WORKERS, MACHINES, AND ECONOMIC GROWTH*

JOSEPH ZEIRA

This paper analyzes a model of economic growth, with technological innovations that reduce labor requirements but raise capital requirements. The paper has two main results. The first is that such technological innovations are not everywhere adopted, but only in countries with high productivity. The second result is that technology adoption significantly amplifies differences in productivity between countries. This paper can, therefore, add to our understanding of large and persistent international differences in output per capita. The model also helps to explain other growth phenomena, like divergence or periods of rapid growth.

I. INTRODUCTION

Economists usually view technological innovations as new methods of production, which enable producers to increase output without increasing inputs. This is equivalent to saying that technological innovations enable producers to reduce the required amounts of all inputs per unit of output. But this view on technical progress does not fit many important innovations, which have contributed to economic growth since the industrial revolution. One type of such innovations are machines that have replaced workers in production, such as the steam engine, the train, the automobile, the computer, and many more, which reduced labor inputs, but at the same time increased capital inputs. Another type of such innovations replaces nonskilled with skilled labor. This paper examines how removing the standard assumption, and instead considering innovations, which replace one input by another, affects the analysis of technology adoption and of economic growth. The paper has two main results. First, technological innovations are not everywhere adopted by producers. Technology adoption depends on the prices of the factors of production, and when these are endogenized it depends on the productivity of the country and on the discount rate. Second, technology adoption amplifies differences in underlying parameters and contributes to

* I wish to thank Roland Bénabou and Stanley Fischer for their encouragement. I have benefited from discussions with Avner Ahituv, Kenneth Arrow, Samuel Bentolila, Ramon Caminal, Mymom Dahan, Stephen Durlauf, David Levhari, Joel Mokyr, Julio Rotemberg, Oren Sussman, Manuel Trajtenberg, Andrew Warner, Philippe Weil, and Yishai Yafeh. I also thank Olivier Blanchard and two anonymous referees for very helpful comments. Remaining errors are all mine. Thanks are also due to Brandeis University for its hospitality and to the Falk Institute for financial support.

The Quarterly Journal of Economics, November 1998

the explanation of large observed international differences in output per capita.

The main idea of the paper is presented by a model of economic growth and adoption of technologies that replace workers by machines. In this model the final output is produced by many intermediate goods, where each can be produced either by a manual technology or by an industrial technology after it is invented. The choice of technology depends crucially on wages and on interest rates. Higher wages induce adoption of the industrial technology, since it saves labor, while higher interest rates reduce technology adoption. Since we assume full capital mobility, interest rates are equal everywhere, but wages differ across countries and so does technology adoption. More specifically, we show that some countries adopt all new technologies and stay at the technological frontier, while other countries might stop adopting technologies at some point and remain technologically backward. The model then shows that technology adoption amplifies differences in underlying parameters between countries. Countries, which differ slightly in their basic productivity, might follow very different paths of technology adoption and economic growth.

The results of the paper can be explained by the overlap between the decisions of technology choice and of factors' choice. If producers adopt more technologies, they must increase the amount of capital input in order to buy the necessary machines in which these technologies are embodied.[1] Hence, the cost of capital might be too large, and technologies might not be adopted. We can therefore view the decision of technology adoption as a standard decision of allocation along a production function (or along an isoquant), where capital changes with technologies. But note that this production function (or isoquant) differs from standard production functions in being relatively flat. The reason is that increasing the capital input enables adoption of better technologies, and thus the marginal productivity of capital does not fall by much. Hence, the production function is flat, and small changes in its slope can lead to large changes in capital, technology, and output. This is the intuitive reason for the amplification effect of technology adoption in this model.

The issue of international output differentials, to which our model is applied, has always puzzled economists and has gained

1. Note, that in standard models of technology adoption producers can choose to increase capital input as they adopt new technologies, but they do not *have* to, since the new technology increases output for any combination of inputs.

renewed interest lately with other issues in economic growth.[2] As emphasized by Lucas [1990], large international differences in output per capita stand in sharp contrast to standard economic theory, especially in a world of capital mobility. Recent empirical studies have shown that large output differentials not only exist, but tend to persist over time.[3] This paper attempts to reconcile these observations with the theory by focusing on differences in technology across countries.[4] Such differences cannot result from differences in technology creation, since most countries adopt innovations rather than invent them. Hence, this paper concentrates on technology adoption across countries.

Recently a number of explanations for differences in technology adoption across countries have been offered: Grossman and Helpman [1991, Ch. 6], Jovanovic and Lach [1991], and Parente and Prescott [1994]. But these studies share the standard view on technological innovations, namely that they enable production of more with less inputs, and hence should be adopted by all. These studies generate differences in technology adoption by assuming adjustment costs to technology adoption. This paper instead assumes no adjustment costs, and differences in technology adoption are a result of the assumption that technical progress requires greater capital inputs.

The paper can also be related to recent empirical cross-country studies, which have attempted to identify exogenous variables that explain international output differences.[5] These studies find that much of the differences can be attributed to geography, to resource endowments, to infrastructure, and to political regime. Such variables can be viewed as components of what is called productivity in this paper, that differs across countries. These studies therefore find that differences in productivity may cause large differences in output per capita. This paper provides a theoretical explanation for such a strong effect, through the amplification effect of technology adoption.

2. The renewed interest in growth has followed the seminal papers of Baumol [1986], Romer [1986], and Lucas [1988]. The issue of international growth differentials is sometimes referred to as the question of convergence. See Baumol [1986], Barro [1991], and Barro and Sala-i-Martin [1992].

3. See Maddison [1995], Ben-David [1994], Quah [1993, 1996], and Sala-i-Martin [1994].

4. Other explanations of international differences refer to capital market imperfections and nonconvexities of various sorts. See Galor and Zeira [1993], Bénabou [1993], and others.

5. See Sachs and Warner [1997], Hall and Jones [1997], and Sala-i-Martin [1997].

The paper is also related to the debate among economic growth historians, which followed the work of Habbakuk [1962]. He found that technical progress in the United States had been more rapid than in Britain during the nineteenth century. Habbakuk attributed it to higher wages due to greater land abundance in the United States.[6] This paper, therefore, offers a theoretical framework, which formalizes the Habbakuk hypothesis in a general equilibrium model. Two other branches in the growth literature of the sixties that are somehow relevant to this paper are embodied technical progress and induced innovations.[7] But that literature dealt with issues very different from those discussed in this paper.

The paper is organized as follows. Section II presents an expository example with one intermediate good, while Section III presents the full model with many intermediate goods. Section IV describes the equilibrium, and Section V presents the results on economic growth and international output differentials. Section VI explains the main results of the paper, and Section VII presents two examples. Section VIII discusses the empirical implications of the model with respect to the capital-output ratio. Section IX concludes, and the mathematical proofs appear in the Appendix.

