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Machine Vision Computing Project

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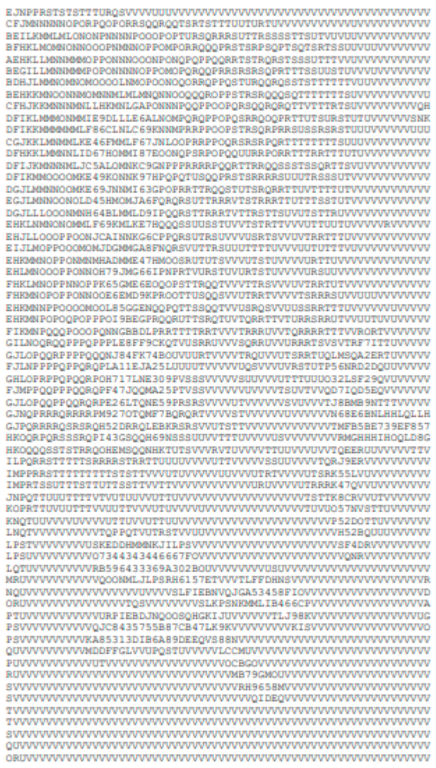
Abstract

Keywords:

Contents

1 Problem statement

In this mini-project, two images are going to be processed. The first image (Fig. 1a) is a 64x64 32-level image, which is a coded array composed of alphanumeric characters. The range of them is 0-9 and A-V, which corresponds to 32 gray levels. Image 2 (Fig. 1b) is a JEPG color image with three lines of characters. The main tasks to complete are listed as follows.



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ORUVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVV
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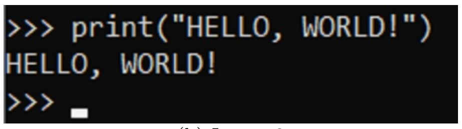
(a) Image 1



```
>>> print("HELLO, WORLD!")
HELLO, WORLD!
>>> _
```

(b) Image 2

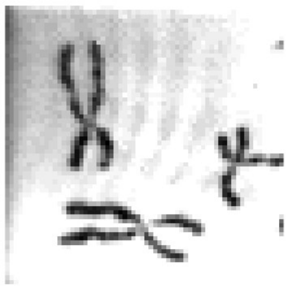
Figure 1: Task images

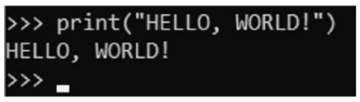
- Display the two original images on the screen and especially extract the middle line of image 2.
- Convert the two images into binary images using different thresholding methods.
- Determine a one-pixel thin image for both two images.
- Edge detection to determine the outlines of the characters of the both images.
- Label different objects for both images, and also sperate image 2 into characters.
- Rotate the original version of image 1 by 30, 60, and 90 degrees respectively.
- Train a classifier using machine learning techniques and test the recognition accuracy on sperated characters in image 2.
- Experiment with pre-processing of the data and discuss the sensitive of the methods in last step.

Since most of tasks for the two images are similar, we will test various methods for each task and make essential comparisons. In each step, the method with best performance is chosen and the result will be used for further processing.

2 Display the original image and preprocessing

Since image 1 is composed of numbers and characters, a decoder is need to generate corresponding gray levels. $0 - 9$ and $A - V$ can be seen as 32 gray levels and the decoded result is shown in Fig. 2a. As for image 2, we need to convert the 3-channel RGB color image to an 1-channel image with 256 gray levels, and the result is illustrated in Fig. 2b.


(a) Image 1



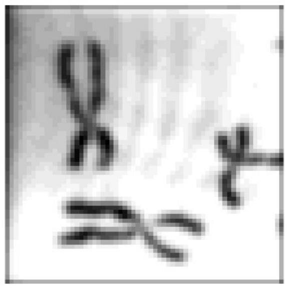
```
>>> print("HELLO, WORLD!")
HELLO, WORLD!
>>> _
```

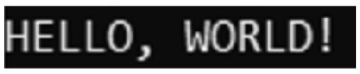
(b) Image 2

Figure 2: Original images

In order to achieve better performance for later tasks, both of the two images should be preprocessed. For image 1, a gaussian filter is applied to smooth the image. On the other side, image 2 contains too much useless information, so the middle line with words "HELLO,WORLD!" is extracted.

The images after preprocessing are shown in Fig. 3.


(a) Image 1



```
HELLO, WORLD!
```

(b) Image 2

Figure 3: Images after preprocessing

3 Binarizing image with thresholding methods

A raw image usually contains the target object, the background and the noise. When processing images, we need to extract the target object that we are interested and eliminate the noise or useless background pixels. This procedure is called binarizing and has been widely used in image processing. Binary images have only two gray levels that represent black and white respectively. The two gray levels are often set as 0 and 255, or logical value 0 and 1. Therefore, they have small data volume, fast processing speed, low cost and high practicality, and also make it more convenient to define various concepts of geometry. Binary image is generally associated with specific algorithms, many of which require binary data as input.

A common method is to set a threshold T to divide the image pixels into two parts. With this thresholding method, we could convert the original image to binary one and sperate the object from the background. Then the difficulty lies in the choice of thresholding methods, which should be robust given different environment conditions and input images. Moreover, thresholds can be more than one to realize both global and local binarization. Global binarization, like those reliant on histograms, use information from the whole image to find one threshold which will make a binary image. Local binarization determines a different threshold value for every pixel based on characteristics of their surrounding area. Localized techniques are sometimes seen as a panacea for images with small details.

In this section, we apply six methods that includes both global and local thresholding techniques. We also compare the different performance between them and select the best method fit for later tasks.

3.1 Mannually threshold setting

It is easy to think out the idea to simply give a global threshold. Here we mannually test different thresholds around the average gray level intensity of the whole image, and set the threshold as 18 for image 1 while 100 for image 2. It is the simplest method but obviously setting threshold for every image mannually is impossible, and the binarizing performance for different input images will be bad if the global threshold always remains unchanged.

3.2 Iterative method

The main idea for iterative method is that the sum of two mean values for part A (the gray value higher than threshold) and B (the gray value lower than threshold) after image segmentation remains basically stable. Therefore, as the iteration proceeds, the final convergence value $\frac{mean(A)+mean(B)}{2}$ can be taken as the segmentation threshold. The flow chart for this iterative method is given in Fig. 4.

