

Topological Born-Infeld charged black holes in Einsteinian cubic gravity

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In this paper, we study four-dimensional topological black hole solutions of Einsteinian cubic gravity in the presence of nonlinear Born-Infeld electrodynamics and a bare cosmological constant. First, we obtain the field equations which govern our solutions. Employing Abbott-Deser-Tekin and Gauss formulas, we present the expressions of conserved quantities namely total mass and total charge of our topological black solutions. We disclose the conditions under which the model is unitary and free of ghost with asymptotically A)dS and flat solutions. We find that, for vanishing bare cosmological constant, the model is unitary just for asymptotically flat solutions. Then, we discuss the behavior of temperature for latter case. We reveal that the temperature of planar and hyperbolic for positive cubic coupling) asymptotically flat solutions is negative as in Einstein-Maxwell gravity case. However, spherical solutions may have positive temperature and are physically reliable. Next, we turn to calculate entropy and electric potential. We show that the first law of thermodynamics is satisfied for spherical asymptotically flat solutions. Finally we peruse the effects of model parameters on thermal stability of these solutions in both canonical and grand canonical ensembles.

I. INTRODUCTION

General relativity has passed all tests successfully The latest one was the detection of gravitational wave 1, almost one hundred years after Einstein predicted it. Despite these achievements, at situations where the space-time is extremely curved, it is unavoidable to modify general relativity The most natural modification is taking into account the higher-order curvature terms. The well-known higher-order Lovelock terms provide this kind of modification while they respect the constraints of early version of general relativity 23. However, the latter has no contribution in four dimensions. Recently a cubic order curvature model with contribution in four dimensions called Einsteinian cubic gravity ECG) has been proposed 4. This model could attract many researcher's attention very soon 5–19. ECG respects the constraints that Lovelock theories do i.e. on a maximally symmetric background, it just propagates a transverse and massless graviton and in all dimensions it has the same relative coefficients of the different curvature invariants involved. Possessing the contribution in four dimensions along with other features have made this model intriguing and important. The latter property makes us able to see the effects of higher-order curvature modifications on $(2 + 1)$ -dimensional holographic duals of gravity theory solutions.

The solutions in the context of ECG have been explored from different points of view. In 5, by constructing perturbative five-dimensional black hole solutions of ECG, the holographic entanglement Renyi entropy has been computed in the dual field theory The first examples of black hole solutions in ECG have been obtained in 6, and thermal behaviors of them have been

explored. In 7, the static and spherically symmetric generalizations of four-dimensional linearly charged and uncharged black hole solutions in ECG have been constructed and thermodynamics of them has been studied. The most general theory of gravity to cubic order in curvature called Generalized Quasi-Topological Gravity GQTG) whose static spherically symmetric vacuum solutions are fully described by a single field equation has been constructed in 8. In latter, the ECG as well as Lovelock and quasi-topological gravities have been recovered in four dimensions as special cases. General results corresponding to static and spherically symmetric black hole solutions of general higher-derivative gravities including GQTG have also been established 9. It has been proved as well that the four-dimensional black hole solutions corresponding to an infinite family of ghost-free higher-order theories are universally stable below a certain mass 10. In 11, by employing the continued fraction approximation, some interesting properties of ECG black hole solutions such as the innermost stable circular orbit of massive test bodies near a black hole and the shadow of an black hole have been computed. Some properties of a nonsupersymmetric conformal field theory in three dimensions which could be a holographic dual to four-dimensional ECG model has been explored in 12. Euclidean AdS-Taub-NUT and bolt solutions with various base spaces in four- and six-dimensions constructed respectively in the context of ECG and GQTG have been studied as well and thermodynamics features of them have been explored 13.

In this paper, we study four-dimensional topological black hole solutions of ECG in the presence of nonlinear Born-Infeld BI) electrodynamics and a bare cosmological constant. To our knowledge, this is the first consideration of nonlinearly charged topological solutions in ECG. The importance of considering nonlinear BI electrodynamics is at least two fold. On the one hand, it resolves the singularity problem of Maxwell electrodynamics at the place of point charge 20. On the other

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hand, it comes from the low energy limit of open superstring theory 21–23. In addition, photon-photon interaction experiments have suggested that there is a nonlinear theory of electrodynamics in vacuum 24. Black holes with different horizon's topologies show drastically different thermodynamical properties as well. For instance, whereas Schwarzschild black holes with spherical horizon are not thermally stable, it has been argued that Schwarzschild-AdS black holes with planar or hyperbolic topologies at horizon are thermally stable and do not underlie Hawking-Page phase transition 25. From holographic point of view, topological solutions are described as duals to thermal states of conformal field theories as well 26.

This paper proceeds as follows. In next section, we will introduce the action of theory and find field equations. Then, we will calculate conserved quantities in section III. In section IV we will turn to compute thermodynamical quantities and check the satisfaction of thermodynamics first law. We will explore thermal stability of our solutions in section V. Last section is devoted to summary and concluding remarks.

II. ACTION AND FIELD EQUATIONS

The Einsteinian cubic gravity (ECG) which is the most general dimension-independent gravity theory that consists of metric and Riemann tensor contractions for which linearized spectrum coincides with Einstein gravity one, in the presence of nonlinear electrodynamics could be written as 47

$$S = \frac{1}{16\pi} \int_{\mathcal{M}} d^4x \sqrt{-g} \left(R + \sum_{i=2}^3 \alpha_i L_i - 2\Lambda_0 - \lambda \mathcal{P} + \mathcal{L} F \right) \quad (1)$$

up to cubic order in curvature. We write action 1 in Planck units where we set $G = c = 1$. In above action, $\Lambda_0 \lambda$ and α_i 's are bare cosmological constant, cubic coupling constant and Lovelock coefficients, respectively. We will assume $\lambda \geq 0$ throughout this paper. Also, L_i 's stand for i th-order Lovelock terms 23. The additional cubic contribution \mathcal{P} is defined as 4

$$\mathcal{P} = 12R_{ab}^c{}^d R_c^e{}^f{}^g R_e^a{}^b{}^c + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} - 12R_{abcd} R^{ac} R^{bd} + 8R_a^b R_b^c R_c^a \quad (2)$$

In this paper, we intend to consider Born-Infeld (BI) nonlinear electrodynamics for which

$$\mathcal{L} F = b^2 \left(1 - \sqrt{1 + \frac{F}{2b^2}} \right) \quad (3)$$

where b is nonlinear parameter and $F = F_{ab} F^{ab}$ in which $F_{ab} = 2\partial_a A_b$ and A_a is the electromagnetic potential. As b tends to infinity \mathcal{L} reduces to the linear Maxwell case i.e. $-F/4$. Whereas the second- and third-order

Lovelock terms are respectively topological and trivial in four-dimensions, the new cubic term \mathcal{P} has contribution in field equations 7.