II. An Expository Example

Consider an economy with one final good Y which is used both for consumption and for investment. The final good is produced by an intermediate good X, which is produced by labor and capital. The output of the final good is described by

$$Y = aX, \tag{1}$$

where a is a productivity parameter, which might differ across countries. The intermediate good is produced by labor and capital in fixed proportions. There are two potential technologies. One is a manual technology, according to which production of one unit of X requires l_0 units of labor and k_0 units of capital (tools). The second is an industrial technology, with unit requirements of l_1 units of

6. This debate is discussed in David [1975].

7. Vintage models of embodied technical progress began with Solow [1960] and are reviewed in Burmeister and Dobell [1970, Ch. 3]. For discussions of induced innovations see Hicks [1932], Kennedy [1962], Burmeister and Dobell [1970, Ch. 3], and Champernowne [1963].

labor and k_1 units of capital (a machine). We assume that

$$l_1 < l_0 \text{ and } k_1 > k_0, \tag{2}$$

namely, the industrial technology saves labor but requires more capital. This is the main assumption of the paper, and it is the point of departure from standard models of technology adoption. While the manual technology exists from time immemorial, the industrial technology is invented at some time T, and is available only afterward. It is further assumed that capital in both technologies must be invested one period ahead of production and that it fully depreciates after one period. Assume that there is a continuum of workers in the economy, of size 1. Assume further, that the interest rate is fixed and equal to r. We denote the gross interest rate by $R = 1 + r$.

In this example workers maximize their net income: $Y - R\,K$, where K is total capital input. Figure I presents the production functions of both technologies, where F_0 describes the manual technology and F_1 the industrial technology. Note that until period T only the manual technology is used, and the economy is at A (if $a/R > k_0$). Once the industrial technology is invented, producers can choose whether to adopt it or to continue to use the manual technology. The new technology is adopted, and producers

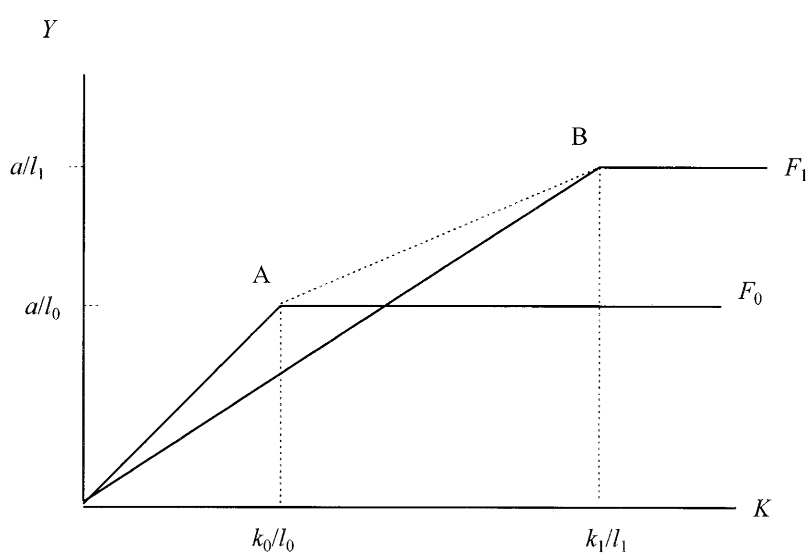

FIGURE I

move from A to B, if

$$a \geq a_0 = R(k_1 l_0 - k_0 l_1)/(l_0 - l_1). \tag{3}$$

Diagrammatically, the new industrial technology is adopted if the slope of the segment AB in Figure I is greater than or equal to R. Note that this result depends crucially on the assumption that capital requirements are greater under the new technology, since otherwise the F_1 curve would be everywhere above the F_0 curve and the new technology would be adopted for all a and R. Hence, this example shows that if technological innovation increases capital requirements, it is not everywhere adopted. This is the first main result of the paper.

Figure II describes the level of aggregate output as a function of productivity a. Productivity affects output through two channels. One is the direct effect, which is shown by the positive slope of the curve. The other channel is through technology adoption, which leads to the sharp discontinuous rise in output at a_0, which is the threshold of adopting the industrial technology. Hence, technology adoption amplifies differences between countries, which is the second result of the paper. The reason for the strong effect of

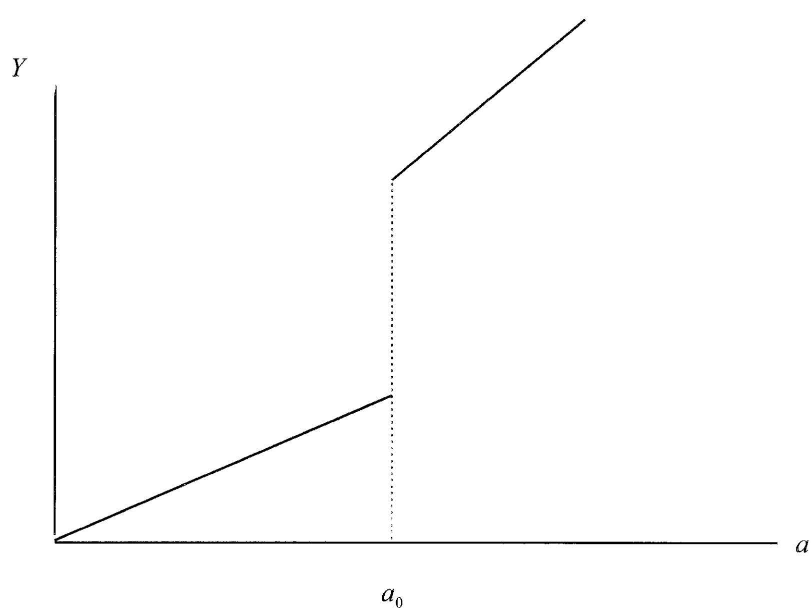

FIGURE II

a/R on output is the relative flatness of the production function in the long run, since the envelope of production functions F_0 and F_1 in Figure I is flatter than both.

III. The Model

The model is set in a discrete time framework. We consider a small open economy in a world with only one final good, which is used both for consumption and for investment. This final good is produced by a continuum of intermediate goods. The set of intermediate goods is the interval [0,1]. Output of the final good is described by

$$\log Y_t = \log a + \int_0^1 \log (x_{j,t})\, dj, \tag{4}$$

where Y_t is output in period t, $x_{j,t}$ is input of the intermediate good j, and a is a productivity parameter, which is country specific. This coefficient may reflect geography like land quality, climate and access to sea, resource endowments, like land abundance and natural resources, or even infrastructure, and should therefore differ across countries. The specific Cobb-Douglas function (4) is chosen for simplicity only, and the results carry through with other production functions as well.