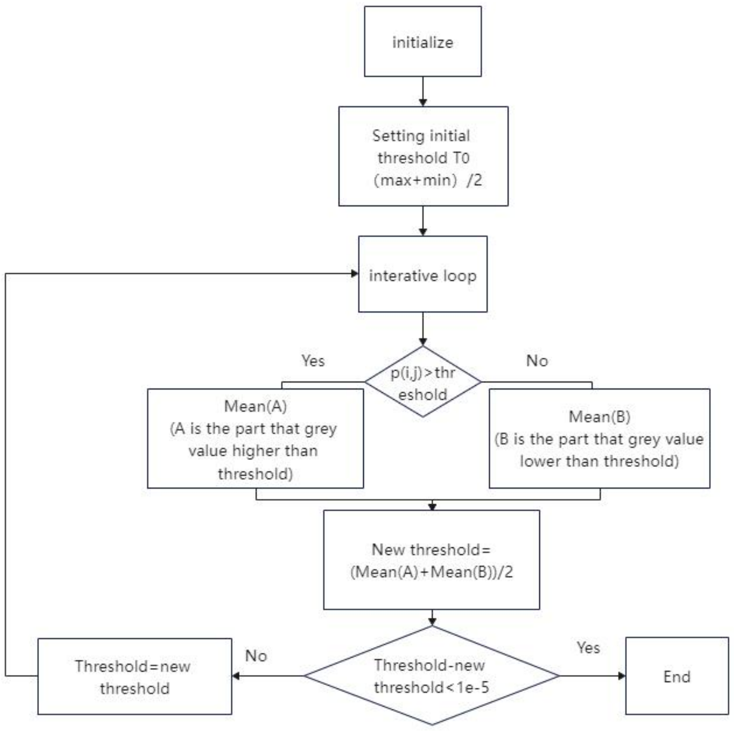

initialize
Setting initial threshold T0 (max+min) /2
interative loop
Yes
p(i,j)>threshold
No
Mean(A) (A is the part that grey value higher than threshold)
Mean(B) (B is the part that grey value lower than threshold)
New threshold= (Mean(A)+Mean(B))/2
Threshold=new threshold
No
Threshold-new threshold<1e-5
Yes
End

Figure 4: Flow chart for iterative method

3.3 OTSU

OTSU method is an image grayscale adaptive thresholding algorithm, which was proposed by Japanese scholar Otsu in 1979 and thus named by his name. The OTSU method divides the image into two parts according to the distribution of gray values on the image, and the foreground is the part that we are interested in. In the procedure, the intra-class variance between the background and foreground is calculated by iterating through different thresholds. When the intra-class variance achieves the maximum value, the corresponding threshold is the estimed boundary.

According to the OTSU method, we assume the threshold T_h could divide the pixels into two parts: C_1 (higher than T_h) and C_2 (lower than T_h). Suppose that the mean values of the two parts are m_1 and m_2, the global mean value of the image is m_G, and the probability of a pixel being classified into C_1 and C_2 classes is p_1 and p_2 respectively. Then they satisfy the following equation:

$$\begin{gathered} p_1 m_1 + p_2 m_2 = m_G \\ p_1 + p_2 = 1 \end{gathered} \tag{1}$$

According to the concept of variance, the intra-class variance can be written and simplified as:

$$\begin{aligned} \sigma^2 &= p_1(m_1 - m_G)^2 + p_2(m_2 - m_G)^2 \\ &= p_1 p_2 (m_1 - m_2)^2 \end{aligned} \tag{2}$$

Because that $p_1 = \sum_{i=0}^{k} p_i$, we get:

$$m_1 = \frac{1}{p_1} \cdot \sum_{i=0}^{k} i p_i, \quad m_2 = \frac{1}{p_2} \cdot \sum_{i=K+1}^{L-1} i p_i \tag{3}$$

where K is the evaluated gray level in this iteration, and L is the total gray levels. Then the cumulative mean value m of the gray level K and the global mean value m_G of the image are:

$$\begin{gathered} m = \sum_{i=0}^{k} i p_i, \quad m_G = \sum_{i=0}^{L-1} i p_i \\ m_1 = \frac{1}{p_1} m, \quad m_2 = \frac{1}{p_2}(m_G - m) \end{gathered} \tag{4}$$

Combine the expression in Eq. 4 and Eq. 2, we finally get the intra-class variance:

$$\sigma^2 = \frac{(m_G p_1 - m)^2}{p_1(1 - p_1)} \tag{5}$$

The gray level k that maximizes the above expression is the OTSU threshold. Since the variance is a measure of the uniformity of gray distribution, the larger the interclass variance between the background and foreground, the greater the difference between the two parts of the image, and the smaller the difference between the two parts will be when part of the foreground is mis-segmented into the background or part of the background is mis-segmented into the foreground. Therefore, the segmentation with the largest interclass variance implies the smallest probability of misclassification.

OTSU is considered as the best algorithm in image binarization because it is simple to compute and independent of image brightness and contrast so it has been widely used in digital image processing. However, it also has certain disadvantages including the sensitivity to image noise and only available for single target. Moreover, it may loss efficacy when the target and background size ratio is disparate, and the inter-class variance function may present double or multiple peaks at this time.

3.4 Niblack

The Niblack algorithm is to calculates a threshold for a pixel based on the points in the neighborhood centered on the pixel point. The threshold for each pixel is calculated by Eq. 6.

$$T_{Niblack} = m + ks \tag{6}$$

In the expression, m is the mean value and s is the standard deviation in the kernel. We use the convolution method, which convolves the image matrix with the all-1 matrix, which is actually the sum of the pixels within this window again. This is more than 10 times faster than traversing pixel by pixel.

The drawback of the Niblack algorithm is that on the one hand, the kernel size will lead to the inability to find the threshold value within the $\frac{r-1}{2}$ pixels in the boundary region. On the other hand, if the $r \times r$ kernel is all background, then the algorithm will inevitably make some of the pixel points become foreground and form pseudo-noise. Therefore, the choice of kernel size is very critical. If the kernel is too small, the noise cannot be effectively suppressed, on the other side it will lead to loss of details.

3.5 Bernsen

The main idea of Bernsen is using the maximum and the minimum value from the kernel area of a pixel to set the threshold. Let the current pixel be the point $P(i, j)$, calculate the maximum value p_{max} and the minimum value p_{min} of all pixels in the window of size $k \times k$ centered on P. If the value of difference is similar, the part belongs to the target or the background. Then we compare the grayscale with the global threshold to determine whether it is the target or the background, if the difference between the two is larger, it may belong to the edge of the intersection of the target and the background, then the average of the two as the kernel threshold.

3.6 Kittler

The main idea for Kittler method is to calculate the average value of the gradient grayscale of the whole image and use this average value as a threshold. The Kittler algorithm is close to the OTSU method in effect, but is faster (depending on the application scenario) when applied in images with high pixel quality.

