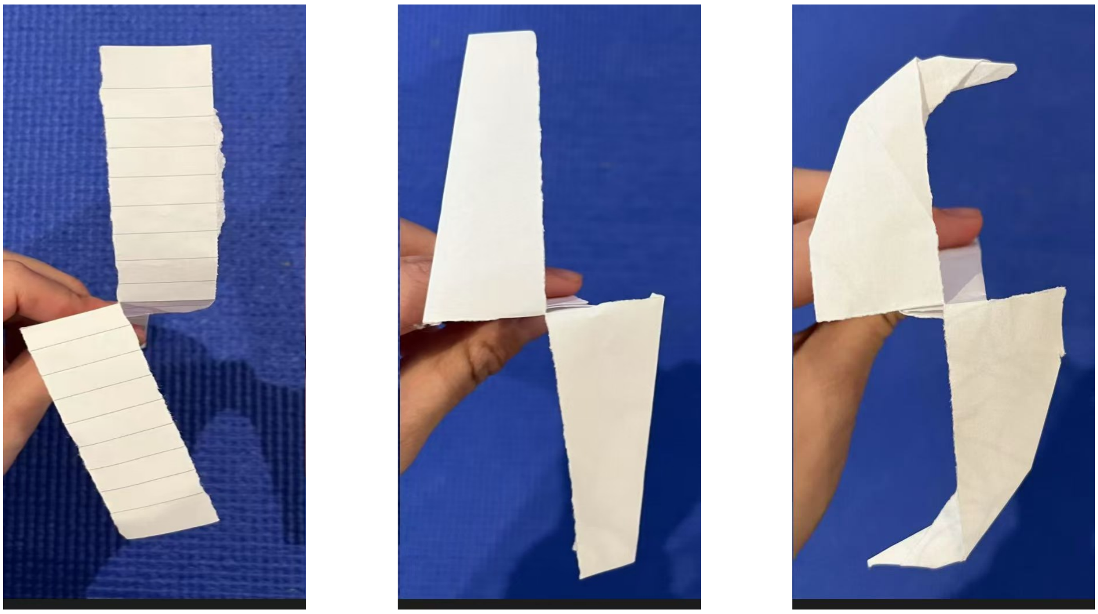
2 Methods

2.1 Experimental Design

2.1.1 Variables

Independent Variables(Factors)

- Blade shape (Qualitative Variable): This variable includes three levels, rectangular, tapered, and curved. The shape of the blades will be altered by folding the wings in different ways, as shown in the figure below:



(a) Rectangular (b) Tapered (c) Curved

Figure 1: The appearance of the blade

- Blade length (Quantitative Variable): This variable is measured in centimeters(cm) and will be controlled by precisely measuring and cutting the blades to different lengths. It is worth noticing that the total weight of the helicopter will remain the same after cutting.

- Added weight (Quantitative Variable): This variable is measured in grams(g) and will be adjusted by attaching different numbers of paperclips to the body of the helicopter. The weight of each paperclip will be measured through calculating the mean weight of a large amount of paperclips.

Dependent variables (Response)

- Flight time (Quantitative): This variable is measured in seconds(s).

Controlled Variables

- Release height: The release height was fixed besides the top edge of the door for each experiment, which is 2.07 meters high.
- Experimental environment: All experiments were conducted in an indoor windless environment to minimize external influences that could affect the flight time.
- Release method: The same person released the helicopter in each experiment, trying to ensure that the wings remained parallel to the ground during release.
- Paper material: All helicopters in the experiment were made of standard A4 paper to eliminate the difference in material properties.

Why these variables?

- The shape of the blades is a key design element that can influence lift generation and aerodynamic efficiency. Different shapes tend to create varying airflow patterns, which may impact the helicopter's ability to stay aloft.
- Different lengths of blades may bring different lift, drag and instability, which will affect the flight time of the paper helicopter.
- The weight of the helicopter's body is likely to affect the required lift needed to keep it airborne, which may affect the result of the experiment.

2.1.2 Experimental Methods

Factorial Design

A factorial design was employed, with each variable tested at multiple levels. Blade length was tested at three levels ($5cm$, $7cm$, and $9cm$), blade shape at three levels (curved, rectangular, and tapered), and weight at three levels ($0g$, $0.5g$ and $1g$). This resulted in a total of 27 unique combinations, each tested 4 times to ensure reliability, yielding 108 experimental runs.

Randomization

To randomize the order of helicopter flight trials, a random number was generated for each trial using the "RAND()" function in Excel. The entire dataset, including these random numbers, was then sorted in ascending order based on the random number column. This effectively shuffled the trial order, ensuring that any uncontrolled factors were distributed randomly across the experiment.

2.1.3 Hypothesis

- H_1 (Blade Shape): there will be a significant difference in flight time between helicopters with rectangular, tapered, and curved blades.
- H_2 (Blade Length): longer blade lengths will result in longer flight times.
- H_3 (Body Weight): heavier body weights will result in shorter flight times.
- H_4 (Interaction): there will be an interaction effect between blade shape and blade length/body weight on flight time.

2.1.4 Materials

- Standard A4 paper
- Scissors
- Ruler (for measuring blade length)
- Retractable Steel Tape Measure (for measure release height)
- Paper clips (for adding weight)
- Stopwatch on smart phones (for measuring time)

2.2 Data Collection

2.2.1 Experimental Procedure

First, a statistical table will be created based on the 27 unique experimental groups according to the factorial design, and the experimental sequence was randomized. Next, 108 paper helicopters were conducted according to the selected blade shape, blade length, and body weight, to try to ensure the consistency in folding. Since each trial causes wear and tear on the paper helicopters, which may affect the results, each helicopter will be tested only once.

Then, the appropriate paper helicopters will be selected according to the statistical table and released under controlled conditions, with the stopwatch started simultaneously. To minimize errors caused by reaction time, the researcher responsible for releasing the helicopter will proceed a three-second countdown, ensuring simultaneity in time measurement. The stopwatch was immediately stopped when the paper helicopter touched the ground, and the flight time was recorded. Each set of experiments is performed four times and the average value is taken. Each experiment will be measured by two researchers at the same time and calculate the average to reduce the error caused by manual measurement.

2.2.2 Construction Process

Figure 2 shows the construction process of a paper helicopter step by step. First, fold the A4 paper into eight equal parts as shown in step (a), and cut along the dotted line. Fold in half along the dotted line as shown in step (b) and unfold to form a crease. Measure the required length (5 cm, 7 cm, 9 cm) from the top of the crease with a ruler and mark it. Then, cut along the crease to the mark to form two parts. Spread the upper part to both sides (as shown in step (c)) as the blade. Keep the lower part vertical, measure the width, and mark at the left and right one-third positions respectively. Then cut to the mark as shown in step (d), and fold it along the dotted line in step (e) as the body of the paper helicopter. In this way, a paper helicopter with a rectangular blade shape is made as shown in step (f).

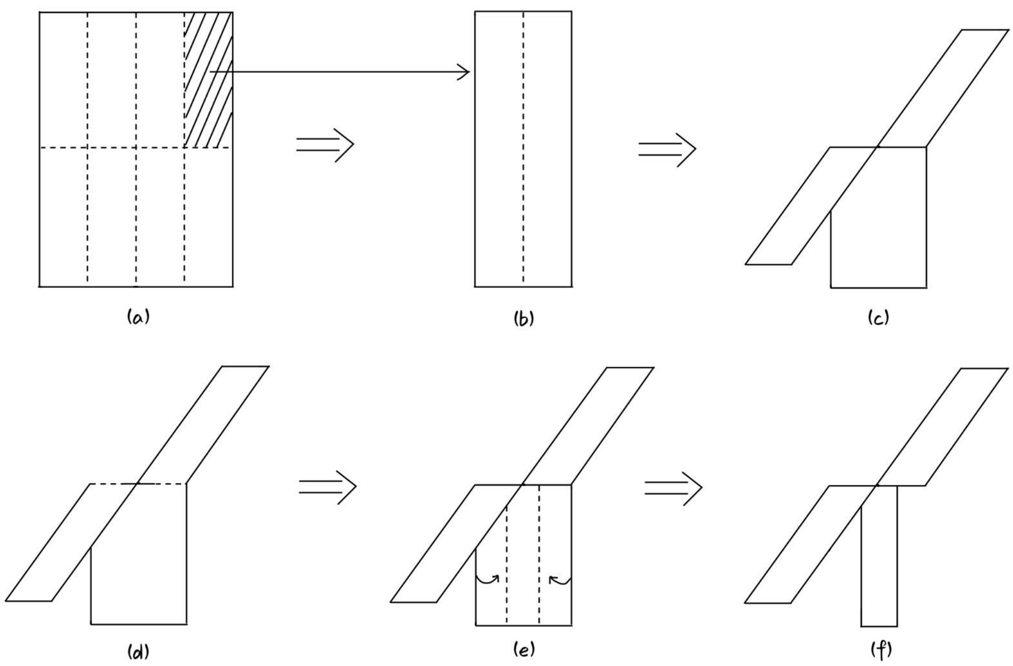


Figure 2: Construction process

On this basis, the blades are folded once or multiple times to make the paper helicopter looks like those in Figure 1(b) and 1(c), which are tapered and curved respectively.

Finally, adjust the angle of the blade to keep it as perpendicular to the body as possible to ensure flight balance.

2.3 Statistical Analysis

This report will employ three statistical methods for analysis. First of all, perform ANOVA to assess the statistical significance of the shape and length of the blade and the added weight on flight time. Next, perform regression analysis to model the relationship between the added weight of the blade length and flight time. Additionally, this report will investigate potential interaction effects among variables using interaction plots and statistical tests, providing further insights into how different factors collectively influence flight time.

