AN ATYPIC 3D SIMILARITY MEASURE FOR INTUITIONISTIC FUZZY SETS AND ITS APPLICATIONS

A Thesis Submitted in
Partial Fulfillment of the Requirements for the Degree of

Master of Science

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DEPARTMENT OF MATHEMATICS

NATIONAL INSTITUTE OF TECHNOLOGY SILCHAR
SILCHAR- 788010, ASSAM, INDIA

May, 2023

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Under The Supervision of

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Certificate

It is certified that the work contained in this thesis entitled "**An Atypic 3D Similarity Measure for Intuitionistic Fuzzy Sets and its Applications**" submitted by Chinmaya Pradhan, Registration No. 21-48-125 for the award of M.Sc. is absolutely based on his own work carried out under my supervision and that this thesis has not been submitted elsewhere for any degree.

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Abstract

The study includes the discussion over the fundamental concepts about the Fuzzy sets, Intuitionistic Fuzzy sets and their properties. The existing similarity measures in the field of IFS are not efficient enough. It produces counter-intuitive results and have 'zero divisor problems' that fails to satisfy similarity measure axioms, and are incapable of detecting with a slight change either in membership or in non-membership degrees. To overcome these shortcomings, an atypic similarity measure has been introduced to detect the similarity between two IFSs. To overcome the drawbacks of the existing Similarity measures "An Atypic 3D Similarity Measure for Intuitionistic Fuzzy Sets" has been proposed in this report. Finally, we compute its efficiency in various fields such as pattern detection, software quality evaluation and detection of diseases. It is observed that the proposed similarity measure is superior than other existing similarity measure.

Key words: Intuitionistic Fuzzy Sets, Similarity Measure, Pattern Recognition, Software Quality Evaluation, Medical Diagnosis.

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Place: Silchar **Chinmaya Pradhan**
Date: May 20, 2023 **Reg. No. 21-48-125**

Heartfully Dedicated to....

Maa & Bapa

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ϱ	Rho (used as membership function)
φ	Phi (used as non-membership function)
ζ	Zeta (used as hesitancy degree)
α	Alpha
β	Beta
L^c	Complement of L
$\forall$	For all
$\Rightarrow$	Implies that
$\rightarrow$	Tends to
$\cup$	Union
$\cap$	Intersection
$\leq$	Less than or equal to
$\geq$	Greater than or equal to
$<$	Strictly less than
$>$	Strictly greater than
$\subseteq$	Subset
$\in$	Belongs to
$=$	Equal to
$\neq$	Not equal to
$\therefore$	Therefore
$i = 1(1)k$	i=1, 2,..., k

Chapter 1

Introduction

Set theory, a branch of mathematical logic that is a discrete collection of things that exclusively accepts the binary value $\{0, 1\}$ has been studied since its inception. It states that if a function on a set L is defined to be between 0 and 1, then an element is a member of that set L if its value is 1, else it is 0, and values in between 0 and 1 are not members of a set L. Crisp set or classical set is the name of this set theory, which cannot address issues that arise in daily life such as decision-making, medical diagnosis, artificial intelligence, etc. We need our system to be able to deal with inaccurate and incomplete data. Later, mathematicians found that some sets have functions that not only accept values between 0 and 1, but also have values that fall within the range [0, 1]. Fuzzy sets [37] are those kinds of sets where an element's membership depends on a real number in the range [0, 1]. The phrase "fuzzy" mimics the ambiguities or "vague" nature of data or terms, meaning that no definitive or conclusive inference might be drawn about an object. The foundations of traditional set theory could not handle these kinds of uncertainties. In order to address this, Zadeh [37] proposed the idea of *fuzzy sets* in 1965.

Many general life terms such as cuteness or hatred etc. are fuzzy in nature as they could not be framed totally under one complete category. The concept of fuzzy terms is that they don't have a convincing idea of the boundary. Hence, we can say that the fuzzy sets have a distinguishable feature than the classical sets which is that the crisp or classical set has fixed inclusions in terms of the boundary which clearly separates the interior objects in the set from the exterior one. Fuzzy Logic Systems (FLS) respond to partial, ambiguous, distorted, or erroneous (fuzzy) input by producing an acceptable but definitive result. A form of reasoning called fuzzy logic is similar to human reasoning. The FL method mimics how humans make decisions by considering all middle-ground options between the digital values YES and NO. The typical logic unit that a computer can comprehend accepts precise input and generates a clear output as True or False, which is equivalent to human responses of Yes and No. Fuzzy logic's creator, Lofti Zadeh, noted that, in contrast to computers, humans may make decisions based on a variety of options between Yes and No, including CERTAINLY YES, POSSIBLY YES, CANNOT SAY, POSSIBLY NO, CERTAINLY NO. The fuzzy logic uses various degrees of input possibilities to produce a clear result. Fuzzy logic is helpful for practical and commercial purposes since it may assist engineers deal with uncertainty even though its reasoning isn't always precise.

Let us explain fuzzy sets with an example: Suppose an air conditioning system has 5- level fuzzy logic system. This system adjusts the temperature of air conditioner by comparing the room temperature and the target temperature value.