We make the following ansatz for four-dimensional topological nonlinearly charged black solutions

$$ds^2 = -N^2 r f r dt^2 + \frac{dr^2}{f r} + r^2 d\Omega_k^2 \quad (4)$$

$$A = h r dt, \quad (5)$$

where

$$d\Omega_k^2 = \begin{cases} d\theta^2 + \sin^2 \theta d\phi^2 & k = 1 \\ d\theta^2 + d\phi^2 & k = 0 \\ d\theta^2 + \sinh^2 \theta d\phi^2 & k = -1 \end{cases} \quad (6)$$

represents a 2-dimensional hypersurface with constant curvature $2k$ and area A_k . $k = 1, 0$ and -1 represent two-sphere S^2 , plane \mathbb{R}^2 and hyperbolic H^2 topologies for the event horizon, respectively. Varying the action 1 with respect to Nr , fr and hr , we are left with three field equations. One of these field equations which arises from variation with respect to fr , is satisfied by $Nr = \text{const}$. Then, we have two field equations

$$\begin{aligned} 0 &= -r^4 \mathcal{H} - 2r^2 (k - \Lambda_0 r^2 - r f' - f) \\ &\quad - \frac{12\lambda}{r} r^3 f f''^2 + 2k r^2 f f''' - 4k r f f'' \\ &\quad + r^3 f f' f'' f'''^2 + 4k f f' + 4r f f'^2 - k r f'^2 \\ &\quad - 4f^2 f' - 2r f^2 r f''' - 2f''^2 \quad (7) \\ 0 &= 2h'^3 - b^2 2h' + r h''^8 \quad (8) \end{aligned}$$

where $\mathcal{H} = b^2 - b^2 (1 - h'^2 b^2)^{-12}$ which is reduced to $-h'^2/2$ for Maxwell case. Note that prime denotes the derivative with respect to r . One could immediately solve the electrodynamic field equation 8 as

$$hr = -\frac{q}{r} \mathbf{F} \left(\frac{11}{2}, \frac{5}{4}, \frac{1}{4} - \frac{q^2}{b^2 r^4} \right) \quad (9)$$

where $\mathbf{F}(x, y, z, w)$ is the Gauss hypergeometric function and q is an integration constant related to total electric charge of black hole. Expanding hr about infinity b the potential of Maxwell electrodynamics can be reproduced as $hr = -qr + \mathcal{O}(b^{-1})$. Substituting hr from Eq. 9 to Einstein field equation 7 and then dividing expressions by $2r^2$ one can integrate Einstein field equation once. After some manipulation, the result could be simplified as follows

$$\begin{aligned} &kr - m - \frac{r^3 \Lambda_0}{3} - r f + \frac{1}{6} b^2 r^3 \\ &\times \left[1 - \mathbf{F} \left(-\frac{1}{2}, -\frac{3}{4}, \frac{1}{4} - \frac{q^2}{b^2 r^4} \right) \right] \\ &+ \frac{\lambda}{r^2} \left[6r f f'' 2k + r f' - 2f \right. \\ &\left. - 2r f'^2 3k + r f' - 12f' f k - f \right] = 0 \quad (10) \end{aligned}$$

where m is an integration constant related to total mass of black hole. For BI-(A)dS gravity in four dimensions ($\lambda = 0$), fr could be obtained from above equation as 27

$$fr = k - \frac{m}{r} - \frac{r^2 \Lambda_0}{3} + \frac{1}{6} b^2 r^2 \left[1 - \mathbf{F} \left(-\frac{1}{2} - \frac{3}{4} \frac{1}{4} - \frac{q^2}{b^2 r^4} \right) \right] \quad (11)$$

In this case, the total mass per unit area A_k is 2728

$$M_E = \frac{m}{8\pi} \quad (12)$$

In next section, we turn to calculate the conserved quantities related to our solutions namely total mass and total charge.

III. CONSERVED QUANTITIES

In present section, we intend to compute total mass and total charge of our black solutions. In order to find total mass, we follow the Abbott-Deser-Tekin method 29 of finding conserved quantities for higher-order curvature gravities 30. According to that, we have to find equivalent quadratic curvature action of the form

$$\mathcal{S}_{\text{EQCA}} = \int d^4x \sqrt{-g} \left[\tilde{\kappa}^{-1} (R - 2\tilde{\Lambda}_0) + \tilde{\alpha} R^2 + \tilde{\beta} R_{ab} R^{ab} + \tilde{\gamma} L_2 \right] \quad (13)$$

where $L_2 = R_{abcd} R^{abcd} - 4R_{ab} R^{ab} + R^2$ is the second-order Lovelock term known as Gauss-Bonnet term. Note that equivalent quadratic curvature action has the same vacuum solution and the same linearized field equations as corresponding higher-order gravity theory. We can write the gravity part of our action 1 which is cubic-order as

$$\mathcal{S}_G = \int_{\mathcal{M}} d^4x \sqrt{-g} f(R_{cd}^{ab})$$

$$f(R_{cd}^{ab}) = \kappa^{-1} R + \alpha_2 L_2 - 2\Lambda_0 - \lambda \mathcal{P} \quad (14)$$

in which additional cubic contribution \mathcal{P} defined in Eq. 2 can be re-written as

$$\mathcal{P} = 12R_{cd}^{ab} R_{af}^{ce} R_{be}^{df} + 4R_{cd}^{ab} R_{ab}^{ef} R_{ef}^{cd} - 12R_{cd}^{ab} R_a^c R_b^d + 8R_b^a R_a^c R_c^b \quad (15)$$

