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<div style="text-align:center">Artworks in a four-dimensional world</div>

I have always wondered what artworks could be like in a of a four-dimensional space. Due to the use of perspective, people utilized it to create realistic artwork. However, it is also what limits their imagination, limits us from breaking through our cage. By realizing the mathematical theoretical background of the dimensional world, we could understand how a four-dimensional world should be interpreted. And I believed in the history of art, men must have made works that involved similar concepts in their very own way. Even though art tends to be abstract, I believe abstract is not the word for a higher dimensional world. It is also often seen the concept of a higher dimensional world or object implemented in popular culture such as movies and games. I have played games that manipulates the hierarchy of dimensional world, and therefore it gave me insights on coming up with this paper.

This paper aims to explore how mathematics and arts are applied on artworks in a four-dimensional world through theoretical research that includes mathematical theories and references from existing art concepts. The main research focus is on the four-dimensional space and its objects, exploring the relationship between existing artistic concepts and mathematical concepts.

Let us begin by imagining a four-dimensional object. A four-dimensional object has four spatial dimensions, representing in a Cartesian coordinate system as (x, y, z, w). Therefore, in the Mathematics of a 4D object, the four-dimensional space can be represented by a 4D vector. From

a basic zero-dimensional object, there is a point only. There are no properties for any direction nor it contains anything. In a one-dimensional world, points of different values of x are connected together to form a linear space. It also has the possibility to move within the linear space. In a two-dimensional world, lines are connected along the new axis of the dimension to form a plane. A being on a two-dimensional world can see objects that are not inside an object. It is not possible to interact with an object (e.g. a circle) that is inside another object (e.g. a square). This situation is similar to a Pokémon game that a tree is blocking your way to the next town, but it is not possible to pass through unless you cut it down. It is also impossible to jump over it in a two-dimensional world. If a one-dimensional object is included in a two-dimensional world, a two-dimensional being can see through everything in that one-dimensional world as how we are able to look at the whole painting. It also implied that a two-dimensional being can only see lines of different lengths perhaps in different shapes and sense its two-dimensionality by looking from different "perspectives". In a three-dimensional world, we can image extending a square along the z-axis in a range equal to its width, thus we have a cube. As a three-dimensional being, it is easy to see through the path behind the tree in the Pokémon game, but the character would have no idea about what is behind the tree. It is also observable to us that a circle is inside a square because we can look at the whole 2d plane along the third extra dimension we have.

Despite living in the three-dimensional world, are we actually seeing the three-dimensional world? This is a problem I never thought about, but it actually made sense. For example, we are only experiencing a three-dimensional object by depth, and in order to do that, it must be either shaded or viewed from multiple perspectives. And what about a cube inside a sphere? Certainly, it is impossible for us to see through what is inside the box. Nonetheless, it becomes interesting that we could apply the same theory to this situation. Since a higher

dimensional being could easily look at everything from the lower dimension, theoretically a four-dimensional being can look through the hidden sphere and the whole world. And it is believed that our space is spatial consistent, which implied our world could simply be a paper in analogy that could be folded and space-wrappable. If a three dimensional being is trapped in a cell or a box, a four-dimensional being might suggest you to "jump" out of it.

In a four-dimensional space, two cubes are connected along the w-axis with the range equal to its height and its width. A tesseract is formed, and it is the cube equivalent in a four-dimensional space. A four-dimensional being can simply see "through" our body in one of their perspective and the take a sphere out without breaking the box.

Before explaining the interaction of a four-dimensional object in our world, I would first introduce a book called *Flatland: A Romance of Many Dimensions* by Edwin Abbott Abbott. In this book, the slicing method was used to determine the three-dimensional object passing through a two-dimensional world. Flatland was a two-dimensional world where its people were two-dimensional shaped. One day a sphere from the three-dimensional world visited the place. When the sphere entered the two-dimensional world, people form Flatland only saw a circle appear out of nowhere, expanding and collapsing. Astonished, a square discovered more about the world beyond its dimensions and got arrested for spreading ideas of "upwards".