According to the definition of partial differentiation of a two-dimensional function, the partial derivatives in the x, y direction can be expressed as:

$$\begin{aligned}\frac{\partial f(x,y)}{\partial x} &= \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon} \\ \frac{\partial f(x,y)}{\partial y} &= \lim_{\varepsilon \to 0} \frac{f(x, y+\varepsilon) - f(x,y)}{\varepsilon}\end{aligned} \tag{7}$$

The image is a discrete two-dimensional function, ε cannot be infinitely small, and our image is discretized by pixel, and the smallest ε is 0 pixels. Therefore, the image discretization above becomes again of the following form ($\varepsilon = 1$).

$$\begin{aligned}\frac{\partial f(x,y)}{\partial x} &= f(x+1, y) - f(x,y) = gx \\ \frac{\partial f(x,y)}{\partial y} &= f(x, y+1) - f(x,y) = gy\end{aligned} \tag{8}$$

This is the gradient of the image in the x-direction and y-direction at the point respectively. From the above expression, it can be seen that the gradient of the image is equivalent to the difference between 2 adjacent pixels. The gradient could be calculated by:

$$M(x,y) = \sqrt{(gx)^2 + (gx)^2} \tag{9}$$

By using the Eq. 9 to calculate the gradient, the threshold is the average value of the gradient.

3.7 Comparison and discussion

The algorithm we applied above include both global binarization and local binarization techniques, the feature of which have been given in the beginning of this part. So we can analyze that mannual set method, iterative method, OTSU and Kittler algorithm are global binarization while Bernsen and Niblack are local binarization.

The mannually set is the simplest but they could be affected by changes in image contrast and brightness and other factor so the effects are not excellent. The iterative method has well performance when distinguishing the foreground and background of the image. However, it can't distinguish the fine points of the image sometimes, and the time complexity of the algorithm is relatively high. As for OTSU, for images with different brightness conditions, as long as the brightness is uniform, the processing effect is better and the algorithm is more stable with low time complexity. We can also find from the comparation between different algorithm when handling the image 1 in Fig. 5, the effect of OTSU is excellent. Kittler algorithm has a similar effect to the OTSU in many scenarios but it is susceptible to noise so maybe it is not quite suitable for our image.

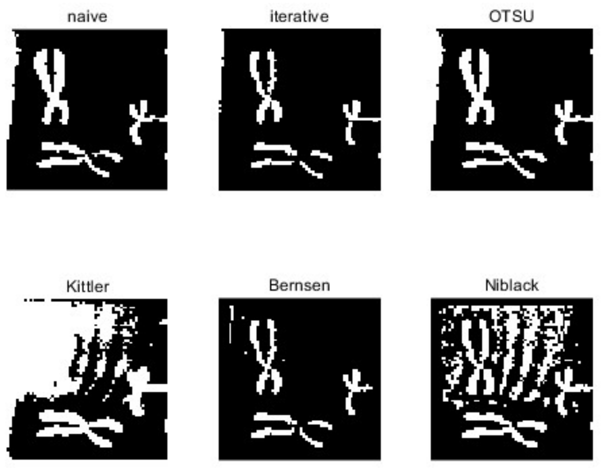


Figure 5: The performance of different binarizing algorithms for image 1

The advantage of local binarization is that it could process the image with uneven illumination better. For example if we want to deal with Fig. 6, when the global threshold is performed directly, the upper left half of the sushi is revealed while the lower right half is still black. The problem of global binarization is that no matter what threshold parameters are set, the full map requirement cannot be met.

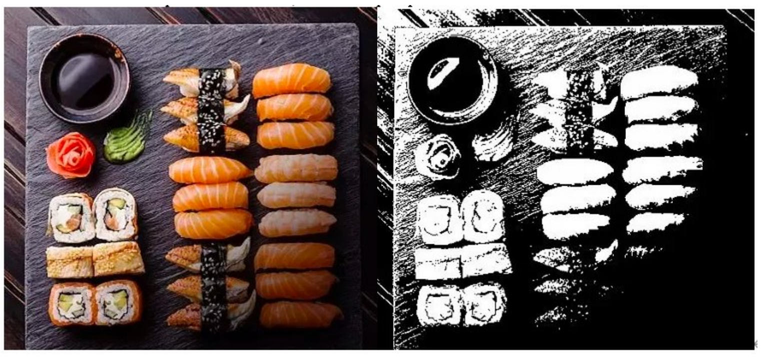


Figure 6: Example to introduce the disadvantage of global binarization

By using the local binarization we could solve the problem above. However, the effect of the algorithm has a large relationship with the select of windows or kernel size, the choice of kernel size is very critical. So it may take long time to select a proper parameter. As shown in Fig. 5, the OTSU algorithm already has quite excellent performance and it is more stable with low time complexity. The image also does not have obviously uneven illumination problem, so we choose OTSU algorithm to binarize image 1.

As for image 2, the performance of different algorithms are shown in Fig. 7, where OTSU algorithm also shows the best result.

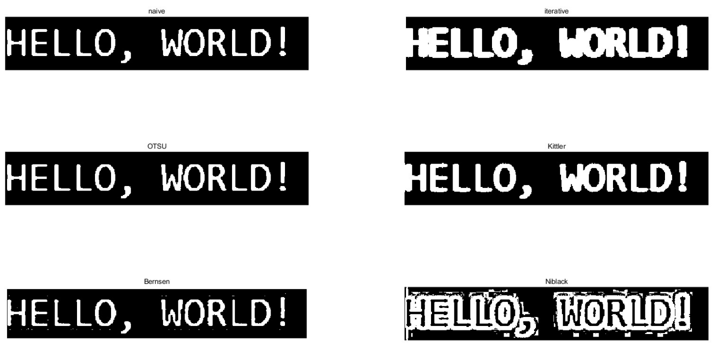


Figure 7: The performance of different binarizing algorithms for image 2

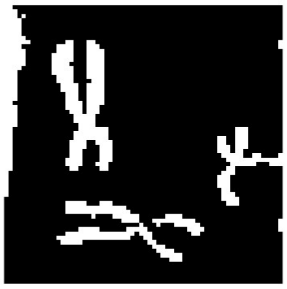
3.8 Chosen algorithm

Since OTSU algorithm show excellent performance in both two images, we finally choose it as our binarization algorithm. The algorithm implementation is shown below as a form of pseudocode.