In appendix A we show how one can obtain Eq. 15. We suppose that action 14 has an asymptotically A)dS solution around \bar{g}_{ab} for which

$$\bar{R}_{cd}^{ab} = \frac{\Lambda}{3} (\delta_c^a \delta_d^b - \delta_d^a \delta_c^b) \quad (16)$$

where Λ is the effective cosmological constant. Indeed, for $\Lambda = 0$, we have an asymptotically flat solution. Comparing Eq. 14, with the most general cubic curvature

action constructed from Riemann tensor contractions 31

$$\mathcal{S}_{\text{GCCA}} = \int d^4x \sqrt{-g} \left[\kappa^{-1} R - 2\Lambda_0 + \alpha R^2 + \beta R_{ab} R^{ab} + \gamma L_2 + F(R_{cd}^{ab}) \right] \quad (17)$$

in which

$$F(R_{cd}^{ab}) \equiv a_1 R_{cd}^{ab} R_{af}^{ce} R_{be}^{df} + a_2 R_{cd}^{ab} R_{ab}^{ef} R_{ef}^{cd} + a_3 R_b^a R_{ea}^{cd} R_{cd}^{eb} + a_4 R R_{cd}^{ab} R_{ab}^{cd} + a_5 R_b^a R_d^c R_{ac}^{bd} + a_6 R_b^a R_a^c R_c^b + a_7 R R_b^a R_a^b + a_8 R^3$$

one can find that

$$\begin{aligned} \gamma &= \frac{\alpha_2}{\kappa} & a_1 &= -\frac{12\lambda}{\kappa} \\ a_2 &= -\frac{4\lambda}{\kappa} & a_5 &= \frac{12\lambda}{\kappa} & a_6 &= -\frac{8\lambda}{\kappa} \\ \alpha &= \beta = a_3 = a_4 = a_7 = a_8 = 0 \end{aligned} \quad (18)$$

In addition, parameters in actions 13 and 17 are related to each other, so that, in four dimensions 30

$$\begin{aligned} \frac{1}{\tilde{\kappa}} &\equiv \frac{1}{\kappa} - \frac{\Lambda^2}{3} a_1 + 4a_2 + 6a_3 + 4a_4 \\ &\quad + 9a_5 + a_6 + 4a_7 + 16a_8 \quad (19) \\ \tilde{\Lambda}_0 &\equiv \frac{\tilde{\kappa}}{\kappa} \Lambda_0 + \frac{2\Lambda}{3} \left(1 - \frac{\tilde{\kappa}}{\kappa} \right) \quad (20) \\ \tilde{\alpha} &\equiv \alpha + \frac{\Lambda}{3} 3a_1 - 6a_2 - 8a_4 + a_5 \\ &\quad + 3 - a_3 + 2a_7 + 12a_8 \quad (21) \\ \tilde{\beta} &\equiv \beta + \frac{\Lambda}{3} - 9a_1 + 24a_2 + 16a_3 + 5a_5 \\ &\quad + 316a_4 + 3a_6 + 4a_7 \quad (22) \\ \tilde{\gamma} &\equiv \gamma + \frac{\Lambda}{3} - 3a_1 + 6a_2 + 3a_3 + 4a_4 \quad (23) \end{aligned}$$

Using Eq. 18, one receives

$$\begin{aligned} \frac{\kappa}{\tilde{\kappa}} &= 1 - \frac{8\lambda\Lambda^2}{3} \quad (24) \\ \Lambda_0 &= \tilde{\Lambda}_0 \left(1 - \frac{8\lambda\Lambda^2}{3} \right) + \frac{16\lambda\Lambda^3}{9} \quad (25) \\ \kappa\tilde{\gamma} &= \alpha_2 + 4\lambda\Lambda \quad (26) \\ \tilde{\alpha} &= \tilde{\beta} = 0 \quad (27) \end{aligned}$$

Moreover, in four dimensions, we have $\Lambda = \tilde{\Lambda}_0$ 30. Therefore, Eq. 25 leads to

$$\frac{8\lambda}{9} \Lambda^3 - \Lambda + \Lambda_0 = 0 \quad (28)$$

whose solution is the effective cosmological constant Λ . As Eq. 28 shows, $\Lambda = \Lambda_0$ for BI-(A)dS gravity $\lambda =$

0). One also needs to calculate the equivalent effective Newton's constant as

$$\frac{1}{\tilde{\kappa}_e} \equiv \frac{1}{\tilde{\kappa}} + 8\Lambda\tilde{\alpha} + \frac{4\Lambda}{3}\tilde{\beta} \quad (29)$$

Putting $\tilde{\alpha} = \tilde{\beta} = 0$ from Eq. 27 to above relation, we find

$$\tilde{\kappa}_e = \tilde{\kappa}. \quad (30)$$

The total mass per unit area A_k is given by 30

$$M = \frac{\kappa}{\tilde{\kappa}_e} M_E \quad (31)$$

where M_E is the total mass in Einstein gravity which can be found as $M_E = m/8\pi$ in four dimensions see Eq. 12. Thus, using Eqs. 24 and 30, we have

$$\frac{\kappa}{\tilde{\kappa}_e} = \frac{\kappa}{\tilde{\kappa}} = 1 - \frac{8\lambda\Lambda^2}{3} \quad (32)$$

which leads to

$$M = \left(1 - \frac{8\lambda\Lambda^2}{3}\right) \frac{m}{8\pi} \quad (33)$$

For model to be unitary and free of ghost, it is necessary to have $\kappa/\tilde{\kappa}_e > 0$ 30. Here, it is remarkable to discuss about the restricts imposed by Eq. 28 as well. This equation is cubic in Λ and its discriminant Δ is

$$\Delta = \frac{32\lambda}{9} (1 - 6\lambda\Lambda_0^2)$$

So, it has three real roots if

$$\Delta \geq 0 \rightarrow \lambda(1 - 6\lambda\Lambda_0^2) \geq 0$$

One of the possibilities is $\Lambda_0 = 0$ for which

$$\Lambda = 0 \text{ and } \Lambda = \pm \frac{3}{2\sqrt{2\lambda}}$$

Note that we have assumed λ to be positive throughout the paper. For $\Lambda = \pm 32\sqrt{2\lambda}\kappa/\tilde{\kappa}_e$ given by Eq. 32 is negative. As a result, for vanishing bare cosmological constant Λ_0 we have just a unitary model with asymptotically flat solution ($\Lambda = 0$). Another possibility is $\Lambda_0^2 = 16\lambda$ for which $\Delta = 0$. In this case, we have

$$\begin{aligned} \text{For } \Lambda_0 &= +\frac{1}{\sqrt{6\lambda}} \quad \Lambda = -\sqrt{\frac{3}{2\lambda}} \text{ and } \Lambda = +\frac{1}{2}\sqrt{\frac{3}{2\lambda}} \\ \text{For } \Lambda_0 &= -\frac{1}{\sqrt{6\lambda}} \quad \Lambda = +\sqrt{\frac{3}{2\lambda}} \text{ and } \Lambda = -\frac{1}{2}\sqrt{\frac{3}{2\lambda}} \end{aligned}$$

where the second one in both cases is a double root. One could check that for all these roots $\kappa/\tilde{\kappa}_e \leq 0$ and therefore the model is not unitary. For some other values of parameters including ones for which $\Delta < 0$, one can find some vacuum solutions within a unitary model, as

well. Note that for $\Delta < 0$ ($1 - 6\lambda\Lambda_0^2 < 0$), the cubic equation 28 has two imaginary complex conjugate roots and just one real root for Λ . In our thermodynamical studies which will be presented in next sections, we focus on vanishing bare cosmological constant case ($\Lambda_0 = 0$). As discussed above, in latter case, an asymptotically flat solution ($\Lambda = 0$) is just allowed. So, the mass per unit area reads

$$M = \frac{m}{8\pi} \quad (34)$$

according to Eq. 33.

