



# Problem D Set Game

Time limit: 6 seconds

Memory limit: 2048 megabytes

#### **Problem Description**

Alice and Bob are playing a game related to set operations. The goal of the game is simple: Alice wants to maximize the sum of the elements in the set, while Bob wants to minimize it.

The game starts with a set containing some elements, and two positive integers p and q are given.

At Alice's turn, she will receive a number k. Each time, she can select two distinct numbers (say a and b) from the set, and add ap + bq to the set. She can perform this operation k times.

When it's Bob's turn, he will also receive a number k. Each time, he can select a number from the set and remove it. He can perform this operation k times as well. We guarantee that after removing elements, the set will always contain at least two elements.

Note that each element in the set appears only once; if an element is generated multiple times, it is counted only once.

We assume that both Alice and Bob play using the optimal strategy. Given the initial set, the values of p and q, and the record of the game process, write a program to calculate the sum of the elements in the set after each round. Because the sum can be very large, output the result modulo  $10^9 + 7$ .

### **Input Format**

The first line contains two integers n and m — the number of set elements and the number of rounds.

The second line contains two integers p and q, which have the same meaning as described.

The third line contains n integers  $a_1, a_2, ... a_n$  — the elements in the initial set.

Each of the following m lines contains the information of rounds. Each line of the round is one of the following:

- 1 k Alice's turn, with k operations.
- 2 k Bob's turn, with k operations.

#### **Output Format**

Output m lines. In the i-th line, output the sum of the elements in the set after i-th round, with modulo  $10^9 + 7$ .



#### **Technical Specification**

- $2 \le n \le 10^5$
- $1 \le m \le 10^5$
- $1 \le p, \ q \le 10^9$
- $1 \le a_i \le 10^9$
- $\forall i \neq j, \ a_i \neq a_j$
- $1 \le k \le 10^9$

#### Sample Input 1

```
5 5 2 3 1 4 2 5 7 1 2 1 1 2 2 2 2 3 1 3 1 3
```

## Sample Output 1

```
157
540
50
7
279
```

#### Note

Here are some properties of modulo operations:

- 1.  $(a+b) \mod m = ((a \mod m) + (b \mod m)) \mod m$
- 2.  $(a-b) \mod m = ((a \mod m) (b \mod m)) \mod m$
- 3.  $(a \times b) \mod m = (a \mod m) \times (b \mod m) \mod m$

To compute modulo division, we need to first compute the modular inverse. Here's an introduction to the modular inverse:

The expression  $a/b \mod m$  can be viewed as  $a \times b^{-1} \mod m$ , where we need to compute the value of  $b^{-1} \mod m$ , also known as the modular inverse.



According to Fermat's Little Theorem:  $b^{m-1} = 1 \mod m$  if b is not a multiple of m and m is a prime number. This implies that  $b^{m-2} = b^{-1} \mod m$ . Therefore, to compute  $b^{-1} \mod m$ , we can calculate  $b^{m-2} \mod m$ . Since the modulo m mentioned in the problem is  $10^9 + 7$  which is a prime number, we have  $b^{m-1} = 1 \mod m$  for every  $b \in [1, m)$ .

To compute  $b^{m-2} \mod m$ , we can use fast exponentiation and be cautious of integer overflow issues during implementation.

```
// Compute a^n mod m
int fpow(int a, int n, int m) {
  // a^0 = 1 \mod m
  if (n == 0) return 1;
  // a<sup>1</sup> = a mod m
  if (n == 1) return a;
  // Compute k = a^{n/2}
  int k = fpow(a, n / 2, m);
  // If n is even, a^n = k * k \mod m
  if (n \% 2 == 0)
    return 111 * k * k % m;
  // Else if n is odd, a^n = k * k * a mod m
  // Be aware of the integer overflow issue
  return 111 * k * k % m * a % m;
}
int main() {
  // Define modulo
  const int m = 1'000'000'007;
  // Any value in [1, m)
  int b = 186265;
  // Compute b_inv (b^{-1}) by Fermat's Little Theorem
  // b^{m-2} = b^{-1} \mod m
  int b_{inv} = fpow(b, m - 2, m);
  // b * b inv modulo m should be 1
  assert(111 * b * b_inv % m == 1);
}
```