3 Results

3.1 Descriptive Statistics

3.1.1 Data Summary

The "summary()" function in R calculates descriptive statistics for each column (variable) in the data frame. It provides a quick summary of the data distribution and key values. For the Run Number column, the statistics show the distribution of Run Numbers. The values range from 1 to 27, indicating that there are 27 data points. The mean and median are both 14, which suggests a symmetrical distribution. The output also shows significant variability in flight times across trials (see the range between the minimum and maximum values for each trial). For trials 3 and 4, the mean is higher than the median, suggesting that the data may be skewed to the right. The average flight times vary from 0.905 to 2.277 seconds, showing differences in performance between helicopters.

```
   Run.Number   Blade.Shape        Blade.Length..cm. Added.Weight..g.
 Min.   : 1.0   Length:27          Min.   :5         Min.   :0.0
 1st Qu.: 7.5   Class :character   1st Qu.:5         1st Qu.:0.0
 Median :14.0   Mode  :character   Median :7         Median :0.5
 Mean   :14.0                      Mean   :7         Mean   :0.5
 3rd Qu.:20.5                      3rd Qu.:9         3rd Qu.:1.0
 Max.   :27.0                      Max.   :9         Max.   :1.0
  Trial.1..s.     Trial.2..s.     Trial.3..s.     Trial.4..s.
 Min.   :0.830   Min.   :0.850   Min.   :0.790   Min.   :0.910
 1st Qu.:1.090   1st Qu.:1.095   1st Qu.:1.155   1st Qu.:1.130
 Median :1.240   Median :1.180   Median :1.290   Median :1.250
 Mean   :1.327   Mean   :1.284   Mean   :1.434   Mean   :1.366
 3rd Qu.:1.500   3rd Qu.:1.430   3rd Qu.:1.645   3rd Qu.:1.560
 Max.   :2.670   Max.   :2.040   Max.   :2.340   Max.   :2.400
  random.order       TrialAverage
 Min.   :0.008864   Min.   :0.905
 1st Qu.:0.160003   1st Qu.:1.119
 Median :0.299235   Median :1.260
 Mean   :0.367376   Mean   :1.353
 3rd Qu.:0.533908   3rd Qu.:1.474
 Max.   :0.996367   Max.   :2.277
```

Figure 3: Summary data

3.1.2 Data Visualization-Box Plot

The box plots will show the distribution of the average flight times for different variables.

Blade Shape

The box plot compares trial averages for three blade shapes: Curved, Rectangular, and Tapered. The X-axis represents blade shape, and the Y-axis represents average flight time.

Rectangular blades have the highest average flight time, with their box positioned significantly higher on the Y-axis than Curved and Tapered blades. Their median is also the highest, confirming this trend. Curved and Tapered blades show similar average flight times, with closely aligned medians, indicating no substantial difference between them. In terms of variability, Rectangular blades have the largest interquartile range (IQR), showing greater spread in flight times. Curved and Tapered blades have more concentrated distributions, indicating less variability. However, the Tapered blade has an outlier, possibly due to an anomaly or experimental error or a deformation of the paper helicopters. The median positions suggest different distribution patterns: Rectangular blades show a slight negative skew, Curved blades have a nearly symmetrical distribution, and Tapered blades show a slight positive skew.

Overall, Rectangular blades maximize flight time, while Curved and Tapered blades perform similarly. The outlier in the Tapered blade group contributes to its higher variability.

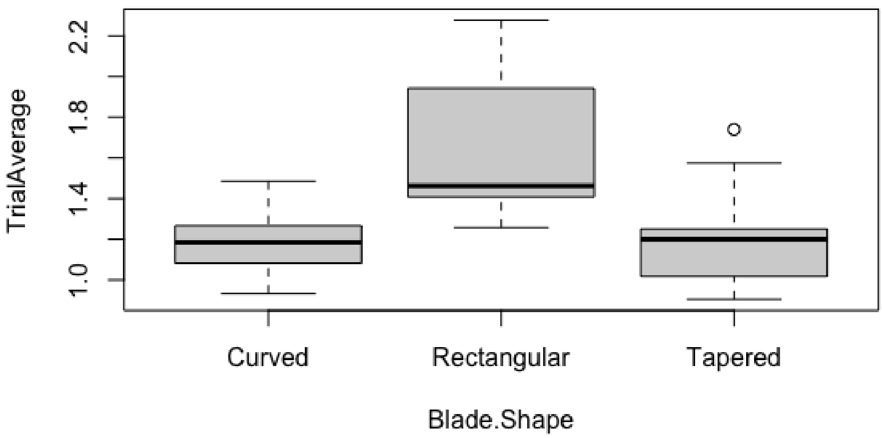


Figure 4: Box plot of different blade shape

Blade Length

The blade length box plot shows blade lengths of 5 cm, 7 cm, and 9 cm on the x-axis and average flight time on the y-axis, which also shows the .

The boxes indicate a trend of increasing Trial Average with longer blade lengths. The 7 cm box is higher than the 5 cm box, and the 9 cm box is slightly higher than the 7 cm box, suggesting longer blades generally result in longer flight times. The median line for 9 cm is the highest, indicating the highest median flight time, while the 7 cm median is slightly lower but still higher than 5 cm. The 5 cm box is narrow, showing less variability, while the 7 cm and 9 cm boxes are wider, indicating greater variability. The median lines for 5 cm and 9 cm are near the center, suggesting symmetrical distributions, while the 7 cm median is slightly below center, indicating a slight negative skew.

Overall, the box plot suggests longer blades lead to longer flight times, with a generally linear relationship, and greater variability in flight times for 7 cm and 9 cm blades compared to 5 cm.

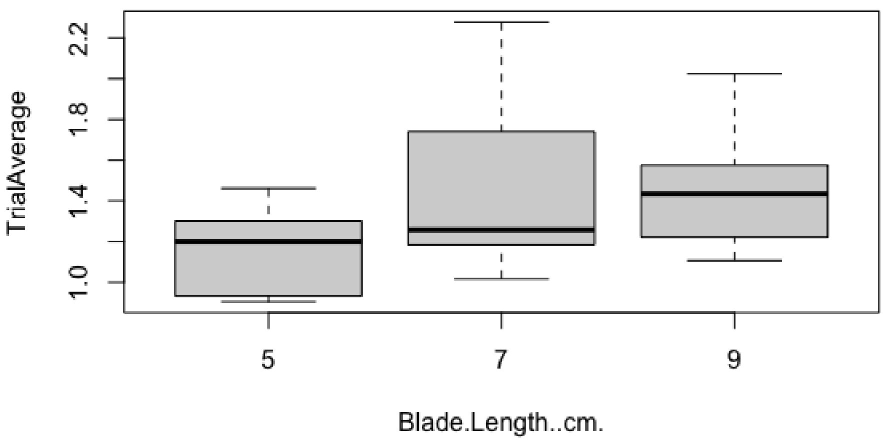


Figure 5: Box plot of different blade length

Added Weight

The added weight box plot shows weights of 0 g, 0.5 g, and 1 g on the x-axis and average flight time on the y-axis.

The boxes show a trend of decreasing Trial Average as weight increases. The 0 g box is the highest, followed by 0.5 g and 1 g, indicating that added weight reduces flight times. The median line for 0 g is the highest, showing the longest median flight time, while the medians for 0.5 g and 1 g are lower and close to each other. The 0 g box is wider, indicating greater variability, while the 0.5 g and 1 g boxes are narrower, showing less variability. The 0.5 g category has an outlier above the upper whisker, representing a trial with a significantly longer flight time. The median lines for 0 g and 0.5 g are near the center, suggesting symmetrical distributions, while the 1 g median is slightly below center, indicating a slight negative skew.

Overall, the box plot confirms that added weight reduces flight time, with the greatest variability at 0 g. The outlier at 0.5 g warrants further investigation.

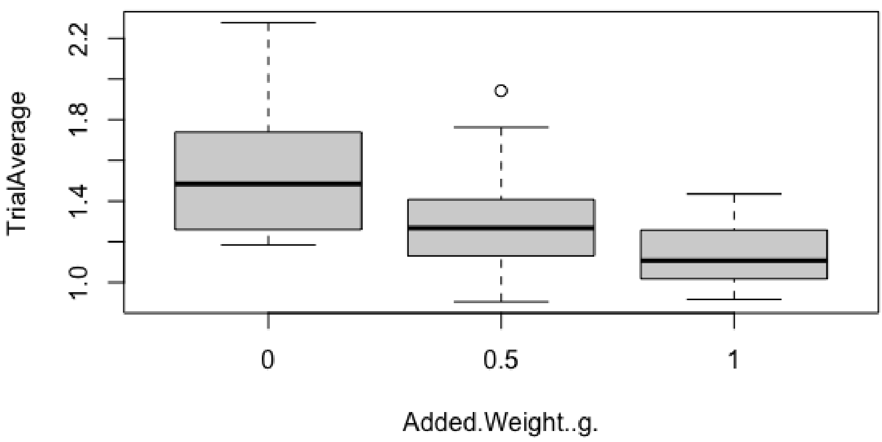


Figure 6: Box plot of different added weight

3.1.3 Data Visualization-Scatter Plot

Blade Length

There is a general trend of increasing average flight time with longer blade lengths, which shows a positive relationship, though the relationship is not perfectly linear, possibly due to interacting factors. At 5 cm, flight times are clustered at the lower end, indicating shorter durations. At 7 cm, flight times are more spread out, with some higher values, suggesting improved performance. At 9 cm, flight times also show variability, with both high and low values. The vertical spread of data points indicates greater variability at 7 cm and 9 cm compared to 5 cm. Some points at 7 cm and 9 cm are noticeably higher, potentially representing outliers or exceptional performance. While longer blades generally result in longer flight times, the relationship is influenced by other factors, as shown by the variability and potential outliers. The scatter plot suggests an optimal blade length for maximizing flight time, likely around 7 cm or 9 cm, but further analysis is needed to confirm this.