Here, we turn to calculate the total charge of our black solutions via Gauss law. Using 9, one can show that $F_{rt} = h'r$ is

$$F_{rt} = \frac{q}{r^2 \sqrt{1 + q^2 b^2 r^4}} \quad (35)$$

Then, using Gauss law, the total charge is given by

$$Q = \frac{1}{4\pi} \int r^2 \mathcal{L}_F F_{\mu\nu} n^\mu u^\nu d\Omega_k \quad (36)$$

where $\mathcal{L}_F = \partial\mathcal{L}\partial F$ and \mathcal{L} is the Lagrangian of BI non-linear electrodynamics expressed in Eq. 3. Also, n^μ and u^ν are the unit spacelike and timelike normals to the hypersurface of radius r given as $n^\mu = \sqrt{-g_{tt}}^{-1} dt = (\sqrt{fr})^{-1} dt$ and $u^\nu = (\sqrt{g_{rr}})^{-1} dr = \sqrt{fr} dr$. Using Eqs. 35 and 36, we can obtain the total charge of black solutions per unit area as

$$Q = \frac{q}{16\pi} \quad (37)$$

In next section, we will obtain thermodynamical quantities corresponding to our solutions and check the first law of thermodynamics for them.

IV. THERMODYNAMICAL QUANTITIES AND THERMODYNAMICS FIRST LAW

In this section, we intend to calculate thermodynamical quantities and check the first law of thermodynamics. For this purpose, we first have to compute field equations near the black hole horizon. Taylor expansion of metric function $f(r)$ near the black hole horizon r_h is

$$f(r) = \sum_{n=0}^{\infty} a_n r^n - r_h^n \quad (38)$$

in which $a_n = f^{(n)}(r_h)/n!$. Note that $a_0 = f(r_h) = 0$ and $a_1 = f'(r_h) = 2\kappa_g$ where κ_g is surface gravity on the horizon and $f'(r_h) \geq 0$. Plugging above expansion into field equation 10, one receives following equation up to

quadratic order of $r - r_h$

$$\begin{aligned}
& kr_h - m - 8\lambda\kappa_g^2 2\kappa_g + \frac{3k}{r_h} - \frac{\Lambda_0}{3} r_h^3 \\
& + \frac{1}{6} r_h^3 b^2 \left[1 - \mathbf{F} \left(-\frac{1}{2} - \frac{3}{4} \frac{1}{4} - \frac{q^2}{b^2 r_h^4} \right) \right] + \left[k - \Lambda_0 r_h^2 \right. \\
& - 2\kappa_g r_h - 24\lambda k \frac{\kappa_g^2}{r_h^2} + \frac{1}{2} r_h^2 b^2 \left. 1 - \sqrt{1 + \frac{q^2}{b^2 r_h^4}} \right] r - r_h \\
& + \left[72a_3 \lambda \kappa_g \kappa_g + \frac{k}{r_h} + 24a_2^2 \lambda \kappa_g - a_2 r_h - 2\kappa_g - \Lambda_0 r_h \right. \\
& - \frac{24a_2 \lambda \kappa_g}{r_h} \left(4\kappa_g + \frac{3k}{r_h} \right) + \frac{1}{2} r_h b^2 \left. \left[1 - \left(1 + \frac{q^2}{b^2 r_h^4} \right)^{-\frac{1}{2}} \right] \right. \\
& \left. + \frac{72\lambda \kappa_g^2}{r_h^3} k + \frac{96\lambda \kappa_g^3}{r_h^2} \right] r - r_h^2 + \mathcal{O} r - r_h^3 = 0
\end{aligned} \tag{39}$$

Solving above equation order by order in terms of $r - r_h$ powers, up to second term, we get:

$$\begin{aligned}
& kr_h - m - 8\lambda\kappa_g^2 \left(2\kappa_g + \frac{3k}{r_h} \right) - \frac{\Lambda_0}{3} r_h^3 \\
& + \frac{1}{6} b^2 r_h^3 \left[1 - \mathbf{F} \left(-\frac{1}{2} - \frac{3}{4} \frac{1}{4} - \frac{q^2}{b^2 r_h^4} \right) \right] = 0 \tag{40} \\
& k + \frac{1}{2} r_h^2 b^2 \left[1 - \sqrt{1 + \frac{q^2}{b^2 r_h^4}} \right] - \Lambda_0 r_h^2 \\
& - 2\kappa_g r_h - 24\lambda \frac{\kappa_g^2}{r_h^2} k = 0 \tag{41}
\end{aligned}$$

As expected, these relations reproduce the following ECG-Maxwell results if $b \rightarrow \infty$

$$kr_h - m - 8\lambda\kappa_g^2 \left(2\kappa_g + \frac{3k}{r_h} \right) - \frac{\Lambda_0}{3} r_h^3 + \frac{q^2}{4r_h} = 0 \tag{42}$$

$$k - \frac{q^2}{4r_h^2} - \Lambda_0 r_h^2 - 2\kappa_g r_h - 24\lambda \frac{\kappa_g^2}{r_h^2} k = 0 \tag{43}$$

It is notable to mention that since the third term in 39 is linear in terms of a_3 we can compute the latter in terms of a_2 . Also, other higher order terms are linear with respect to other coefficients and therefore, all of them could finally be determined in terms of a_2 . Consequently the family of solutions have only one free parameter, a_2 . This fact shows that the present model allows black solutions which have regular horizons where the regularity condition reduces the number of solutions from a two-parameter family to a one-parameter one.

