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1 Introduction

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$$\theta_{(\ell)} = 0,$$

$$\theta_{(n)} < 0.$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Δ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 Δ cc 1$\mathcal{L}_n\theta_{(\ell)}$ cc 1 cc 1 cc 1$\mathcal{L}_n\theta_{(\ell)} < 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 12 cc 1 cc 1 cc 1t^a cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Δ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Δ

$$\mathcal{L}_t\,\theta_{(\ell)} \triangleq 0. \tag{1}$$

cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1m^a cc 1$\bar{m}^a$ cc 1 cc 1 cc 1 cc 12 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$^2\epsilon = im \wedge \bar{m}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t^a cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\mathcal{L}_t\,{}^2\epsilon = -C\,\theta_{(n)}\,{}^2\epsilon \tag{2}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$C < 0$ cc 1 cc 1 cc 1 cc 1$C = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$C > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Δ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$G_{ab} \equiv R_{ab} - (1/2)R\,g_{ab} = T_{ab}$[1] cc 1 cc 1 cc 1 cc 1 cc 1Δ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[65] cc 1(1) cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$C = \frac{\mathcal{L}_\ell\,\theta_{(\ell)}}{\mathcal{L}_n\,\theta_{(\ell)}}. \tag{3}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\theta_{(\ell)} = 0$ cc 1 cc 1 cc 1 cc 1 cc 1$G_{ab} = T_{ab}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} \mathcal{L}_\ell\,\theta_{(\ell)} &= -T_{ab}\ell^a\ell^b, && (4)\\ \mathcal{L}_n\,\theta_{(\ell)} &= -(\mathcal{R}/2) + T_{ab}\ell^a n^b. && (5) \end{aligned}$$

cc 1$\mathcal{R}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 12 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\mathcal{R} = (8\pi/\mathcal{A})$ cc 1$\mathcal{A}$ cc 1 cc 12 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$C = \frac{T_{ab}\ell^a\ell^b}{(4\pi/\mathcal{A}) - T_{ab}\ell^a n^b} \tag{6}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Δ cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 *cc 1* cc 1 cc 1 [67, 68] cc 1 cc 1 cc 1 cc 1 cc 1(6) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1T_{ab} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[1] cc 1 cc 1 cc 1$c = 1$ cc 1$8\pi G = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$8\pi G$ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R = 2m(v)$ cc $1v$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\dot{m}(v) > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[60, 61, 70] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[43, 44, 52, 69, 71, 72, 73, 74, 75, 76, 77, 78] cc 1

3 Spherical Symmetric Collapse formalism

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$ds^2 = -e^{2\alpha(r,t)}dt^2 + e^{2\beta(r,t)}dr^2 + R(r,t)^2 d\theta^2 + R(r,t)^2 \sin^2\theta\, d\phi^2, \tag{7}$$

cc $1\alpha(r,t)$ cc $1\beta(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1θ cc 1ϕ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $u^a = u\,(\partial/\partial t)^a$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$e^0 = e^{\alpha(r,t)}\, dt, \quad e^1 = e^{\beta(r,t)}\, dr, \quad e^3 = R\, d\theta, \quad e^4 = R \sin\theta\, d\phi. \tag{8}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $de^I + \omega^I_J \wedge e^J = 0$ cc $1I, J$ cc 1 cc 1 cc 1 cc 1 cc 1 cc $1I, J, K, \cdots = 0, 1, 2, 3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} \omega^0{}_1 = \left(\alpha'\, e^{-\beta}\right)\, e^0 + \left(\beta'\, e^{-\alpha}\right)\, e^1, \qquad & \omega^0{}_2 = (\dot{R}/R)\, e^{-\alpha}\, e^2, \\ \omega^1{}_2 = -(R'/R)\, e^{-\beta}\, e^2, \qquad & \omega^2{}_3 = -(\cot\theta/R)\, e^3. \end{aligned} \tag{9}$$

cc 1 cc 1 cc 1 cc 1 cc $1\Omega_{IJ}$ cc 1 cc 1 cc 1 $\Omega_{IJ} = d\omega_{IJ} + \omega_{IK} \wedge \omega^K{}_J$ cc 1 cc 1 cc 1 cc 1 cc $1i, j = 2, 3$ cc 1

$$
\begin{aligned}
\Omega^0{}_1 &= \left[\left\{e^{-\alpha}\,(e^{\beta})_{,t}\right\}_{,t} - \left\{e^{-\beta}\,(e^{\alpha})_{,r}\right\}_{,r}\right] e^{-(\alpha+\beta)}\; e^0 \wedge e^1, \\
\Omega^0{}_i &= \left[\frac{e^{-\alpha}}{R}\left\{(R)_{,t}\; e^{-\alpha}\right\}_{,t} - \frac{R_{,r}}{R}(e^{\alpha})_{,r}\; e^{-(\alpha+2\beta)}\right] e^0 \wedge e^i \\
&\qquad + \left[\frac{e^{-\beta}}{R}\left\{(R)_{,t}\; e^{-\alpha}\right\}_{,r} - \frac{R_{,r}}{R}(e^{\beta})_{,t}\; e^{-(\alpha+2\beta)}\right] e^1 \wedge e^i \\
\Omega^1{}_i &= \left[-\frac{e^{-\alpha}}{R}\left\{(R)_{,r}\, e^{-\beta}\right\}_{,t} - \frac{R_{,t}}{R}(e^{\alpha})_{,r}\; e^{-(2\alpha+\beta)}\right] e^0 \wedge e^i \\
&\qquad + \left[-\frac{e^{-\beta}}{R}\left\{(R)_{,r}\, e^{-\beta}\right\}_{,r} - \frac{R_{,t}}{R}(e^{\beta})_{,t}\; e^{-(2\alpha+\beta)}\right] e^1 \wedge e^i \\
\Omega^2{}_3 &= \left[\left(\frac{R_{,t}}{e^{\alpha}R}\right)^2 - \left(\frac{R_{,r}}{e^{\beta}R}\right)^2\right] e^2 \wedge e^3 + (1/R^2)\, e^2 \wedge e^3. \qquad (10)
\end{aligned}
$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\Omega_{IJ} = (1/2)\,\Omega_{IJKL}\, e^K \wedge e^L$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R_{IJ} = \Omega^K{}_{IKL}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$G_{IJ} = R_{IJ} - (1/2)\eta_{IJ}R$ cc 1$R = \eta_{IJ}R^{IJ}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$u^I = (1, \vec{0}\,)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$e^I{}_a\, u^a = u^I$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$u^a = e^{-\alpha}\,(\partial/\partial t)^a$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$
T_{ab} = (p_t + \rho)\, u_a u_b + p_t\, g_{ab} + (p_r - p_t)\, X_a\, X_b - 2\eta\, \sigma_{ab} - \zeta\theta h_{ab}, \qquad (11)
$$

cc 1η cc 1ζ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1X^a cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 u^a cc 1 cc 1 cc 1 cc 1 cc 1$X_aX^a = 1$ cc 1 cc 1 cc 1 cc 1σ_{ab}, θ cc 1h_{ab} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1ρ cc 1p_t cc 1p_r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$
\begin{aligned}
\theta &= \nabla_a u^a, \qquad h^a{}_b = (\delta^a{}_b + u^a u_b\,) && (12) \\
\sigma^{ab} &= \frac{1}{2}\left(h^{ac}\,\nabla_c\, u^b + h^{bc}\,\nabla_c\, u^a\right) - \frac{1}{3}\theta P^{ab}, && (13) \\
X^a &= e^{-\beta(r,t)}\,(\partial/\partial r)^a && (14)
\end{aligned}
$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$
\begin{aligned}
&\theta = e^{-\alpha}(\dot\beta + 2\dot R/R), && (15) \\
&h_{ab} = e^{2\beta(r,t)}dr^2 + R(r,t)^2 d\theta^2 + R(r,t)^2 \sin^2\theta\, d\phi^2, && (16) \\
&\sigma^1{}_1 = (2/3)\,(\dot\beta - \dot R/R)e^{-\alpha}, && (17) \\
&\sigma^2{}_2 = \sigma^3{}_3 = (-1/3)\,(\dot\beta - \dot R/R)e^{-\alpha}. && (18)
\end{aligned}
$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\bar{\sigma}^2 = \sigma_{ab}\,\sigma^{ab}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\bar{\sigma}^2 = (2/3)e^{-2\alpha}(\dot{\beta} - \dot{R}/R)^2. \tag{19}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(2/3) cc 1 cc 1 cc 1$\sigma = e^{-\alpha}(\dot{\beta} - \dot{R}/R)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$T^0{}_0 = -\rho, \quad T^1{}_1 = p_r - \frac{4}{3}\eta\sigma - \theta\zeta, \quad T^2{}_2 = T^3{}_3 = p_t + \frac{2}{3}\eta\sigma - \theta\zeta. \tag{20}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\nabla_a\,T^{ab} = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t cc 1 cc 1

$$\dot{\rho}e^{-\alpha} + (\rho + p_t)\theta + (p_r - p_t)\dot{\beta}e^{-\alpha} - (4/3)\,\eta\sigma^2 - \zeta\theta^2 = 0, \tag{21}$$

cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}(p_t - \zeta\theta)' + \alpha'(\rho + p_t - \zeta\theta) - (4/3)\,\eta\sigma' - (4/3)\eta\sigma\,(\alpha' + 3R'/R)& \\ + (p_r - p_t)' + (\alpha' + 2R'/R)(p_r - p_t) = 0.&\end{aligned} \tag{22}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(21) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\dot{\beta}$ cc 1

$$\dot{\beta} = -\frac{\dot{\rho}}{\rho + p_r - (4/3)\,\eta\sigma} - \frac{2\dot{R}}{R}\,\frac{\rho + p_t + (2/3)\eta\sigma - \zeta\theta}{\rho + p_r - (4/3)\eta\sigma - \zeta\theta}\,. \tag{23}$$

cc 1 cc 1 cc 1 cc 1 cc 1(22) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\alpha' = \frac{2R'}{R}\frac{p_t - p_r + 2\eta\sigma}{\rho + p_r - (4/3)\eta\sigma - \zeta\theta} - \frac{(p_r - 4/3\,\eta\sigma - \zeta\theta)'}{\rho + p_r - (4/3)\eta\sigma - \zeta\theta}\;. \tag{24}$$

cc 1 cc 1 cc 1 cc 1R_{01} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\alpha'\dot{R} + \dot{\beta}R' - \dot{R}' = 0. \tag{25}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1α' cc 1 cc 1 cc 1(24) cc 1$\dot{\beta}$ cc 1 (23) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(25) cc 1 cc 1 cc 1 cc 1R^2 cc 1 cc 1 cc 1

$$[(p_r - 4/3\,\eta\sigma - \zeta\theta)\,R^2\,\dot{R}\,]_{,r} + [\rho R^2 R']_{,t} = 0 \tag{26}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r,t)$ cc 1 cc 1

$$F' \propto \rho\,R^2 R', \tag{27}$$
$$\dot{F} \propto -(p_r - 4/3\,\eta\sigma - \zeta\theta)\,R^2\,\dot{R}, \tag{28}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r,t)$ cc 1 *cc 1 cc 1* cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1F cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 G_{00} cc 1G_{11} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\left(\frac{2R'\beta'}{R}-\frac{2R''}{R}-\frac{R'^2}{R^2}\right)e^{-2\beta}+\left(\frac{2\dot{R}\dot{\beta}}{R}+\frac{\dot{R}^2}{R^2}\right)e^{-2\alpha}+\frac{1}{R^2}=\rho, \quad (29)$$

$$\left(\frac{2\alpha'R'}{R}+\frac{R'^2}{R^2}\right)e^{-2\beta}+\left(\frac{2\dot{\alpha}\dot{R}}{R}-\frac{\dot{R}^2}{R^2}-\frac{2\ddot{R}}{R}\right)e^{-2\alpha}-\frac{1}{R^2}=p_r-\frac{4}{3}\eta\sigma-\zeta\theta. \quad (30)$$