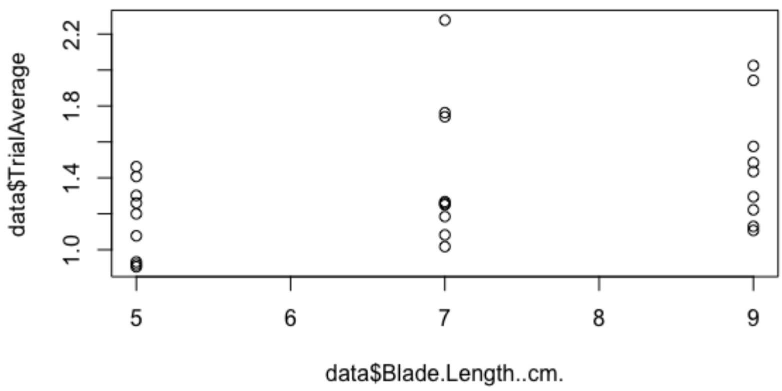


Figure 7: Scatter plot of different blade length

Added Weight

There is a clear trend of decreasing average flight time as added weight increases, which shows a strong negative correlation. At 0 g, flight times are clustered at the higher end, indicating longer durations. At 0.5 g, flight times are more spread out, with both higher and lower values, while at 1 g, they are clustered at the lower end, showing shorter durations. The vertical spread of data points reveals greater variability at 0.5 g compared to 0 g and 1 g. One data point at 0.5 g is noticeably higher, potentially an outlier or an instance of exceptional performance. The scatterplot confirms a consistent negative trend, with added weight reducing flight time. However, the variability at 0.5 g and the presence of a potential outlier suggest that other factors may influence flight time at this weight.

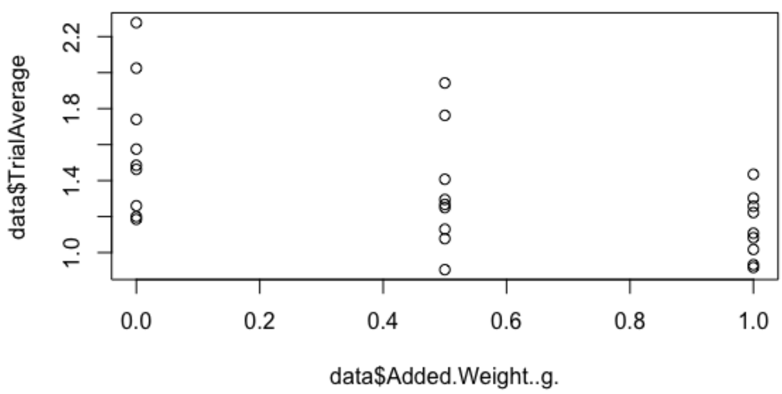


Figure 8: Scatter plot of different added weight

3.2 ANOVA Analysis

While the box plot provides a visual comparison, it's essential to perform statistical tests (e.g., ANOVA) to determine if the differences observed are statistically significant (As shown in Figure 9), following the procedures outlined in Montgomery [6].

```
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Blade.Shape        2 1.2174  0.6087   21.95 5.75e-06 ***
Blade.Length..cm.  1 0.4209  0.4209   15.18 0.000778 ***
Added.Weight..g.   1 0.8602  0.8602   31.02 1.34e-05 ***
Residuals         22 0.6102  0.0277                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 9: ANOVA Table

The P-value is very small (less than 0.001) with F-value of 21.95 for Blade.Shape, indicating strong statistical significance. This means there is a significant difference in average flight time between at least two of the blade shapes. Similar to Blade shape, the p-value is also very small for Blade.Length with F-value of 15.18, indicating strong statistical significance. This means there is a significant effect of blade length on average flight time. For Added Weight, the p-value is again very small with F-value of 31.02, indicating a strong statistical significance. This means there is a significant effect of added weight on average flight time. All three factors (Blade shape, Blade length, and added weight) have a statistically significant impact on the average flight time of the helicopters. This ANOVA supports the findings from the box plots and scatter plots, confirming the observed trends.

3.3 Correlation and Regression Analysis

While the box plot and scatter plot provides a visual representation, it's essential to perform statistical analysis (e.g., correlation, regression) to quantify the relationship between blade length and average flight time and determine its statistical significance.

```
        Pearson's product-moment correlation

data:  data$Blade.Length..cm. and data$TrialAverage
t = 1.9786, df = 25, p-value = 0.05898
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.01401408  0.65621506
sample estimates:
      cor
0.3679595


        Pearson's product-moment correlation

data:  data$Added.Weight..g. and data$TrialAverage
t = -3.0927, df = 25, p-value = 0.004828
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.7551053 -0.1825081
sample estimates:
       cor
-0.5260383
```

Figure 10: Correlation Result

3.3.1 For Blade Length

The correlation coefficient (0.3679595) suggests a moderate positive correlation. The p-value (0.05898) is slightly greater than 0.05. This means that the correlation is not statistically significant at the 0.05 level. However, it is close to the significance threshold and may be considered marginally significant. The confidence interval includes 0 (-0.01401408 to 0.65621506). This further supports the idea that the correlation might not be significantly different from zero. There is a moderate positive correlation between blade length and average flight time, but it is not statistically significant at the 0.05 level.

3.3.2 For Added Weight

The correlation coefficient (-0.5260383) suggests a moderate negative correlation. The p-value (0.004828) is less than 0.05, indicating that the correlation is statistically significant at the 0.05 level. The confidence interval does not include 0 (-0.7551053 to -0.1825081). This supports the idea that the correlation is significantly different from zero. There is a moderate negative correlation between added weight and average flight time, and it is statistically significant at the 0.05 level.

3.3.3 Regression Analysis

To further explore the relationships between the independent variables (blade length and added weight) and the dependent variable (Trial Average), a linear regression model was fitted. The model included the main effects of blade length (Blade.Length..cm.) and added weight (Added.Weight..g.).

The regression model was specified as:

$$TrialAverage \sim Blade.Length..cm. + Added.Weight..g. \tag{1}$$

```
Call:
lm(formula = TrialAverage ~ Blade.Length..cm. + Added.Weight..g.,
    data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.38630 -0.16894 -0.06449  0.15836  0.70620

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        1.03609    0.24264   4.270 0.000266 ***
Blade.Length..cm.  0.07646    0.03252   2.351 0.027271 *
Added.Weight..g.  -0.43722    0.13008  -3.361 0.002594 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.276 on 24 degrees of freedom
Multiple R-squared:  0.4121,    Adjusted R-squared:  0.3631
F-statistic: 8.412 on 2 and 24 DF,  p-value: 0.001704
```

Figure 11: Regression Result

The Estimate is the estimated coefficient for each predictor which represents the change in Trial Average for a one-unit increase in the predictor, holding other variables constant. The intercept (1.03609) is the estimated Trial Average when both Blade Length and Added Weight are zero. For each 1 cm increase in blade length, Trial average is estimated to increase by 0.07646 seconds (holding added weight constant). For each 1 gram increase in added weight, Trial Average is estimated to decrease by 0.43722 seconds (holding blade length constant). The p-value is the probability of observing the t-value or a more extreme value if the null hypothesis were true. The intercept (p¡0.001) shows highly significance and the blade length $p = 0.027271$ is significant at 0.05 level, also for the added weight the $p = 0.002594$ shows highly significance.

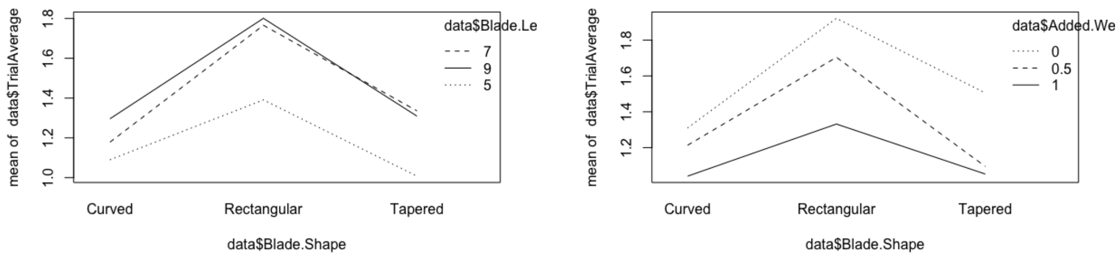
The results show both blade length and added weight are statistically significant predictors of Trial Average. Longer blade lengths are associated with longer average flight times, but the effect is relatively small (0.07646 seconds per cm). Increased added weight has a negative impact on average flight time. The model explains approximately 41.21% of the variance in Trial Average (Multiple R-squared). The overall regression model is statistically significant.

3.4 Interaction Effects

To investigate the potential interaction effects between the independent variables, interaction plots and a linear regression model with interaction terms were utilized.

3.4.1 Interaction between Blade Shape and Blade Length/Added Weight

Interaction plots were generated to visualize the potential interactions between blade shape and blade length, as well as blade shape and added weight. These plots, presented in Figure 12, suggest potential interactions.



(a) blade shape * blade length (b) blade shape * added weight

Figure 12: Interaction Plots

- Blade Shape and Blade Length: The interaction plot revealed that the effect of blade length on Trial Average appeared to differ across blade shapes. Specifically, rectangular blades showed a more pronounced increase in Trial Average with increasing blade length compared to curved and tapered blades. This suggests that the optimal blade length for maximizing flight time may depend on the blade shape.
- Blade Shape and Added Weight: The interaction plot also suggested a potential interaction between blade shape and added weight. The decrease in Trial Average with increasing added weight appeared more pronounced for rectangular blades compared to curved and tapered blades. This indicates that rectangular blades might be more sensitive to changes in added weight.

3.4.2 Interaction between Blade length and Added Weight

A linear regression model was fitted to assess the interaction between blade length and added weight on Trial Average. The model included the main effects of blade length and added weight, as well as their interaction term (Blade.Length..cm * Added.Weight..g.).

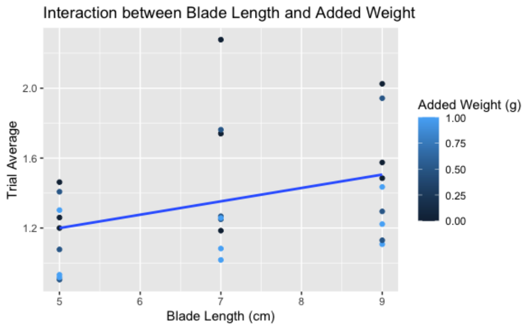
```
Call:
lm(formula = TrialAverage ~ Blade.Length..cm. * Added.Weight..g.,
    data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.38630 -0.15310 -0.09449  0.18127  0.70620

