

# INDEX

- $\Gamma$ -representation, 213
- admissible, 293, 326
  - $p$ -admissible, 332
  - $*-q$ -admissible, 332
  - $*-$ admissible, 327
- algorithm
  - $\nu$ -SVM, 285
  - Chaikin, 34
  - classification, 275
  - corner cutting, 137
  - de Boor, 135
  - de Casteljau, 134
  - Doo-Sabin, 173
  - learning
    - ERM, 258, 260
    - multi-task, 284
    - online, 285
    - tree-based, 240
- antilinearity, 333
- approximation
  - error, 42, 265
  - least square, 86
  - order, 43
  - with regularization, 86
- Bézier points, 126
- basis
  - $\Gamma$ -basis, 213
  - B-spline, 136
  - B-spline like, 136
  - Bernstein, 133
  - Bernstein-like, 117
  - piecewise, 144
  - biorthogonal, 89
  - dual, 89
  - Gröbner basis, 213
  - H-basis, 213
  - hierarchical, 70, 93
  - homogeneous, 218
  - Newton, 203
  - of an ideal, 213
  - Riesz, 88
  - wavelet, 88
- Bayes rule, 275
- Bessel property, 294
- BHS, 51
- blossom, 126
  - for polynomials, 131
- cancellation property, 88
- central surface, 180
- characteristic map, 164
  - complex, 169
  - normalized, 165
- classifier
  - associated with RKHS, 276
  - binary, 275
  - support vector machine, 276
  - SVM
    - $q$ -norm soft margin, 278
    - hard margin, 276
    - soft margin, 277
- coherent state, 292
  - from a unitary representation, 293
- commuting property, 329
- complete monotonicity, 5
- condition number, 238
- consistency condition, 152
- control point, 133, 158
- coordinate system
  - local, 179
- covering number, 261
  - $\ell^p$ -empirical, 264
- cross validation, 95
  - generalized, 95

- multilevel version of, 96
  - potential, 95
  - multilevel version of, 95
- curvature continuity
  - non-trivial, 177
  - trivial, 177
- data
  - generic, 164
- data compression, 92
- data fidelity term, 86
- DFT, 168
- difference
  - divided, 207
  - multivariate, 204
  - finite, 203
  - forward, 5
- dimension
  - $V_\gamma$  dimension, 264
- distance
  - fill, 237
  - Hausdorff, 42
  - separation, 237
- distribution
  - conditional, 258
  - marginal, 258
- domain decomposition, 241
- edge
  - convex, 28
  - straight, 28
- eigencoefficient, 163
- eigenfunction, 163
  - subsubdominant, 178
- eigenvalue
  - subdominant, 163
  - subsubdominant, 177
- eigenvector
  - ineffective, 160
- ENO, 29
  - weighted, 31
- entropy
  - maximum discrimination, 285
  - metric, 264
- ERM, 258

- error
  - $\phi$ -error, 279
  - empirical, 279
  - approximation, 42, 265
  - empirical, 259, 260
    - minimization of, 260
    - penalized, 268
  - ERM decomposition, 260
  - excess generalization, 269
  - generalization, 259
  - misclassification, 275
  - regularization, 269
    - decomposition, 270
  - sample, 260
- evaluation point, 239
- expansion
  - far field, 239
  - multipole, 239
- exponent
  - separation, 282
  - Tsybakov's noise, 282
  - variancing, 271, 280
- extraordinary point, 150
- feature selection, 284
- Fourier index, 168
- Fourier transform, 234
  - discrete, 168
- FSI, 58
- function
  - $\psi$ -regression, 265
  - central height, 154
  - classifying loss, 278
  - conditionally positive definite, 236
  - empirical target, 260
  - harmonic, 101
  - Hilbert, 217
    - affine, 218
    - homogeneous, 217
  - Lipschitz, 262
  - loss, 258
  - nodal, 68
  - piecewise, 137
  - piecewise  $C^n$ , 138
  - refinable, 58