The same slicing method can be applied in our three-dimensional world. When a four-dimensional object or being pass through our world, we could a morphing shape that pops out of nowhere and perhaps a floating object if it is "on the ground" in their four-dimensional world. Before exploring the three-dimensional slices from a four-dimensional object, one could question the validity of the fourth spatial dimension. The quantum Hall effect experiment (Zilberberg) that validate the existence of the fourth spatial dimension. The experiment shows evidence that

electrons are moving as if a fourth spatial dimension exists. Furthermore, the famous string theory also suggested that our universe has more than three spatial dimensions.

In order to get a better understanding of our world in comparison to the forth dimension, we could image our world is a plane that the forth dimension w is perpendicular to our plane just as the Flatlanders were treated. Since our world is three dimensional, we could name our world as the hyperplane. Only when w is at a certain value would our world be intersecting with the fourth dimensional object. Let us assume the w coordinate of our world is 0, we would have the following figure. (See Fig. 1)
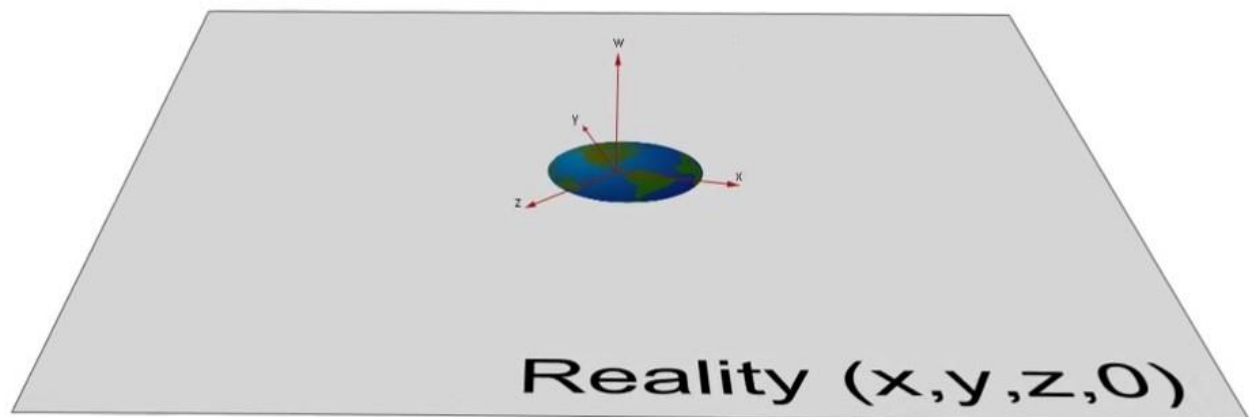

Fig. 1. Earth is on the three-dimensional hyperplane where our w coordinate is 0

Due to lack of four-dimensional perspective and distortion in our three-dimensional perspective, it would be difficult to determine a correct three-dimensional slice of a 4D object with the corresponding aspect and perspective. Even though it is impossible to comprehend a four-dimensional object, but it is possible to analyze the slice of a 4D Object when its w value is 0. The Flatlanders will never be able to see the whole 2D object because they could only see lines of different lengths. A three dimensional being like us could never look at the whole 3D object from a single perspective. We construct a 3D object by analyzing different shades and perspectives.

Since a sphere will first show from nothing to expanding to its full size and shrinking to nothing when it intersects the plane. We can image a 4D object passing through our world, with the change in W we can calculate the respective 3D object from the 4D shape's equation.

In a 2-dimensional world, take the example of a circle that is at the origin of a Cartesian coordinate system, $x^2 + y^2 = R^2$ where R is a Constant. A circle on a coordinate system that has a center is at the origin, the radius is always the square root of (x^2 + y^2).

In a 3-dimensional world, take the example of a sphere that is at the origin of a Cartesian coordinate system, $x^2 + y^2 + z^2 = R^2$ where R is a Constant. An extra variable z is added for the z-axis. The radius would be the square root of (x^2+y^2+z^2).