The intermediate goods are produced by labor and capital in fixed proportions, where labor is a variable input and capital has to be invested one period ahead of time and it fully depreciates after one period of production. There are two potential technologies for production of intermediate goods. The first is a manual technology, in which one unit of intermediate good j is produced by $l_0(j)$ units of labor and $k_0(j)$ units of capital (tools). The second technology is industrial, namely a machine that enables production of the intermediate good with much less labor. Such technologies exist only for some of the intermediate goods. For each such intermediate good j production of one unit requires $l_1(j)$ units of labor and a machine made of $k_1(j)$ units of capital, where

$$l_1(j) < l_0(j), \text{ and } k_1(j) > k_0(j), \tag{5}$$

since a machine is costlier than the tools needed for manual production. This is the basic assumption of the paper. It is useful to define the following function h over intermediate goods, which measures the capital required to reduce labor by one unit; namely,

$$h(j) = (k_1(j) - k_0(j))/(l_0(j) - l_1(j)). \tag{6}$$

Our main assumption is, therefore, that $h(j)$ is positive for all j. We now assume that the intermediate goods are ordered by increasing capital cost of labor reduction, so that h is nondecreasing in j. In order to ensure existence and uniqueness of equilibrium, we also assume that the functions l_0, l_1, k_0, k_1, and h are continuous.

As mentioned above, not all intermediate goods have industrial technologies at each point in time, as these are invented over time. Let us assume that these technologies are invented by the order of increasing capital cost.[8] Namely, in each period t the set of intermediate goods, for which labor-replacing machines have been invented, is $[0,f_t]$, where f_t can be viewed as the technological frontier at period t. Obviously, f_t increases with t. In each period $t - 1$, producers of intermediate goods $0 \leq j \leq f_{t-1}$, for which machines have already been invented, can choose between the old manual technology or the new industrial technology for production in period t. If they choose the industrial technology, we say that this technology is adopted for production in period t. Note that producers of the other intermediate goods, for which machines have not been invented yet, have no choice and they continue to use the manual technology.

The economy has overlapping generations of individuals who live two periods each, and their utility is

$$U = U(c_1,c_2), \tag{7}$$

where c_1 and c_2 are consumption in the first and second periods of life, and U is a standard utility function. Each individual supplies one unit of labor only when young. We assume for simplicity that there is no population growth and the size of each generation is normalized to 1.[9]

All markets are assumed to be perfectly competitive. The final good is assumed to be perfectly tradable, but the intermediate goods and labor are not tradable, and their markets are domestic. Capital is assumed to be fully mobile. The world's interest rate is r, and the gross interest rate is $R = 1 + r$. There are two reasons why we adopt the assumption of full capital mobility. The first is to abstract from issues of saving, so that

8. This assumption is made to simplify the presentation only. Zeira [1996] shows that the results are similar even if invention does not follow any specific order.

9. The overlapping generations framework is not necessary and is used for simplicity only. A representative consumer model would yield the same results, since the model assumes full capital mobility for a small open economy.

growth is affected only by technology adoption. The second reason is the challenge presented by Lucas [1990], who claims that international output differentials are more difficult to explain in a world of capital mobility.

IV. Equilibrium

Let the final good serve as a numeraire. Denote wages in period t by w_t and the prices of the intermediate goods by $p_{j,t}$. Denote by M_t the set of all technologies that are adopted in period t, namely, $M_t \subseteq [0,f_{t-1}]$ is the set of all intermediate goods that use machines in period t.

A technology is adopted if it reduces the costs of producing j, namely if

$$w_t l_1(j) + Rk_1(j) \leq w_t l_0(j) + Rk_0(j), \tag{8}$$

or if

$$h(j) \leq w_t/R. \tag{9}$$

The set of adopted technologies is therefore equal to

$$M_t = [0,f_{t-1}] \cap \{j\colon Rh(j) \leq w_t\}. \tag{10}$$

Hence, if wages are higher, the set of adopted technologies M_t is larger. Wages have a positive effect on technology adoption since adoption involves purchasing machines that replace workers. Higher wages provide an incentive to adopt more technologies.[10]

Since h is nondecreasing, we conclude that the set of adopted technologies is an interval, that is

$$M_t = [0,m_t], \tag{11}$$

where m_t is the most recent technology adopted in the economy for production in period t.

There are two cases.

1. Adoption of all existing technologies, so that $m_t = f_{t-1}$, and $w_t \geq Rh(m_t)$.
2. Adoption of some, but not all, existing technologies, when $Rh(m_t) \geq w_t$, and $0 \leq m_t < f_{t-1}$.

We now turn to real wage determination. Profit maximization

10. This point was made by Habbakuk [1962] in his comparison of technical progress in the United States and in Britain. Our model formalizes this point within a general equilibrium model.

by firms, which produce the final good, leads to the following first-order condition:

$$p_{j,t} = \frac{\partial Y_t}{\partial x_{j,t}} = \frac{Y_t}{x_{j,t}}. \tag{12}$$

Each intermediate good is produced with constant marginal productivities, and hence excess profits are driven to zero. The zero profit conditions are

$$p_{j,t} = Rk_1(j) + w_t l_1(j) \qquad \text{if} \qquad 0 \le j \le m_t, \tag{13}$$

and

$$p_{j,t} = Rk_0(j) + w_t l_0(j) \qquad \text{if} \qquad j > m_t. \tag{14}$$

From equations (4), (12), (13), and (14), we derive the following condition:

$$\int_0^{m_t} \log\,[Rk_1(j) + w_t l_1(j)]\,dj + \int_{m_t}^{1} \log\,[Rk_0(j) + w_t l_0(j)]\,dj = \log a. \tag{15}$$

Note that the left-hand side of equation (15) is an increasing function of wages w_t and due to boundedness of the l_i functions, it is also continuous and differentiable. Hence, equation (15) uniquely determines the wage level, namely it determines a unique function w, where $w_t = w(m_t)$. Furthermore, this function is both continuous and differentiable.

From equations (10) and (15) we see that technology adoption is affected by the level of wages on the one hand, but wages are affected by technology adoption on the other hand, as it affects both the technologies used and the amount of capital in use. We next prove the existence of equilibrium and examine its properties.

LEMMA 1. The function w is increasing wherever $w(m) > Rh(m)$, and decreasing wherever $w(m) < Rh(m)$.

Proof. Define a function G by

$$G(m,w) = \int_0^{m} \log\,[Rk_1(j) + wl_1(j)]\,dj + \int_{m}^{1} \log\,[Rk_0(j) + wl_0(j)]\,dj.$$