- regressing loss, 259
- regression, 258
- regularizing, 269
- scaling, 88
- target, 260
- FVS, 76
- Gauss map, 154
- GCV, 95
- generator, 57
  - for  $\rho(G)$ , 316
  - linearly independent, 58
    - locally, 58
    - over a set, 58
  - local, 58
  - refinable, 58
  - stable, 58
  - symmetric, 65
- geometric characterization, 199
- grading, 210
  - monomial, 212
  - Newton basis for, 218
  - strict, 212
- Haar measure, 299
  - for  $SU_{1,1}$ , 300
- half domain, 170
- half segment, 170
- hyperplane
  - in general position, 200
  - optimal, 277
- ideal, 212
  - basis of, 213
  - generated by  $F$ , 213
  - homogeneous, 213
  - monomial, 213
- inequality
  - probability, 260
  - Bernstein, 13, 14, 261
  - Hoeffding, 261
  - Jackson-Favard, 16
- inner product
  - native, 235
- integral representation
  - Koornwinder's, 8
  - Laplace, 9
- interpolation
  - $C^2$  cubic spline, 40
  - almost poisedness of, 197
  - BHS scheme, 51
  - correct for  $P$ , 194
  - geometric, 50
  - Hermite, 47, 114, 195
    - cubic two-point, 47
    - piecewise, 140
    - quintic two-point, 48
  - Hermite-Birkhoff, 195
  - ideal, 195
  - Lagrange, 195
  - poised for  $P$ , 194
  - polynomial, 194
  - Taylor, 113
    - with multiplicities, 195
- irreducibility, 308
- Jacobi weight, 3
- Jacobian determinant, 164
- K-functional, 17, 267
- kernel
  - conditionally positive definite, 236
  - Gaussian, 264
  - Mercer, 268
  - radial, 234
  - translation invariant, 234
- knot vector, 134
- lattice
  - natural, 200
  - principal, 201
- least interpolant, 209
- leave-one-out technique, 274
- locality, 88
- loss
  - exponential, 278
  - hinge, 277
  - least-square, 259, 278
- lower set, 223
- Möbius transformation, 299

- margin, 277
  - maximal, 277
- mask, 63
- matrix
  - $G$ -equivalent, 160
  - central Hessian, 177
  - cross product, 91
  - discretized Gramian, 91
  - Fourier block, 168
  - influence, 95
  - observation, 91
  - subdivision, 161
  - unimodular, 59
- matrix coefficients, 293, 303
  - orthogonality relations for, 295
- mesh, 56
  - refinement of, 56
- multilevel method, 246
- multiresolution analysis
  - intertwining, 74
- nested iteration, 93
- norm
  - $\epsilon$ -insensitive, 259
- normal
  - central, 154
- normal equation, 91
- normal form, 216
- offset, 278
- operator
  - analysis, 292, 293
  - Bernstein-Durrmeyer, 3
  - Bernstein-Durrmeyer quasi-interpolant, 10
  - dilation, 318
  - Durrmeyer, 3, 284
  - projection, 280
  - shape, 181
  - subdivision, 32
  - synthesis, 292, 293
- optimization
  - convex, 268
- orthogonality relation, 312
- outlier, 97
  - finding procedure, 99
- parameterization, 40
  - centripetal, 41
  - chordal, 41
  - Foley-Nielson, 41
  - improvement, 47
  - uniform, 41
- partition of unity, 243
- patch, 152
- path of length  $n$ , 205
- polygon
  - control, 25
  - edges of, 25
  - convex, 28
  - strictly convex, 28
- polynomial
  - Bernstein basis, 3, 117
  - Jacobi, 8
  - Lagrange fundamental, 199
  - Laurent, 59
  - leading coefficient of, 207
  - Legendre, 9
  - multidegree of, 209
  - Newton fundamental, 202
  - monomial, 207
  - reduced, 214
  - total degree of, 199
- Powell-Sabin 12-split, 71
- prewavelet, 61
  - mask of, 63
- proximity relation
  - 0-proximity, 35
  - 1-proximity, 35
- pseudoaffinity, 128
- quasi-interpolant
  - Bernstein-Durrmeyer, 10
- reduction, 215
- refinement
  - rule, 25, 27
  - equation, 58
- refitting criterion
  - global, 99
  - local, 101