From 40 and 41, we could determine m and surface

gravity κ_g as functions of model parameters:

$$\begin{aligned}
\kappa_g &= \frac{r_h^3}{24\lambda k} \left[1 + \frac{12k\lambda b^2}{r_h^2} \left(1 - \sqrt{1 + \frac{q^2}{b^2 r_h^4}} \right) \right. \\
&\quad \left. - \frac{24k\lambda\Lambda_0}{r_h^2} + \frac{24k^2\lambda}{r_h^4} \right]^{\frac{1}{2}} - \frac{r_h^3}{24\lambda k} \tag{44} \\
\frac{m}{r_h} &= k - \frac{24\lambda\kappa_g^2}{r_h^2} k - \frac{\Lambda_0 r_h^2}{3} - \frac{16\lambda\kappa_g^3}{r_h} \\
&\quad + \frac{b^2 r_h^2}{6} \left[1 - \mathbf{F} \left(-\frac{1}{2} - \frac{3}{4} \frac{1}{4} - \frac{q^2}{b^2 r_h^4} \right) \right] \tag{45}
\end{aligned}$$

It is notable to point out that m is related to total mass M via Eq. 33. If λ and b tends to zero and infinity respectively the above equations reduce to RN-(A)ds black hole ones:

$$\kappa_g = \frac{k}{2r_h} - \frac{q^2}{8r_h^3} - \frac{\Lambda_0 r_h}{2} \tag{46}$$

$$\frac{m}{r_h} = k - \frac{\Lambda_0 r_h^2}{3} + \frac{q^2}{4r_h^2} \tag{47}$$

The Hawking temperature of our solution can be written in terms of surface gravity 32

$$T = \frac{\kappa_g}{2\pi} \tag{48}$$

where κ_g has been given in Eq. 44. Using 44 and 48, one could find the temperature as

$$\begin{aligned}
T &= \frac{r_h^3}{48\pi\lambda k} \left[1 + \frac{12k\lambda b^2}{r_h^2} \left(1 - \sqrt{1 + \frac{q^2}{b^2 r_h^4}} \right) + \frac{24k^2\lambda}{r_h^4} \right]^{\frac{1}{2}} \\
&\quad - \frac{r_h^3}{48\pi\lambda k} \tag{49}
\end{aligned}$$

where we set $\Lambda_0 = 0$. Note that, according to discussions presented in section III we study thermodynamics of asymptotically flat solutions ($\Lambda = 0$) with vanishing bare cosmological constant Λ_0 . Since a non-negative temperature T is physically reliable, we have to look for conditions under which $T \geq 0$. In the case of hyperbolic solutions ($k = -1$), for positive values of λ as we have assumed in present work), Eq. 49 shows that the temperature is negative. For planar solutions ($k = 0$), the temperature 49 reads as

$$T(k=0) = -\frac{b^2 r_h}{8\pi} \sqrt{1 + \frac{q^2}{b^2 r_h^4}} - 150$$

It is obvious that the temperature of planar BI charged black solutions ($k = 0$) is negative, as well. So, these cases $k = 0$ and -1 are ruled out from our thermodynamical discussions. It is remarkable to mention that linearly charged hyperbolic and planar black solutions in Einstein gravity have negative temperatures, as well see

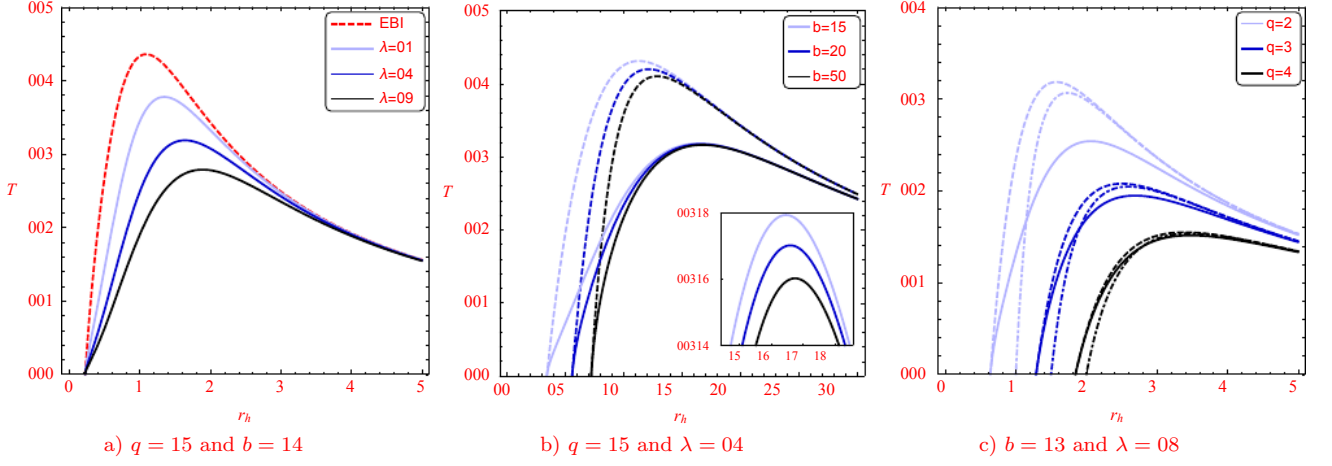


FIG. 1: The behavior of T versus r_h for spherical asymptotically flat solutions with various values of λb and q . Note that the solid (dashed) curves show the temperature in the presence (absence) of cubic terms. In the right panel, dot-dashed curves show the temperature for Einstein-Maxwell regime.

Eq. 46. For spherical solutions ($k = 1$), the temperature 49 may be non-negative. Therefore, we study the thermodynamics of asymptotically flat solutions with spherical topology on horizon.

Now, we are going to disclose the effects of each model parameter λb and q on temperature of spherical asymptotically flat solutions. From Eq. 49, one can see that for $k = 1$, the first term is positive while the second term is negative. Increasing parameters b and q enhances the magnitude of

$$\frac{12k\lambda b^2}{r_h^2} \left(1 - \sqrt{1 + \frac{q^2}{b^2 r_h^4}} \right)$$

in Eq. 49. The latter term is negative for $k = 1$ and appears in first term which is positive. So, as b or q enhances, T lowers for $k = 1$ case. In order to exhibit these effects, we have plotted T versus r_h for different values of bq and λ in Fig. 1. This figure shows that as each one of parameters bq and λ grows, the temperature value becomes lower. It means that as influences of cubic terms nonlinearity of electrodynamics) grow fades), the temperature of spherical black holes decreases. Fig. 1 shows that for spherical asymptotically flat solutions, there is a maximum temperature T_{\max} . The behavior of T_{\max} in terms of model parameters is exhibited in table I as well. One can find that the value of T_{\max} decreases as each of parameters λb and q increases.