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
(Intercept)                         0.87567    0.37495   2.335   0.0286 *
Blade.Length..cm.                   0.09938    0.05216   1.905   0.0693 .
Added.Weight..g.                   -0.11639    0.58086  -0.200   0.8430
Blade.Length..cm.:Added.Weight..g. -0.04583    0.08081  -0.567   0.5761
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2799 on 23 degrees of freedom
Multiple R-squared:  0.4202,    Adjusted R-squared:  0.3446
F-statistic: 5.557 on 3 and 23 DF,  p-value: 0.005107
```

(a) Interaction Table



(b) blade length * added weight

Figure 13: Interaction Plots

The regression results, presented in Figure 13 Interaction Table, showed that the interaction term ($Blade.Length..cm : Added.Weight..g.$) was not statistically significant ($p = 0.5761$). This suggests that the effect of blade length on Trial Average does not significantly depend on the level of added weight, and vice versa. This finding was supported by the interaction plot (Figure[13 ,(b)]), which showed relatively parallel trends across different levels of added weight.

While the interaction plots suggested potential interactions between blade shape and blade length/added weight, the regression model indicated no significant interaction between blade length and added weight. This discrepancy highlights the importance of combining visual inspection of interaction plots with statistical testing. The potential interactions involving blade shape warrant further investigation with more specific interaction tests or more complex models.

3.5 Post-Hoc Tests

Following the significant main effect of blade shape observed in the ANOVA, Tukey's Honestly Significant Difference (HSD) post-hoc test was conducted to determine which specific blade shapes differed significantly in their mean Trial Average. The results of the Tukey HSD test are presented in full in Appendix D.

The post-hoc test revealed a highly significant difference in mean Trial Average between rectangular blades and curved blades ($diff = 0.46444444, p\,adj = 0.0000171$). Specifically, rectangular blades exhibited significantly higher average trial averages than curved blades. Similarly, a highly significant difference was found between rectangular blades and tapered blades ($diff = -0.43500000, p\,adj = 0.0000414$), with rectangular blades again showing higher average trial averages. However, no statistically significant difference in mean Trial Average was observed between curved and tapered blades ($diff = 0.02944444, p\,adj = 0.9256539$).

In summary, the Tukey HSD test demonstrated that rectangular blades resulted in significantly longer average flight times compared to both curved and tapered blades. There was no statistically significant difference in flight times between curved and tapered blades.

4 Conclusions

In conclusion, this project used ANOVA, regression analysis and interaction plots to provide empirical evidence supporting the hypothesis, which shows the importance of blade shape and added weight in optimizing paper helicopter flight time. However, limitations may affect the flight time of the paper helicopter. Therefore, future research should focus on further exploring the potential interaction effects, considering a wider range of design parameters, and accounting for uncontrolled factors to refine our understanding of the complex dynamics influencing flight performance. The insights gained from this study can serve as a foundation for further investigations, and practical applications in the design and optimization of small-scale flying devices.

5 General Discussion and Recommendation

This project aimed to investigate factors that affect the flight time of paper helicopters, specifically blade shape, blade length, and added weight. The findings provide valuable insights into the design and optimization of paper helicopters.

The rectangular blades across different conditions possess a consistent advantage of maximizing flight time, highlights the importance of blade shape in generating air resistance (or lift). This finding is consistent with basic aerodynamic principles, where larger surface areas are generally subject to more air resistance.

The significant negative correlation between added weight and flight time reinforces the fundamental concept that minimizing weight is critical to flight performance. This is consistent with the laws of physics, where added weight requires more lift to overcome gravity.

The effects of blade length are more complex, since the interaction plot suggests that the optimal blade length may depend on blade shape. This highlights the importance of considering interactions when designing experiments and interpreting results. Despite the visual cues in the interaction plot, the lack of a significant interaction between blade length and added weight in the regression model emphasizes the importance of combining visual analysis with statistical tests. This discrepancy may be due to limitations of the linear regression model or the specific levels of the variables used in this study.

This study has several limitations. For example, the paper helicopters were handmade, resulting in imperfect consistency in each experimental sample, which are not as fine as those made by machines. Secondly, despite the method of taking the average value of multiple experiments, there will still be inevitable measurement errors. Next, the study used a limited range of blade lengths and added weights, which may reduce the difference due to different samples. Last but not least, while randomization was used, uncontrolled factors such as air currents and paper inconsistencies likely influenced the results.

Future research should address these limitations by utilizing more sophisticated experimental designs, incorporating a wider range of variables, and considering more complex helicopter models. As for future directions, variables can be added on the basis of eliminating enough limitations. For example, the material can be changed, such as cardboard and plastic can be used for experiments. Different paper materials will produce different air resistance, which may affect the experimental results. Also, changing the number of blades can also change the air resistance to affect the experimental results. Optimizing the experiment based on the above ideas may better improve the performance of the paper helicopter, make the experiment more ideal and extend the flight time of the helicopter.

Despite these limitations, this project has made a valuable contribution to the understanding of paper helicopter flight dynamics. The results can be used to optimize the design of paper helicopters for educational purposes, recreational activities, and other applications.

In conclusion, this project successfully demonstrated the significant effects of blade shape and attached weight on the flight time of paper helicopters. The study also highlights the potential for interactions and the importance of combining visual analysis with statistical tests. Future research should build on these findings to further explore the complex dynamics that affect flight performance and develop more advanced models for predicting and optimizing helicopter flight.

References

[1] Crawley, M. J. (2012). *The R book*. John Wiley & Sons.

[2] Field, A., Miles, J., & Field, Z. (2012). *Discovering statistics using R*. Sage Publications.

[3] Fox, J., & Weisberg, S. (2019). *An R companion to applied regression* (3rd ed.). Sage Publications.

[4] Fox, J., Weisberg, S., & Price, B. (2023). *car: Companion to Applied Regression* (Version 3.1-2) [R package]. Retrieved from https://CRAN.R-project.org/package=car

[5] Kutner, M. H., Nachtsheim, C. J., Neter, J., & Li, W. (2005). *Applied linear statistical models*. McGraw-Hill Irwin.

[6] Montgomery, D. C. (2017). *Design and analysis of experiments*. John Wiley & Sons.

A Appendix A: Raw Data

See submitted data set.

B Appendix B: R Code

See submitted R file.

C Appendix C: ANOVA Assumptions Check-Normality of Residuals

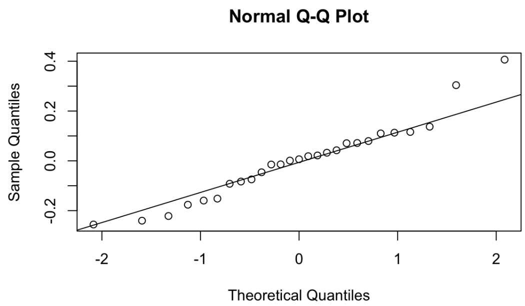


Figure 14: Q-Q Plot

The normality of residuals assumption was assessed using a Normal Q-Q plot. The plot shows the sample quartiles of the residuals plotted against the theoretical quartiles of a normal distribution.

The points in the Q-Q plot generally follow the straight line, suggesting that the residuals are approximately normally distributed. There are some minor deviations from the line at the tails, which is common. Additionally, one point in the upper right corner appears to be a potential outlier.

Despite the minor deviations, the overall pattern indicates that the residuals are reasonably close to a normal distribution, supporting the assumption of normality for the ANOVA model. The potential outlier warrants further investigation to ensure it does not unduly influence the results.

This histogram shows a distribution that is roughly bell-shaped, with a central peak around 0.0. However, the distribution exhibits a slight positive skew, indicating a greater number of residuals with positive values. The tails of the distribution are relatively short, suggesting that there are few extreme residuals. Two bars on the far right are separated from the main distribution and might indicate potential outliers.

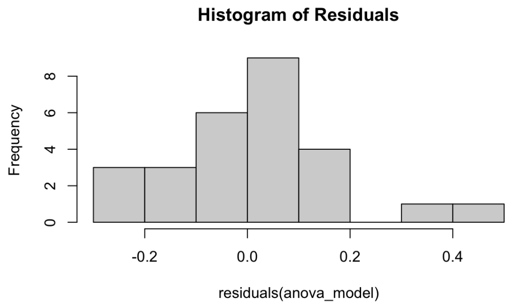


Figure 15: Histogram

Despite the slight positive skew, the histogram suggests that the residuals are reasonably close to a normal distribution. This provides support for the normality assumption of the ANOVA model. The potential outliers indicated by the two separated bars should be further investigated to ensure they do not unduly influence the results.

