№4 - 32

$E_1' = (1\ 2\ -1\ -2)^T \quad X = (7\ 14\ -1\ 2)^T$

$E_2' = (2\ 3\ 0\ -1)^T$

$E_3' = (1\ 2\ 1\ 4)^T$

$E_4' = (1\ 3\ -1\ 0)^T$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 2 & 3 & 2 & 3 & 14 \\ -1 & 0 & 1 & -1 & -1 \\ -2 & -1 & 4 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 3 & 4 & 1 & 12 \end{array}\right) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 3 & 2 & 7 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -1 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

rang = 4 ⇒ доказано

$X' = (0\ 2\ 1\ 2)^T$

№4.53

$x_1 = (1, 1, 1, 1, 0)^T$ $\quad x_4 = (1, 1, 5, 5, 2)^T$

$x_2 = (1, 1, -1, -1, -1)^T$ $\quad x_5 = (1, -1, -1, 0, 0)^T$

$x_3 = (2, 2, 0, 0, -1)^T$

$$\begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 & -1 \\ 1 & -1 & 0 & 5 & -1 \\ 1 & -1 & 0 & 5 & 0 \\ 0 & -1 & -1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & -1 & -1 & 2 & -1 \\ 0 & -2 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & -2 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$x_1, x_2, x_5$ – лин. нез. вектора

$x_3 = x_1 + x_2$ ; $x_4 = 3x_1 - 2x_2$

№ 4.37

$E_1 = (1\ 2\ 1)^T \quad E_2 = (2\ 3\ 3)^T \quad E_3 = (3\ 7\ 1)^T$

$E_1' = (3\ 1\ 1)^T \quad E_2' = (5\ 2\ 1)^T \quad E_3' = (1\ 1\ -6)^T$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 7 \\ 1 & 3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \end{pmatrix}$$ rang = 3 $\Rightarrow$ $\Rightarrow E_1, E_2, E_3$ - базис

$$\begin{pmatrix} 3 & 5 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & -1 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -5 \end{pmatrix}$$ rang = 3 $\Rightarrow$ $\Rightarrow E_1', E_2', E_3'$ - базис

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 3 & 5 & 1 \\ 2 & 3 & 7 & 1 & 2 & 1 \\ 1 & 3 & 1 & 1 & 1 & -6 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 3 & 5 & 1 \\ 0 & -1 & 1 & -5 & -8 & -1 \\ 0 & 1 & -2 & -2 & -4 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 3 & 5 & 1 \\ 0 & 1 & -1 & 5 & 8 & 1 \\ 0 & 0 & -1 & -7 & -12 & -8 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 3 & 5 & 1 \\ 0 & 1 & -1 & 5 & 8 & 1 \\ 0 & 0 & 1 & 7 & 12 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -18 & -31 & -23 \\ 0 & 1 & 0 & 12 & 20 & 9 \\ 0 & 0 & 1 & 7 & 12 & 8 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -42 & -71 & -41 \\ 0 & 1 & 0 & 12 & 20 & 9 \\ 0 & 0 & 1 & 7 & 12 & 8 \end{array}\right) \Rightarrow T_{\beta \to \beta'} = \begin{pmatrix} -42 & -71 & -41 \\ 12 & 20 & 9 \\ 7 & 12 & 8 \end{pmatrix}$$