Let us now calculate the entropy of our black holes. The entropy of black solutions in higher-order gravities could be computed by Wald's formula given by 33

$$S = -2\pi \int_H d^2x \sqrt{h} \frac{\delta \mathcal{L}_G}{\delta R_{abcd}} \epsilon_{ab} \epsilon_{cd} \quad (51)$$

where h is the determinant of the induced metric on the horizon, $\delta \mathcal{L}_G \delta R_{abcd}$ is the Euler-Lagrange derivative of

TABLE I: The behavior of maximum temperature T_{\max} for various values of model parameters.

λ	b	q	T_{\max}
0.01			0.10
0.02	0.2	0.5	0.08
0.05			0.06
0.01	0.01		0.10
	0.50	1	0.08
	0.95		0.07
0.01	0.6	0.8	0.08
		1.2	0.07
		1.6	0.06

gravitational Lagrangian and ϵ_{ab} is the binormal of the horizon normalized as $\epsilon_{ab} \epsilon^{ab} = -2$. Applying above formula on our theory introduced by action 1, we receive

$$S = \frac{1}{4} \int_H d^2x \sqrt{h} \left[1 + 2\alpha_2 R_2 + \lambda \left(36 R_{ab}^{ef} R_{cd}^{fg} R_{acef} + 3 R_{ab}^{ef} R_{cdef} - 12 R_{ac} R_{db} - 24 R^{ef} R_{ebfc} g_{bd} + 24 g_{bd} R_{ce} R_a^e \right) \epsilon^{ab} \epsilon^{cd} \right] \quad (52)$$

in which R_2 is the Ricci scalar of the induced metric on the horizon. This term comes from Gauss-Bonnet term $L_2 = R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd}$ in action 1. As pointed out before, this term is topological and hence has no effect in our calculations so far. However, it has contribution in entropy. Using the metric 4 with $N = 1$, one determines the entropy per unit area as

$$S = \frac{r_h^2}{4} \left[1 - 24\lambda \frac{\kappa_g^2}{r_h^2} \left(\frac{2k}{\kappa_g r_h} + 1 \right) \right] + k\alpha_2 \quad (53)$$

in which κ_g could be replaced by Eq. 44. It is remarkable to mention that, above relation reduces to $r_h^2/4$ 27 if higher-order terms disappear ($\lambda = \alpha_2 = 0$).

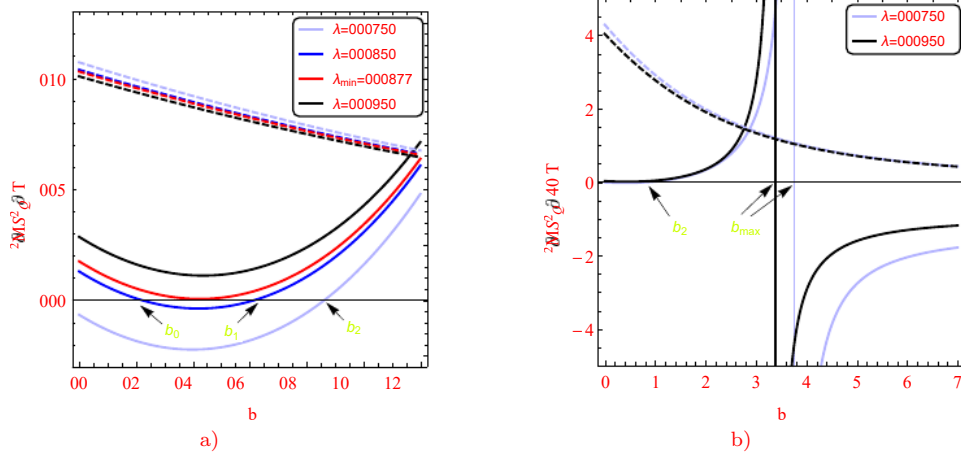


FIG. 2: The behaviors of $\partial^2 M \partial S^2_Q$ and T (dashed) versus b for different values of λ with $q = 1$ and $r_h = 0.5$.

Now, we turn to calculate the electric potential. The electric potential U measured at infinity with respect to horizon is defined by

$$U = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r=r_h} \quad (54)$$

where $\chi = \partial_t$ is the null generator of the horizon. Using 9 and 54, one finds

$$U = \frac{q}{r_h} \mathbf{F} \left(\frac{11}{2} \frac{5}{4} \frac{1}{4} - \frac{q^2}{b^2 r_h^4} \right) \quad (55)$$

In order to check the first law of thermodynamics, we first write a Smarr-type formula. Using Eqs. 34, 37 and 45, the Smarr-type formula $M r_h Q$ can be written as

$$M r_h Q = \frac{1}{8\pi} \left[k r_h - \frac{4\lambda \kappa_g^2}{r_h} 6k + 4\kappa_g r_h + \frac{1}{6} b^2 r_h^3 \left(1 - \mathbf{F} \left(-\frac{1}{2} - \frac{3}{4} \frac{1}{4} - \frac{16\pi Q^2}{b^2 A_k^2 r_h^4} \right) \right) \right] \quad (56)$$

According to Eq. 53, $r_h = r_h(S, Q)$ and in general $M = M(S, Q)$. We can then consider S and Q as a complete set of extensive quantities for mass. Therefore, temperature T and electric potential U are defined as conjugate intensive quantities for S and Q respectively and

$$T = \left(\frac{\partial M}{\partial S} \right)_Q \quad U = \left(\frac{\partial M}{\partial Q} \right)_S$$

These intensive quantities can be calculated using Eqs.

37, 53 and 56 where¹

$$\begin{aligned} T &= \left(\frac{\partial M}{\partial S} \right)_Q = \left(\frac{\partial M}{\partial r_h} \right)_Q \left(\frac{\partial S}{\partial r_h} \right)_Q^{-1} \quad (57) \\ U &= \left(\frac{\partial M}{\partial Q} \right)_S = \left(\frac{\partial M}{\partial Q} \right)_{r_h} + \left(\frac{\partial M}{\partial r_h} \right)_Q \left(\frac{\partial r_h}{\partial Q} \right)_S \\ &= \left(\frac{\partial M}{\partial Q} \right)_{r_h} - \frac{\left(\frac{\partial S}{\partial Q} \right)_{r_h} \left(\frac{\partial M}{\partial r_h} \right)_Q}{\left(\frac{\partial S}{\partial r_h} \right)_Q} \\ &= \left(\frac{\partial M}{\partial Q} \right)_{r_h} - T \left(\frac{\partial S}{\partial Q} \right)_{r_h} \quad (58) \end{aligned}$$

Our calculations show that T and U computed by Eqs. 57 and 58 coincide with ones obtained from Eqs. 49 and 55 for nonlinearly charged spherical ($k = 1$) asymptotically flat solutions. Thus, the first law of thermodynamics

$$dM = T dS + U dQ, \quad (59)$$

is satisfied by these quantities.