cc 1 cc 1$G_{01}=R_{01}$ cc 1 cc 1 cc 1 cc 1 cc 1G_{00} cc 1RR' cc 1G_{11} cc 1 cc 1$R\dot{R}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}\left[R(1+\dot{R}^2e^{-2\alpha}-R'^2e^{-2\beta})\right]_{,r}&=\rho R^2R',\\ \left[R(1+\dot{R}^2e^{-2\alpha}-R'^2e^{-2\beta})\right]_{,t}&=-(p_r-\frac{4}{3}\eta\sigma-\zeta\theta)R^2\dot{R}. \quad (31)\end{aligned}$$

cc 1 cc 1(27) cc 1(31) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$F(r,t)=R(1+\dot{R}^2e^{-2\alpha}-R'^2e^{-2\beta}). \quad (32)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$H=e^{-2\alpha(r,t)}\dot{R}^2$ cc 1$G=e^{-2\beta(r,t)}R'^2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}&\rho=\frac{F'}{R^2R'}\ , &(33)\\ &p_r=-\frac{\dot{F}}{R^2\dot{R}}+\frac{4}{3}\eta\sigma+\zeta\theta\ , &(34)\\ &\alpha'=\frac{2R'}{R}\frac{p_t-p_r+2\eta\sigma}{\rho+p_r-\frac{4}{3}\eta\sigma-\zeta\theta}-\frac{(p_r-\frac{4}{3}\eta\sigma-\zeta\theta)'}{\rho+p_r-\frac{4}{3}\eta\sigma-\zeta\theta}\ , &(35)\\ &2\dot{R}'=R'\frac{\dot{G}}{G}+\dot{R}\frac{H'}{H}\ , &(36)\\ &F=R(1-G+H). &(37)\end{aligned}$$

cc 1 cc 1 cc 1 cc 1G_{00} cc 1G_{11} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(31) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(22) cc 1 cc 1 cc 1 cc 1(25) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(32) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\alpha(r,t)$ cc 1$\beta(r,t)$ cc 1$R(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1p_t, p_r, $\rho,\eta\sigma$ cc 1$\zeta\theta$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t_i = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t_i = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1 cc 1 cc 1 cc 1$R(r,t_i) = r$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = r\,a(r,t)$ cc 1 cc 1 cc 1 cc 1$a(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = t_i$ cc 1$a(t_i) = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t_s cc 1$a(r,t_s) = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\dot{a} < 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(33) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\rho = F'/r^2$ cc 1 cc 1 cc 1 cc 1 cc 1F cc 1$r = 0$ cc 1 cc 1 cc 1 r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r,t) = r^3\, m(r,t)$ cc 1 $m(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(34) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1p_r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$m(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(33) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1 $R' = 0$ cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R' = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

4 Pressureless Collapse: OSD and LTB models

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1ζ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$p_r = p_t = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$F' = \rho\, R^2 R'\ , \quad \dot{F} = 0\,, \tag{38}$$

$$\alpha' = 0\ , \qquad \frac{\dot{R}'}{R'} = \dot{\beta}. \tag{39}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\dot{F} = 0$ cc 1 cc 1$F = F(r)$ cc 1$F(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\alpha' = 0$ cc 1 cc 1 cc 1$\alpha = \alpha(t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\bar{t}$ cc 1 cc 1$d\bar{t} = e^{\alpha}\, dt$ cc 1 cc 1 cc 1 cc 1 cc 1$\alpha(t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\alpha = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(\dot{R}'/R') = \dot{\beta}$ cc 1 cc 1$R' = e^{\beta(r,t)+h(r)}$ cc 1 cc 1$h(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$e^{2h(r)} = 1 - k(r)$ cc 1$k(r)$ cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\dot{R}^2 = \frac{F(r)}{R} - k(r)\,, \tag{40}$$

cc 1 cc 1 cc 1 cc 1

$$ds^2 = -dt^2 + \frac{R'(r,t)^2}{1-k(r)}\, dr^2 + R(r,t)^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \tag{41}$$

cc 1 cc 1 cc 1 cc 1 cc 1$\dot{R}$ cc 1 cc 1 cc 1 cc 1 cc 1+ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1− cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r) = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r) > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $k(r) < 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r)$ cc 1 cc 1 cc 1$k(r) = r^2 K(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 140 cc 1

$$t \;\;=\;\; t_s - \frac{R^{\frac{3}{2}}}{\sqrt{F}}\, Y\left[\frac{R\, k(r)}{F}\right] \tag{42}$$

cc 1t_s cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1

$$t_s \;\;=\;\; \frac{r^{\frac{3}{2}}}{\sqrt{F}}\, Y\left[\frac{rk(r)}{F}\right] \tag{43}$$

cc 1 cc 1$Y(y)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[3] cc 1

$$\begin{aligned} Y(y) &= \frac{\sin^{-1}\sqrt{y}}{y^{3/2}} - \frac{\sqrt{1-y}}{y}, && 1 \geq y > 0 \\ &= (2/3), && y = 0 \\ &= -\frac{\sinh^{-1}\sqrt{-y}}{(-y)^{3/2}} - \frac{\sqrt{1-y}}{y}, && 0 > y > -\infty. \end{aligned} \tag{44}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 (6) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1T_{ab} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(33) cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1

$$C = \frac{2F(r)'}{2R(r,t)' - F(r,t)'} \tag{45}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

4.1 Homogeneous collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r) = m\, r^3$ cc 1 cc 1 cc 1$m(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1$a(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t cc 1 cc 1 cc 1 cc 1$K(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1K cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$K = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$K = 1$ cc 1 cc 1 cc 1 cc 1 cc 1$K = -1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

4.1.1 Marginally bound collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(40) cc 1 cc 1 cc 1(42) cc 1

$$R(r,t) = r\left(1 - \frac{3}{2}\frac{\sqrt{F}}{r^{3/2}}\,t\right)^{\frac{2}{3}}. \tag{46}$$

cc 1 cc 1 cc 1 cc 1 cc 1(46) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\dot{R} < 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t_s cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(46) cc 1 cc 1$R = 0$ cc 1 cc 1 cc 1$t_s = (2/3\sqrt{m})$ cc 1m cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t_s cc 1 cc 1 cc 1$t_s = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(r,t) = \left[(3/2)\sqrt{F}\;(-t)\right]^{\frac{2}{3}}. \tag{47}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = r_H$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = r_H$ cc 1 cc 1 $F(r_H) = 2M = r_H^3 m$ cc 1 cc 1 cc 1(171) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1t_H cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(47) cc 1

$$t_H = -\frac{2}{3}F(r_H) = -\frac{4M}{3}. \tag{48}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $g^{ab}\,\nabla_a R\,\nabla_b R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R_{ah} = F(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(47) cc 1 $R = F$ cc 1 cc 1

$$R_{ah}(r) = -\frac{3}{2}\,t.^2 \tag{49}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(49) cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = t_s$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1t_H cc 1 cc 1 cc 1 cc 1 cc 1$\dot{R}_{ah} = -3/2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 0$ cc 1$t = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc

[2]From the equation (46), the time curve of apparent horizon is given by $t_{ah}(r) = (2/3)\sqrt{r^3/F} - (2/3)R_{ah}$. This clearly shows that the AH starts to form at exactly the same time when shell reaches the Schwarzschild radius, $R = 2M$.

1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 (49) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 11 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

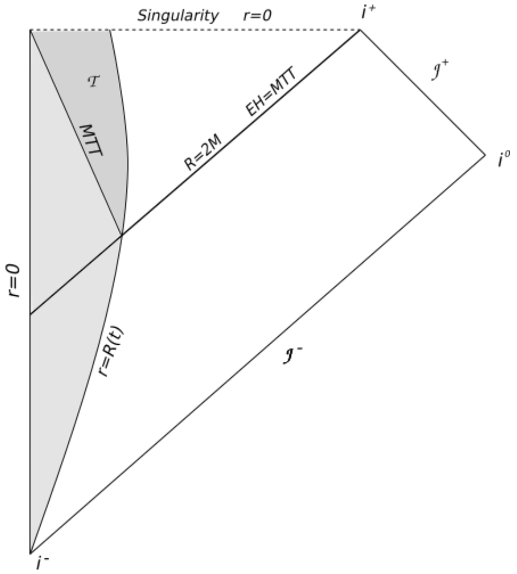


Figure 1: cc 1 cc 1 cc 1 cc 1 $k = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $r = R(t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $t = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $t = -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $t = -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $\dot{R}_{ah} = -3/2$ cc 1 cc 1 cc 1 cc 1 cc 1 $t = 0$ cc 1 cc 1 τ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\left(\frac{dr}{dt}\right)_{Null} = \frac{1}{R'}. \tag{50}$$

cc 1 cc 1 $dR/dt = [R' \, (dr/dt)_{Null} + \dot{R}]$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR}{dt} \;=\; 1 - \sqrt{\frac{F}{R}} \tag{51}$$

cc 1 $R(t)^{3/2} = (3/2)\sqrt{m}r^{3/2}(-t)$ cc 1 $F(r) = m\, r^3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR}{dt} = 1 + \frac{2}{3}\frac{R}{t}. \tag{52}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t^{-2/3}$ cc 1 cc 1

$$R_{eh}(t) = 3t + C'(-t)^{2/3}. \tag{53}$$

cc 1 cc $1C'$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(47) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R = 2M$ cc $1t = -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(53) cc 1 cc $1C' = 3(9M/2)^{1/3}$ cc 1

cc 1 cc 1 cc 1(53) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(53) cc 1 cc 1 cc $1t = t^i_{eh}$ cc $1R_{eh} = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $t^i_{eh} = -(9M/2)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(53) cc 1

$$\dot{R}_{eh} = 3 - 2(9M/2)^{1/3}(-t)^{-1/3}. \tag{54}$$

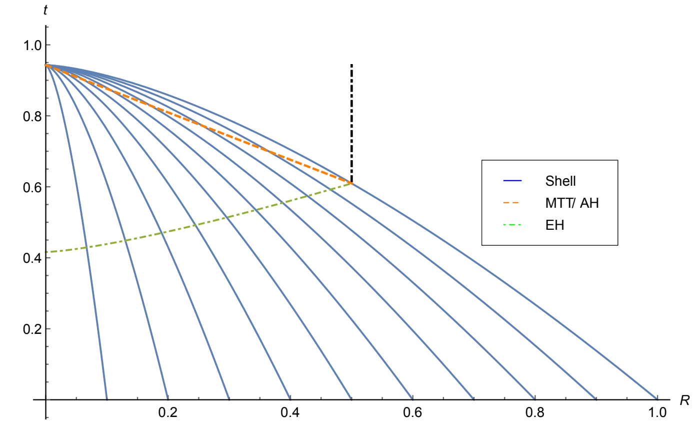
cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t^i_{eh}$ cc $1\dot{R}_{eh} = 1$ cc 1 cc $1t = -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\dot{R}_{eh} = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t = -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 11 cc 1

Example

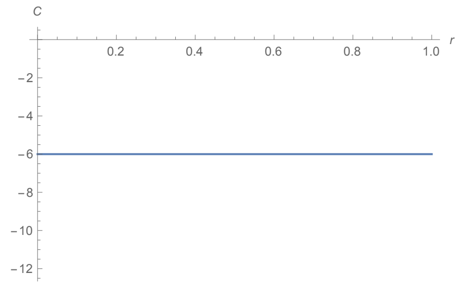
cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1F(r) = mr^3$ cc $1m = (1/2)$ cc 1 cc $1t - R(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 12 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t = 0.94$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R(r,t) = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t = 0.61$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1C$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 11 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

4.1.2 Bounded collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(41) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $r = \sin\chi$ cc 1



(a)



(b)