The function G satisfies

$$G_m(m,w) = \log\,[Rk_1(m) + wl_1(m)] - \log\,[Rk_0(m) + wl_0(m)].$$

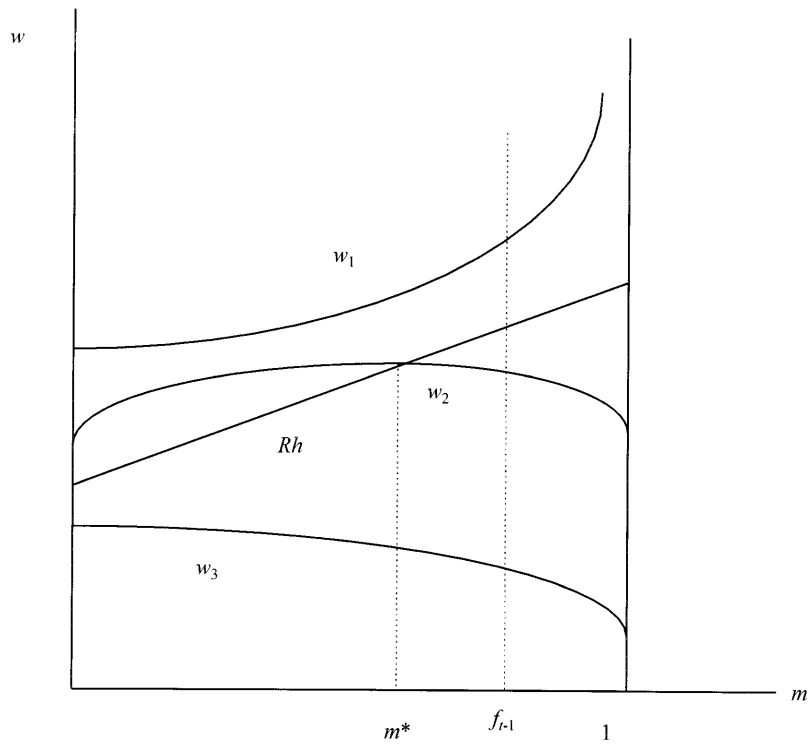

FIGURE III

Hence, if $w(m) > Rh(m)$, we get $G_m(m,w) < 0$. Hence, w is increasing in this region. The proof for the other region is similar. ■

COROLLARY. The functions w and Rh intersect at most once.

From Lemma 1 and Figure III we can derive Proposition 1 which proves the existence and uniqueness of equilibrium.

PROPOSITION 1. For any technological frontier f_{t-1}, there exists a unique equilibrium in the economy, namely a wage rate w_t and a level of technology adoption m_t, which satisfy the equilibrium conditions (10) and (15).

Proof. It follows immediately from Figure III. ■

Formally, the equilibrium level of technology adoption in the economy is

$$m_t = \min\{m^*, f_{t-1}\}, \tag{16}$$

where

$$m^* = \max \{m: w(m) \geq Rh(m)\}. \tag{17}$$

Proposition 2 shows that the equilibrium is welfare maximizing. This proposition also characterizes the equilibrium in a way that is useful for some of the following discussions.

PROPOSITION 2. The equilibrium allocation maximizes net income in the economy: $Y_t - R\,K_t$, where K_t is the aggregate amount of capital used in production of intermediate goods in period t.

Proof. See the Appendix.

V. TECHNOLOGY AND OUTPUT ACROSS COUNTRIES

A. Differences in Technology Adoption

We now turn to analyze the equilibrium, which is described in Figure III by the w and Rh curves. From that figure we deduce that countries can be classified into three types of growth paths, due to differences in the parameter a.

1. In the first case, described by curve w_1 in Figure III, w is everywhere above Rh, and all new technologies are adopted. This case occurs when $w(1) \geq Rh(1)$, or when $a \geq a_1$, where a_1 is defined by

$$\log a_1 = \log R + \int_0^1 \log\,[k_1(j) + h(1)l_1(j)]\,dj.$$

2. In the second case, described by the w_2 curve, w intersects with Rh at one point, m^*. In this case, the economy adopts technologies, as long as $f_{t-1} \leq m^*$. But once the technical frontier passes m^*, the economy stops adopting new technologies and remains stagnant. This case occurs when $w(1) < Rh(1)$ and $w(0) > Rh(0)$, or when $a_0 < a < a_1$, where a_0 is defined by

$$\log a_0 = \log R + \int_0^1 \log\,[k_0(j) + h(0)l_0(j)]\,dj.$$

3. In the third case, described by w_3, w lies everywhere below Rh, and no technology is adopted; namely, $m_t = 0$. This case occurs if $w(0) \leq Rh(0)$, or when $a \leq a_0$.

The above analysis leads to the first main result of the paper, that countries differ in technology adoption. This is not due to adjustment costs or to slow diffusion, since there are no such costs in our framework. Some countries adopt fewer technologies than others because they have lower wages. Therefore, technology adoption, which saves labor but requires more capital, is not profitable. These countries have lower wages because their productivity is lower.[11]

B. Amplification by Technology Adoption

As shown above, differences in technology adoption contribute to output differences across countries. We next show that technology adoption can greatly amplify differences in productivity across countries. To demonstrate it, we first calculate output by adding up labor inputs in all sectors and equating to 1. Equilibrium output is

$$(18)\quad Y_t = \left[\int_0^{m_t} \frac{l_1(j)}{Rk_1(j) + w_t l_1(j)}\, dj + \int_{m_t}^{1} \frac{l_0(j)}{Rk_0(j) + w_t l_0(j)}\, dj\right]^{-1}.$$

It is easy to verify that output Y_t is increasing with technology adoption m_t and with wages w_t.

Figure IV describes how output changes with country productivity a. The solid curve describes this relationship when all machines have already been invented and $f = 1$, while the broken curve describes this relationship when the technology frontier f is less than 1. When a changes between a_0 and a_1, more technologies are adopted, and this adds to the direct effect of productivity. As a result, the curve is much steeper within the interval $[a_0, a_1]$ than outside it, as shown in Proposition 3.

PROPOSITION 3. The slope dY/da in the interval $[a_0,a_1]$, where technology adoption changes, is higher by

$$\frac{Y^3}{aR^2}\frac{1}{h'(m)}\left[\frac{1}{k_0(m)/l_0(m) + h(m)} - \frac{1}{k_1(m)/l_1(m) + h(m)}\right]$$

than outside the interval, due to the effect of technology adoption.

Proof. See the Appendix.

Hence, technology adoption amplifies the effect of productivity. From Proposition 3 it follows that this amplification effect is stronger the flatter h is, and the larger the increase in capital-

11. Note that this result can also be related to Habbakuk [1962]. He claimed that technical progress in the United States exceeded that in Britain due to higher wages in the United States, as a result of greater land abundance.

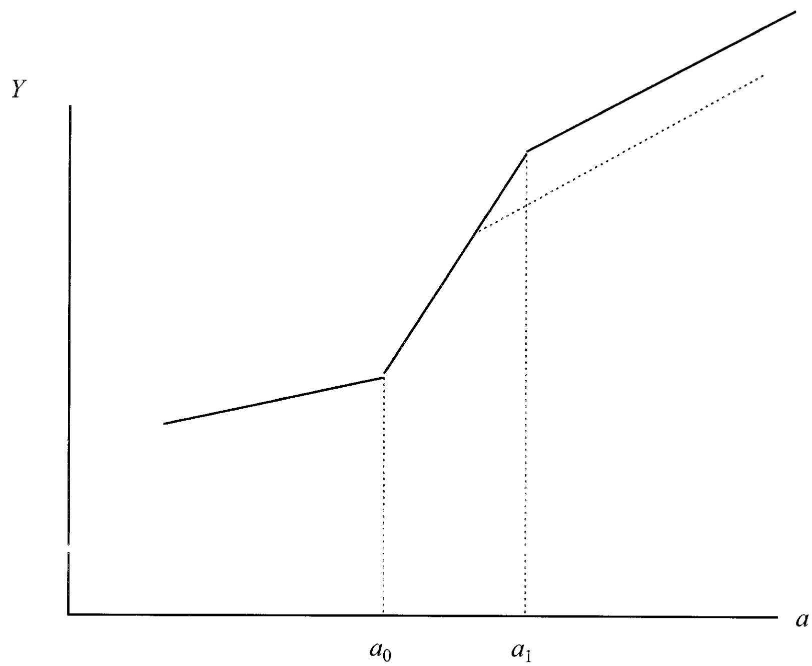

FIGURE IV

labor ratio k/l when adopting the industrial technology. Our model can, therefore, help to explain large differences in output, even if the underlying differences in productivity are not large.