Algorithm 1 OTSU Algorithm

```
Input: A gray image
Output: A binary image
 1: Initialize $bestThreshold = 0,\ g_{max} = 0$
 2: for $k$ from 0 to $grayLevel - 1$ do
 3:   for each $pixel$ do
 4:     Classify pixels according to the current threshold $k$ into 2 classes.
 5:     Calculate the number of pixels for each class and the sum of pixel values.
 6:     Calculate inner-class variance $g$ according to Eq. 5.
 7:     if $g > g_{max}$ then
 8:       $g_{max} = g$
 9:       $bestThreshold = k$
10:     end if
11:   end for
12: end for
13: for each $pixel$ do
14:   if $pixel > bestThreshold$ then
15:     $pixel = 1$
16:   else
17:     $pixel = 0$
18:   end if
19: end for
```

The final binarizing result for both images using OTSU algorithm is shown in Fig. 8.


(a) Image 1

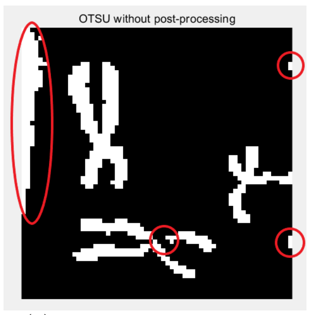
(b) Image 2

Figure 8: Binarized image

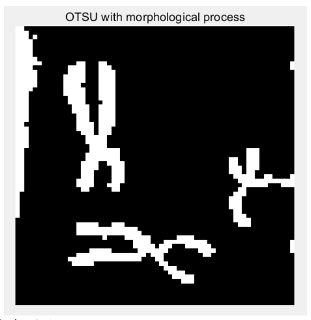
The binary image result of OTSU algorithm (Fig. 9a) still has the following defects: (1) the section of the lower chromosome has a fracture; (2) the noise on the both left and right sides of the image cannot be removed. To improve the threshold quality, we adopt some post-processing.

For the fracture of the chromosome graph, we adopt the morphological operations in Image Processing Toolbox in the lower section of the image. The 'bridge' operation is employed to bridge the unconnected pixels, while the 'fill' operation is used to fill the isolated internal pixel points. The fracture-repaired section will be put back into the original image (Fig. 9b).

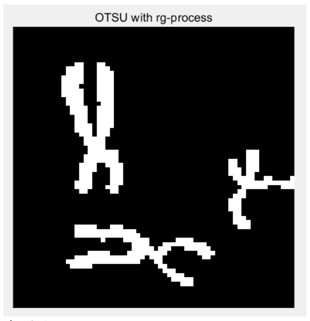
To remove the noise points on the sides of the image, the region growing operation is adopted. Manually choose four points to push into the stack as the seeds, then process the region growing until all the points is labeled in 0 and 1. Thus, the noise points are removed (Fig. 9c).



(a) Without post-processing



(b) After morphological processing



(c) After region growing processing

Figure 9: Post-processing for binarized image 1

4 Image Thinning

Image Thinning generally refers to an operation of Image Skeletonization of binary images. Thinning is the abbreviation of the process of reducing the foreground region of an image from multi-pixel width to unit pixel width. The process largely preserves the extent and connectivity of the original region while throwing away most of the original foreground pixels, which is also one of the main difficulties in algorism design.

Thinning provides a compact yet effective representation of key topologic and geometric features in an object. The compactness feature of skeletons facilitates efficient assessment of local structural metrics including scale, orientation, topology, geometry, etc.

4.1 Hilditch thinning algorithm

The basic principle of the Hilditch algorithm is to determine whether a point in the image is a boundary point or a connected point, and if it is a boundary point, the point can be removed. The whole process is to iterate over each pixel of an image from the top left corner to the bottom right corner, and each iteration removes the outermost layer of pixels from the image, leaving a line in the middle after the layers are peeled off to get the final refined image, i.e., the skeleton line.

The 8 neighbours of P is shown as Fig. 10. Then the determination of whether a point P should be deleted in an iterative operation is based on whether it satisfies the following six conditions:

P4	P3	P2
P5	P	P1
P6	P7	P8

Figure 10: The 8 neighbors of point P in Hilditch algorithm

- P is a foreground pixel.
- $P1$, $P3$, $P5$ and $P7$ are not all foreground pixels, i.e., P is a boundary point.
- There are at least two foreground pixels in $P1 \sim P8$, indicating that P is not an endpoint or isolated point. Endpoints and isolated points cannot be deleted.
- The connections parameter of the eight connected regions of P is 1, which ensures that the refinement of the skeleton line will not be broken.
- If $P3$ is marked for deletion and $P3$ is 0, the number of linked connections of P is 1, ensuring that the two pixel-wide horizontal bars will not be eroded away.
- If $P5$ is marked for deletion and $P5$ is 0, the number of linked connections of P is 1, ensuring that the two pixel-wide vertical bars will not be corroded off.

where the connections parameter of the eight connected regions of P can be calculated as following equation.

$$N_c^8(P) = \sum_{i=1}^{4}(\bar{P}_{2i-1} - \bar{P}_{2i-1}\bar{P}_{2i}\bar{P}_{2i+1}) \tag{10}$$

4.2 Zhang-Suen thinning algorithm

Zhang-Suen thinning algorithm is an iterative algorithm, and the whole iterative process is divided into the following two steps. The sketch map for the 8 neighbors around point $P1$ is shown in Fig. 11.

P9	P2	P3
P8	P1	P4
P7	P6	P5

Figure 11: The 8 neighbors of point $P1$ in Zhang-Suen algorithm

Step 1. Loop through all non-zero pixels points and mark as deleted those that meet the following conditions:

- The number of foreground pixels in the eight connected region of $P1$ is 2-6.

- Cumulative number of occurrences of 0 to 1 in one lap of $P2 \sim P9 \sim P2$ is 1.
- $P2 * P4 * P6 = 0$
- $P4 * P6 * P8 = 0$

Step 2. Loop through all non-zero pixels points and mark as deleted those that meet the following conditions:

- The number of foreground pixels in the eight connected region of $P1$ is 2-6.
- Cumulative number of occurrences of 0 to 1 in one lap of $P2 \sim P9 \sim P2$ is 1.
- $P2 * P4 * P8 = 0$
- $P2 * P6 * P8 = 0$

Loop the above two steps until no pixel is marked for deletion in both steps, and the output is the refined skeleton of the binary image.

4.3 Rosenfeld thinning algorism

To explain the Rosenfeld thinning algorism, the following concepts are necessary to be introduced.

- Azimuthal boundary point

  For the current pixel P1:

  - If P2 = 0, P1 is the northern boundary point;
  - If P6 = 0, P1 is the southern boundary point;
  - If P8 = 0, P1 is the western boundary point;
  - If P4 = 0, P1 is the eastern boundary point.
- Isolated point
  - If the value of all 8 connected pixels of P1 is 0, P1 is an isolated point.
- Endpoint
  - If there is one and only one pixel of the 8 connected pixels of P1 has the value of 1, P1 is an endpoint.
- 8-simple
  - If setting the value of P1 to 0 does not change the 8 connectivity of the 8 surrounding pixels, P1 is said to be 8-simple.