Figure 2: cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1$C-r$ cc 1

$$ds^2 = -dt^2 + a^2(t)\,d\chi^2 + R(r,t)^2(d\theta^2 + \sin^2\theta\,d\phi^2). \tag{55}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(40) cc 1$k > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(r,t) = \frac{F(r)}{k(r)}\cos^2(\eta/2) = rm\cos^2(\eta/2)\,, \tag{56}$$

$$t = \frac{F(r)}{2k(r)^{3/2}}(\eta + \sin\eta) = \frac{m}{2}(\eta + \sin\eta)\,, \tag{57}$$

cc 1$k(r) = r^2$ cc 1$F(r) = mr^3$ cc 1 cc 1 cc 1(56) cc 1 cc 1 $a(t) = m\cos^2(\eta/2)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(56) cc 1(57) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1$t_i = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 $\eta = \pi$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$a(t=0) \equiv a_0 = m$ cc 1 $\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1$a = 0$ cc 1$\eta = \pi$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[3] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta \to \pi - \eta$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(56) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(r,t) = (R_0/2)(1 + \cos\eta). \tag{58}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R_0 = (rm)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R = 2M$ cc 1 cc 1

$$\eta_{2M} = \cos^{-1}\left(4M/R_0 - 1\right). \tag{59}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$K_{\theta\theta}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(t) = a(t)\sin\chi \tag{60}$$
$$2M = F(r), \tag{61}$$

cc 1M cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(t=0) \equiv R_0 = m\sin\chi_0, \qquad 2M = F(r_0), \tag{62}$$

cc 1$r_0 = \sin\chi_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\chi_0 = \sin^{-1}\left(2M/R_0\right)^{1/2}, \qquad a_0 = m = \left[R_0^3/(2M)\right]^{1/2}. \tag{63}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1

$$t_s = \frac{\pi F(r)}{2k(r)^{\frac{3}{2}}} = \frac{\pi}{2}m. \tag{64}$$

cc 1 cc 1 cc 164 cc 1$t_s = (\pi m/2)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(t,r) = F(r)$ cc 1 cc 1 cc 156 cc 1 cc 1$\eta = 2\cos^{-1}[R(r,t)k/F(r)]^{1/2}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\eta_{AH} = 2\cos^{-1} r = 2\cos^{-1}\left(\,\sin\chi\,\right) = \pi - 2\,\chi. \tag{65}$$

cc 1 cc 1 cc 1 cc 1 cc 1r_0 cc 1χ_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\eta_{AH} = \pi - 2\chi_0 = 2\cos^{-1}\left(2M/R_0\right)^{1/2} = \cos^{-1}\left(4M/R_0 - 1\right), \tag{66}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\cos^{-1}x = (1/2)\cos^{-1}(2x^2-1)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 R_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(59) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$da/d\eta$ cc 1 cc 1 cc 1 cc 1 cc 1$a(\eta) = m\cos^2(\eta/2)$ cc 1 cc 1 cc 1(65) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 157 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t_{ah} = (m/2)\,(\pi - 2\chi + \sin\,2\chi)\,. \tag{67}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1 cc 1 cc 1 cc 1(57) cc 1 cc 1$(\eta-\chi)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(d\chi/d\eta) = 1$ cc 1 cc 1 cc 1 cc 155 cc 1 cc 1 cc 1 cc 14π cc 1 cc 1 cc 1 cc 1$A = a^2\sin^2\chi$ cc 1 cc 1 cc 1 cc 1 cc 156 cc 1 cc 1 cc 1 cc 1$da = -(m/2)\sin\eta\,d\eta$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(\partial/\partial\eta)$ cc 1 cc 1 cc 1 cc 1 cc 1η cc 1

$$\begin{aligned}
\pounds_\eta\, A &= \frac{da}{d\eta}\sin\chi + a\cos\chi\frac{d\chi}{d\eta} \\
&= -m\sin\chi\sin[\eta/2] + m\cos[\eta/2]\cos\chi = m\cos\left[\frac{\eta}{2}+\chi\right].
\end{aligned} \tag{68}$$

cc 1$\cos(\pi/2) = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(dA/d\eta) \leq 0$ cc 1 cc 1 [47] cc 1

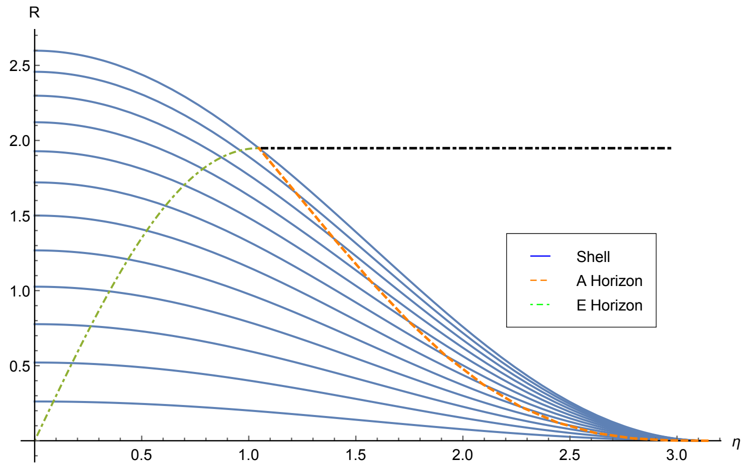
$$\eta \geq \pi - 2\chi, \tag{69}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(69) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(65) cc 1 cc 1 cc 1 cc 1 cc 1

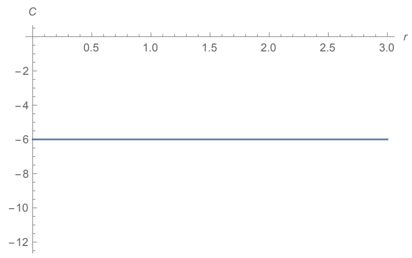
cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\mathcal{I}^+$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(r,t) cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dr}{dt} = \frac{\sqrt{1-k(r)}}{R'} = \frac{\sqrt{1-r^2}}{m\cos^2(\eta/2)}. \tag{70}$$

cc 1 cc 1 cc 1 cc 1 cc 1$(\chi-\eta)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(d\chi/d\eta) = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R_0 = m\sin\chi_0$ cc 1 cc 1 R_0 cc 1r_0 cc 1χ_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\chi = \chi_0$ cc 1 $\eta = \eta_{AH} = \eta_{2M}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1



(a)



(b)

Figure 3: cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 156 cc 1 cc 1 cc 1 cc 1 cc 165 cc 1 cc 1 cc 1 cc 1 cc 172 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\chi_0 = \pi/3$ cc 1$r_0 = 0.866$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$m = 3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(\eta_{2M}, R_{2M}) = (1.0472, 2)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2$ cc 1$\eta_{_{2M}} = 1.0472$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\chi_{EH} = \chi_0 + (\eta - \eta_{2M}). \tag{71}$$

cc 1 cc 1 cc 1(56) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R_0 cc 1 cc 1

$$R_{EH} \quad = \quad m r_{EH} \cos^2(\eta/2) = m \sin(\chi_0 + \eta - \eta_{2M}) \cos^2(\eta/2)\,. \tag{72}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = (\eta_{2M} - \chi_0)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \eta_{2M} = (\pi - 2\chi_0)$ cc 1 cc 1 cc 1$R_{EH} = m \sin\chi_0 \cos^2(\pi/2 - \chi_H) = m \sin^3\chi_0 = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(72) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1

$$\frac{dR_{eh}}{d\eta} = m\cos\left[3\left(\chi_0 + \eta/2\right) - \pi\right]\cos\left(\eta/2\right) \tag{73}$$

cc 1 cc 1 cc 1 cc 1$\eta_{2M} = \pi - 2\chi_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \eta_{2M}$ cc 1 cc 1$dR_{EH}/d\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 13 cc 1

Example

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[44] cc 1

$$\rho(r) = \frac{m_0\,\mathcal{E}(\varsigma)}{r_0^3}\left[1 - \mathrm{Erf}\left\{\varsigma\left(\frac{r}{r_0} - 1\right)\right\}\right], \tag{74}$$

cc 1$m_0 = m(r \to \infty)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 r_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r_0 = 2m_0$) cc 1 cc 1 cc 1ς cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1ς cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\mathcal{E}(\varsigma)$ cc 1 cc 1 cc 1 cc 1 cc 1

$$\mathcal{E}(\varsigma) = 3\varsigma^3\,\left[2\pi\varsigma(2\varsigma^2+3)(1+\mathrm{Erf}\,\varsigma) + 4\sqrt{\pi}\exp(-\varsigma^2)(1+\varsigma^2)\right]^{-1}, \tag{75}$$

cc 1Erf cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\varsigma = 1,5$ cc 115 cc 1 cc 1 cc 1 cc 1 cc 1 cc 14 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1ς cc 1 cc 11 cc 115 cc 1 cc 1 cc 1 cc 1 cc 1ς cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1ς cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t) - t$ cc 1 cc 1$\varsigma = 15$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = 0.99$ cc 1$R = 1.88$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$C - r$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1+ cc 1$r = 1.99$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 2.3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\rho(r) - r$ cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1t cc 1 cc 1 cc 1 cc 1 cc 1ς cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1 cc 1 cc 1 cc 1$\varsigma = 5$ cc 1 cc 1 cc 1$t = 1.88$ cc 1$R = 1.62$ cc 1C cc 1 cc 1 cc 1 $-$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 10 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\varsigma = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1

4.1.3 Unbounded collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k < 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R cc 1$a(t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(41) cc 1 cc 1 cc 1 cc 1$r = \sinh\chi$ cc 1

$$ds^2 = -dt^2 + \frac{R'^2}{1+k(r)}dr^2 + R^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad (76)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(40) cc 1$k < 0$ cc 1

$$R(r,t) = \frac{F(r)}{2|k(r)|}\left(\cosh\eta - 1\right), \quad (77)$$

$$t(\eta) = \frac{F(r)}{2|k(r)|^{3/2}}\left(\sinh\eta - \eta\right) \quad (78)$$

cc 1$k(r) = -r^2$ cc 1$F = mr^3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \eta_0$ cc 1$t = t_0 = (m/2)(\sinh\eta_0 - \eta_0)$ cc 1$R_0 \equiv R(r, t = t_0) = (rm/2)(\cosh\eta_0 - 1) \equiv rm\alpha$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1$t = 0$ cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[3] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1(77) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(t,r) = (R_0/2\alpha)(\cosh\eta - 1), \quad (79)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1

$$\eta_{2M} = -\cosh^{-1}(4M\alpha/R_0 + 1). \quad (80)$$

cc 1 cc 1 cc 1 cc 1$-$ cc 1 cc 1$\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t_s = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(t,r) = a(t)\sinh\chi, \quad 2M = F(r). \quad (81)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R_0 = m\alpha\sinh\chi_0, \qquad 2M = F(r_0) = m\sinh^3\chi_0, \quad (82)$$

cc 1$r_0 = \sinh\chi_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\chi_0 = \sinh^{-1}(2M\alpha/R_0)^{1/2}, \qquad m = [R_0^3/2M\alpha^3]^{1/2}. \quad (83)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 177 cc 1 cc 1$\eta = \cosh^{-1}[(2R(r,t)k/F) + 1]$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R(r,t) = F(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc

$1\eta = \eta_{AH} = \cosh^{-1}(2r^2+1) = \cosh^{-1}(2\sinh^2\chi + 1) = -2\chi$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1r_0$ cc $1\chi_0$ cc 1 cc 1 cc 1 cc 1 cc $1R_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1{-2\chi_0}$ cc 1

$$\eta_{AH} = -2\sinh^{-1}(2M\alpha/R_0)^{1/2} = -\cosh^{-1}(4M\alpha/R_0+1), \tag{84}$$

cc 1 cc 1 cc 1 cc 1 cc 1(83) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $12\sinh^{-1}x = \cosh^{-1}(2x^2+1)$ cc $1x > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(80) cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 12 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 176 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 14π cc $1A = a^2(\eta)\sinh^2\chi$ cc 1

cc 1 cc 1 cc 177 cc $1a(\eta) = m\sinh^2(\eta/2)$ cc 1 cc 1 $da = (m/2)\sinh\eta\, d\eta$ cc 1 cc $1(d\chi/d\eta) = 1$ cc 1

$$\frac{dA}{d\eta} = \frac{da}{d\eta}\sinh\chi + a\cosh\chi\frac{d\chi}{d\eta} = \sinh(\eta/2+\chi) \le 0 \tag{85}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\eta_{AH} = -2\chi$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $(\eta-\chi)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1(d\chi/d\eta) = 1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1(\chi = \chi_0)$ cc $1\eta = \eta_{2M}$ cc 1 cc 1 cc 1 cc 1 cc 1

$$\chi_{EH} = \chi_0 + (\eta - \eta_{2M}). \tag{86}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} R_{EH} &= m r_{EH}\sinh^2(\eta/2) = m\sinh(\chi_{EH})\sinh^2(\eta/2) \\ &= m\sinh(\chi_0 + \eta - \eta_{2M})\sinh^2(\eta/2). \end{aligned} \tag{87}$$

cc 1 cc 1 cc $1\eta = \eta_{2M} = -2\chi_0$ cc $1R_{EH} = m\sinh^3\chi_0 = 2M$ cc 1 cc 1 cc 1 cc $1\eta = -2\chi_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\eta = \eta_{2M} = -2\chi_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR_{eh}}{d\eta} = m\sinh(\eta/2)\ \sinh[(3\eta/2) + \chi_H - \eta_{2M}]. \tag{88}$$

cc 1 cc 1 cc 1 cc 1$\eta = \eta_{2M} = -2\chi_0$ cc 1$(dR_{EH}/d\eta) = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

