N = 10

M =2 i.e. 2 people share birthday
Group Size = 1 to M-1,
Number of group = 10/1 = 10

10

Favourable ways for no Collison = 365.364.363.362.361.360.359.358.357.356
This can be written as permutation of 365 days taken 10 at a time = P(365,10)
Total Ways = 365 ^10 i.e. every one can take 365 birthdays.

Probability of no 2 people share birthday = $\frac{P(365,10)}{365^\wedge 10}$……………………..$Q_1$

Probability of 2 people share birthday = 1 - $\frac{P(365,10)}{365^\wedge 10}$ =

N = 10

2*1=2	8
+	
2*2 =4	6
+	
2*3 = 6	4
+	
2*4 = 8	2
+	
2*5 =10	0

M =3 i.e. 3 people share birthday
Group Size k = 1 to M-1 i.e. 2,
Number of group = N/k
=10/2 = 5

Because group size is 2, rest of the people can form group of size 1 only.

1. Number of ways *i* group of 2 can be chosen ${}^{n}C_2 * {}^{n-2}C_2 \ldots\ldots {}^{n-2(i-1)}C_2 = \frac{n!}{(2!)^i * (n-2i)!}$
2. *i* birthdays for these group can be chosen in ${}^{365}C_i = \frac{365!}{i! * (365-i)!}$
3. There will be *n-2i* people left and they can take *365 – i* birthdays = P (365-i, n-2i) = $\frac{(365-i)!}{(365-(n-i))!}$

Combining everything = $\frac{n!}{(2!)^i * (n-2i)!} \quad \frac{365!}{i! * (\cancel{365-i})!} \quad \frac{(\cancel{365-i})!}{(365-(n-i))!} = \frac{P(N,2i) * P(365,n-i)}{(2!)^i * i!}$

Summing for *N/k pairs* H(N, 365, 2) = $\sum_{i=1}^{N/k} \frac{P(N,2i) * P(365,n-i)}{(2!)^i * i!}$ this is all possible ways where no collision can happen

$Q_2 =$ H(N,365, 2) / 365^n ....

Probability of 3 people have same birthday = $1 - \sum_{i=1}^{M-1} Q_i$

Generalized solution M
Group Size k = 1 to M-1,
Number of group = N/k

1. Number of ways *i* group of k can be chosen $\quad {}^{n}C_k * {}^{n-k}C_k \ldots\ldots {}^{n-k(i-1)}C_k = \frac{n!}{(k!)^i*(n-ki)!}$
2. *i* birthdays for these group can be chosen in $\quad {}^{365}C_i = \frac{365!}{i!*(365-i)!}$
3. There will be *n-ki* people left and they can further for group of size from 1 to *k-1* and this can be calculated in similar way

$\sum_{j=1}^{M-1} H(n-ki, 365-i, j)$ ………….Check previous slide

Combining everything **H(365, n, k)** = $\sum_{i=1}^{n/k} \frac{n!}{(k!)^i*(n-ki)!} \frac{365!}{i!*(365-i)!} \sum_{j=1}^{i-1} H(n-ki, 365-i, j)$