```
        Shapiro-Wilk normality test

data:  residuals(anova_model)
W = 0.95498, p-value = 0.2823

Loading required package: carData
Warning: group coerced to factor.Levene's Test for Homogeneity of Variance (center
= median)
      Df F value Pr(>F)
group  2  1.5559 0.2316
      24

        Bartlett test of homogeneity of variances

data:  TrialAverage by Blade.Shape
Bartlett's K-squared = 4.5344, df = 2, p-value = 0.1036
```

Figure 16: ANOVA Check

The assumptions of normality of residuals and homogeneity of variances were assessed using the following statistical tests:

- Shapiro-Wilk Normality Test: The Shapiro-Wilk test on the residuals of the ANOVA model yielded a p-value of 0.2823, which is greater 0.05. This indicates that the residuals are not significantly different from a normal distribution, supporting the assumption of normality.

- Levene's test for homogeneity of variances resulted in a p-value of 0.2316, which is greater than 0.05. This suggests that the variances of the residuals are not significantly different across the groups defined by Blade Shape, supporting the assumption of equal variances.

- Bartlett's test also resulted in a p-value of 0.1036, which is greater than 0.05, further supporting the assumption of equal variances across the groups.

Based on these test results, the assumptions of normality of residuals and homogeneity of variances appear to be met for the ANOVA model. This strengthens the validity of the ANOVA results.

D Appendix D: Tukey HSD Output (Full)

This appendix contains the complete output of the Tukey HSD post-hoc test, which was used to determine which specific group means of Blade.Shape was significantly different from each other.

```
Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = TrialAverage ~ Blade.Shape + Blade.Length..cm. +
Added.Weight..g., data = data)

$Blade.Shape
                           diff        lwr        upr     p adj
Rectangular-Curved   0.46444444  0.2672278  0.6616611 0.0000171
Tapered-Curved       0.02944444 -0.1677722  0.2266611 0.9256539
Tapered-Rectangular -0.43500000 -0.6322166 -0.2377834 0.0000414
```

Figure 17: Tukey HSD

D.1 Explanation of Columns

- diff: The difference between the means of the two groups being compared.
- lwr: The lower bound of the 95% confidence interval for the mean difference.
- upr: The upper bound of the 95% confidence interval for the mean difference.
- p adj: The adjusted p-value for the comparison. This is the p-value after adjusting for multiple comparisons, controlling for the family-wise error rate.

D.2 Interpretation

- Rectangular vs. Curved: The mean Trial Average for rectangular blades is 0.46444444 higher than for curved blades. The adjusted p-value (0.0000171) is highly significant (p¡0.001), indicating a statistically significant difference between these two groups.
- Tapered vs. Curved: The mean Trial Average for tapered blades is 0.02944444 higher than for curved blades. However, the adjusted p-value (0.9256539) is not significant (p¿0.05), indicating no statistically significant difference between these two groups.
- Tapered vs. Rectangular: The mean Trial Average for tapered blades is 0.43500000 lower than for rectangular blades. The adjusted p-value (0.0000414) is highly significant (p¡0.001), indicating a statistically significant difference between these two groups.

D.3 Conclusion

The Tukey HSD test demonstrates that rectangular blades have significantly higher average trial averages compared to both curved and tapered blades. There is no statistically significant difference between curved and tapered blades.

E Appendix E: Regression Model Diagnostics - Homoscedasticity

The assumptions of linear regression were checked using diagnostic plots, as described in Fox and Weisberg [3].

The assumption of homoscedasticity (equal variances) was assessed using Residuals vs. Fitted Values plot.

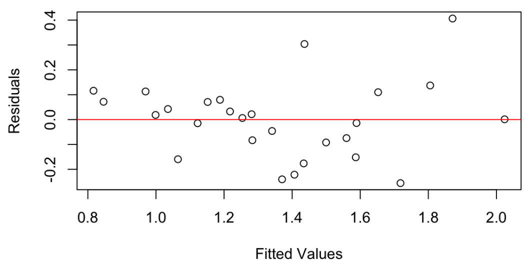


Figure 18: Fitted Plot

The plot shows the residuals plotted against the fitted values from the regression model. The points are randomly scattered around the horizontal line at zero, indicating that the residuals are randomly distributed. The spread of the points appears to be reasonably constant across the range of fitted values, with no clear patterns or funnel shapes. There are a few points that are slightly further away from the line than the others, but they are not extremely far.

The Residuals vs. Fitted Values plot suggests that the assumption of homoscedasticity is reasonably met for the regression model. There are no clear indications of unequal variances or other patterns that would indicate a serious violation of the assumption.

The assumption of homoscedasticity was further assessed using a Scale-Location plot (Figure 6).

The plot shows the square root of the absolute standardized residuals plotted against the fitted values from the regression model. The red line, representing a smooth fit to the points, is roughly horizontal, indicating that the spread of the residuals is relatively consistent across the range of fitted values. The points are somewhat scattered around the red line, with no strong patterns or trends. There is one point at the far right that

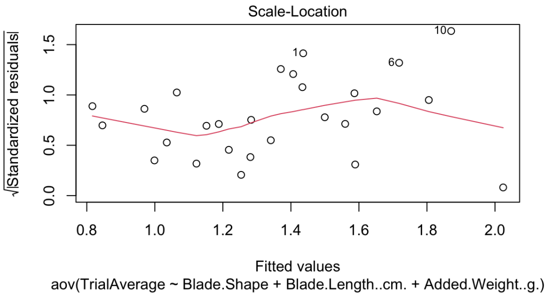


Figure 19: Scale-location

is relatively low, which might be a potential outlier.

The Scale-Location plot suggests that the assumption of homoscedasticity is reasonably met for the regression model. While the red line is not perfectly horizontal, there are no strong indications of unequal variances or other patterns that would indicate a serous violation of the assumption. The potential outlier at the far right should be considered further.

F Appendix F: Figures

Figure 1: The appearance of rectangular blade

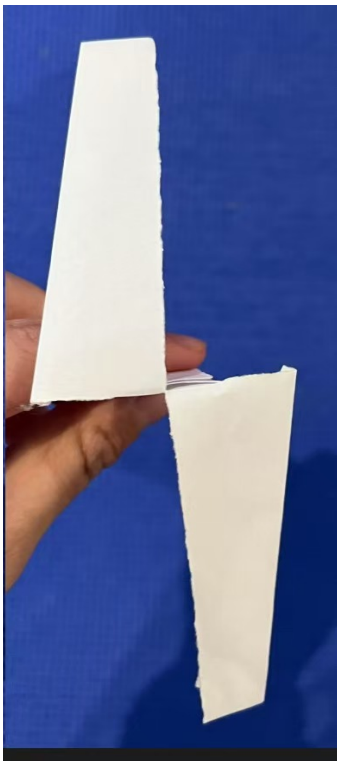


Figure 2: The appearance of tapered blade

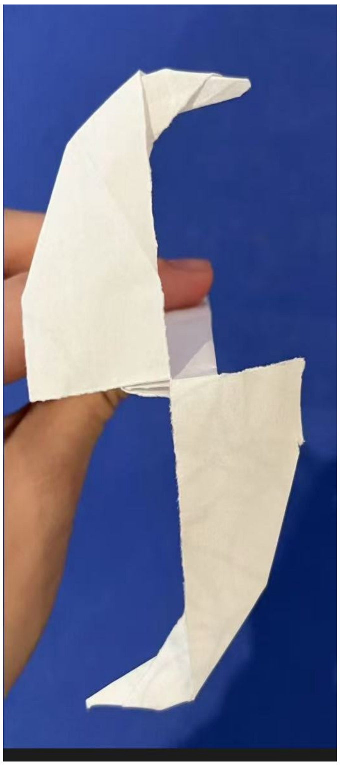


Figure 3: The appearance of curved blade

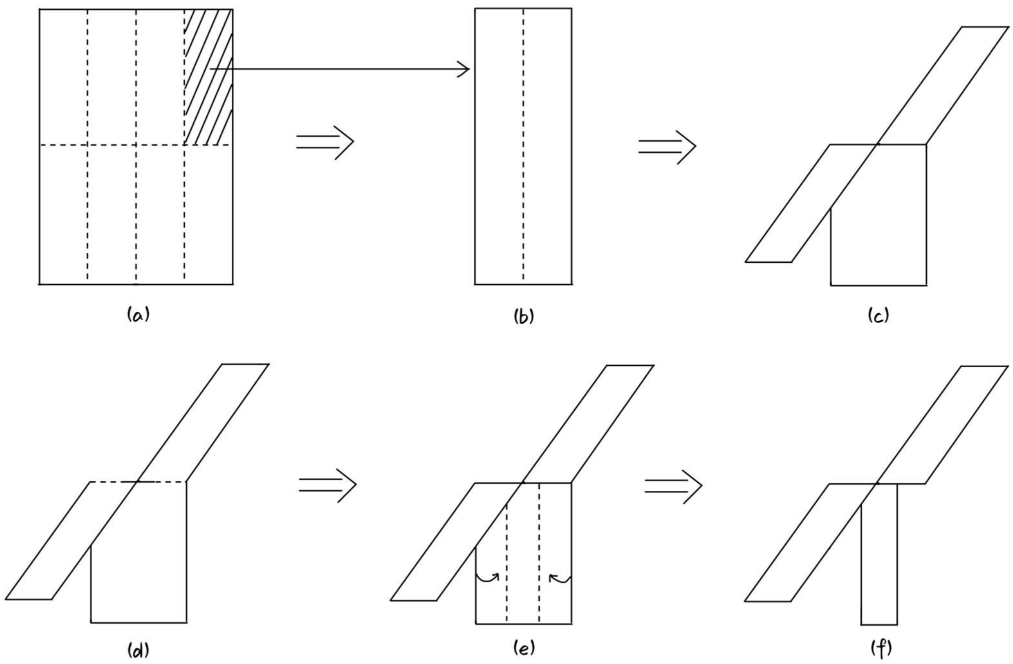


Figure 4: Construction process