4.2 Inhomogeneous collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\alpha = \alpha(t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 138 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r,t) = r^3 m(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r) = K(r)\, r^2$ cc 1

4.2.1 Marginally bound collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$K = 0$ cc 1 cc 1 cc 1 cc 1

$$ds^2 = -dt^2 + R'^2(r,t)\, dr^2 + R^2(r,t)(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{89}$$

cc 1$R(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\dot{R}^2 = F(r)/R. \tag{90}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t = (2/3)\left[\sqrt{r^3/F} - \sqrt{R(r,t)^3/F}\right], \tag{91}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t_i = 0$ cc 1 $R(r,t_i) = r$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t_s = (2/3) r^{3/2}/\sqrt{F}. \tag{92}$$

cc 1 cc 1 cc 1$F(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t = t_s - (2/3)\sqrt{R(r,t)^3/F}. \tag{93}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t_s = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1$t_{2M} = (-4M/3)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t) = F(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R_{AH}(r,t) = -(3/2)\, t. \tag{94}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\dot{R}_{AH} = -(3/2)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R_{AH}(r,t) - t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc

1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R(r,t)-t$ cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[44, 67] cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R[r,t_n(r)]$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR}{dt} = \dot{R} + R' \left(\frac{dr}{dt}\right)_{null}, \tag{95}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k=0$ cc 1 cc 1$(dr/dt)_{null} = (1/R')$ cc 1 cc 1 cc 1 $(dR/dt) = (1+\dot{R})$ cc 1 cc 1 cc 1 cc 1(90) cc 1 cc 1 cc 1

$$\frac{dR}{dt} = 1 - \sqrt{\frac{F}{R}}, \tag{96}$$

cc 1 cc 1 cc 1 cc 1(93) cc 1 cc 1 cc 1$F(r) = (2/3)R^3(-t)^{-2}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR}{dt} = 1 + \frac{2R}{3t}. \tag{97}$$

cc 1 cc 1 cc 1$R(r,t) = 3t + C' t^{2/3}$ cc 1C' cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 C' cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R=2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R_{EH} = 3t + 3(9M/2)^{1/3}(-t)^{2/3}. \tag{98}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R=2M$ cc 1$t=-4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1R_{EH} cc 1 cc 1 cc 1$t \geq -4M/3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$2M$ cc 1

Examples:

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} \rho_1(r) &= (3M/2500)(10-r)\,\Theta(10-r), \\ \rho_2(r) &= (3M/40\sqrt{10})(10-r^2)\,\Theta(10-r^2), \end{aligned} \tag{99}$$

cc 1$\Theta(x)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)=2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$M=1$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R-t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 15 cc 16 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1ρ_1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t) - t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[44, 67] cc 1

4.2.2 Bounded collapse

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$k(r) > 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(r,t) = \frac{F(r)}{k(r)} \cos^2(\eta/2) = r\cos^2(\eta/2) \tag{100}$$

$$t = \frac{F(r)}{2k(r)^{3/2}}(\eta + \sin\eta) = \frac{r^{3/2}}{\sqrt{F}}(\eta + \sin\eta), \tag{101}$$

cc 1 cc 1 cc 1 cc 1 cc 1$k(r)$ cc 1 cc 1 cc 1 $k(r) = F(r,t)/r$ cc 1$F(r,t) = m(r)r^3$ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1$t = t_i = 0$ cc 1$R(r,t_i) = r$ cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \pi$ cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 *cc 1* cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1

$$t_s = \frac{\pi F(r)}{2\,k(r)^{3/2}} = \frac{\pi}{2m(r)^{1/2}} \tag{102}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1102 cc 1t_s cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1100 cc 1 cc 1$\eta = 2\cos^{-1}(Rk/F)^{1/2}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta_{2M} = 2\cos^{-1}(2M/r)^{1/2}$ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(t,r) = F(r,t)$ cc 1 cc 1 cc 1100 cc 1 cc 1101 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R_{ah} = r_{ah}\cos^2(\eta_{ah}/2) \tag{103}$$

$$t_{ah} = \frac{1}{2[m(r_{ah})]^{1/2}}(\eta_{ah} + \sin\eta_{ah}). \tag{104}$$

cc 1 cc 1 cc 1r_{ah} cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = F$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dr_{ah}}{dt} = \frac{\dot{R}}{F' - R'}. \tag{105}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 (100) cc 1(101) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1

$$\frac{dr_{ah}}{d\eta} = -\frac{(\sin\eta)/2 + (1-k/k)^{1/2}\cos^2(\eta/2)}{D}, \tag{106}$$

cc 1 cc 1 cc 1 cc 1D cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} D &= (kF'/F) - [(F'/F) - (k'/k)]\cos^2\eta/2 \\ &\qquad + [(1-k)/4k]^{1/2}\,(F'/F - 3k'/2k)\,[\eta + \sin(\eta)]\,. \end{aligned} \tag{107}$$

cc 1 cc 1 cc 1 cc 1106 cc 1103 cc 1104 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1104 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2M = F$ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dr_{eh}}{dt} = \frac{[1-k(r)]^{1/2}}{R'} \tag{108}$$

cc 1 cc 1 cc 1 cc 1 cc 1100 cc 1 cc 1 cc 1 cc 1 cc 1108 cc 1

$$\frac{dr_{eh}}{d\eta} = -\frac{(\sin\eta/2) + (1-k/k)^{1/2}\cos^2(\eta/2)}{\bar{D}} \tag{109}$$

cc 1 cc 1 cc 1 cc 1$\bar{D}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} \bar{D} &= [(F'/F) - (k'/k)]\cos^2\eta/2 \\ &\qquad - [(1-k)/4k]^{1/2}\,(F'/F - 3k'/2k)\,[\eta + \sin(\eta)]\,. \end{aligned} \tag{110}$$

cc 1 cc 1 cc 1(r_H, η_{2M}) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R_{eh} = r_{eh}\cos\left[\frac{\eta}{2}\right]^2 \tag{111}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1106 cc 1109 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1100 cc 1 cc 1 cc 1 cc 1103 cc 1 cc 1 cc 1 cc 1 cc 1111 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\chi_{_H} = \pi/3$ cc 1$r_H = 0.866$ cc 1$m(r) = (n_0 + rn_1)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(\eta_{2M}, R_{2M}) = (2.1399, 2.0)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2.0$ cc 1$\eta_{_{2M}} = 2.1399$ cc 1 $n_0 = 1/11$ cc 1$n_1 = 1/4$ cc 1

Examples:

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho(r) = (3M/5000)(100 - r^3)\,\Theta(100 - r^3). \tag{112}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$C - r$ cc 1 cc 1 cc 1 cc 1 cc 17 cc 1 cc 1$R - t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R - t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[44] cc 1

$$\rho(r) = \frac{m_0}{\pi^{3/2} r_0^3} \exp(-r^2/r_0^2), \tag{113}$$

cc 1m_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[$\rho\,(0)/e$] cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r_0 = 100\, m_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 200$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 18 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 200$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 380$ cc 1 cc 1$R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho(r) = \frac{m_0}{8\pi r_0^3} \exp(-r/r_0), \tag{114}$$

cc 1m_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[$\rho\,(0)/e$] cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 70$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 70$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 100$ cc 1 cc 1$R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 19 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1M cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho(r) = \frac{8\,(m_0/\,r_0^3\,)\,[(r/r_0) - \varsigma]^{\,2}}{[2\varsigma + (3 + 2\varsigma^2)\sqrt{\pi}e^{\varsigma^2}\{1 + \mathrm{Erf}(\varsigma)\}]}\; \exp[(2r/r_0)\varsigma - (r/r_0)^{\,2}\,], \tag{115}$$

cc 1$m_0 = M/2$ cc 1 cc 1 cc 1 cc 1 cc 12r_0 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\bar{r} = 2M$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r (r > \bar{r})$ cc 1$m(r) = M + \int_{\bar{r}}^{r} \rho(\hat{r})\hat{r}^2\, d\hat{r}$ cc 1 cc 1 cc 1σ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$M = 1$ cc 1$r_0 = 10$ cc 1$\varsigma = 10$ cc 1 cc 1$r = 65$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2.4$ cc 1 cc 1 cc 1$t = 1014$ cc 1 cc 1$r = 100$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc

1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 110 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $R = 3$ cc $1t = 1500$ cc 1 cc 1 cc 1 cc 1 cc $1r = 140$ cc 1 cc 1 cc 1

5 Spacetimes admitting viscous matter fields

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(33) cc 1(37) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 *cc 1 cc 1 cc 1* cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\psi_2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[79, 80] cc 1 cc 1 cc $1\psi_2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}\psi_2 &= \frac{e^{-2\beta}}{6}\left[\alpha'' + {\alpha'}^2 - \alpha'\beta' + R'^2/R^2 - R''/R + (R'\beta')/R - (R'\alpha')/R\right] - \frac{1}{6R^2}\\ &- \frac{e^{-2\alpha}}{6}\left[\ddot\beta + \dot\beta^2 - \dot\alpha\dot\beta + \dot R^2/R^2 - \ddot R/R - (\dot R\dot\psi)/R + (\dot R\dot\alpha)/R\right]. \qquad (116)\end{aligned}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(116) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $F(r,t)$ cc 1

$$F(r,t) = (\rho + \bar p_\theta - \bar p_r + 2\eta\sigma)\,(R^3/3) - (\psi_2/2)\,R^3, \qquad (117)$$

cc $1\bar p_r = (p_r - \zeta\theta)$ cc $1\bar p_\theta = (p_t - \zeta\theta)$ cc 1 cc 1 cc $1\mathcal{F}(r,t) = -\psi_2\,R^3$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[79] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1F(r,t)$ cc 1 (31) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}\dot{\mathcal{F}} &= -(1/6)\left[R^3\left\{\rho + \bar p_\theta + (2/3)\eta\sigma\right\}\right]_{,t} - (R^3/6)\left[\bar p_r - (4/3)\eta\sigma\right]_{,t} \qquad (118)\\ \mathcal{F}' &= -(1/6)\,R^3\rho' - (1/6)\left[R^3\left(\bar p_\theta - \bar p_r + 2\eta\sigma\right)\right]' \qquad (119)\end{aligned}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\dot\rho$ cc 1

$$\dot\rho e^{-\alpha} + \left[\rho + \bar p_r - (4/3)\eta\sigma\right](\Theta - \sigma) = 0. \qquad (120)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\dot\rho$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(21) cc 1(22) cc 1 cc 1 cc 1

$$\begin{aligned}\dot\rho &= -\dot\beta\left[\rho + \bar p_r - (4/3)\eta\sigma\right] - (2\dot R/R)\left[\rho + \bar p_\theta + (2/3)\eta\sigma\right], \qquad (121)\\ p_r' &= \left\{(4/3)\eta\sigma\right\}' + (2R'/R)\left(\bar p_\theta - \bar p_r + 2\eta\sigma\right) - \alpha'\left\{\rho + \bar p_r - (4/3)\eta\sigma\right\}. \qquad (122)\end{aligned}$$

cc 1 cc 1 cc 1 cc 1 cc 1(121) cc 1 cc 1 cc 1(120) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$(\rho + \bar{p}_r) = \frac{8}{3}\eta\sigma - \frac{4}{3}\eta\theta + e^{-\alpha}\,\frac{2\dot{R}}{R\sigma}\,(\bar{p}_\theta - \bar{p}_r)\,. \tag{123}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $(\bar{p}_\theta - \bar{p}_r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1σ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(123) cc 1(121) cc 1 cc 1