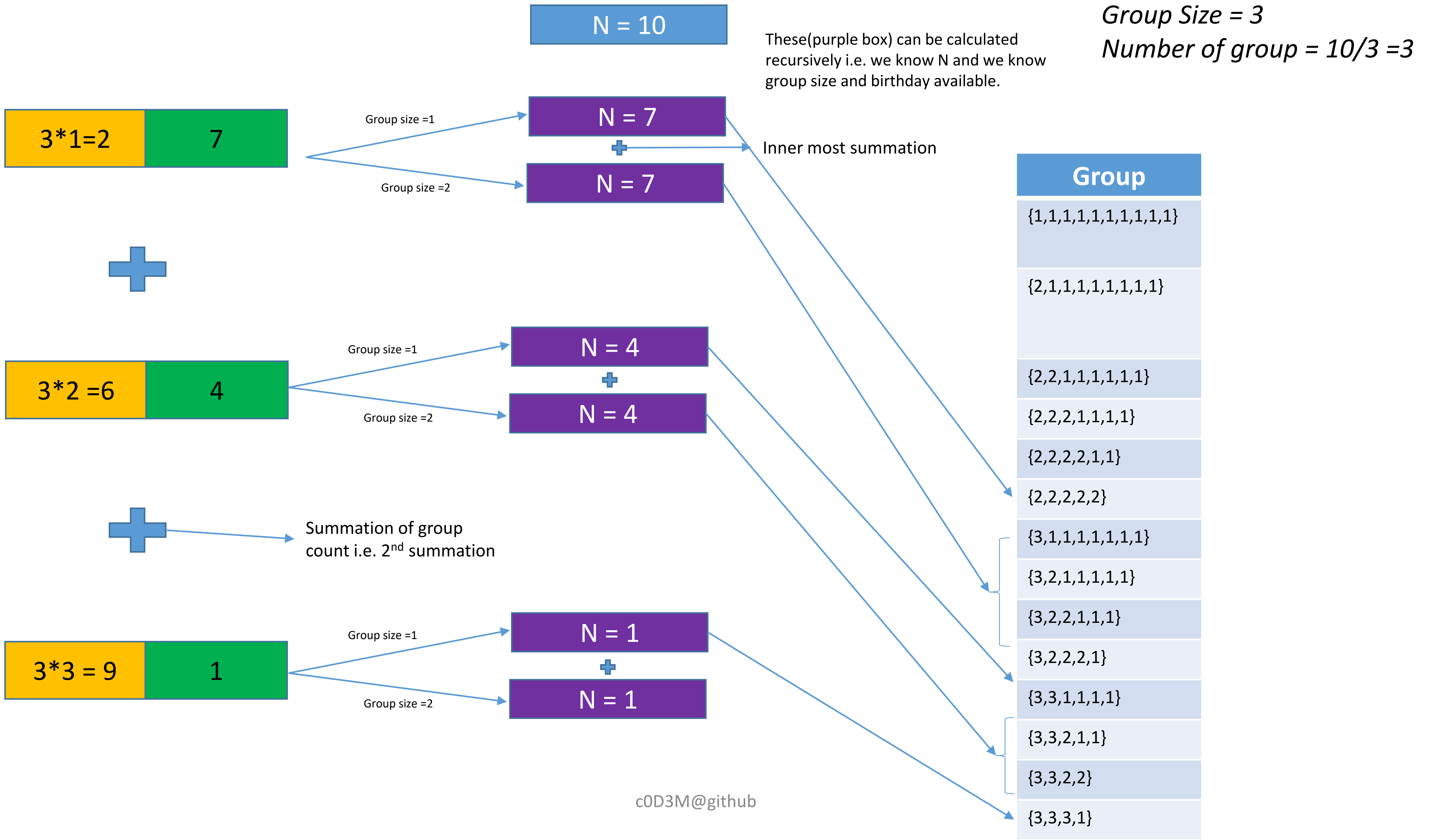
This is total ways for which no collision can occur if M people share birthday out of N.

Probability Q(365, n, k) = H(365, n, k) / 365^n

Replacing that in above equation accordingly we get

$$\sum_{k=1}^{M-1} \sum_{i=1}^{n/k} \frac{n!}{365^{ki}(k!)^i * (n-ki)!} \frac{365!}{i! * (365-i)!} \sum_{j=1}^{i-1} \frac{Q(n-ki, 365-i, j) * (365-i)^{n-ki}}{365^{n-ki}}$$

This is the probability when in a group of N, M people **not share** a birthday, subtracting from 1 will give the probability of sharing birthdays.


N = 10
These(purple box) can be calculated recursively i.e. we know N and we know group size and birthday available.
Group Size = 3
Number of group = 10/3 =3
3*1=2
7
Group size =1
N = 7
Inner most summation
Group size =2
N = 7
3*2 =6
4
Group size =1
N = 4
Group size =2
N = 4
Summation of group count i.e. 2nd summation
3*3 = 9
1
Group size =1
N = 1
Group size =2
N = 1
c0D3M@github
Group
{1,1,1,1,1,1,1,1,1,1}
{2,1,1,1,1,1,1,1,1}
{2,2,1,1,1,1,1,1}
{2,2,2,1,1,1,1}
{2,2,2,2,1,1}
{2,2,2,2,2}
{3,1,1,1,1,1,1,1}
{3,2,1,1,1,1,1}
{3,2,2,1,1,1}
{3,2,2,2,1}
{3,3,1,1,1,1}
{3,3,2,1,1}
{3,3,2,2}
{3,3,3,1}