№2.729

(!) $[a,[b,c]] = b(a,c) - c(a,b)$

Док-во:

$V \triangleq b\times c \quad u \triangleq a\times(b\times c) = a\times V$

$$V = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = (b_y c_z - b_z c_y)\bar{i} + (b_z c_x - b_x c_z)\bar{j} +$$

$$+ (b_x c_y - b_y c_x)\bar{k}$$

$$u = a\times V = (a_y b_x c_y - a_y b_y c_x - a_z b_z c_x + a_z b_x c_z)\bar{i} +$$

$$+ (a_z b_y c_z - a_z b_z c_y - a_x b_x c_y + a_x b_y c_x)\bar{j} +$$ прибавим и вычтем

$$+ (a_x b_z c_z - a_x b_x c_z - a_y b_y c_z + a_y b_z c_y)\bar{k} + a_x b_x c_x \bar{i} +$$

$$+ a_y b_y c_y \bar{j} + a_z b_z c_z \bar{k} - a_x b_x c_x \bar{i} - a_y b_y c_y \bar{j} -$$

$$- a_z b_z c_z \bar{k} = b_x(a_y c_y + a_z c_z)\bar{i} + a_x b_x c_x \bar{i} + b_y(a_z c_z + a_x c_x)\bar{j} +$$

$$+ a_y b_y c_y \bar{j} + b_z(a_x c_x + a_y c_y)\bar{k} + a_z b_z c_z \bar{k} -$$

$$- \big(c_x(a_y b_y + a_z b_z)\bar{i} + a_x b_x c_x \bar{i} + c_y(a_z b_z + a_x b_x)\bar{j} +$$

$$+ a_y b_y c_y \bar{j} + c_z(a_x b_x + a_y b_y)\bar{k} + a_z b_z c_z \bar{k}\big) =$$

$$= (b_x \bar{i} + b_y \bar{j} + b_z \bar{k})(a_x c_x + a_y c_y + a_z c_z) -$$

$$- (c_x \bar{i} + c_y \bar{j} + c_z \bar{k})\cdot(a_x b_x + a_y b_y + a_z b_z) =$$

$= b(a\cdot c) - c(a\cdot b)$, что и т.д.