Rosenfold algorithm is an iterative algorithm, the operations performed in one iteration are as follows.

Step 1. Scan all pixels and delete if the pixel is a northern boundary point and is 8simple but not an isolated point and endpoint.

Step 2. Scan all pixels and delete if the pixel is a southern boundary point and is 8simple but not an isolated point and endpoint.

Step 3. Scan all pixels and delete if the pixel is an eastern boundary point and is 8simple but not an isolated point and endpoint.

Step 4. Scan all pixels and delete if the pixel is a northern boundary point and is 8simple but not an isolated point and endpoint.

After executing the above 4 steps, an iteration is completed. Repeat the above process until there are no more pixel points in the image that can be deleted, then exit the loop.

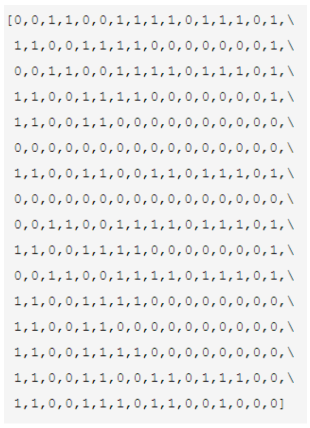
4.4 Look up table Algorithm

For each current point P, there are 256 combinations of values for its surrounding connected eight pixels. If a table with the values of P under these 256 cases was made, whether to delete each point or not can be decided by looking up the table.

The bases for the tabulation process are:

- Internal points should not be deleted.
- Isolated points should not be deleted.
- Endpoints should not be deleted.
- For a boundary point P, if the connected component does not increase after removing P, then P can be deleted.

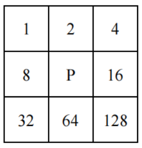
According to these bases, a table shown in Fig. 12 was generated.



```
[0,0,1,1,0,0,1,1,1,1,0,1,1,1,0,1,\
 1,1,0,0,1,1,1,1,0,0,0,0,0,0,0,1,\
 0,0,1,1,0,0,1,1,1,1,0,1,1,1,0,1,\
 1,1,0,0,1,1,1,1,0,0,0,0,0,0,0,1,\
 1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,\
 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\
 1,1,0,0,1,1,0,0,1,1,0,1,1,1,0,1,\
 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\
 0,0,1,1,0,0,1,1,1,1,0,1,1,1,0,1,\
 1,1,0,0,1,1,1,1,0,0,0,0,0,0,0,1,\
 0,0,1,1,0,0,1,1,1,1,0,1,1,1,0,1,\
 1,1,0,0,1,1,1,1,0,0,0,0,0,0,0,0,\
 1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,\
 1,1,0,0,1,1,1,1,0,0,0,0,0,0,0,0,\
 1,1,0,0,1,1,0,0,1,1,0,1,1,1,0,0,\
 1,1,0,0,1,1,1,0,1,1,0,0,1,0,0,0]
```

Figure 12: Table for image thinning

The index of the P point in the table can be generated by assigning different values to the surrounding eight points. The assignment factors for the different positions are shown in Fig 13. The final index of the P is the sum of the surrounding eight pixels' values multiplied by the assignment factor.



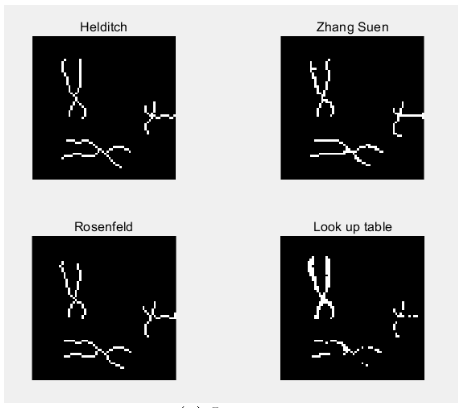
1	2	4
8	P	16
32	64	128

Figure 13: Assignment factors for different positions around P

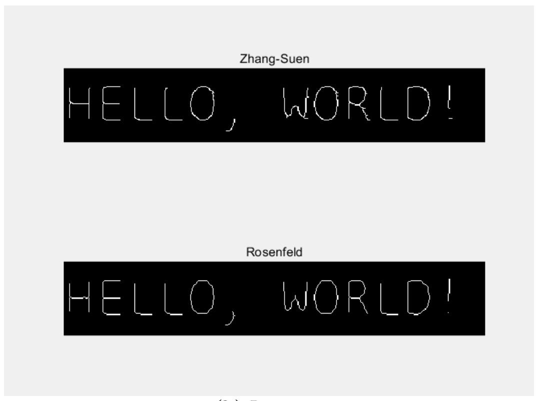
Each iteration will iterate through all the foreground pixels, calculate the index, and look up the table to decide whether the point is deleted or not. The iteration is stopped when no more new points are deleted.

4.5 Comparison and discussion

We implemented the above four iterative refinement algorithms and obtained the results as shown in the Fig. 14. The Helditch algorithm generally performs well, but the final skeleton is slightly distorted compared to the chromosome shape in the original image. The Zhang Suen algorithm can also accomplish the task of thinning, but the quality of the final skeleton is not very good and has more deformation. The Rosenfeld algorithm performs the best, the object is thinned to 1 pixel image and there is no breakage. The skeleton shape maintains the object form in the original picture. Although the Look up Table algorithm has the advantage of fast speed, but it cannot thin the object well, and there are a lot of fracture defects that make the skeletal form not well preserved. So finally, we choose Rosenfeld algorithm to process image thinning.



(a) Image 1



(b) Image 2

Figure 14: Comparison results of different thinning algorithm

4.6 Chosen algorithm

Finally Rosenfeld method is chosen as the thinning algorithm. The flow of implemented algorithm is shown as below.

Algorithm 2 Rosenfeld Algorithm

Input: A binary image

Output: A binary image showing the skeleton of the objects

```
1: repeat
2:     if the pixel is an azimuthal boundary point then
3:         if the pixel is 8-simple then
4:             if the pixel is not isolate point or endpoint then
5:                 set the pixel value to 0
6:             end if
7:         end if
8:     end if
9: until no longer a pixel deleted
```

5 Edge detection

Edge is one of the most basic features of an image and it is generally used when distinguishing the object and the background. Edge could be defined as a discontinuity in the local characteristics of an image. The idea underlying most edge-detection techniques is the computation of a local derivative operator.