$$\dot{\rho} = \frac{6\dot{R}^2}{R^2}e^{-\alpha}\left[2\eta + \frac{\bar{p}_\theta - \bar{p}_r}{\sigma}\right], \tag{124}$$

cc 1 cc 1(119) cc 1 cc 1 $\mathcal{F}(r,t) = -(R^3/6)\,(\bar{p}_\theta - \bar{p}_r + 2\eta\sigma)$ cc 1 cc 1 cc 1(118) cc 1

$$\dot{\rho} = -(3\dot{R}/R)\,[\rho + \bar{p}_r - (4/3)\eta\sigma]\,. \tag{125}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(121) cc 1 cc 1 cc 1 cc 1(15) cc 1 cc 1 cc 1

$$(\rho + \bar{p}_r)\,\theta = -\dot{\rho}e^{-\alpha} - \frac{2\dot{R}}{R}\,(\bar{p}_\theta - \bar{p}_r)\,e^{-\alpha} + \frac{4}{3}\eta\sigma^2, \tag{126}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(124) cc 1

$$\dot{\rho}e^{-\alpha} = \left[(4/3)\eta\sigma^2 - (\rho + \bar{p}_r)\,\Theta\right]\left[1 - \frac{(2/3)\,(\bar{p}_\theta - \bar{p}_r)}{\rho + \bar{p}_r - (4/3)\eta\sigma}\right]^{-1}. \tag{127}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(126) cc 1 cc 1(127) cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}(\rho + \bar{p}_r)\,\theta' + (\rho + \bar{p}_r)'\,\theta \;\; &= \;\; \left(\dot{\rho}e^{-\alpha}\right)'\left[1 + \frac{(2/3)\,(\bar{p}_\theta - \bar{p}_r)}{\rho + \bar{p}_r - (4/3)\eta\sigma}\right] + \left[(4/3)\eta\sigma^2\right]' \\ &\quad - \dot{\rho}e^{-\alpha}\left[\frac{(2/3)\,(\bar{p}_\theta - \bar{p}_r)}{(\rho + \bar{p}_r - (4/3)\eta\sigma)}\right]', \end{aligned} \tag{128}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(122) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$
\begin{aligned}
(\rho+\bar{p}_r)\,\theta' \;&= -\Big[(4/3)\eta\sigma' - (4/3)\eta\sigma\,\alpha^{'} + (2R^{'}/R)\,(\bar{p}_\theta-\bar{p}_r+2\eta\sigma)\Big]\,\theta \\
&\quad + \left[\frac{(2/3)}{\left(\rho+\bar{p}_r-\frac{4}{3}\eta\sigma\right)-\frac{2}{3}\,(\bar{p}_\theta-\bar{p}_r)}\right]\Bigg[\alpha^{'}\,(\rho+\bar{p}_r)\,\{2\eta\sigma^2-\theta\,(\bar{p}_\theta-\bar{p}_r)\} \\
&\quad + \big\{(\rho+\bar{p}_r)\Theta+(4/3)\eta\sigma^2\big\}\Bigg\{(\bar{p}_\theta-\bar{p}_r)^{'} - \frac{2\,(\bar{p}_\theta-\bar{p}_r)^2}{\left(\rho+\bar{p}_r-\frac{4}{3}\eta\sigma\right)} \\
&\quad - \frac{4\eta\sigma\,(\bar{p}_\theta-\bar{p}_r)}{\left(\rho+\bar{p}_r-\frac{4}{3}\eta\sigma\right)}\Bigg\} - \frac{8}{3}\alpha'\eta^2\sigma^3\Bigg] + \big\{(4/3)\eta\sigma^2\big\}'. \qquad (129)
\end{aligned}
$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(129) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$(p_t - p_r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1η cc 1ζ cc 1 cc 1 cc 1 cc 1 $p_r = p_t \equiv p$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$(\rho + p)\,\theta' = 0. \qquad (130)$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[80, 81, 82] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[80] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $(p_t - p_r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(129) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 *cc 1 cc 1* cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(123) cc 1(124) cc 1(129) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $(p_t - p_r) \propto \sigma$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $p_t = k_t\rho$ cc 1$\sigma = k_\sigma\rho$ cc 1$\theta = k_\theta\rho$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1k_t, k_σ cc 1k_θ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

5.1 Time independent mass function

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $p_r = (4/3)\eta\sigma + \zeta\theta$ cc 1 cc $1[p_t + (2/3)\eta\sigma - \zeta\theta]$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$
\begin{aligned}
F' &= \rho\, R'\, R^2 && (131)\\
\dot F &= -(R^2 \dot R)\,(p_r + \frac{4}{3}\eta\sigma - \zeta\theta) = 0, && (132)\\
\alpha' &= (2R'/\rho R)\,[p_t + (2/3)\eta\sigma - \zeta\theta]\,, && (133)\\
(\dot G/G) &= 2\alpha'(\dot R/R'), && (134)\\
F(r,t) &= R(r,t)(1 - G + H), && (135)
\end{aligned}
$$

cc $1H(r,t) = e^{-2\alpha(r,t)}\dot R^2$ cc $1G(r,t) = e^{-2\beta(r,t)}R'^2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(132) cc 1(135) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1p_t = k_t\rho$ cc $1\sigma = k_\sigma\rho$ cc 1 cc $1\theta = k_\theta\rho$ cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 (133) cc 1(134) cc 1

$$\exp(\,2\alpha) = R^{4a_1}, \qquad \exp(\,2\beta) = \frac{R'^2}{b(r)R^{4a_1}}, \tag{136}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1a_1 = k_t + (2/3)\eta k_\sigma - \zeta k_\theta$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(33) cc 1 cc 1 cc 1 cc 1

$$ds^2 = -R^{4a_1}dt^2 + \frac{R'^2}{b(r)R^{4a_1}}dr^2 + R(r,t)^2 d\theta^2 + R(r,t)^2 \sin^2\theta\, d\phi^2 \tag{137}$$

cc 1 cc 1 cc 1 cc 1(135) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\dot R = -R^{2a}\left[\frac{F(r)}{R} - 1 + b(r)R^{4a_1}\right]^{1/2}. \tag{138}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(135) cc 1 cc 1 cc 1 cc 1 cc 1 cc $1a_1 = -(1/4)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(135) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$dt \;=\; -\frac{R\,dR}{[F(r) + b(r) - R(r,t)]^{1/2}}. \tag{139}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R(r,t)$ cc $1F(r)$ cc $1b(r)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R = (F/b)\cos^2(\eta/2)\,. \tag{140}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$dt = \left(\frac{F^2}{2b^2}\right)\frac{\sin\eta\,\cos^2(\eta/2)}{[F+b-(F/b)\cos^2(\eta/2)]^{1/2}}\;d\eta. \tag{141}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} t &= \frac{4}{3}[F+b-(F/b)\cos^2(\eta/2)]^{1/2}\left[F+b+(F/2b)\cos^2(\eta/2)\right] \\ &\quad - (4/3)\{F+b-(F/b)\}^{1/2}\left[F+b+(F/2b)\right]. \end{aligned} \tag{142}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \pi$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1 $R(t_i, r) = [F(r)/b(r)]$ cc 1$t_i = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \pi$ cc 1 cc 1 cc 1 cc 1$R = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t_s = \frac{4}{3}\left[(F+b)^{\frac{3}{2}} - \{F+b-(F/b)\}^{1/2}\{F+b+(F/2b)\}\right]. \tag{143}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1 cc 1(6) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$u^\mu = \chi l^\mu + (2\chi)^{-1}n^\nu$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$x^\mu = \chi l^\mu - (2\chi)^{-1}n^\nu$ cc 1

$$T_{\mu\nu}l^\mu l^\nu = (1/4\chi)\left[\rho + p_t - (4/3)\eta\sigma - \zeta\theta + (p_t - p_r)\right], \tag{144}$$
$$T_{\mu\nu}l^\mu n^\nu = (1/2)\left[\rho - (p_t - (4/3)\eta\sigma - \zeta\theta + (p_t - p_r))\right]. \tag{145}$$

cc 1$p_r = (4/3)\eta\sigma + \zeta\theta$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1

$$C = (1/2\chi)\left[\frac{\rho + 2\{p_t - (4/3)\eta\sigma - \zeta\theta\}}{4\pi/\mathcal{A} - (1/2)\left[\rho - 2\{p_t - (4/3)\eta\sigma - \zeta\theta\}\right]}\right]. \tag{146}$$

Examples

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho(r) = \frac{m_0}{\pi^{3/2}r_0^3}\exp(-r^2/r_0^2), \tag{147}$$

cc 1m_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1r_0 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[$\rho\,(0)/e$] cc 1

cc 1 cc 1 cc 1 cc 1 cc 1$r_0 = 100\,m_0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 200$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 200$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 380$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R = 2$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 200$ cc 1 cc 1 cc 1$t = 3215$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$a_1 = -(1/4)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = 1912$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1p_r cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 111 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1p_t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho(r) = \frac{m_0}{8\pi r_0^3}\exp(-r/r_0), \tag{148}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = 70$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $t = 406$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$t = 660$ cc 1 cc 112 cc 1

5.2 Time dependent mass function

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\rho = \frac{F'}{R^2R'};\quad p_r = -\frac{\dot F}{R^2\dot R} + (4/3)\eta\sigma + \zeta\theta \tag{149}$$

$$\alpha' = \frac{2R'}{R}\frac{p_t - p_r + 2\eta\sigma}{\rho + p_r - (4/3)\eta\sigma - \zeta\theta} - \frac{p_r' - (4/3)\eta\sigma' - \zeta\theta'}{\rho + p_r - \frac{4}{3}\eta\sigma - \zeta\theta} \tag{150}$$

$$(\dot G/G) = (2\alpha')(\dot R/R')\,;\qquad F(r,t) = R(1 - G + H). \tag{151}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $p_r = k_r\rho$ cc 1$p_t = k_t\rho$ cc 1$\sigma = k_\sigma\rho$ cc 1$\theta = k_\theta\rho$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\exp(2\alpha) = \frac{R^{4a_1}}{\rho^{2a_2}},\qquad \exp(2\beta) = \frac{R'^2}{1 + r^2B(r,t)}, \tag{152}$$

cc 1 cc 1 cc 1 cc 1a_1 cc 1a_2 cc 1 cc 1 cc 1 $a_1 = [k_t - k_r + 2\eta k_\sigma]/[1 + k_r - (4/3)\eta k_\sigma - \zeta k_\theta]$ cc 1$a_2 = [k_r - (4/3)\eta k_\sigma - \zeta k_\theta]/[1 + k_r - (4/3)\eta k_\sigma - \zeta k_\theta]$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$ds^2 = -\frac{R(r,t)^{4a_1}}{\rho(r,t)^{2a_2}}dt^2 + \frac{R(r,t)'^2}{1 + r^2 B(r,t)}dr^2 + R(r,t)^2\,[\,d\theta^2 + \sin^2\theta\, d\phi^2] \tag{153}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(151) cc 1 cc 1 cc 1 cc 1

$$\dot{R} = -R^{2a_1}\rho^{-a_2}\left[\frac{F(r,t)}{R} + r^2 B(r,t)\right]^{1/2}. \tag{154}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$F(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1$B(r,t)$ cc 1 cc 1 cc 1$\rho(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1

$$F(r,t) = F_1(r)F_2(t), \quad B(r,t) = B_1(r)B_2(t), \quad \rho(r,t) = \rho_1(r)\rho_2(t), \tag{155}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $B_1(r) = k(r)/r^2$ cc 1$B_2(t) = -F_2(t) = -\rho_2(t)^{2a_2}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$R(r,t)$ cc 1 cc 1

$$R = [F_1(r)/k(r)]\,\cos^2{(\eta/2)}, \tag{156}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(154) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$dt = \frac{[F_1(r)\,\cos^2(\eta/2)]^{(1-2a_1)}\,\rho_1^{a_2}}{k(r)^{(3/2-2a_1)}}\,d\eta. \tag{157}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned} t_{shell} \;\; = \;\; & \frac{2F_1(r)^{1-2a_1}\rho_1(r)^{a_2}\cos(\eta/2)^{3-4a_1}}{(4a_1-3)k(r)^{3/2-2a_1}}\;{}_2F_1\left[\frac{1}{2},\frac{3}{2}-2a_1;\frac{5}{2}-2a_1;\cos^2(\eta/2)\right] \\ & - \frac{2\sqrt{\pi}F_1(r)^{1-2a_1}\rho_1(r)^{a_2}}{(4a_1-3)k(r)^{3/2-2a_1}}\frac{\Gamma[5/2-2a_1]}{\Gamma[2-2a_1]}, \end{aligned} \tag{158}$$

cc $1{}_2F_1(a,b;c;z)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\Gamma(x)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = 0$ cc 1 cc 1$R(t_i, r) = [F(r)/k(r)]$ cc 1$t = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$\eta = \pi$ cc 1$R = 0$ cc 1 cc 1t cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$t_s = \frac{2\sqrt{\pi}F_1(r)^{1-2a_1}\rho_1(r)^{a_2}}{(3-4a_1)k(r)^{3/2-2a_1}}\frac{\Gamma[5/2-2a_1]}{\Gamma[2-2a_1]}. \tag{159}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1C cc 1 cc 1 cc 1

$$C \;\; = \;\; \frac{1}{2\chi}\left[\frac{\rho + p_t - (4/3)\eta\sigma - \zeta\theta + (p_t - p_r)}{(4\pi/\mathcal{A}) - (1/2)\{\rho - \{p_t - (4/3)\eta\sigma - \zeta\theta + (p_t - p_r)\}\}}\right]. \tag{160}$$