In next section, we will study the thermal stability of spherical solutions in both canonical and grand canonical ensembles.

V. THERMAL STABILITY

In this section, we intend to study thermal stability of the four-dimensional nonlinearly charged asymptotically flat solutions of ECG with spherical horizon in both canonical and grand canonical ensembles. In canonical ensemble where the charge is a

¹ To obtain Eq. 58, we use cyclic rule as $\left(\frac{\partial r_h}{\partial Q} \right)_S = -\frac{\partial S / \partial Q_{r_h}}{\partial S / \partial r_h Q}$

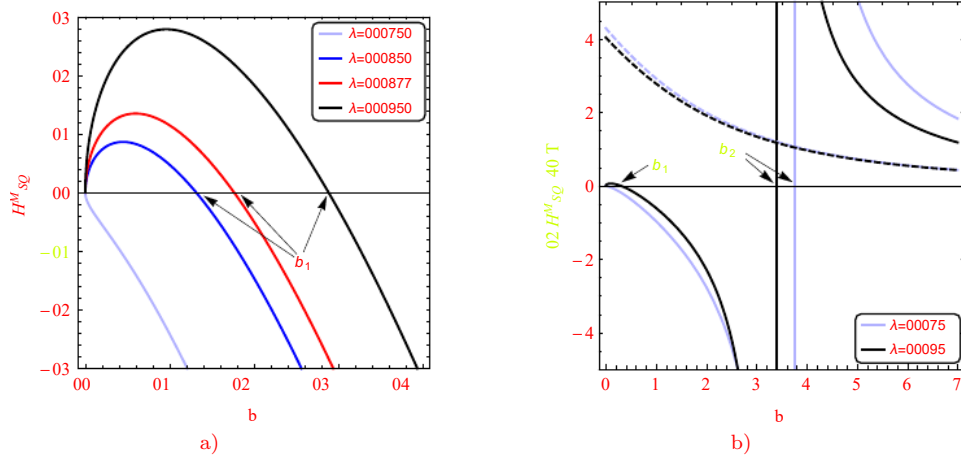


FIG. 3: The behavior of $\mathbf{H}^M_{S,Q}$ and T (dashed) versus b for different values of λ with $q = 1$ and $r_h = 0.5$.

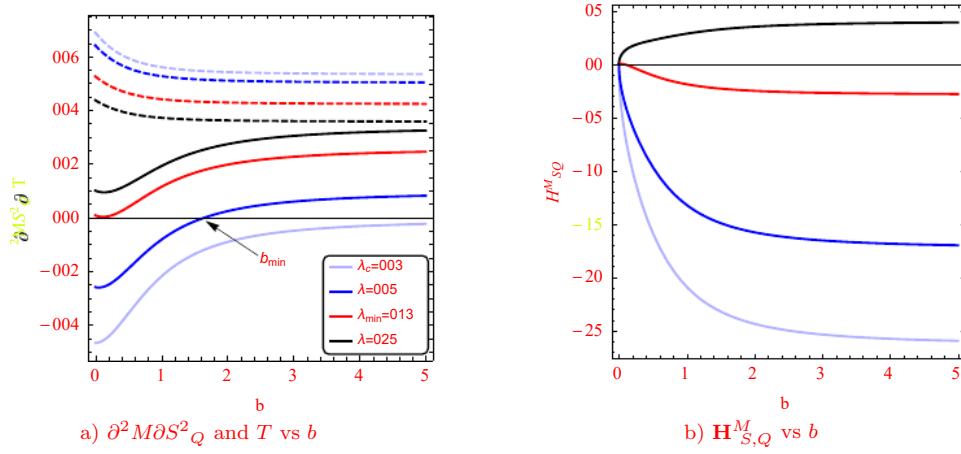


FIG. 4: The behaviors of $\partial^2 M \partial S^2_Q \mathbf{H}^M_{S,Q}$ and T (dashed) versus b for different values of λ with $q = 1$ and $r_h = 1$.

fixed parameter, the positivity of heat capacity $C = T \partial T \partial S_Q = T \partial^2 M \partial S^2_Q$ guarantees the local stability [34]. Therefore, it is sufficient to check that $\partial^2 M \partial S^2_Q$ is positive in order to explore the stability of solutions in the ranges where temperature is positive as well. Charge Q is no longer fixed in grand canonical ensemble. In latter ensemble, the system is locally stable provided the determinant of Hessian matrix $\mathbf{H}^M_{S,Q} = [\partial^2 M \partial S \partial Q]$ is positive, in positive temperature ranges. Since the explicit forms of $\partial^2 M \partial S^2_Q$ and $\mathbf{H}^M_{S,Q}$ are complicated, we avoid writing them here. However, the main results are depicted in Figs. 2-4

Let us now turn to study the stability of black solutions in canonical and grand canonical ensembles. Our investigations show that the values of charge q and horizon radius r_h determine how thermal stability is influenced by other parameters. We study thermal stability in canonical ensemble for two different types of solutions Figs. 2 and 4(a). Let us focus on the first type exhib-

ited in Fig. 2. As Fig. 2(a) shows, in highly nonlinear electrodynamics regime small nonlinearity parameter b values), there is a minimum value for cubic coupling λ λ_{\min} that for values greater than it, the system is stable up to b_{\max} Fig. 2(b). For λ values a bit smaller than λ_{\min} the system is stable except for b values between b_0 and b_1 Fig. 2(a). As λ becomes smaller, b_0 tends to zero and b_1 becomes greater. So, the instability interval becomes larger. In this case, the system is stable for b values between b_2 and b_{\max} Fig. 2(b). b_2 and b_{\max} increase as λ decreases. Also, for b values greater than b_{\max} including linear Maxwell case), the system is unstable. Note that we plot the curves in Fig. 2(b) just for two λ values in order not to have a messy plot. For other λ values, the qualitative behavior is the same. In addition, we plot the temperature in Figs. 2-4 as well, to ensure it is positive in the regions under study. One could see that the behavior of temperature in terms of λ and b coincides with what discussed before i.e. tem-

perature decreases as each of latter parameters increases. In second type displayed in Fig. 4(a) we again have a minimum value for λ . However, in this case, there is no upper bound for b and the system is stable for all b values if $\lambda \geq \lambda_{\min}$. For λ values a bit smaller than λ_{\min} there is a minimum value for b b_{\min} as well, that for $b > b_{\min}$ we have a stable system. As λ becomes smaller, we have a critical value λ_c that for $\lambda < \lambda_c$ the system is unstable for all b values.

