1.

$$p(x_{new}|q_c) = \prod_{j=1}^{D} p_{cj}^{x_{new,j}} (1 - p_{cj}^{1-x_{new,j}})$$

$$x_p = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$x_s = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$x_{new} = [1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0]$$

$$s_{new} = \sum x_i = 5$$

$$\Rightarrow q_{cj} = \frac{\sum_{i=1}^{N_c} x_{ij}}{\sum_{j'=1}^{D} \sum_{i=1}^{N_c} x_{ij'}}$$

$$p_p = \frac{1}{24}[2 \quad 1 \quad 1 \quad 5 \quad 5 \quad 1 \quad 4 \quad 5]$$

$$p_s = \frac{1}{22}[5 \quad 5 \quad 2 \quad 5 \quad 2 \quad 1 \quad 1 \quad 1]$$

$$p(x_{new}|C = p) = \left(\frac{2}{24}^1 * \frac{23}{24}^1 * \frac{23}{24}^1 * \frac{5}{24}^1 * \frac{5}{24}^1 * \frac{1}{24}^1 * \frac{4}{24}^1 * \frac{19}{24}^1\right)$$

$$= \frac{2010200}{24^8}$$

$$p(x_{new}|C = s) = \left(\frac{5}{22}^1 * \frac{17}{22}^1 * \frac{20}{22}^1 * \frac{5}{22}^1 * \frac{2}{22}^1 * \frac{1}{22}^1 * \frac{1}{22}^1 * \frac{21}{22}^1\right)$$

$$= \frac{357000}{22^8}$$

$$p(C = p) = \frac{6}{6+7}, p(C = s) = \frac{7}{6+7}$$

$$P(C = p|x_i) = \frac{p(x_{new}|C = p) * p(C = p)}{p(x_{new}|C = p) * p(C = p) + p(x_{new}|C = s) * p(C = s)}$$

$$\approx 0.706$$

$$P(C = s|x_i) = \frac{p(x_{new}|C = s) * p(C = s)}{p(x_{new}|C = p) * p(C = p) + p(x_{new}|C = s) * p(C = s)}$$
$$\approx 0.294$$

2.

$$p(x_i|q_c) = (\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}) \prod_{j=1}^{D} q_{cj}^{x_{ij}}$$

$$\log p(x_i|q_c) = \log\left\{\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) \prod_{j=1}^{D} q_{cj}^{x_{ij}}\right\} = \log\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) + \log \prod_{j=1}^{D} q_{cj}^{x_{ij}}$$

$$= \log\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) + \sum_{j=1}^{D} \log q_{cj}^{x_{ij}}$$

$$= \log\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) + \sum_{j=1}^{D} x_{ij} \log q_{cj}$$

$$\log \prod_{i=1}^{N_c} p(x_i|q_c) = \sum_{i=1}^{N_c} \log p(x_i|q_c) = \sum_{i=1}^{N_c} \log\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) + \sum_{j=1}^{D} x_{ij} \log q_{cj}$$

$$\mathcal{L}(q_{cj}, \lambda) = \sum_{i=1}^{N_c} \log\left(\frac{s_i!}{\prod_{j=1}^{D} x_{ij}!}\right) + \sum_{j=1}^{D} x_{ij} \log q_{cj} + \lambda\left(\sum_{j=1}^{D} q_{cj} - 1\right)$$

$$\frac{\partial \mathcal{L}}{\partial q_{cj}} = \sum_{i=1}^{N_c} x_{ij} \frac{1}{q_{cj}} + \lambda = 0$$

$$\Rightarrow q_{cj} = -\frac{\sum_{i=1}^{N_c} x_{ij}}{\lambda}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{j=1}^{D} q_{cj} - 1 = \sum_{j'=1}^{D} -\frac{\sum_{i=1}^{N_c} x_{ij'}}{\lambda} - 1 = 0$$

$$\Rightarrow \lambda = -\sum_{j'=1}^{D} \sum_{i=1}^{N_c} x_{ij'}$$

$$\Rightarrow q_{cj} = -\frac{\sum_{i=1}^{N_c} x_{ij}}{\lambda} = \frac{\sum_{i=1}^{N_c} x_{ij}}{\sum_{j'=1}^{D} \sum_{i=1}^{N_c} x_{ij'}} \blacksquare$$