Note that technology adoption amplifies not only the effect of productivity, but of interest rates as well. This model assumes that capital is fully mobile and hence, interest rates are equal across countries. But countries that restrict capital mobility may face different interest rates, and that might affect technology adoption as well. Hence, restrictions on capital mobility can also contribute toward explaining international differences in technology adoption and output.

C. Other Results

The model can account not only for output differentials, but also for divergence between countries and regions across time as well, such as recorded by Maddison [1995, Ch. 1], Pritchett [1997], and many others. Consider two countries, one with high a as in curve w_1 in Figure III, and one with a lower a, as in curve w_2 in

Figure III. Once the technology frontier passes m^*, the first country continues to adopt new technologies and to grow, while the second country does not adopt new technologies and stagnates. Hence, the two countries diverge from one another.

The model can also explain changes in economic growth across time for the same country. If a country changes its productivity a, by improving its infrastructure, or if it reduces its interest rate, by liberalizing its capital markets, it can initiate a period of rapid technology adoption and economic growth, by increasing m^*. Hence, the model can also explain how some countries can experience periods of rapid growth. The model can therefore shed some light on the Southeast Asia experience.

VI. Explanation

We next turn to an explanation of the intuition behind the main results of the model, by introducing aggregate production functions of the short, medium, and long run. The short-run production function F is defined as the maximum amount of output produced by capital input K and labor input 1, where the technologies adopted are fixed at [0, m]. Namely, the short-run production function is

(19)

$$F(K,m) = \max\left\{a \exp\left(\int_0^1 \log x_j \, dj\right): K = \int_0^1 x_j k_j \, dj,\ 1 = \int_0^1 x_j l_j \, dj\right\},$$

where $k_j = k_1(j)$ for $j \leq m$, and $k_j = k_0(j)$ for $j > m$, and similarly for l_j. The medium-run production function H is defined as the maximum amount of output produced by K units of capital, where technology is a choice variable, namely,

$$H(K,f) = \max\{F(K,m): 0 \leq m \leq f\}, \tag{20}$$

where f is the technology frontier. The equilibrium output and capital are derived by maximizing net income: $H(K,f) - RK$, as implied by Proposition 2. The long-run production function is $H(K,1)$, which describes the relationship between capital and output along time, as the technology frontier keeps growing.

Proposition 4. The aggregate production functions F and H satisfy the following.

(a) Technology adoption raises output if capital is large but reduces output if capital is small, namely $F_m > 0$ for large K and $F_m < 0$ for small K.

(b) Denote by $m(K)$ the level of technology that maximizes output at a given amount of capital K. Then $m(K)$ is increasing with K, if it is not 0 or 1, and is independent of a.

(c) The medium-run production function is a combination of the long-run and short-run production functions in the following way: $H(K,f) = H(K,1)$ for $m(K) \leq f$, and $H(K,f) = F(K,f)$ for $m(K) > f$.

(d) Both F and H are concave functions of K.

(e) Let $\overline{F}$ and $\overline{H}$ be the aggregate production functions for $a = 1$. Then $F = a\overline{F}$, and $H = a\overline{H}$.

Proof. See the Appendix.

Proposition 4 presents a new look at the main results of the paper. First, it shows that technologies do not always raise output, but can also reduce it. Second, it shows that technologies are not everywhere adopted. To show this, note that producers reach a level of capital K at which the marginal product of capital is equal to R, namely where

$$\overline{H}_K(K,f) = R/a, \tag{21}$$

due to Proposition 4(e). Let K^* be the amount of capital for which the long-run marginal product of capital is equal to R, namely,

$$\overline{H}_K(K^*,1) = R/a. \tag{22}$$

If at this level $f \geq m(K^*)$, the economy remains stagnant and does not adopt any further technologies as f increases. If $f < m(K^*)$, the economy continues to adopt technologies and grow until capital reaches K^* or until $f = 1$.

Figure V shows two countries, country A with low productivity a_A and country B with high productivity a_B. Country A does not adopt all technologies and remains at K_A^* and at a technology level $m_A = m(K_A^*) < f$. Country B adopts all existing technologies and continues to adopt new ones. This is the first main result of the paper; namely, that not all countries adopt all technologies. Furthermore, the $\overline{H}$ curve is very flat, since it is an envelope of intersecting production functions, due to Proposition 4(a). Hence, even a small difference between productivities a_A and a_B is sufficient to create a large difference in capital and output. This is the second main result of the paper, on the amplification effect of technology adoption.

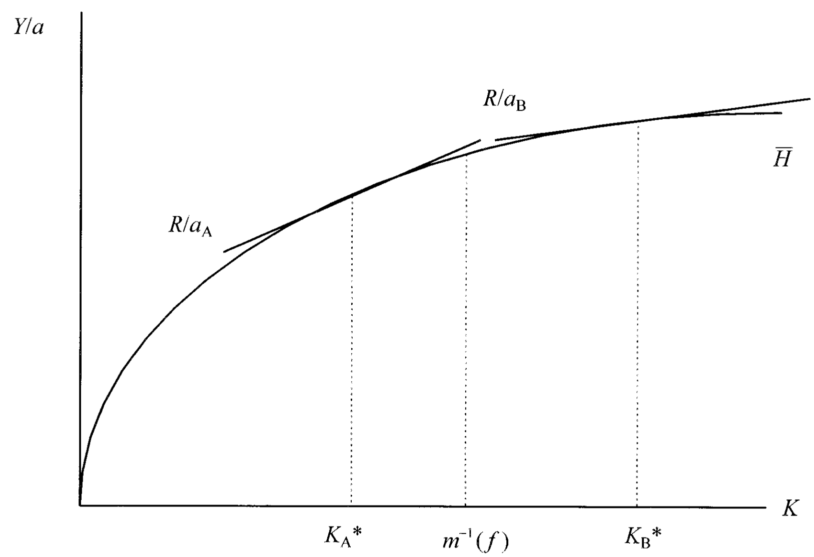

FIGURE V

It is important to compare these results with those of standard models of technological innovations, where output increases for any combination of inputs. In such models, if the level of technology m is ordered by increasing productivity, output is maximized at the technological frontier $m = f$, and the medium-run production function becomes identical to the short-run production function; namely, $H(K,f) = F(K,f)$. Hence, under standard models the two main results of the paper do not hold: existing technologies are adopted by all, and the long-run production function is not flat, so that productivity has a relatively small effect on output.