We continue by discussing the thermal stability in grand canonical ensemble. For the first type represented in Fig. 3 in highly nonlinear electrodynamics regime Fig. 3(a), the system is unstable for some cubic couplings, however for some greater ones, the system becomes stable up to a specific value of b b'_1 . For b values greater than b'_1 we have an unstable system between b'_1 and b'_2 Fig. 3(b). For $b > b'_2$ including linear Maxwell case), the system becomes thermally stable. b'_1 b'_2 is greater smaller) for greater values of λ . In second type displayed in Fig. 4(b) whereas the system is unstable for some values of λ it becomes totally stable as λ grows.

VI. SUMMARY AND CONCLUDING REMARKS

In present paper, we studied four-dimensional topological Born-Infeld BI) charged black hole solutions in the context of Einsteinian cubic gravity ECG) in the presence of a bare cosmological constant. ECG is the most general gravity up to cubic order in curvature which is independent of dimension and its linearized spectrum coincides with general relativity one. Also, both theoretical open superstring theory) and experimental photon-photon interaction experiments) evidences suggest nonlinear theories of electrodynamics such as BI model. Topological solutions are known as duals to conformal field theories' thermal states as well and could show extremely different thermodynamical features. To our knowledge, this is the first study of topological solutions in ECG with nonlinear electrodynamics.

First, we introduced the action of theory and obtained field equations. Integrating field equations, we received the electromagnetic potential as well as a second order differential equation for metric function. Then, we presented the total mass M formula by employing Abbott-Deser-Tekin method and showed that total mass depends on cubic coupling in general. We obtain total charge Q via Gauss formula as well. Moreover, we discussed the conditions under which the model is unitary and ghost-free and explore different possible cases. We found that if bare cosmological constant vanishes, the model is unitary just with an asymptotically flat solution. As one of the possibilities, we focused on study the thermodynamics of latter case. Putting the expansion of metric function near horizon in second order field equation, we received the temperature T and total mass as functions of horizon radius r_h and charge q . We showed that, with

a vanishing bare cosmological constant, the temperature is physically reliable ($T \geq 0$) just for spherical solutions and it is negative for planar and hyperbolic solutions as in the Einstein-Maxwell gravity case. Next, we revealed the influences of cubic coupling λ nonlinearity parameter b and black hole charge on the behavior of temperature. We found out that the temperature decreases as each of latter parameters enhances. It is remarkable to mention that, as b grows, the linear Maxwell electrodynamics is reproduced. In addition, the temperature of spherical asymptotically flat solutions have a maximum value as horizon radius changes. This maximum temperature lowers for greater values of λb and q . In order to check the satisfaction of thermodynamics first law, we turned to compute entropy and electric potential. We used the Wald formula in order to obtain the entropy of our solutions. We showed that the first law of thermodynamics is satisfied for spherical BI charged asymptotically flat solutions of ECG. Finally we analyzed thermal stability of our solutions in both canonical and grand canonical ensembles. We showed that the values of q and r_h specify how the stability is affected by other parameters, in both ensembles. For one type of solutions, the system is unstable for b values larger than a maximum value b_{\max} including linear Maxwell case) in canonical ensemble. In small b regime highly nonlinear electrodynamics), there is a minimum λ value, as well, that system is stable if $\lambda \geq \lambda_{\min}$ for b values from 0 to b_{\max} . For another type, we have λ_{\min} which is defined before, as well. However, in this case, black solutions are stable if $\lambda \geq \lambda_{\min}$ with no upper bound for b . Furthermore, for λ values a bit smaller than λ_{\min} the system is stable for $b > b_{\min}$. There is also a critical value for λ that if $\lambda < \lambda_c$ the black solutions are thermally unstable for all b values. In grand canonical ensemble, the first type system is stable for large b values including linear Maxwell case). The latter is unstable for some λ values, in small b regime, whereas for some greater ones, it is stable for b values between 0 and an upper bound. For second type, the system is totally unstable for some λ values whereas it is totally stable for some greater ones.

In present work, we focused on asymptotically flat solutions with vanishing bare cosmological constant according to discussions given in Sec. III. In latter case, the higher curvature terms have no contribution in total mass formula. However, if one considers possible A)dS vacuum solutions in a unitary model, a correction due to cubic curvature terms appears in total mass formula see Eq. 33. This fact may affect the thermodynamical studies, drastically. It is fascinating to explore these kinds of charged solutions in future works. In addition, here, we considered Born-Infeld model as nonlinear electrodynamics. It is interesting to study the effects of other known nonlinear models on ECG solutions. Moreover, it is worthwhile to explore the critical behavior of nonlinearly charged solutions of ECG in both extended 35 and non-extended 36 thermodynamics phase spaces. Some of these issues are under investigation and the results will

appear elsewhere.

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Appendix A: Calculation of \mathcal{P}

Here, we present the detailed calculations for obtaining \mathcal{P} in Eq. 15. From Eq. 2, we have

$$\mathcal{P} = 12R_{ab}^c R_c^e R_e^f R_f^a + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} - 12R_{abcd} R^{ac} R^{bd} + 8R_a^b R_b^c R_c^a \quad \text{A1}$$

Above equation, can be re-written as

$$\mathcal{P} = 12R^{abcd} R_{cedf} R_a^e R_b^f + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} - 12R_{cd}^{ab} R_a^c R_b^d + 8R_a^b R_b^c R_c^a \quad \text{A2}$$

Then, using

$$R^{abcd} R_{cedf} R_a^e R_b^f = R^{abcd} R_a^e R_b^f R_{c f d e} + \frac{1}{4} R^{abcd} R_{ac}{}^{ef} R_{b d e f}$$

Eq. A2 can be achieved as

$$\mathcal{P} = 12R^{abcd} R_a^e R_b^f R_{c f d e} + 3R^{abcd} R_{ac}{}^{ef} R_{b d e f} + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} - 12R_{cd}^{ab} R_a^c R_b^d + 8R_a^b R_b^c R_c^a \quad \text{A3}$$

Rearranging above equation yields

$$\mathcal{P} = 12R_{cd}^{ab} R_{af}^{ce} R_{be}^{df} + 4R_{cd}^{ab} R_{ab}^{ef} R_{ef}^{cd} - 12R_{cd}^{ab} R_a^c R_b^d + 8R_b^a R_a^c R_c^b \quad \text{A4}$$

which is one expressed in Eq. 15.

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