In a 4-dimensional world, take the example of a hypersphere that is at the origin of a Cartesian coordinate system, $x^2 + y^2 + z^2 + w^2 = R^2$ where R is a Constant. An extra variable w is added for the w-axis. The radius is the square root of (x^2+y^2+z^2+w^2) and $R^2 - w^2$ is a sphere. Hence, when a hypersphere is passing through the w-axis, the slices are always a sphere. Since we can determine the circle slice of a sphere at different z value, we can also determine the sphere slice of a hypersphere at different w value. (See Fig. 2)
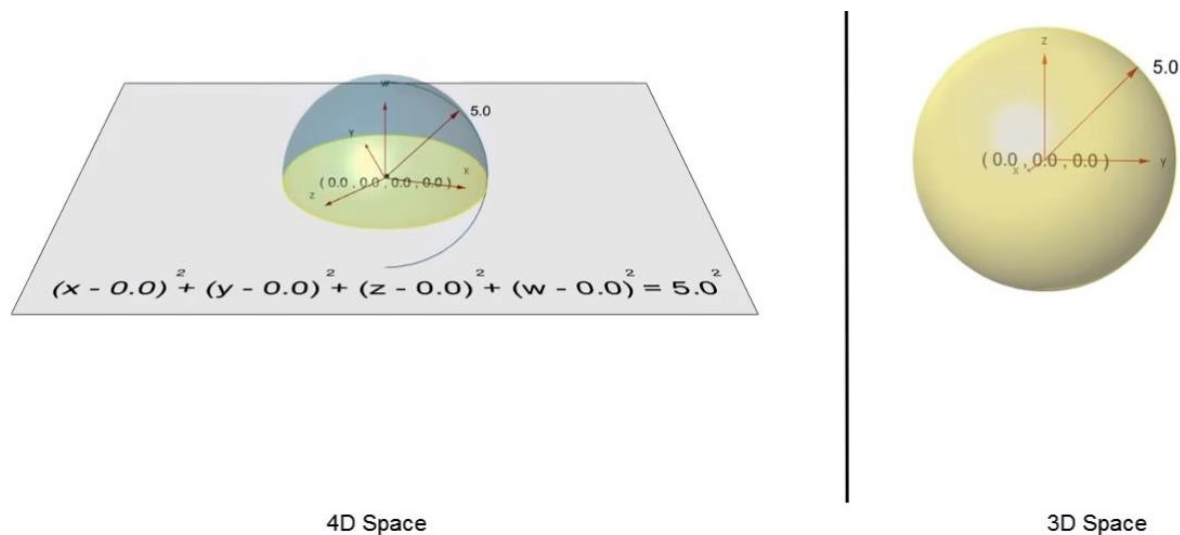

$$(x - 0.0)^2 + (y - 0.0)^2 + (z - 0.0)^2 + (w - 0.0)^2 = 5.0^2$$

4D Space

3D Space

Fig. 2. The equation of a hypersphere passing though the hyperplane.

Therefore, in the case of a hypersphere, a sphere would be expanding and shrinking as it passes our world. Things are more interesting when studying a non-uniform shape like a hypercone or a cube entering the hyperplane at different angle.

In a three-dimensional space, if a cone with the circle bottom facing the z axis enters a 2D plane at z=0, a circle of its radius will first show up and the radius shall decrease at the rate equals to the slope of the slant. Similarly, when a 4-dimensional object passes through the 3-dimensional world, we can understand its shape from the equation, which is $x^2 + y^2 + z^2 - w^2 = 0$. Therefore, w^2 is a sphere as long as the axis of the hypercone is parallel to the w-axis where the radius depends on the w value of the hypercone. However, when the hypercone is rotating through the 4th dimension, different shapes are observed. (See Fig. 3)
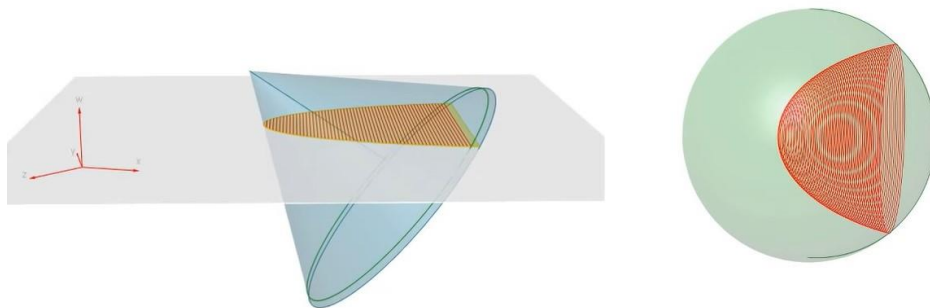

Fig. 3. A cone intersect the hyperplane forming a paraboloid.