```
  Run.Number    Blade.Shape          Blade.Length..cm. Added.Weight..g.
Min.   : 1.0   Length:27          Min.   :5          Min.   :0.0
1st Qu.: 7.5   Class :character   1st Qu.:5          1st Qu.:0.0
Median :14.0   Mode  :character   Median :7          Median :0.5
Mean   :14.0                      Mean   :7          Mean   :0.5
3rd Qu.:20.5                      3rd Qu.:9          3rd Qu.:1.0
Max.   :27.0                      Max.   :9          Max.   :1.0
 Trial.1..s.      Trial.2..s.      Trial.3..s.      Trial.4..s.
Min.   :0.830   Min.   :0.850   Min.   :0.790   Min.   :0.910
1st Qu.:1.090   1st Qu.:1.095   1st Qu.:1.155   1st Qu.:1.130
Median :1.240   Median :1.180   Median :1.290   Median :1.250
Mean   :1.327   Mean   :1.284   Mean   :1.434   Mean   :1.366
3rd Qu.:1.500   3rd Qu.:1.430   3rd Qu.:1.645   3rd Qu.:1.560
Max.   :2.670   Max.   :2.040   Max.   :2.340   Max.   :2.400
 random.order       TrialAverage
Min.   :0.008864   Min.   :0.905
1st Qu.:0.160003   1st Qu.:1.119
Median :0.299235   Median :1.260
Mean   :0.367376   Mean   :1.353
3rd Qu.:0.533908   3rd Qu.:1.474
Max.   :0.996367   Max.   :2.277
```

Figure 5: Summary data

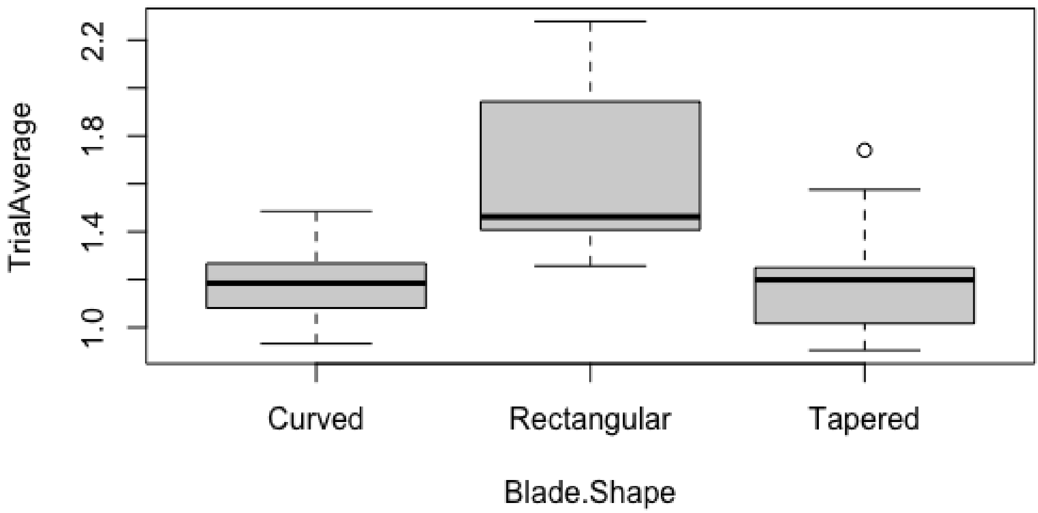


Figure 6: Box plot of different blade shape

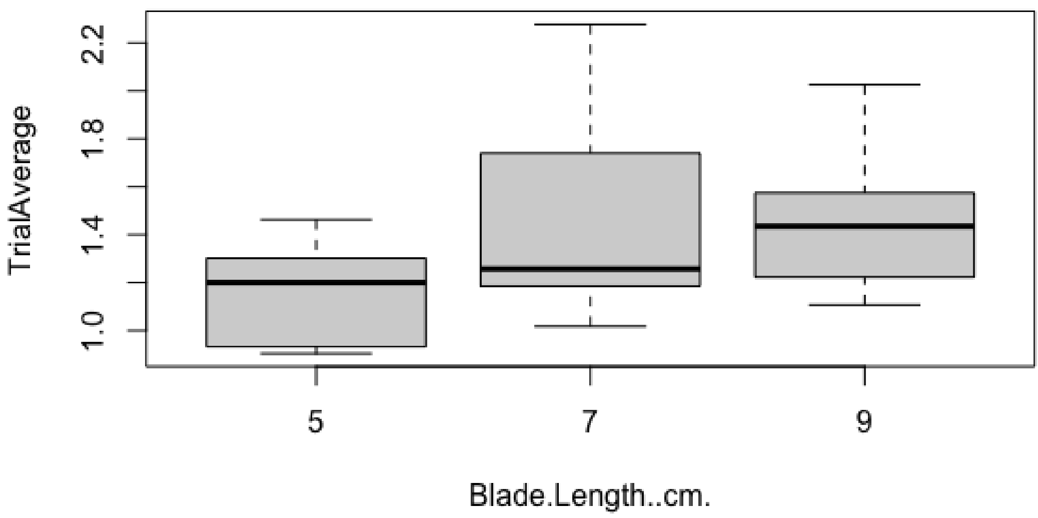


Figure 7: Box plot of different blade length

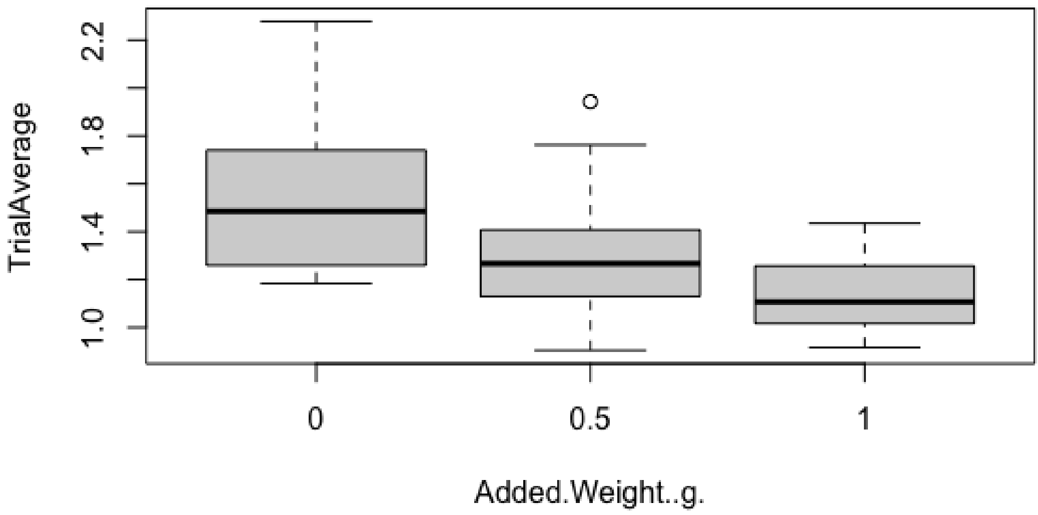


Figure 8: Box plot of different added weight

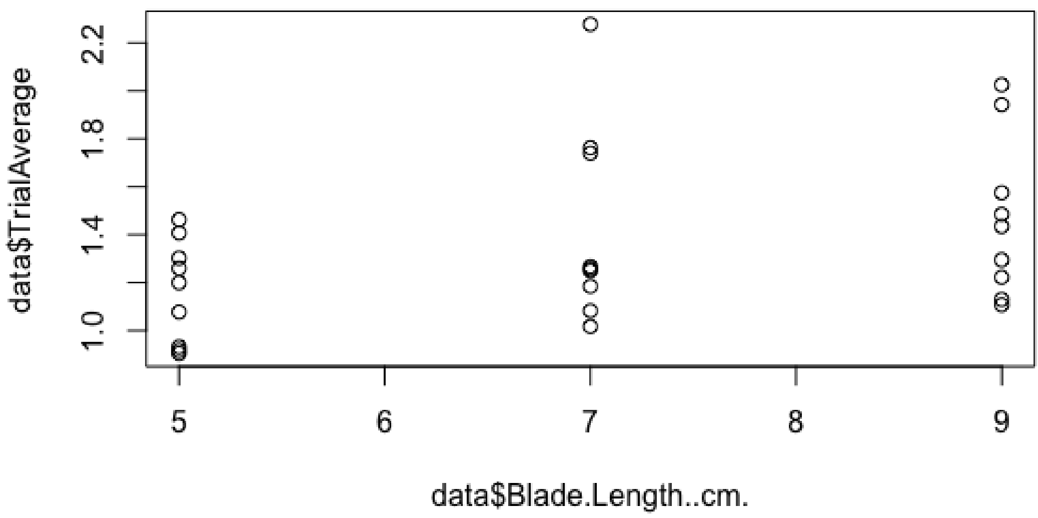


Figure 9: Scatter plot of different blade length

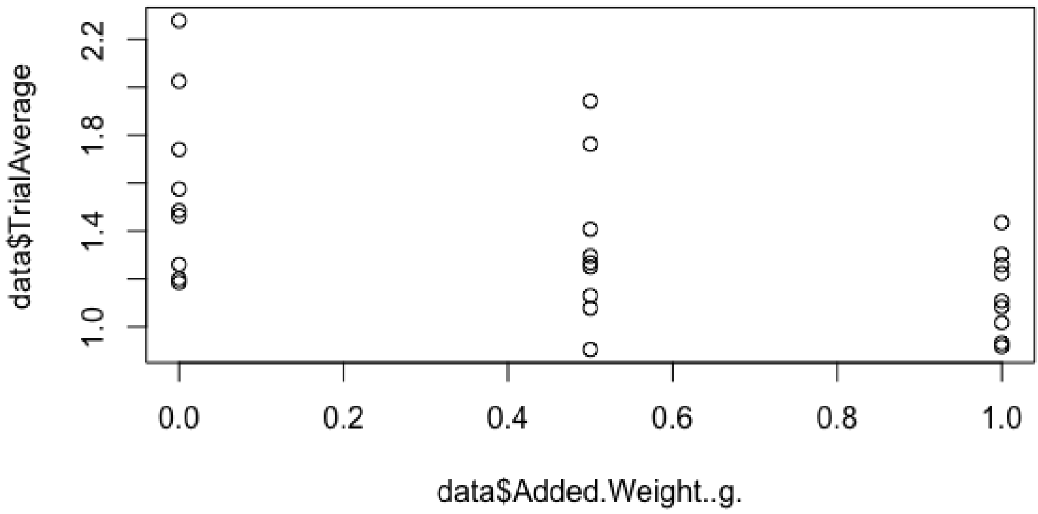


Figure 10: Scatter plot of different added weight