- regression
  - logistic, 278
  - problem, 258
- regularization, 86
  - by smoothing, 249
  - parameter, 268
  - scheme, 268
    - error decomposition, 270
    - multi-kernel, 284
  - term, 86
- remainder of division, 215
- representation, 292
  - $p$ -integrable, 332
  - \*- $q$ -integrable, 332
  - \*-square integrable, 327
  - bounded, 321
  - completely reducible, 296
  - continuous, 293, 321
  - contragredient, 298, 302, 322
  - irreducible, 293
  - nonunitary, 302
  - reducible, 293
  - separately continuous, 321
  - square integrable, 293, 326
  - unitary, 292, 301
  - weak\*-continuous, 321
  - weakly continuous, 321
- reproducing property, 314, 329
- ring, 194
  - graded, 210
- risk, 260
  - empirical risk minimization, 258
- RKHS, 233, 268
- segment, 156
- semi-orthogonality, 89
- separable
  - strictly, 281
  - weakly, 282
- set
  - $V_\gamma$  shattered, 264
  - uniform Glivenko-Cantelli, 263
- smoothing parameter, 233
- source point, 239
- space
  - Beppo-Levi, 235
  - coorbit, 333
  - CW-Pspace, 142
  - CW-space, 115
  - D-invariant, 196
  - Dirichlet, 312
  - EC-piecewise, 140
  - EC-Pspace, 140
  - EC-space, 114
  - extended Chebyshev, 114
  - fractional Sobolev, 94
  - generator of, 57
  - Haar, 197
  - hypothesis, 260
  - interpolation, 208, 267
    - degree reducing, 209, 212
    - of minimal degree, 208
  - Korobov, of periodic distributions, 308
  - local finitely generated, 58
  - multiplicity of, 59
  - native, 236
  - piecewise, 140
  - refinable, 57
  - reproducing kernel Hilbert, 233, 268
  - shift-invariant, 57
  - Sobolev, 234
  - Sobolev, of periodic distributions, 308
  - spline, 134
  - W-piecewise, 140
    - complete, 142
  - W-Pspace, 140
  - W-space, 113
    - complete, 115
  - wavelet, 56
- spline, 150
  - $C_0^k$ , 153
  - box, 59
  - in subdivision form, 156
  - multi-box, 64
  - planar, 153
  - real valued, 153
  - simplex, 204
  - thin-plate, 235

- spline ring, 151, 156
- spline surface, 153
  - $C_r^k$ , 154
  - normal continuous, 154
  - single-sheeted, 154
- square integrability, 305
- standard simplex, 3, 204
- stencil
  - central, 31
  - left, 31
  - regular, 26
  - right, 31
- subdivision
  - mid-point, 57
    - of the quadrilateral, 57
- subdivision operator
  - data dependent, 32
- subdivision scheme
  - $(\lambda, \mu)$ -scheme, 180
  - $C_0^k$ -scheme, 161
  - B-spline, 34
  - Chaikin algorithm, 34
  - convexity-preserving, 28
  - curve, 32
  - displacement-safe, 26
  - Doo-Sabin algorithm, 173
  - ENO-based interpolatory, 29
  - essentially non oscillatory, 29
  - factorizable, 33
  - flip invariant, 167
  - geodesic, 34
  - linear 4-point interpolatory, 25
  - linear for curves, 33
  - linear stationary, 159
  - nonlinear 4-point interpolatory, 26
  - normal continuous, 164
  - projection, 34
  - quasi-linear, 32
  - shift invariant, 167
  - single-sheeted, 164
  - standard, 163
  - standard  $C_1^k$ , 165
  - symmetric, 167
- subdivision surface, 156
- support vector, 277
- SVM, 276
- Sylvester's identity, 119
- system
  - complete W-, 142
  - CW-Psystem, 142
  - generating, 158
- tangent
  - convex, 28
  - straight, 28
- tangent plane
  - central, 154
- tension parameter, 25
- tensor product sites, 223
- term order, 211
- thresholding, 90
- total positivity, 136
- trade-off principle, 246
- tree, 90
- triangulation
  - Fraeijns de Veubeke-Sander, 76
  - type-1, 60
  - type-2, 66
- two-scale equation, 58
- UGC, 263
- uncertainty, 246
  - relation, 248
- uniform convergence
  - theory of, 261
- unisolvent set, 235
- Voronovskaja type result, 16
  - strong, 17
- wavelet, 61
  - adaptive wavelet method, 88
  - basis, 88
  - decomposition, 56
  - space, 56
- Weingarten map, 181
- WENO, 31
- Wronskian, 113
- Zwart-Powell element, 75