According to different intersection, different shapes are formed. (See Fig. 4)
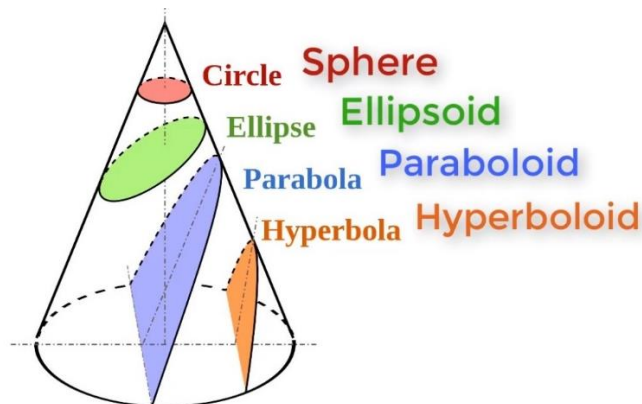
Fig. 4. Different possible 3D slice of hypercone.

It is inferred a shadow would also be a lower dimensional image and share the same concepts. A shadow is also what the dimensional being could see from their point of view. Considering from a one-dimensional world, it is not possible to have observable shadows. For the two-dimensional world, a shadow of a line is produced. For a three-dimensional world, a shadow would be a two-dimensional image.

One of the important features is that a four-dimensional being can see through everything in our 3d world. It could be imagined as we are able to see through everything on a 2D plane, for instance, a maze drawn on a paper. If we are trying to escape a 3D Maze, we wouldn't be able to see through every path but to draw a map in order to decipher a way out. In this case, a 4D being can instantly see through every possible path behind the wall and have a "global" view on a real 3D maze, and therefore, solve the solution quicker than as without having to move to be able to see through walls.

There are limitations on dimensions. Simply said, it's impossible to present a real 4D object projection/shadow accurately because different dimensions have different perspectives in viewing. Considering from the fact that even with correct shading and colors, a painting without perspective will merely be inaccurate and not proportional and looks completely flat.

Perspective played an important part in art history and it is now introduced. The central projection and section is a process (Bodish) that an artist would transfer an object onto a canvas by imagining that the image is traced onto a window, parallel the artist's eye, was written in a book by Leone Batista Alberti in 1435.

Fig. 5. "Tracing on Glass, After Nature", from Frederic Goupil, *La perspective experimentale, artistique, methodique et attruyante ou l'orthographie des formes* (1860) (Henderson, plate 41).

When a person tried to draw an object accurately in space, one must consider the space in lines. There is always a point that converge lines away from the observer, and it is adapted to a visual representation. As a result, objects are placed in a respective position and the painting would make sense. And also, using a perspective technique is representing a side of an object that is simply not really representing the object, but our brain would fill in information to fill the unknown in the invisible area. As a result, one could say a box drawn on a paper is a box, or those without the 3-dimensional sense would say that it is a flat plain square conjoined with two rhombuses.

However, if a cube is not distorted and drawn on a two-dimensional medium, it is difficult for one to determine that would be truly a 3-dimension object. Similar to a 4-dimensional object, it is not possible to represent it in a 3 dimensional medium accurately without distortion, let alone a 2d paper or canvas, we can only express a perspective of it. Still,

one approach might be the expansion of an object. Since a cube can be expanded into 6 squares,

and a tesseract can also be expanded into 8 cells.

Sometimes we couldn't even tell if it's a 3D object as our mind could deceive our spatial

senses, and the perspective of a 3d object won't make sense if there are no vanishing point in a

drawing.