Examples

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 113 cc 1

cc 1 cc 1 cc 1 cc 1 cc $1k_r = (1/2)$ cc $1k_t = 1/4$ cc $1\eta = 1/16$ cc 1 $k_\sigma = 1/4$ cc $1\zeta = (1/2)$ cc $1k_\theta = (3/2)$ cc $1a_1 = -0.3$ cc $1a_2 = -0.37$ cc 1 cc 1 cc 1 cc $1r = 200$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1t = 3215$ cc 1 cc 1 cc 1 cc $1t = 9 \times 10^7$ cc 1 cc 1 cc 1 cc 1 cc 110^3 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1F(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1r = 1800$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1C$ cc 114 cc 1 cc $1k_r = (1/80)$ cc $1k_t = 1/81$ cc $1\eta = 1/16$ cc 1 $k_\sigma = 1/20$ cc $1\zeta = (1/12)$ cc $1k_\theta = (1/8)$ cc $1a_1 = 0.006$ cc $1a_2 = -0.002$ cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 $k_r = 1/2$ cc $1k_t = 1/10$ cc $1\eta = 1/8$ cc $1k_\sigma = 1/4$ cc 1 $\zeta = 1/2$ cc $1k_\theta = 3/2$ cc $1a_1 = -0.48$ cc $1a_2 = -0.41$ cc 1 cc 1 15 cc 1

cc 1 cc 1 cc $1k_r = (1/80)$ cc $1k_t = 1/81$ cc $1\eta = 1/16$ cc 1 $k_\sigma = 1/20$ cc $1\zeta = (1/12)$ cc $1k_\theta = (1/8)$ cc $1a_1 = 0.006$ cc $1a_2 = -0.002$ cc 1 cc 116 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

6 Discussions

cc 1 [3] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 [44, 52, 69] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R(r,t) - t$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1R(r,t)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc

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cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 *cc 1* cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1[83] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1m(v)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\dot{m}(v) \geq 0$ cc 1 cc 1 cc 1 cc 1[84] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 [71, 72, 77] cc 1

Acknowledgements

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 158/14/25/2019 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1/2019/000916 cc 1

Appendix: Junction Conditions

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1K = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc $1\mathcal{M}^-$ cc 1 cc $1M$ cc 1 cc $1\mathcal{M}^+$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Σ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(τ, θ, ϕ) cc 1 cc 1$\mathcal{M}^-$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Σ cc 1 $f_-(r,t) = r - r_b = 0$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1Σ cc 1

$$ds_-^2 = a(\tau)^2 \left(-d\tau^2 + r_b^2\, d\Omega^2 \right). \tag{161}$$

cc 1 cc 1 cc 1 cc 1 cc 1t cc 1τ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$dt/d\tau = a(\tau)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$r = R(\tau)$ cc 1$t = T(\tau)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$ds_+^2 = -\left(Z\dot{T}^2 - Z^{-1}\dot{R}^2 \right) d\tau^2 + R(\tau)^2 \left(d\theta^2 + \sin^2\theta d\phi \right) \tag{162}$$

cc 1$Z = (1 - 2M/R)$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$R(\tau) = r_b\, a(\tau)\,; \qquad a(\tau)^2 = \left[Z\,(dT/d\tau)^2 - Z^{-1}\,(dR/d\tau)^2 \right]. \tag{163}$$

cc 1 cc 1 cc 1 cc 1n^a cc 1$\mathcal{M}^-$ cc 1 cc 1 cc 1 cc 1 cc 1$\mathcal{M}^+$ cc 1 cc 1 cc 1 cc 1 cc 1

$$n_a = a(\tau)\,(dr)_a, \qquad n_a = -(dR/d\tau)\,(d\tau)_a + (dT/d\tau)\,(dr)_a. \tag{164}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR(r_b,\tau)}{d\tau} = -a(\tau)\,[F(r_b)/R]^{1/2}, \tag{165}$$

cc 1 cc 1$-$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\frac{dR}{dT} = -\,(2M/R)\,(1 - 2M/R). \tag{166}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1165 cc 1 cc 1 cc 1

$$\left(\frac{dT}{d\tau} \right) = a/(1 - 2M/R) \tag{167}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$\begin{aligned}
K^-_{\tau\tau} &= 0, && (168)\\
K^+_{\tau\tau} &= -\sqrt{\frac{2M}{R}}\left(1 - \frac{2M}{R}\right)\ddot{T} + \dot{a}\sqrt{\frac{2M}{R}} + \frac{4M^2a^2}{\left(1 - \frac{2M}{R}\right) r_s^3}, && (169)\\
K^-_{\theta\theta} &= R\left[1 - \frac{F}{R} + \frac{2M}{R}\right]^{1/2}, \qquad K^+_{\theta\theta} = R. && (170)
\end{aligned}$$

cc 1$K_{\tau\tau}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(167) cc 1$K_{\theta\theta}$ cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$F(r_b) \equiv m\, r_b^3 = 2M. \tag{171}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(153) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1$K_{\theta\theta}$ cc 1 cc 1 cc 1 cc 1

$$F(r_b, \tau) \equiv m(r_b, \tau)\, r_b^3 = 2M, \tag{172}$$

cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1(27) cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

$$p_r = \zeta\theta + (4/3)\eta\,\sigma. \tag{173}$$

References

[1] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[2] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[3] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[4] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[5] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**14** cc 1 cc 1

[6] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**1** cc 1 cc 1 [Gen. Rel. Grav. **34**, 1141 (2002)] cc 1

[7] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[8] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[9] cc 1 cc 1 cc 1 cc 1 cc 1**29** cc 1 cc 1

[10] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**20** cc 1 cc 1 [Gen. Rel. Grav. **29**, 935 (1997)] cc 1

[11] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**107** cc 1 cc 1

[12] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[13] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**143** cc 1 cc 1

[14] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**20** cc 1 cc 1

[15] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**19** cc 1 cc 1

[16] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**93** cc 1 cc 1

[17] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

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[21] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**6** cc 1 cc 1

[22] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**42** cc 1 cc 1

[23] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**43** cc 1 cc 1 cc 1 cc 1

[24] cc 1 cc 1 cc 1 cc 1 cc 1**43** cc 1 cc 1 cc 1 cc 1

[25] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**47** cc 1 cc 1

[26] cc 1 cc 1 cc 1 cc 1 cc 1**140** cc 1 cc 1

[27] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**166** cc 1 cc 1

[28] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**13** cc 1 cc 1

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[30] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**58** cc 1 cc 1 cc 1

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[32] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**81** cc 1 cc 1

[33] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**16** cc 1 cc 1

[34] cc 1 cc 1
cc 1 cc 1 cc 1**73** cc 1 cc 1 cc 1

[35] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**76** cc 1 cc 1 cc 1

[36] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**34** cc 1 cc 1 cc 1 cc 1 *ibid***cc 1** cc 1 cc 1

[37] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1

[38] cc 1 cc 1 cc 1 cc 1 cc 1**149** cc 1 cc 1

[39] cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1 cc 1**17** cc 1 cc 1

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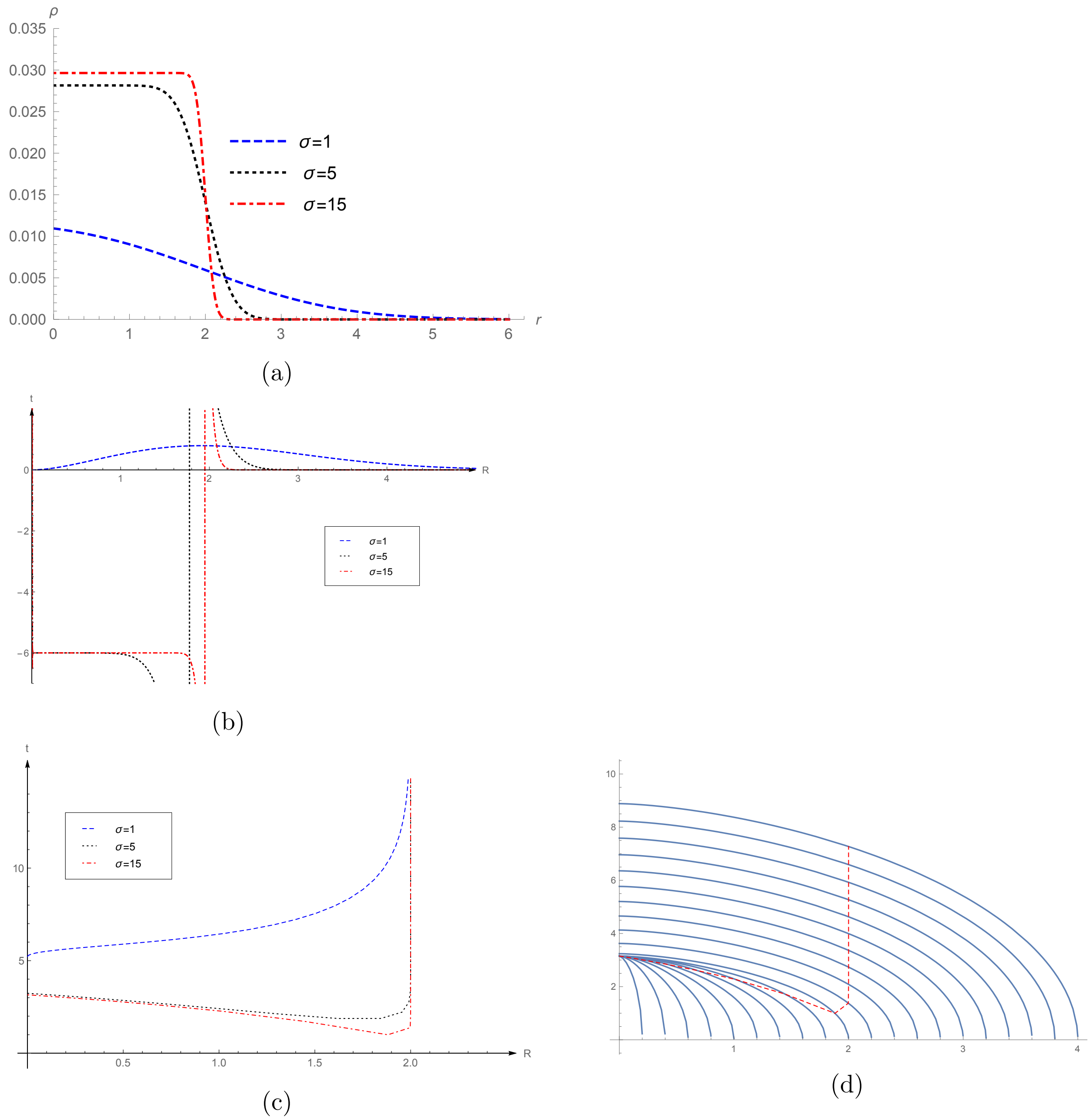