We could define a gradient operator where the gradient is a vector pointing in the direction of the most dramatic change in the grey level of the image.

$$\nabla \vec{f} = \begin{pmatrix} G_x \\ G_y \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \tag{11}$$

The magnificent is $\nabla f = mag(\nabla \vec{f}) = (G_x^2 + G_y^2)^{1/2}$, and the gradient is commonly approximated by the absolute values

$$\nabla f = \mid G_x \mid + \mid G_y \mid \tag{12}$$

Direction of $\nabla \vec{f}$ at (x, y) is

$$\alpha(x, y) = tan^{-1}(G_x, G_y)^T \tag{13}$$

Image edge detection is based on the gradient of an image, and obtaining the gradient of an image translates into using various operators to convolve the image. Therefore core of the image edge detection algorithm therefore lies in the different operators.

5.1 Roberts operator

The Roberts operator, also known as the cross-difference algorithm, is a gradient algorithm based on cross-differencing, where edge lines are detected by local differencing calculations. It is often used to process low noise images with steep edges and is more effective when the image edges are close to plus 45 degrees or minus 45 degrees.

The mask of roberts operator in the two directions are:

$$G_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad G_y = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{14}$$

To convert it into the form of the formula:

$$G_x = f(i, j) - f(i-1, j-1), \quad G_y = f(i-1, j) - f(i, j-1) \tag{15}$$

The formula shows that the roberts operator enhances image edges at plus or minus 45 degrees very well. However, the disadvantage is that the positioning of the edges is not very accurate and the extracted edge lines are thick.

5.2 Prewitt operator

Prewitt operator is another method which uses the difference in grey scale between the upper and lower, left and right neighbors of a pixel is used to detect edges at their extremes and to remove some of the pseudo-edges, which has a smoothing effect on noise. The principle is to use two directional masks in image space to convolve with the image in the neighborhood, one to detect horizontal edges and one to detect vertical edges.

$$G_x = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}, \quad G_y = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 1 \end{pmatrix} \tag{16}$$

Compared with the Roberts operator which uses a 2x2 filtering operator, the Prewitt operator uses a 3x3 operator to compute pixel values in specific regions of the image, which results in the edge detection profile of the Prewitt operator being more pronounced in both the horizontal and vertical parts of the image than the edges detected by the Robert operator. After calculating the value of gradient , we need to set a threshold which is used to distinguish the edge according to the relationship between and the value of threshold.

5.3 Sobel operator

The Sobel operator adds the concept of weighting to the Prewitt operator, considering that the distance between neighboring points has a different impact on the current pixel, with the closer the pixel the greater the impact on the current pixel, thus sharpening the image and highlighting the edge contours. The Sobel operator combines Gaussian blurring and differentiation and calculates an approximation of the degree of lightness or darkness of the image, by comparing the degree of lightness or darkness at the edges of the image and recording the specific pixel points in the region above a threshold as edge points. The two directional masks used in sobel operator are:

$$G_x = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}, \quad G_y = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & 1 \end{pmatrix} \tag{17}$$

Because the Sobel operator combines Gaussian smoothing and differential derivatives, the results will be more noise immune and the Sobel operator is a more commonly used method for edge detection when accuracy is not very important.

5.4 Laplacian operator

The Laplacian operator is an isotropic second order differential operator with rotational invariance. The formula of second order difference is:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{18}$$

The expression could be convert to the discrete form which is suitable for image processing:

$$\nabla^2 f = f[f(x+1, y) + f(x-1, y) + f(x, y+1) - f(x, y-1)] - 4f(x, y) \tag{19}$$

The main idea of Laplacian operator is to find the relationship between the grayscale of the central pixel and the other pixels in the neighborhood by calculating the gradient of the central pixel of the neighborhood in four direction or eight direction and then sum up the gradients. If the central pixel has a higher grey level, the grey level of the central pixel is raised; if not, the grey level of the central pixel is lowered, thus sharpening the image. The mask we used is to calculate the gradient of 8 directions. The mask is:

$$G = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix} \tag{20}$$

When the pixel itself and its neighbors have the same grey value, the result of the Laplacian operator is zero; when the pixel itself has a higher grey value than the mean grey value of its neighbors, the result of the operation is positive; otherwise, it is negative. The edge of the image can therefore be determined by the excess of zero between the positive and negative peaks.

5.5 Canny operator

The Canny operator is a multi-stage edge detection operator that combines filtering, enhancement and detection, with the goal of finding an optimal edge profile. The Canny consists of four steps.

Step 1. Smoothing of the image with a Gaussian filter.

The derivative calculation is very sensitive to noise so it is essential to use gaussian filter to smooth the input to avoid enlarging the noise. The method is to multiply each pixel point and its neighborhood by a Gaussian matrix and take the average with weights as the final grey value.

Step 2. Computation of the value and direction of the gradient using a difference with first-order partial derivatives.

Because an edge is a collection of pixels with a large variation in grey value so this step will calculate the value and direction according to Eq. 12 and Eq. 13.

Step 3. Non-maximal suppression of the gradient amplitude.

The gradient image obtained from step 2 suffers from uneven edge width, blurring and misidentification so the non-maximal value points of the gradient image need to be suppressed to remove those non-edge pixels. The non-maximum suppression method is used to find the local maxima of the gradient image as edge points and to set the grey scale value of the non-maximum points to 0.

The neighbor around specific pixel could be divided into 4 areas according to the direction as shown in Fig. 15. The process could be achieved by comparing each pixel with two neighbor pixels that have the same gradient. If the gradient value is not the maximum of the three pixels, the pixel is suppressed, i.e. the pixel is made zero.

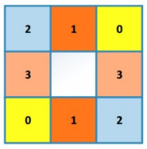


Figure 15: The four areas divided according to direction

Step 4. Detect and connect the edges using a double thresholding algorithm.

The edge image obtained in step 3 still has many pseudo-edges, so the Canny algorithm uses a double-thresholding algorithm to detect and connect the edges. If the gradient amplitude at a pixel position exceeds a high threshold, it is retained as an edge pixel and if it is less than the low threshold, it is excluded. If in between, determine if there are pixels in the 8 neighborhood space of the pixel that are above the high threshold, and if so, retain the pixel.

5.6 Comparison and discussion

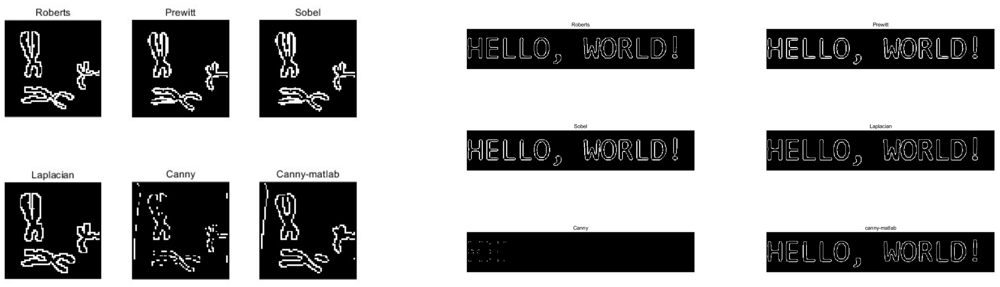
The Roberts operator is the more basic edge detection operator and it is not very accurate. It is suitable for images with steep edges and little noise, especially for images with many plus or minus 45 degrees edges.