Fig. 6.

Without vanishing point in any image/painting 2dimensional work, it would be

completely flat and just like the old religious painting. Therefore, in order to create a slightly

more accurately work of the 3D slice of a 4-dimensional object, one must create a 3-dimensional

work. In is more achievable nowadays in the form of 3D painting.

What kept us from truly exploring the 4D world? Plato's *Allegory of the Cave* provided insights into this question. In the cave, there was a world where its people were born in a cage and never exposed to light directly and could only experience themselves through the shadow of reality. Even though the people were released at a certain time to finally experience light, they would be initially blinded by the true reality.

The situation is similar to our reality since we are accustomed to our visual world that thought the only way to sense to 4th dimension is by visualizing it in our familiar medium or forms. Or we do actually live in a 4-dimensional space, that we chained and observing the world through a three-dimensional world. What If our visuals are slightly different to what we used to? Assumed we do have a pair of compound eyes; our vision range would be increased vastly.

Even though it has been suggested that 4th dimension exist, why must it be correct then? If it does exist, shouldn't we be seeing something floating or object that morphs for no reason?

Cubism was one of the most important art movement in history. Within a decade, Cubism had already run its course and mad its impact. Cubism was described as simultaneity of vision, and it was heavily influenced by the ideas of the fourth dimension. Artists liked theses interesting ideas as well as non-Euclidean geometry. It was said (Fasanelli 946) the interest stemmed from a popular science fiction called *Gestes et Opinions du Docteur Faustroll* (1911) by French Jarry (1873-1907). Jarry was attracted by the novelty of higher-dimensional geometries and therefore wrote about the work of the British mathematician ARUTHER CAYLEY. Then the art form evolved as more new ways of approaching the reality is found. In cubism artists used several view points and depict them on a single perspective.

Cubism was originally formed by Georges Braque and Pablo Picasso in France. It was a way of moving in a different direction from traditional perspective paintings. Since the time

already came photography and artistic value of drawing what is to be seen has decreased.

Guillaume Apollinaire, an art critic and poet, commented they were "moving toward an entirely

new art which will stand, with respect to painting as envisaged heretofore, as music stands to

literature. It will be pure painting, just as music is pure literature" (Apollinaire, Art History,

1036) As a still medium, cubism in a drawing brings dynamics and perspectives from a single

viewpoint, which was revolutionary at that time. Picasso suggested a method of viewing cubist

paintings just as one would approach a musical composition, by analyzing it instead of

questioning its representations. This method is similar to the method employed when one

considers the slicing model of the fourth dimension. (Bodish)

Fig. 7 "The Bride Stripped Bare by Her Bachelors, Even"

Marcel Duchamp "The Bride Stripped Bare by Her Bachelors, Even" 1925-23 also

known as The Large Glass was a great attempt to present higher dimensional world in a 3D

format, even though it is considered as a work of Dadaism. The work is composed of two big glass panes, also influenced by the thought that artists used to just paint whatever they see through the "window". However, when the work was released in 1923 it was still declared unfinished. It wasn't until an accident took place during the shipping to The Brooklyn Museum that the glass was shattered, Marcel claimed it had been finally completed. He reassembled all the shattered pieces and installed it at the Philadelphia Museum of Art which is being exhibited at the current moment still.

The work is not simply a painting, it's a composition of lead wire, lead foil and dust that claimed to have been collected in Marcel's apartment. And the work is not the abstraction that being produced in 1915 which merely is abstracted geometries put together. In this work, it has a familiar outline of a three-dimensional perspective object and some object resembled to the 3d slices of a higher dimensional figures. He also created a book called The Green Box explaining the meaning or ideas in this work, however the book is only composed in any chronological order that it is always in flux. Duchamp wrote that in order to understand the fourth dimension, one must "construct all the three-dimensional states of the four-dimensional figure" (Henderson 140). Despite this, the work is not meant to be definitive as other works created by Marcel Duchamp.