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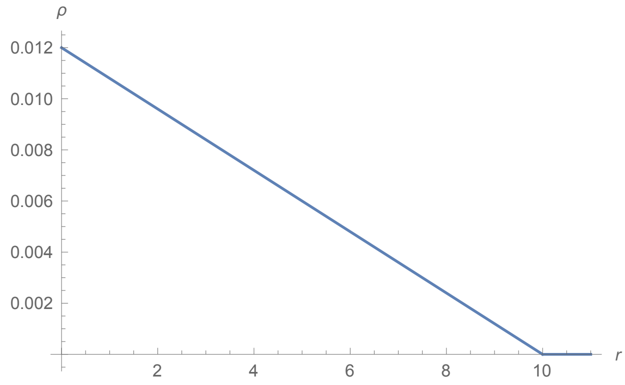

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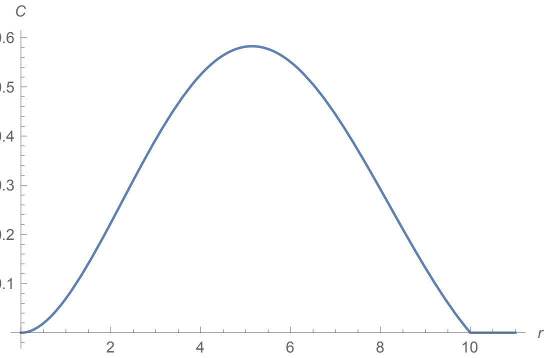

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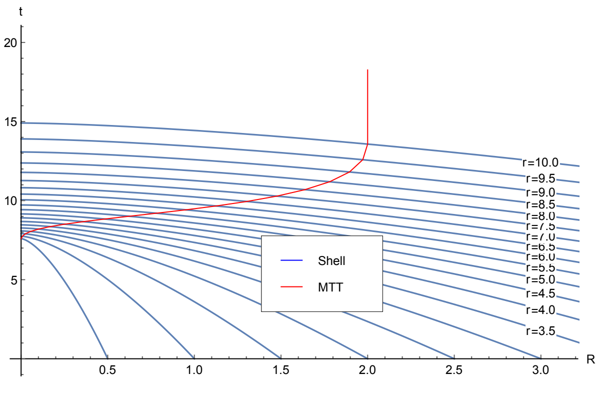

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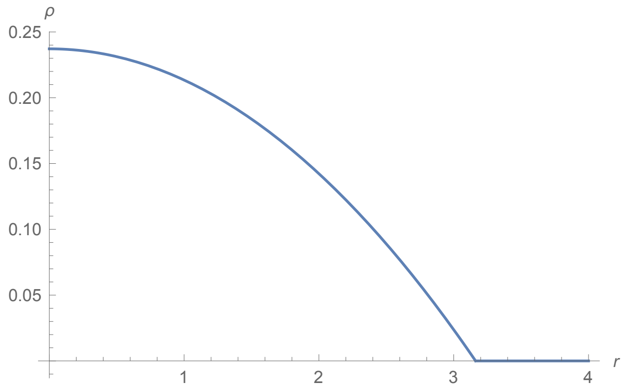

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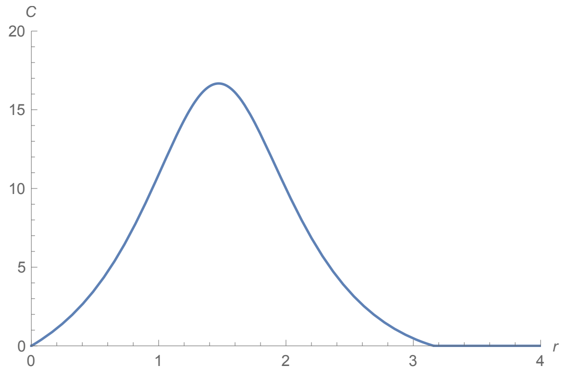

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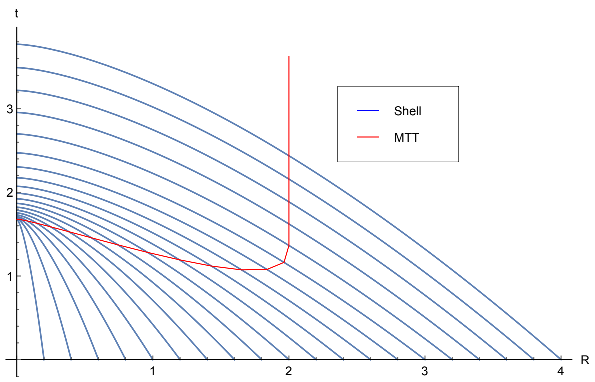

(c)

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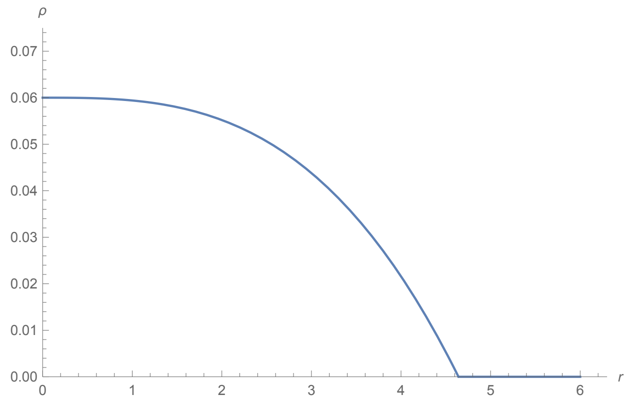

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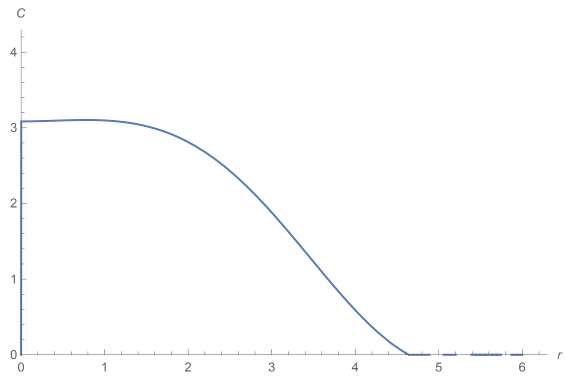

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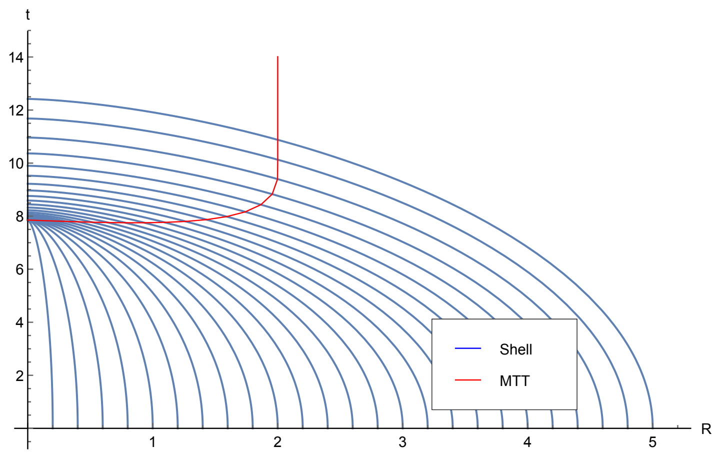

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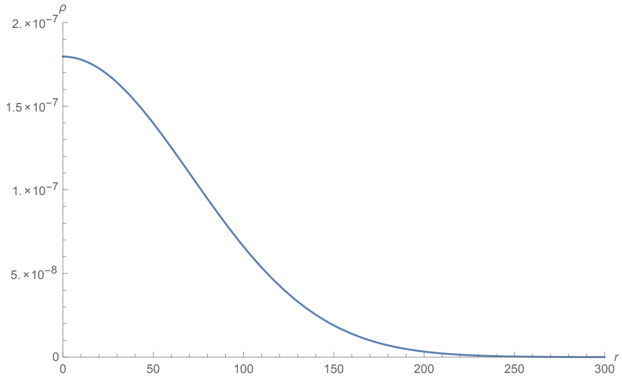

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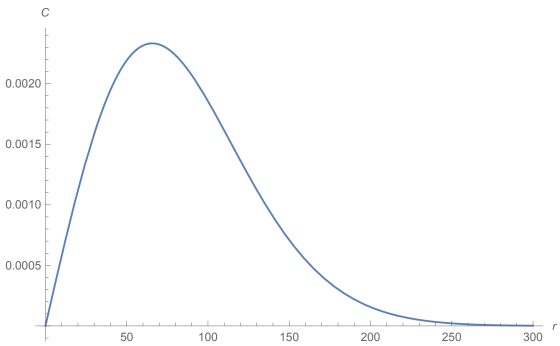

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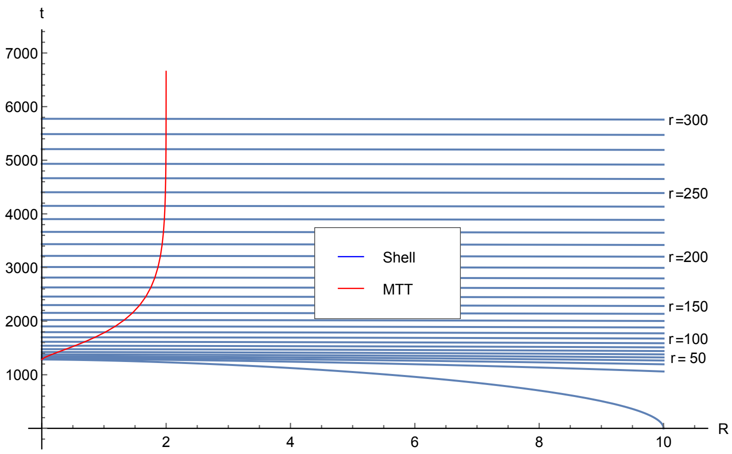

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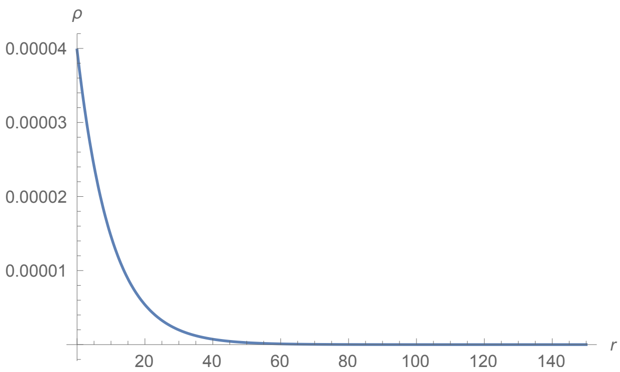

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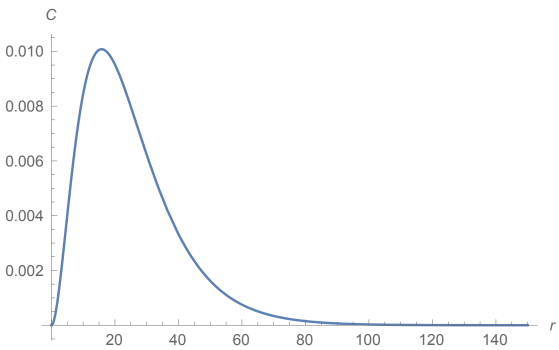

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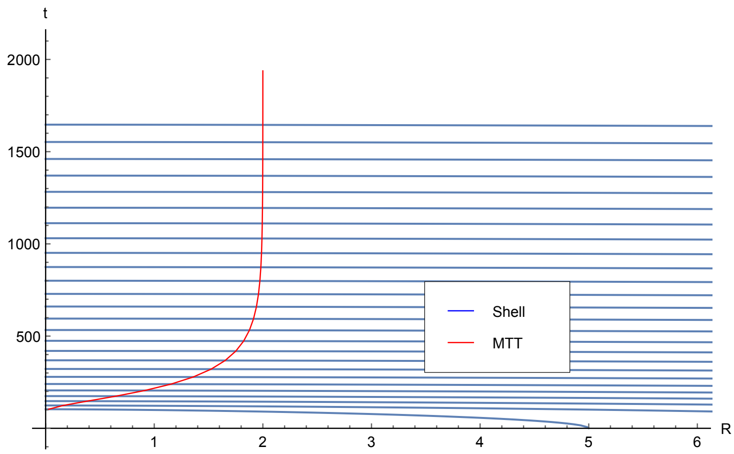