VII. Two Examples

A. Example 1

In this example input requirements for each technology, manual or industrial, are equal for all intermediate goods, namely, $l_0(j) = l_0$, $k_0(j) = k_0$, $l_1(j) = l_1$ and $k_1(j) = k_1$, for all j. Hence, the function h is constant as well and is equal to

$$h(j) = h = (k_1 - k_0)/(l_0 - l_1) . \tag{23}$$

Note that since the Rh curve in Figure III is horizontal countries either adopt all new technologies or do not adopt any technology at all. Hence, in this example the threshold for technology adoption is

$$a_0 = a_1 = R(k_1 + hl_1) = R(k_0 + hl_0). \tag{24}$$

In this example an economy with productivity $a' \geq a_0$ adopts every new technology and grows with the world's technical progress. Its output grows from a'/l_0, when $f_{t-1} = 0$, to a'/l_1, when $f_{t-1} = 1$. An economy with productivity $a'' < a_0$ does not adopt any technology, and its output per worker remains a''/l_0. Hence, these two countries diverge from one another. Note that this example is similar in many aspects to the expository example presented in Section II.

B. Example 2

In this example we assume that the manual technology uses no capital at all, namely $k_0(j) = 0$ for all j; and the industrial technology uses no labor at all, namely $l_1(j) = 0$ for all j. This example is an extreme form of the case of a manual technology that uses very few tools and of machines that need few workers to operate them. In this example wages are described by

$$\log w = (1-m)^{-1}\left[\log a - m \log R - \int_0^m \log k_1(j)\, dj - \int_m^1 \log l_0(j)\, dj\right]. \tag{25}$$

Hence, the curve w_1 of Figure III goes to infinity as m approaches 1, while w_2 and w_3 fall to minus infinity. Hence, economies with low productivity stop growing at some technology level m^*, while economies with high productivity continue to grow unboundedly. The threshold between stagnation and unbounded growth is a_1.

The short-run production function in this example is

$$F(K,m) = a\varphi(m)K^m, \tag{26}$$

where

$$\varphi(m) = \exp\left[-m\log m - (1-m)\log(1-m) - \int_0^m \log k_1(j)\, dj - \int_m^1 \log l_0(j)dj\right]. \tag{27}$$

Hence, F is similar to a standard Cobb-Douglas production function, except that the coefficient m is not fixed, but changes with technical progress.

In order to calculate the long-run production function $H(K,1)$, where m is a choice variable, note that maximization of $F(K,m)$ with respect to m leads to the following first-order condition:

$$K = [m/(1 - m)]h(m). \tag{28}$$

Hence, the optimal technology $m = m(K)$ is an increasing function of capital, and it goes to 1 as K grows to infinity. The long-run production function is

$$Y = H(K,1) = a\varphi[m(K)]K^{m(K)}, \tag{29}$$

and its slope, the marginal productivity of capital, converges to

$$a \exp\left[-\int_0^1 \log k_1(j)\, dj\right], \tag{30}$$

as K grows to infinity. Hence, the slope of H is very flat for large values of K. Technology adoption depends crucially on whether (30) is greater or smaller than R. If it is greater than R, all technologies are adopted, while if it is smaller than R, technology adoption and capital accumulation stop at some point.

VIII. The Capital-Output Ratio

This model describes economic growth as driven by technological innovations, which is a very common view. The novelty of the model is that it assumes that these innovations increase the capital requirement for each unit of output, as they are embodied in new machines that replace labor. Hence, the model has a very strong empirical implication; namely, that the ratio of capital to output tends to rise with technical progress and economic growth. This section examines whether this prediction fits the data on output and capital across time and across countries. We first examine the precise predictions of the model, and then confront them with the stylized facts that emerge from a number of recent studies. We show that indeed these stylized facts support the predictions of our model.

The evolution of the capital-output ratio can be traced along the long-run production function, as it is equal to $K/H(K,1)$. Note that according to Proposition 4(d) the production functions are concave in K. Hence, as an economy adopts new technologies and grows, it moves along the H curve, and the capital-output ratio rises. Though, note that this rise can be quite modest, since the curve is rather flat. The prediction of the model with respect to how output and the capital-output ratio are related across countries is more complex, since countries differ not only with respect to technology and capital but also with respect to productivity a. On the one hand, technology adoption and capital accumulation, namely movement along the H curve, increase the capital-output ratio. On the other hand, higher productivity a, which shifts the H curve upward, reduces the capital-output ratio. This is reflected in the following formalization: $K/Y = a^{-1}\ K/\overline{H}(K,1)$. Hence, richer countries do not necessarily have higher capital-output ratios.

We now turn to present some of the stylized facts. First, note that the traditional view for many years has been that the capital-output ratio is steady. This has been one of the stylized facts described in Kaldor's [1961] influential paper.[12] We now present some recent data and research that lead to different conclusions. We first examine data on output and capital in 88 countries in the years 1960–1994, based on World Bank data and calculated by Collins and Bosworth [1996]. According to the data, the capital-output ratios have risen for most growing economies from 1960 to 1994. Furthermore, the capital-output ratio increased in all developed economies, where data are more reliable, except for the United States and Norway, in which it has remained constant. Interestingly, the East Asian countries, which have experienced the most rapid growth in these years, also had the largest increase in capital-output ratios. From 1970 to 1994 the capital-output ratio has risen in Japan from 2.2 to 4.6, in Indonesia from 1.2 to 2.8, in South Korean from 1.2 to 2.9, in Malaysia from 1.3 to 2.5, in Singapore from 1.4 to 2.9, in Thailand from 1.1 to 1.8, and in Taiwan from 1.2 to 2.2.[13]

As mentioned above, the model predicts a mixed effect of output on the capital-output ratio across countries. Figure VI presents a scatter diagram of capital-output ratios and output per worker in 1988 across 29 countries, for which these data have been available in Summers and Heston [1991]. The scatter diagram shows the two effects, but the overall relationship between output and the capital-output ratio is positive.[14] Hence, these cross-country data also support the view that the capital-output ratio rises with economic growth.

12. See Romer [1989].
13. See Collins and Bosworth [1996, Table 13].
14. A regression analysis of the data shows a significant positive coefficient.