Still, at the bottom part, on the left side stands the bachelors which is named as "malic molds" and malic is a linguistic form of male that Marcel made up. While the upper pane is the stripped bride at the top. The contrast representing a realm of male and female, which create a sense of desire that power the machinery part at the bottom which also what powers both realms. Marcel was being sarcastic to the rationalization of everything in the world, even as basic as our desire. The work was also made in 1915 during the World War I resulted from modern

machinery as the products of rationalization. Certainly, every kind of technology was made for military purposes at first as it is the primary source of funding to scientists.

Apart from that, in every Marcel's works it is very important to have his signature because it's what transforms a ready-made object into art that something we look at differently. The work is itself a signature because Bride and Bachelors in French are *mariée* and *célibataires* respectively, and that the first three words of the two French words are mar and cel. Combining the two would be marcel which is his name and the fact that it is the first three words might suggest that fact that the work itself is a three-dimensional slice observed through the window of a four-dimensional space.

Since the glass was shattered, the work itself cannot be simply looked through a flat plane of glass window, it is refracted and the viewing medium(glass) is not unnoticeable. Somehow it can be viewed as different layers of image assembled to the medium.

There is a limitation on how merely a paper can do. No matter how good or innovative a drawing is, it is still on a 2-dimensional medium. It is certainly not enough to express the true potential of a 4-dimensional art. Because in order to create a 4-dimensional art piece, one must consider from the fact that 4-dimensional being can see at multiple 3 dimensional objects at once, and therefore creating an artwork in the form of a three dimensional medium would best suit the idea. This artwork should be created in a way that we are required to see from multiple perspective but would make sense if all the perspective can all be seen at once. The "putting together" images were a good attempt by the artists, however, an artist is merely an artist, without the mathematical background it is difficult for them to create what transcends the human mind. Instead of using "abstract" to present 4-dimensional art in the wrong way, an mathematical approach should be implemented while creating this artwork. In order to do that, one must study

the perspective in a 4-dimensional world. In order words, to accurately present a 4-dimensional object we must present a work which is an intersection/cut of the 4-dimensional object.

Modern art works also took the fourth dimension in a different approach into artworks. Using the slicing method, Jed Malitz created life-size glass sculptures of human figures within architectural forms. Siren is of the work among the collections. He described his work as a 4D sculpture of cut glass and refracted light. (See Fig. 6)

Fig. 8. *Siren* by Jed Malitz

Even though the work is not in the higher dimensional form, however, it explored the concept of transforming 2 dimensional slices into a 3-dimensional object.

A true 4-dimensional slice of 3D object should not a perspective and can be observed as whole while being still a three-dimensional work. Our vision is certainly not able to achieve it, unless we have a way of implementing viewing the whole 3-dimensional object as a whole in a device installed in our brain, otherwise there are many ways to interact with an object. Perhaps we could start by touching the work.

Another work that merged the concepts of a higher dimension in a playful way is the game FEZ. FEZ is a video game that the character lives in a 2-dimensional world. A golden hexahedron appeared, teared the fabric of spacetime and revealed a third dimension. This accident caused the character to gain the ability to move across higher dimension despite being a two-dimensional creature. The character set out for an adventure to collect all the missing fragments of the golden cube.

Please note that even though the player who plays the character can see through the whole scene at once, it does not mean that the character can see the whole stage at once. This is similar to the concept of a first-person view compared to a third person view. We are able to see the whole 2-dimensional plane at once as the 4-dimensional being can see the whole 3-dimensional "hyperplane" at once. We might be able to know what's behind the door provided the place is visible by light, so is light actually 4 dimensional or 3 dimensional?

All in all, perspective makes the world logical but also limits our experience to explore higher dimensions. Our two-dimensional vision keeps us from exploring higher dimensions. We can never truly experience a 4D object if we are limited to the system that we are used to. In

order to see all sides or perspective at once, as a unified view. One could make use of the optical illusion by simply using a convex lens to see from 360 degree just like what a compressed 360-degree photo looks like.

Even though it is still not clear about the accuracy of a four-dimensional world or object, and my limitations of my knowledge prevent me from going further, I believe with the technology advancement, it would not be far for us to find the truth and realize our imagination. And eventually, we can all be unchained from our cages and explore what we dreamt of.

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