The edge detection results of the Prewitt operator are more pronounced than those of the Roberts operator in both the horizontal and vertical directions. Although the Prewitt operator suppresses noise by averaging the grey values around the pixel, this type of noise suppression is only equivalent to a low-pass filtering operation on the image, which will result in the Prewitt operator being less accurate than the Roberts operator in locating the edges of the image.

The Sobel operator provides a high level of noise immunity to edge detection by combining Gaussian blurring and first order differential. However, the disadvantage of this algorithm is that it does not strictly separate the subject from the background of the image, which can lead to blurred edge contours.

The Laplacian operator has a good overall effect, but the operator is very sensitive to noise and it strengthens the noise component, both of which make the operator prone to losing part of the edge orientation information, resulting in some discontinuous detected edges, while the noise immunity is relatively poor.

The canny operator is considered to be the optimal algorithm for edge detection and is much more accurate in identifying image edges than other edge detection algorithms. Even though the effect of our experiment is not quite excellent, the effect of MATLAB built-in canny function works well. The effect for image 1 using different operators are shown in Fig. 16a.



(a) Effect of different operators for image 1 (b) Effect of different operators for image 2

Figure 16: Comparison of different edge detection algorithms

5.7 Chosen algorithm

Chosen reason

Algorithm 3 Laplacian operator

Input: A binary image

Output: A image with only object's edge

```
1: for all central pixels do
2:     Multiplying matrix $A$ (central pixel and its neighbors) and matrix $G$ (8 neighbor operator)
3:     Sum the gradient value
4:     if $value > laplaceThreshold$ then
5:         pixel=1
6:     else
7:         pixel=0
8:     end if
9: end for
```

6 Labling and segmentation

The connected components labelling operation performs the unit change from pixel to region of segment. All the pixels belong to the same connected component should be given the same identifying label. Based on this, regional features can be obtained and different objects in the picture can be separated, following with regional operations and decision making.

6.1 Classical label propagation algorithm

The basic principle to determine whether two foreground pixels P_0 and P_n belong to the same component is that: if there is a sequence of foreground pixels $(P_0, P_1, ..., P_n)$, and P_i is a neighbor of P_{i-1} for $i = 1, 2, \ldots, n$, then P_0 and P_n belong to the same component.

The classical label propagation makes only one pass through the image. In the ergodic procedure, the image is scanned in rows left to right, top to down. For foreground point P and its 8 connected neighbors shown in Fig. 17, if P_1, P_2, P_3, P_4 all have the value of 0, P will be assigned a new label. If one of the points P_1, P_2, P_3, P_4 is not 0 points, P will be assigned the same label as the non-zero point. If the number of non-zero points among P_1, P_2, P_3, P_4 is more than 1, P will be assigned the minimum label and an equivalence found will be entered in an equivalence table. Each entry in the equivalence table consists of an ordered pair, whose values are the label numbers.

P1	P2	P3
P4	P	

Figure 17: P and its neighbors in classical label propagation algorithm

After the scanning, a median label matrix and a global equivalence table will be generated. Based on the equivalence table initial pairs, equivalent classes can be derived by using the graph search method. Each equivalence class is assigned a unique label by the minimum label in the class. Thus, the final label matrix is generated. Using ‘label2rgb’ function to convert the final label matrix into RGB image. Different components will be colored with different colors.

6.2 Region growing algorithm

The region growing algorithm is an operation that aggregate pixels or subregions into larger regions according to a predefined criterion, which is often used for image segmentation. The regions after segmentation need to meet the following conditions:

- $\cup(R_i = R)$
- R_i is a connected region, $i = 1, 2, ..., n$
- $R_i \cap R_j = \o$, for $\forall i, j(i \neq j)$
- $H(R_i) = True$, $i = 1, 2, ..., n$
- $H(R_i \cup R_j) = False$, for $\forall i, j(i \neq j)$

The specific steps of the region growing algorithm are as follows.

Step1. Traversing the image by row, when a foreground point P is encountered, pushing the current point into the stack as the seed point.

Step2. Determine whether the stack is empty. If the stack is non-empty, assign the label to the top element of the stack and pop the top element of the stack. Then push the foreground point of 8 connected neighbors of the top element into the stack.

Step3. Repeat Step 2 until the stack is empty.

Step4. Continue traversing the image by row until the next foreground point is encountered, assigning a new label to the pixel. Then repeat steps 2 - 3 until the image is traversed.

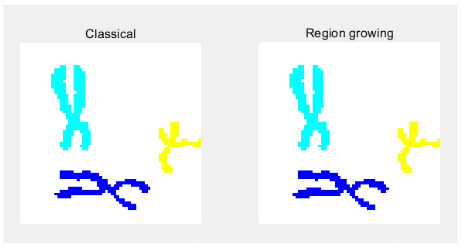
Step5. Using ‘label2rgb’ function to convert the final label matrix into RGB image. Different components will be colored with different colors.

6.3 Comparison and discussion

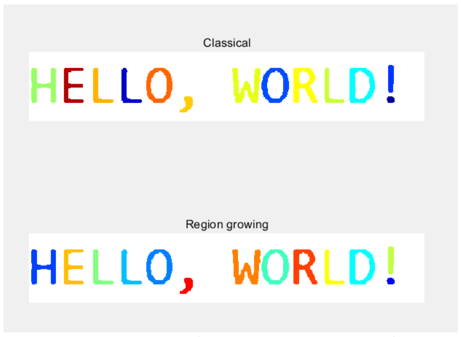
As shown in Fig. 18, both methods can complete the labelling operation very well, and the segmentation quality is about the same. However, the region growing algorithm is significantly faster than the classical label propagation algorithm. It takes 0.164s to complete the labelling using the classical label propagation algorithm, while the region growing algorithm takes only 0.03s. So, we choose the region growing algorithm.

6.4 Chosen algorithm

The finally chosen region growing algorithm is shown below.