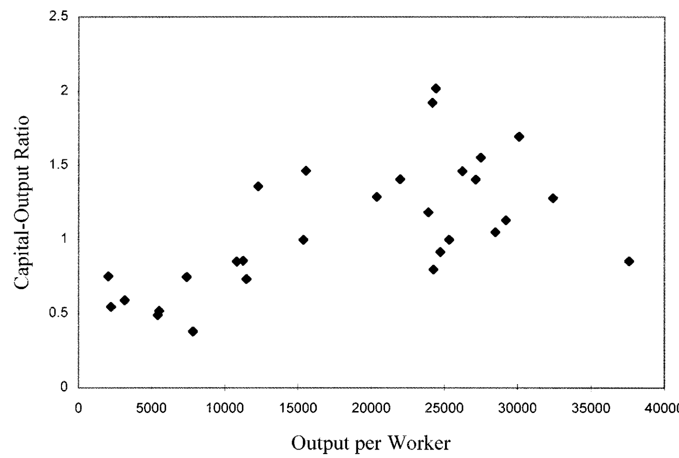

FIGURE VI

Another way to think about the model and its empirical implications is to consider a more specific type of capital, namely equipment and machines. This type of capital fits the spirit of the model more than aggregate capital, which includes structures as well. Since the share of equipment and machines in capital rises with development,[15] considering this type of capital further strengthens the above empirical results. Thus, for example, although the aggregate capital-output ratio in the United States has been rather steady, the ratio of machines and equipment capital to output has increased from 0.28 in 1950 to 0.43 in 1995.[16] In a recent study of economic growth since the industrial revolution, Maddison [1995] presents estimates of the ratio of machines and equipment capital to output in six developed countries (United States, France, Germany, Netherlands, United Kingdom, and Japan) over a long period of time. He finds that this ratio has

15. See Feinstein [1981].
16. *Survey of Current Business* [May 1997, pp. 14 and 92].

risen in all six countries from less than 0.1 in 1820 to more than 0.7 in 1992.[17]

Another piece of evidence on the importance of the specific type of capital of equipment and machines appears in De Long and Summers [1991] and in De Long [1992], who find that investment in equipment and machines has a significant relation to economic growth. Their main finding is that an increase of "three percentage points in the share of GDP devoted to equipment investment leads to an increase in the growth of GDP per worker of 1.02 percent per year" [De Long and Summers 1991, p 454].[18] It follows that the ratio of equipment capital to output rises with economic growth, as long as it is below the level of 3 (which is far above current levels).

A final comment on this issue is that although the paper describes workers replaced by machines, its results are more general and hold for any technological innovation that replaces one input by another. Consider, for example, many recent technological innovations, which require replacing less skilled workers by better trained and better educated workers. Hence, a broader implication of the model is that the ratio of the sum of physical and human capital to output should rise with economic growth. While we do not have comparable estimates of human and physical capital, we do know that education and skills have risen dramatically with economic growth. That also tends to support the main claim of the paper.

IX. Concluding Comments

Technical progress plays an important role in the process of economic growth since the industrial revolution. Recently, economists have turned to further analysis of the endogenous creation of technology and its effect on economic growth. Less attention has been given to the issue of technology diffusion and adoption. Since most countries in the world adopt and use technologies, rather than invent them, the question of international output differences requires a better understanding of technology adoption. This paper suggests that the analysis of technology adoption becomes more realistic and yields interesting results if we abandon the standard assumption of disembodied technical innovations, which

17. Ratio of machinery and equipment to output in the United States has increased from 0.07 in 1820 to 0.86 in 1992, in the United Kingdom from 0.05 in 1820 to 0.65 in 1992, and in Japan from 0.10 in 1890 to 1.07 in 1992.

reduce all required inputs in production, and consider instead innovations that save labor but use more capital (machines) or other inputs.

The paper shows that adoption of such technologies is no longer everywhere beneficial, but it depends crucially on prices of factors of production, which depend on country parameters, namely on productivity and the discount rate. Hence, producers in some countries may find it not profitable to adopt new technologies, while these technologies are adopted in more productive countries. The paper further shows that under this assumption technology adoption amplifies differences in productivity between countries and thus helps to explain some of the observed large differences in GDP per capita across countries.

Appendix

Proof of Proposition 2. Net income in period t, $Y - R\,K$, is maximized subject to the full employment constraint:

$$\int_0^m x_j l_1(j)\,dj + \int_m^1 x_j l_0(j)\,dj = 1,$$

and subject to $m \leq f_{t-1}$.

The Lagrangian is therefore equal to

$$a \exp\left(\int_0^1 \log x_j\,dj\right) - R\left[\int_0^m x_j k_1(j)\,dj + \int_m^1 x_j k_0(j)\,dj\right] + \lambda\left(1 - \int_0^m x_j l_1(j)\,dj - \int_m^1 x_j l_0(j)\,dj\right).$$

The first-order conditions with respect to x_j are

$$Y/x_j = Rk_i(j) + \lambda l_i(j),$$

where $i = 0, 1$ accordingly. The first-order condition with respect to m is

$$R[k_0(m) - k_1(m)] \geq \lambda[l_1(m) - l_0(m)].$$

It is easy to verify that these conditions are the same as the equilibrium conditions, where $m = m_t$ and $\lambda = w_t$. ■

Proof of Proposition 3. The slope is equal to

$$\frac{dY}{da} = \frac{\partial Y dw}{\partial w\, da} + \frac{\partial Y}{\partial m}\frac{dm}{da}.$$

Note that

$$\frac{dw}{da} = \frac{\partial w}{\partial m}\frac{dm}{da} + \frac{\partial w}{\partial a} = \frac{\partial w}{\partial a}\,,$$

since $\partial w/\partial m = 0$ within $[a_0,a_1]$ and $dm/da = 0$ outside it. Hence, the slope in $[a_0,a_1]$ exceeds the slope outside it by

$$\frac{\partial Y}{\partial m}\frac{\partial m}{\partial a}\,.$$

From (18) and from $w(m) = Rh(m)$, it follows that

$$\frac{\partial Y}{\partial m} = \frac{Y^2}{R}\left[\frac{1}{k_0(m)/l_0(m) + h(m)} - \frac{1}{k_1(m)/l_1(m) + h(m)}\right].$$

The level of technology adoption m is determined by

$$\int_0^m \log\,[k_1(j) + h(m)l_1(j)]\,dj + \int_m^1 \log\,[k_0(j) + h(m)l_0(j)]\,dj = \log\left(\frac{a}{R}\right).$$

Hence,

$$\frac{dm}{da} = \frac{Y}{aR}\frac{1}{h'(m)}\,.$$

That completes the proof. ■

Proof of Proposition 4. We first examine the short-run output maximization (19). If λ is the shadow price of labor and μ is the shadow price of capital, the first-order conditions boil down to

$$\text{(A1)}\quad I(\lambda,\mu,m) = \int_0^m \log\,[\lambda l_1(j) + \mu k_1(j)]\,dj + \int_m^1 \log\,[\lambda l_0(j) + \mu k_0(j)]\,dj = \log a,$$

and to

$$\text{(A2)}\qquad K = \int_0^1 \frac{k_j}{\lambda l_j + \mu k_j}\,dj \Big/ \int_0^1 \frac{l_j}{\lambda l_j + \mu k_j}\,dj = \frac{I_\mu}{I_\lambda}\,,$$

if the solution is internal. Equation (A2) does not hold in the two cases of corner solutions, namely if labor is not fully utilized, when K is very small, then $\lambda = 0$, and if capital is not fully utilized, when K is very large, then $\mu = 0$. Since the likelihood of these corner solutions is low, we do not discuss them explicitly from now on. It is easy to see that condition (A1) defines a function ψ, by $\lambda = \psi(\mu)$, which is decreasing and convex. We can rewrite (A2) as follows:

$$-\psi'(\mu) = K. \tag{A3}$$

Hence, as capital K increases, μ decreases, and λ increases. Since μ is equal to the marginal productivity of capital, it follows that F is concave in K.