```
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Blade.Shape        2 1.2174  0.6087   21.95 5.75e-06 ***
Blade.Length..cm.  1 0.4209  0.4209   15.18 0.000778 ***
Added.Weight..g.   1 0.8602  0.8602   31.02 1.34e-05 ***
Residuals         22 0.6102  0.0277                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 11: ANOVA Table

```
        Pearson's product-moment correlation

data:  data$Blade.Length..cm. and data$TrialAverage
t = 1.9786, df = 25, p-value = 0.05898
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.01401408  0.65621506
sample estimates:
      cor
0.3679595


        Pearson's product-moment correlation

data:  data$Added.Weight..g. and data$TrialAverage
t = -3.0927, df = 25, p-value = 0.004828
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 -0.7551053 -0.1825081
sample estimates:
       cor
-0.5260383
```

Figure 12: Correlation Result

```
Call:
lm(formula = TrialAverage ~ Blade.Length..cm. + Added.Weight..g.,
    data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.38630 -0.16894 -0.06449  0.15836  0.70620 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)        1.03609    0.24264   4.270 0.000266 ***
Blade.Length..cm.  0.07646    0.03252   2.351 0.027271 *  
Added.Weight..g.  -0.43722    0.13008  -3.361 0.002594 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.276 on 24 degrees of freedom
Multiple R-squared:  0.4121,	Adjusted R-squared:  0.3631 
F-statistic: 8.412 on 2 and 24 DF,  p-value: 0.001704
```

Figure 13: Regression Result

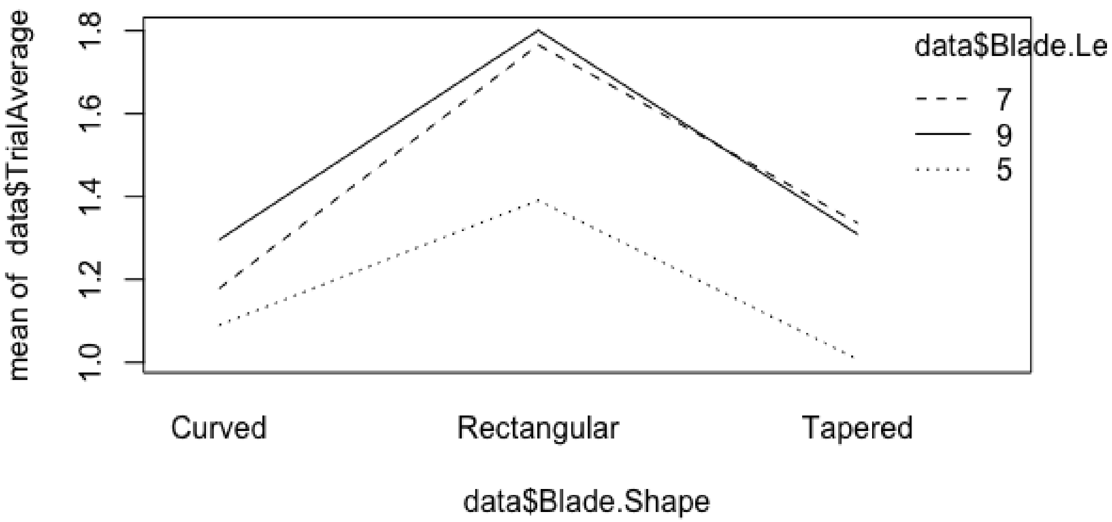


Figure 14: Interaction plots of blade shape and blade length

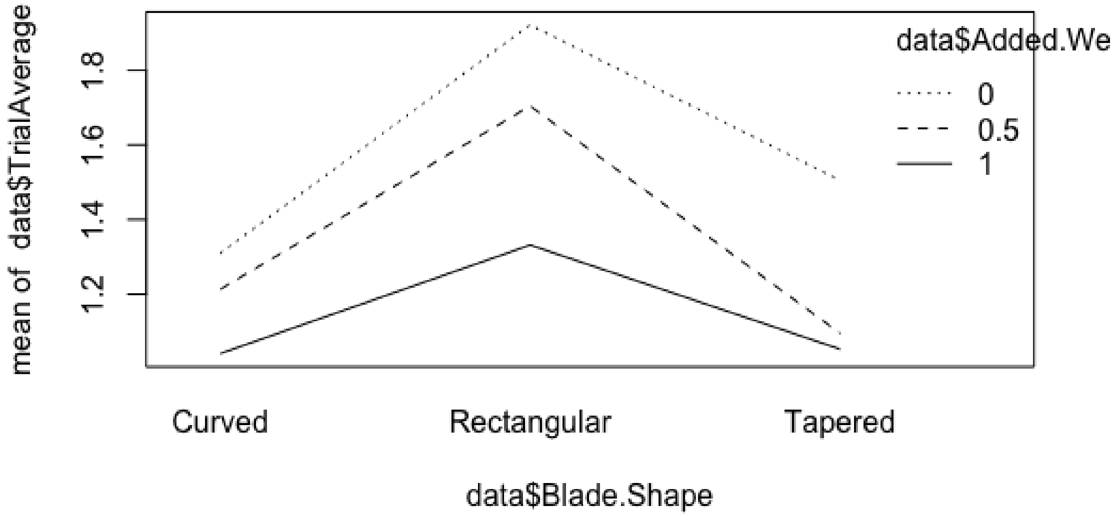


Figure 15: Interaction plots of blade shape and added weight

```
Call:
lm(formula = TrialAverage ~ Blade.Length..cm. * Added.Weight..g.,
    data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.38630 -0.15310 -0.09449  0.18127  0.70620

Coefficients:
                                    Estimate Std. Error t value Pr(>|t|)
(Intercept)                          0.87567    0.37495   2.335   0.0286 *
Blade.Length..cm.                    0.09938    0.05216   1.905   0.0693 .
Added.Weight..g.                    -0.11639    0.58086  -0.200   0.8430
Blade.Length..cm.:Added.Weight..g. -0.04583    0.08081  -0.567   0.5761
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2799 on 23 degrees of freedom
Multiple R-squared:  0.4202,    Adjusted R-squared:  0.3446
F-statistic: 5.557 on 3 and 23 DF,  p-value: 0.005107
```