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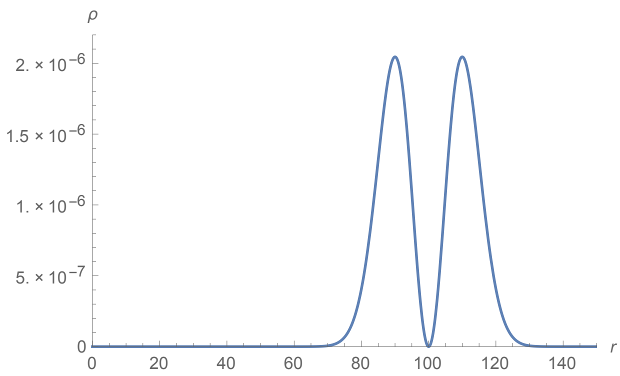

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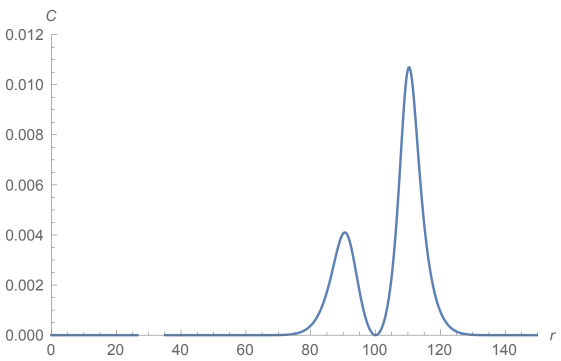

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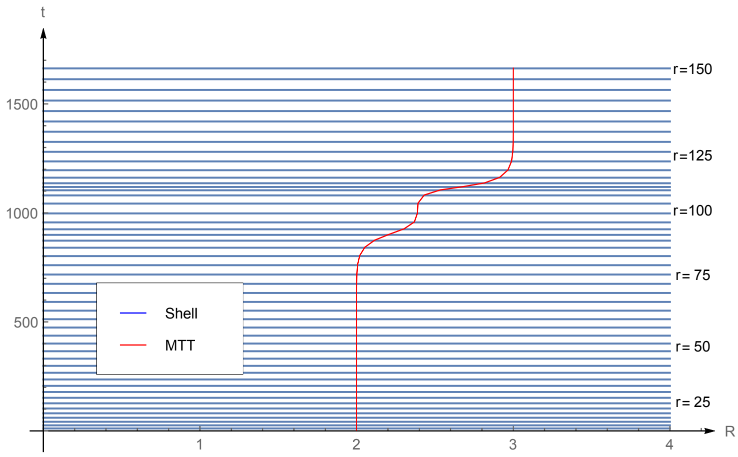

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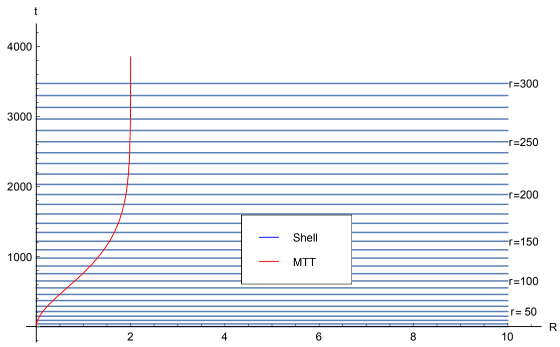

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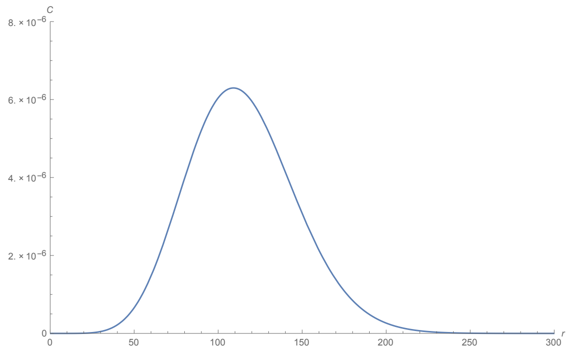

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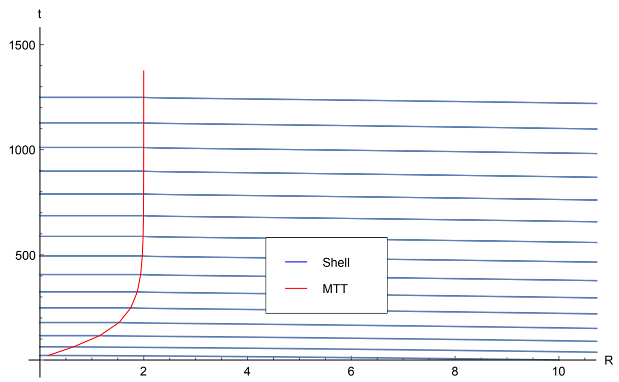

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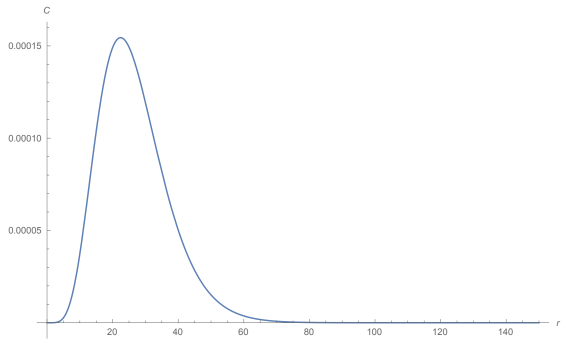

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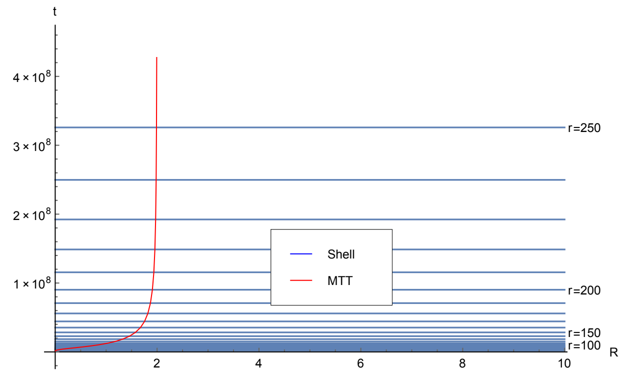

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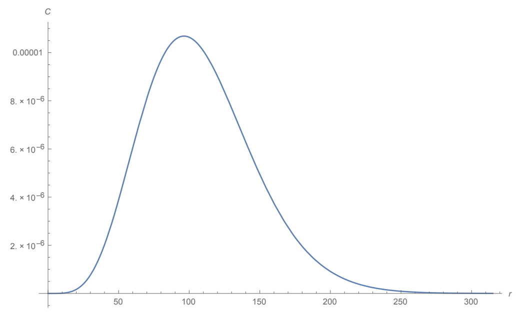

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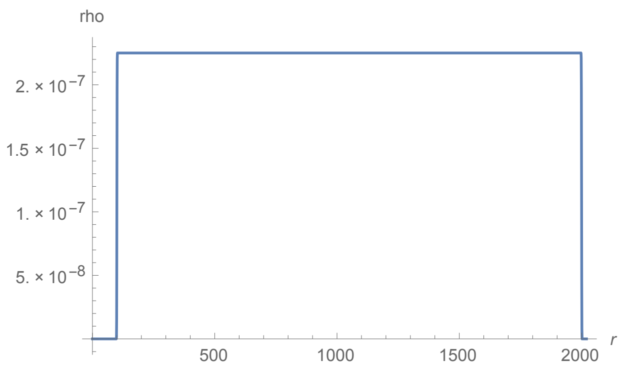

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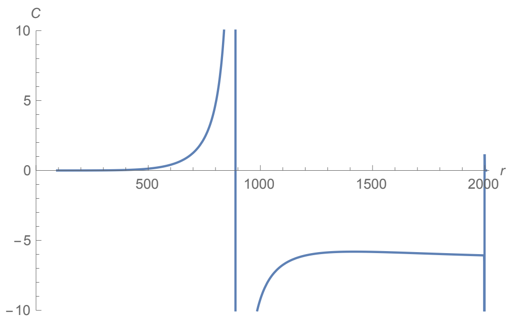

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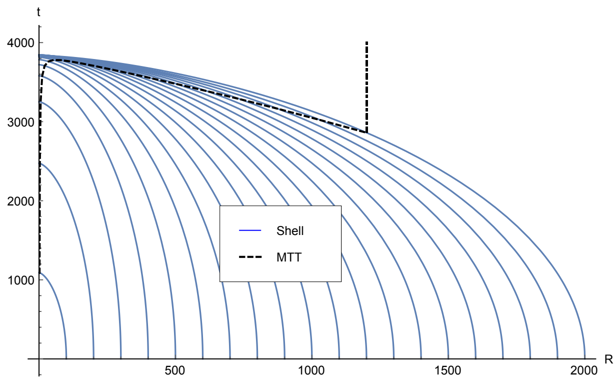

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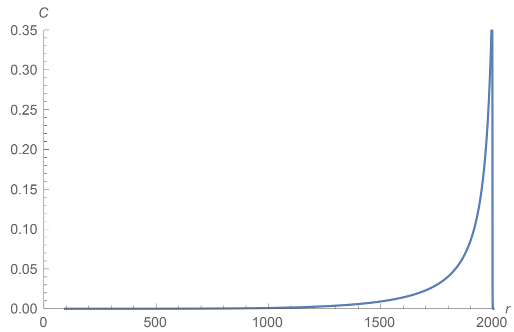

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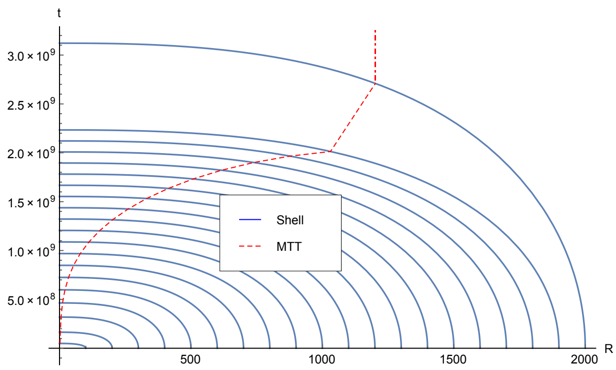

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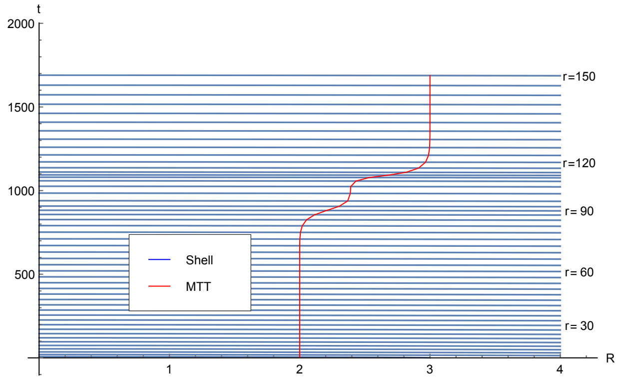

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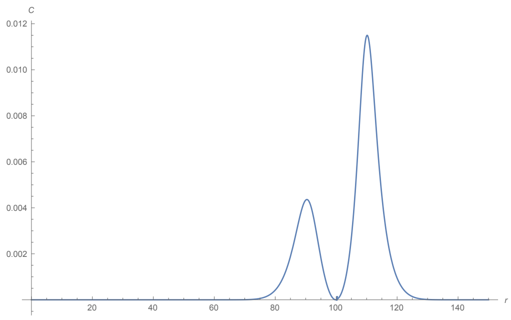

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