To analyze technology choice, note that

$$F_m = x_m\mu[l_0(m) - l_1(m)][\lambda l/\mu - h(m)].$$

As K increases, $\lambda l/\mu$ increases from zero to infinity. Hence, F_m is negative for small K and positive for large K. That proves part (a) of Proposition 4.

We also deduce that output is maximized when

$$\lambda l/\mu = h(m), \tag{A4}$$

or in corner solutions, when $m = 0$, or $m = f$, or $m = 1$. Hence, as K increases, $\lambda l/\mu$ rises, and as a result m increases or remains unchanged (in a corner solution). Note also that from (A2) it follows that $\lambda l/\mu$ depends on K only and is independent of a. This proves part (b) of Proposition 4. Part (c) follows immediately from (b).

We have shown above that F is concave with respect to K. Note that, when technology is variable, both $\lambda l/\mu$ and m increase with K (the corner solution is trivial). But

$$I_m = \log \frac{k_1(m) + (\lambda l/\mu)l_1(m)}{k_0(m) + (\lambda l/\mu)l_0(m)},$$

and it equals 0 at the optimal m, where $\lambda l/\mu = h(m)$. Hence, the rise in $\lambda l/\mu$ increases I, and therefore μ falls. Thus, H is concave in K as well.

Finally, it is clear from (19) and (20) that the optimal x_j and optimal m are independent of a. Furthermore, from the first-order conditions we get that μ and λ are proportional to a. This proves (e). ■

THE HEBREW UNIVERSITY OF JERUSALEM AND CEPR

REFERENCES

Barro, Robert J., "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics,* CVI (1991), 407–443.

Barro, Robert J., and Xavier Sala-i-Martin, "Convergence," *Journal of Political Economy,* C (April 1992), 223–251.

Baumol, William J., "Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show," *American Economic Review,* LXXVI (1986), 1072–1085.

Bénabou, Roland, "Workings of City: Location, Education, and Production," *Quarterly Journal of Economics,* CVIII (1993), 619–652.

Ben-David, Dan, "Convergence Clubs and Diverging Economies," CEPR Working Paper No. 922, 1994.

Burmeister, Edwin, and Rodney A. Dobell, *Mathematical Theories of Economic Growth* (New York: Macmillan, 1970).

Champernowne, David, "A Dynamic Growth Model Involving a Production Function," in F. A. Lutz and D. C. Hague, eds., *The Theory of Capital* (New York: Macmillan, 1963).

Collins, Susan M., and Barry P. Bosworth, "Economic Growth in East Asia: Accumulation versus Assimilation," *Brookings Papers on Economic Activity* (1996, 2), 135–203.

David, Paul A., *Technical Choice, Innovation and Economic Growth* (London: Cambridge University Press, 1975).

De Long, J. Bradford, "Productivity and Machinery Investment: A Long-Run Look, 1870–1980," *Journal of Economic History,* LIII (1992), 307–324.

De Long, J. Bradford, and Lawrence H. Summers, "Equipment Investment and Economic Growth," *Quarterly Journal of Economics,* CVI (1991), 445–502.

Feinstein, Charles H., "Capital Accumulation and the Industrial Revolution," in Roderick Floud and Donald McCloskey, eds., *The Economic History of Britain Since 1700,* Vol. 1 (Cambridge: Cambridge University Press, 1981).

Galor, Oded, and Joseph Zeira, "Income Distribution and Macroeconomics," *Review of Economic Studies,* LX (1993), 35–52.

Grossman, Gene M., and Elhanan Helpman, *Innovation and Growth in the World Economy* (Cambridge, MA: MIT Press, 1991).

Habbakuk, H. J., *American and British Technology in the Nineteenth Century* (Cambridge: Cambridge University Press, 1962).

Hall, Robert E., and Charles I. Jones, "Levels of Economic Activity across Countries, *American Economic Review Papers and Proceedings,* LXXXVII (May 1997), 173–177.

Hicks, John R., *The Theory of Wages* (London: Macmillan, 1932).

Jovanovic, Boyan, and Saul Lach, "The Diffusion of Technology and Inequality among Nations," NBER Working Paper No. 3732, 1991.

Kaldor, Nicholas, "Capital Accumulation and Economic Growth," in F. A. Lutz and D. C. Hague, eds., *The Theory of Capital* (New York: St. Martin's Press, 1961), pp. 177–222.

Kennedy, C., "The Character of Improvements and of Technical Progress," *Economic Journal,* LXXII (1962), 899–911.

Lucas, Robert E., Jr., "On the Mechanics of Economics Development," *Journal of Monetary Economics,* XXII (1988), 3–42.

——, "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review Papers and Proceedings,* LXXX (May 1990), 92–96.

Maddison, Angus, *Monitoring the World Economy: 1820–1992* (Paris: OECD, 1995).

Parente, Stephen L., and Edward C. Prescott, "Barrier to Technology Adoption and Development," *Journal of Political Economy,* CII (1994), 298–321.

Pritchett, Lant, "Divergence, Big Time," *Journal of Economic Perspectives,* XI (Summer 1997), 3–17.

Quah, Danny, "Galton's Fallacy and Tests of the Convergence Hypothesis," *Scandinavian Journal of Economics,* XCV (1993), 427–443.

——, "Convergence Empirics across Economies with (Some) Capital Mobility," *Journal of Economic Growth,* I (1996), 95–124.

Romer, Paul M., "Increasing Returns and Long-Run Growth," *Journal of Political Economy,* XCIV (1986), 1002–1037.

——, "Capital Accumulation in the Theory of Long-Run Growth," in Robert J. Barro, ed., *Modern Business Cycle Theory* (Cambridge, MA: Harvard University Press, 1989), pp. 51–127.

Sachs, Jeffrey D., and Andrew M. Warner, "Fundamental Sources of Long-Run Growth," *American Economic Review Papers and Proceedings,* LXXXVII (May 1997), 184–188.

Sala-i-Martin, Xavier, "Regional Cohesion: Evidence and Theories of Regional Growth and Convergence," CEPR Discussion Paper No. 1075, 1994.

——, "I Just Ran Two Million Regressions," *American Economic Review Papers and Proceedings,* LXXXVII (May 1997), 178–183.

Solow, Robert M., "Investment and Technical Progress," in Kenneth J. Arrow, Samuel Karlin, and P. Suppes, eds., *Mathematical Methods in the Social Sciences* (Stanford: Stanford University Press, 1960).

Summers, Robert, and Alan Heston, "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950–1988," *Quarterly Journal of Economics,* CVI (1991), 327–368.

Zeira, Joseph, "Workers, Machines and Economic Growth," mimeo, Hebrew University of Jerusalem, 1996.