Figure 16: Interaction Table

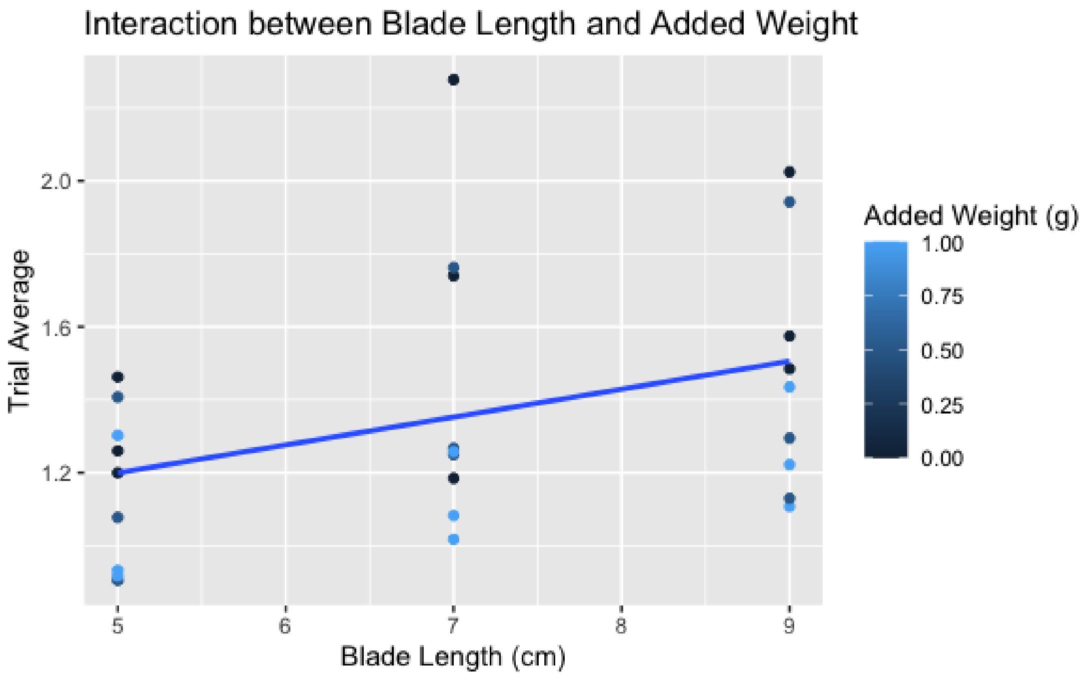


Figure 17: Interaction plots of blade length and added weight

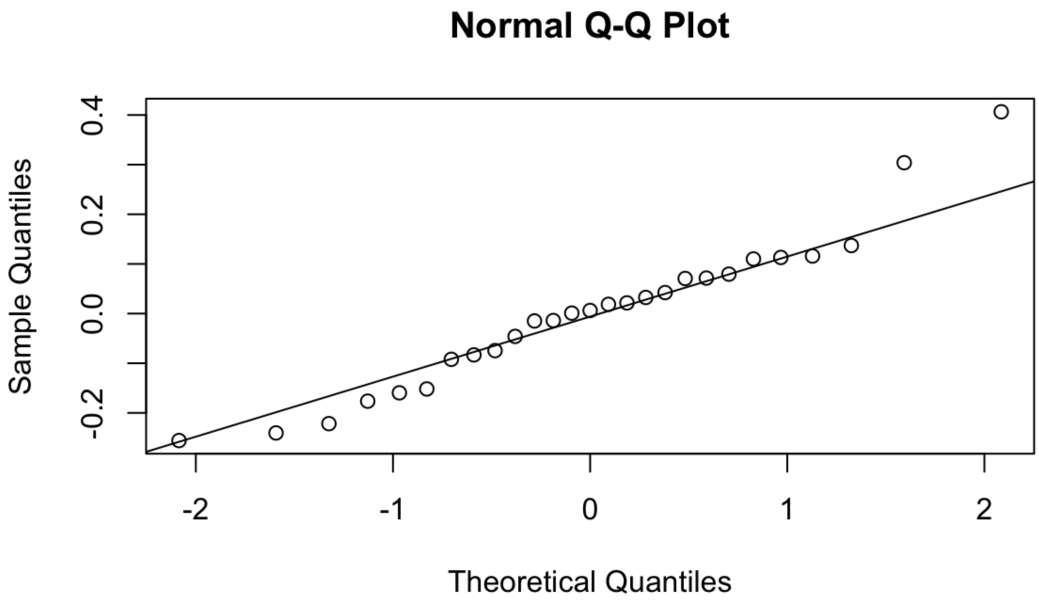


Figure 18: Q-Q Plot

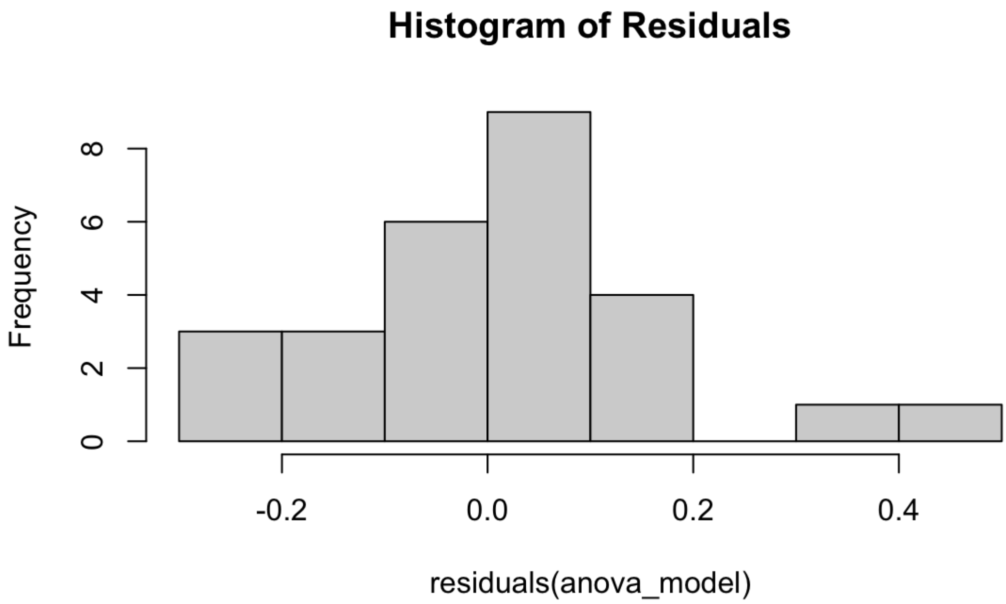


Figure 19: Histogram

```
        Shapiro-Wilk normality test

data:  residuals(anova_model)
W = 0.95498, p-value = 0.2823

Loading required package: carData
Warning: group coerced to factor.Levene's Test for Homogeneity of Variance (center
= median)
      Df F value Pr(>F)
group  2  1.5559 0.2316
      24

        Bartlett test of homogeneity of variances

data:  TrialAverage by Blade.Shape
Bartlett's K-squared = 4.5344, df = 2, p-value = 0.1036
```

Figure 20: ANOVA Check

```
Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = TrialAverage ~ Blade.Shape + Blade.Length..cm. +
Added.Weight..g., data = data)

$Blade.Shape
                           diff        lwr        upr     p adj
Rectangular-Curved   0.46444444  0.2672278  0.6616611 0.0000171
Tapered-Curved       0.02944444 -0.1677722  0.2266611 0.9256539
Tapered-Rectangular -0.43500000 -0.6322166 -0.2377834 0.0000414
```

Figure 21: Tukey HSD

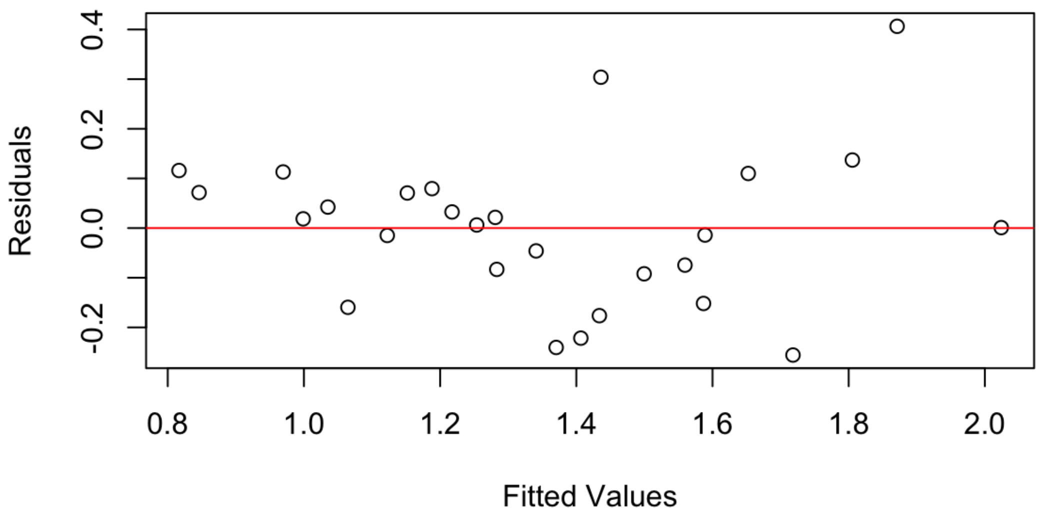


Figure 22: Fitted Plot

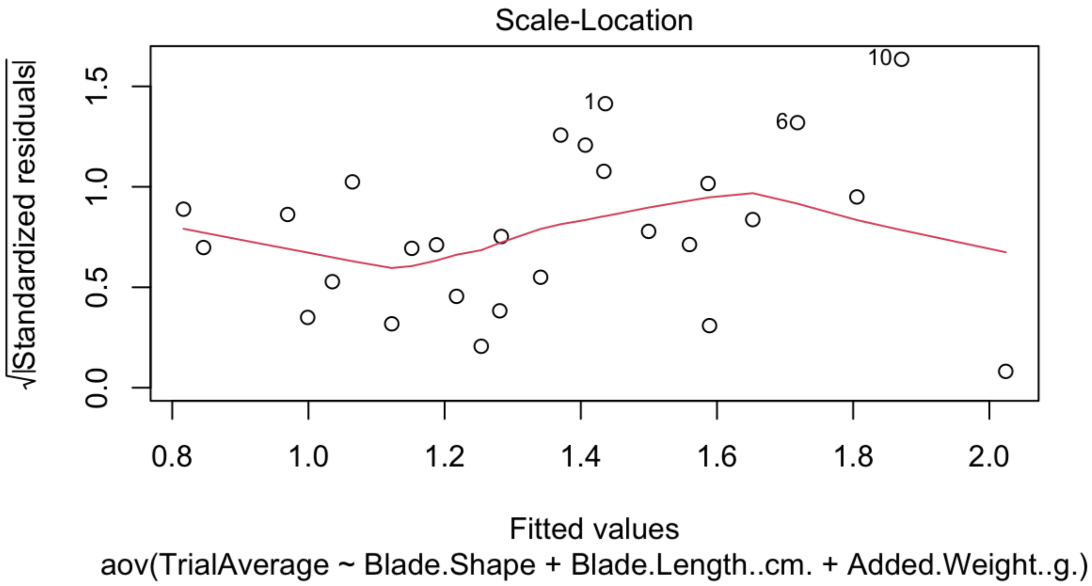


Figure 23: Scale-location