(a) Result comparison of labelling operation for image 1



(b) Result comparison of labelling operation for image 2

Figure 18: Comparison of different label algorithms

Algorithm 4 Region Growing Algorithm

```
Input: A binary image
Output: A RGB image
 1: for each pixel do
 2:   if the pixel is poreground point then
 3:     Push the point into stack
 4:     repeat
 5:       Assign the current label number to the top element and pop it
 6:       Push the foreground point of the 8 neighbors of the top element into stack
 7:     until the stack is empty
 8:     Update label number
 9:   end if
10: end for
11: Convert the final label matrix into RGB image
```

6.5 Object seperation

For image 2, the characters need to be separate. The separate algorithm is designed as follows.

Step 1. For a particular label, find all pixels belong to this component. Calculate the horizontal distance r from each pixel to the leftmost pixel and the vertical distance c from each pixel to the uppermost pixel.

Step 2. Take the segment with length of $(r_{max} + 20)$ and width of $(c_{max} + 20)$ to generate the new image of the corresponding label.

Step 3. Repeat the above steps until all labels have the new image.

The result is shown in Fig. 19.

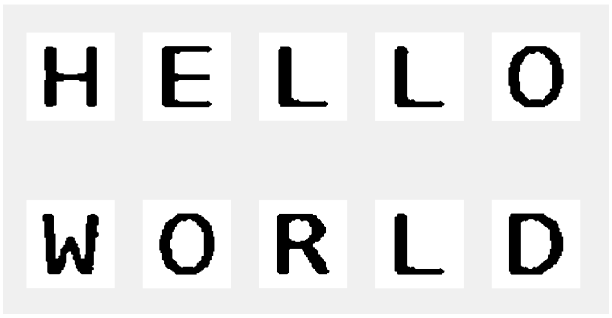


Figure 19: Result of separation for image 2

7 Image rotation

The kind of image rotation method could be divided into forward mapping and backward mapping. Forward mapping uses the coordinated of original image to calculate the new coordinates of rotated image and the grayscale values are passed through the process. This method could generate regular noise. On the contrary, the backward begins from the rotated image to find the point that corresponds to the original image. We applied these methods respectively in the project.

7.1 Forward mapping

The process uses three matrices to rotate image. The first step is to transform the origin of coordinate from the top left corner to the center of the image. Assuming the width and height of image is h and h, the coordinate of a pixel is (x_0, y_0) and the transformed point is (x_1, y_1), the process is:

$$\begin{pmatrix} x_1 & y_1 & 1 \end{pmatrix} = \begin{pmatrix} x_0 & y_0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -0.5w & -0.5h & 1 \end{pmatrix} \tag{21}$$

The rotation angle is set to θ, the point through rotating is (x_2, y_2), the rotation operation is:

$$\begin{pmatrix} x_2 & y_2 & 1 \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & 1 \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{22}$$

The last step is to inverse the coordinate, the point will be (x_3, y_3):

$$\begin{pmatrix} x_3 & y_3 & 1 \end{pmatrix} = \begin{pmatrix} x_2 & y_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0.5w & 0.5h & 1 \end{pmatrix} \tag{23}$$

Because the rotation could convert some value to non-integer, we need to perform rounding operations. The result is shown in Fig. 20. It is obvious there is regular noises in the image.

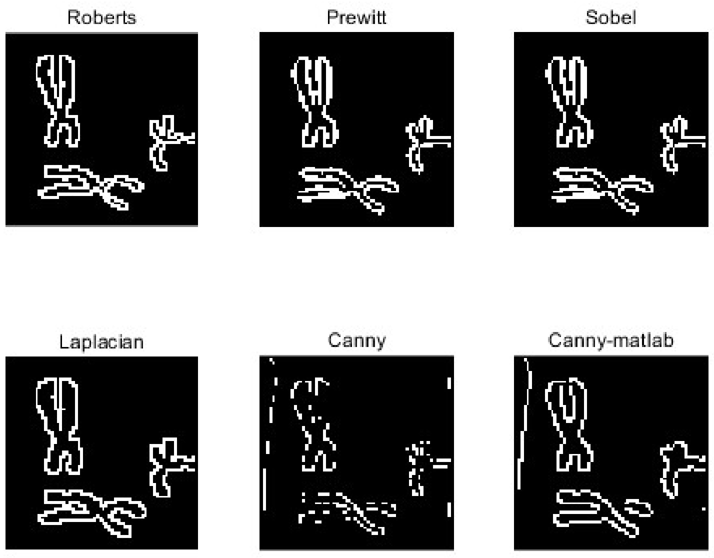


Figure 20: The effect of different operators for image 2

7.2 Backward mapping

Backward mapping begins with the rotated image, find the point corresponding to the original image and pass the grey scale value from the original image, so that each pixel of the rotated image could correspond to a point in the original image after coordinate transformation and interpolation. Interpolation is a very critical step in this process. We applied the nearest neighbor interpolation and bilinear interpolation.

7.2.1 Nearest neighbor interpolation

The main idea for this method is to make the grey value of the transformed pixel equal to the grey value of the nearest input pixel. As shown in Fig. 21, four Q points are from the original image and according to principle of nearest, the value of P point should equal to $Q3$.

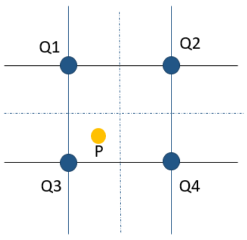


Figure 21: An example of nearest neighbor interpolation

This method only use the value of nearest pixel without considering the influence of other neighboring pixels, resulting in a significant discontinuity in gray value and an obvious mosaic.

7.2.2 Bilinear interpolation

The main idea for Bilinear interpolation is using linear interpolation in two directions to get a point. As shown in Fig. 22 the first linear interpolation in the x direction to obtain $R1$ and $R2$, and then the second linear interpolation in the y direction gives P.

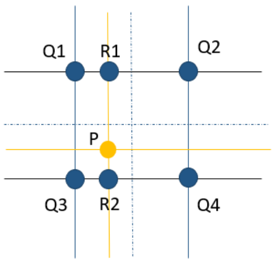


Figure 22: An example of Bilinear interpolation

The bilinear interpolation method is better than the nearest neighbor interpolation method but the computation is a bit larger and the program takes a bit longer to run. The bilinear interpolation could generate higher quality images and basically overcomes the discontinuity of the nearest neighbor interpolation grayscale value because it takes into account the correlation effect of the four direct neighbors around the sample point to be measured on the sample point. The results of image rotation using backward mapping is shown in Fig. 23, the left column uses nearest neighbor interpolation and the right one uses bilinear interpolation.

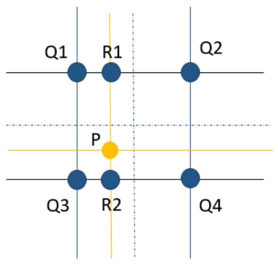


Figure 23: An example of Bilinear interpolation

8 Classifier design using machine learning methods

9 APP design

10 Conclusion