- Define linguistic variables and terms.
- Construct membership funtions for them.
- Construct knowlwdge based rules.
- Convert crisp data into fuzzy data using membership functions(fuzzification).
- Evaluate rules in the rule base(Inference Engine).
- Combine results from each rule(Inference Engine).
- Convert output data into non-fuzzy values.(defuzzification)

Suppose F is the set of the Aussies cricketers. We want to create a set P of 'good batter' by selecting some of Aussies Cricket players {Maxwell, Warner, Smith, Lyon, Stoinis}, in which 'good' is a vague term. So the collection P would be fuzzy in nature. $P=$ {(Maxwell, 0.8), (Warner, 0.9), (Smith, 1), (Lyon, 0.3), (Stoinis, 0.4)}. From here, f_P(Smith) indicates the batting performance of Smith as 1, who is the best batter among the Indian cricketers, and f_P(Warner) $=$ 0.9, who is also a very good batter but f_P(Lyon) $=$0.3 as he is the one who is actually a bowler but he can bat and less good batter from every other Indian cricketer in the set F. These values of membership function are taken just for the sake of understanding that tells us from the membership function of P as to how loosely or strongly an element is a member the set P.
However, in practice, the degree of non-membership (φ) of an element in a fuzzy set could not be given by complement of the degree of membership (1-ϱ) always. Because in certain cases there may be some hesitation involved in stating an element as non-member called hesitation degree. However, when a human being expresses the degree of membership of a particular element in a fuzzy set, they frequently fail to state a corresponding degree of non-membership. For example, Let fzzy.app be an android app that calculates that consider beautiful as the membership function and calculates membership and non-membership of various flower in

a garden based on the various training datasets input in it. To test its accuracy, its beta version is asked to used by gardener working in the building/office/institute greater than 5000sq.feet and they are rewarded accourding to the number of input they give in it. Suppose Assume that 65% of gardener 'like more' the scheme and earn an handsome prize money; then, according to fuzzy set theory, 35% of people 'like less' that scheme and didn't earn accourding to the expectations. Then what about the part of gardener who don't have smartphone or didn't download the app at all? This becomes unrealistic to observe these type of conditions in more detail through the fuzzy set theory. As a result, Atanassov [1] presented the intuitionistic fuzzy set (IFS) as a generalized fuzzy set in 1983, which has proven to be particularly effective in dealing with ambiguity.
So, if we consider F to be the set of people consuming that brand, assume that for every $r \in F$ we know the percentage of people who like more that brand $\varrho(r)$ and let $\varphi(r)$ is the number referring to the people who like less that brand. Then we can show that the part of the people who don't consume cold drinks at all is $1-\varrho(r)-\varphi(r)$. Thus, we have constructed a set $\{t, \varrho(r), \varphi(r) : r \in F\}$ these types of sets where $0 \leq \varrho(r)+\varphi(r) \leq 1$ are referred as IFSs which introduce a new term in logic and formulation of the degree of hesitation called hesitation margin (1 - $(\varrho(r)+\varphi(r))$) giving room of uncertainty. Thus, IFSs are a more appropriate tool to deal with chaotic situations giving a large room for uncertainty.
This project provides a study on the concept of similarity measures between two IFSs, a crucial tool for computing the similarity or difference between two objects and providing a starting point for geometry.

Distance measures such as Hamming distance, Euclidean distance, Normalized Hamming distance, Normalized Euclidean distance are the most known distance functions for fuzzy sets. As IFS is the generalization of fuzzy sets, so distance measure in fuzzy sets such as Hamming and Euclidean distance is also further generalized to distance measure in IFSs by including the two-parameter known as non-membership function and hesitancy degree, which is proposed by Szmidt and Kacprzyk [29] in 2000. In addition, Dengfeng et. al. [9] (2002) independently formulated the similarity measures for IFS & used them for pattern recognition. However, Liang and Shi [17] (2003), as well as Mitchell [22] (2003), pointed out that Li and Cheng's [9] measurements aren't always beneficial in some circumstances and made some modifications respectively. An atypic formulation, for similarity calculation between IFSs is modelled satisfying axioms of the definition. Then the proposed similarity measure is further applied in pattern recognition, software quality evaluation, and diseases detection which will also be discussed in another section.

Motivation and Objective

Several real-life situations are found where the existing similarity measured proposed by various mathematicians provide low accuracy and fails to determine the similarity between two IFSs as they gives counter-intuitive results or has zero-divisor problems in identifying the similarity between two IFSs.
In this thesis, our objective is to propose new similarity measures that can determine the similarity between two IFSs in a more specific way and apply that measure over various fields related to vague boundaries.

Chapter 2

Literature review

2.1 Preliminaries

In this section, we'll take a look over a few fundamental definitions and some mathematical properties related to IFS.

Definition 1 *[37] A classical or Crisp set is a set with fixed and well-defined boundaries.*

2.1.1 Fuzzy Set

Definition 2 *[37],[12] If F denote the universe of discourse, then a fuzzy set P in $F = \{r\}$ which is in ordered paired set is defined as*

$$P = \{\langle r, \varrho_P(r)\rangle : \ r \in F\} \tag{2.1}$$

where $\varrho_P(r) : F \to [0,1]$ is the membership function of the fuzzy set P, $\varrho_P(r) \in [0,1]$ is the membership of $r \in F$ in P.

Definition 3 *[3] On the universe of discourse F, non-membership function for an IFS P is defined by $\varphi_P(r) = 1 - \varrho_P(r)$, where $\varphi_P(r) \in [0,1]$.*

2.1.2 Intuitionistic Fuzzy Set and Its Operations

Definition 4 *[3] A ordered triples set P in F is called IFS when its defined as*

$$P = \{\langle r, \varrho_P(r), \varphi_P(r)\rangle : \ r \in F\} \tag{2.2}$$

where $\varrho_P(r) : F \to [0,1], \quad \varphi_P(r) : F \to [0,1]$ *with the condition*

$$0 \le \varrho_P(r) + \varphi_P(r) \le 1 \quad \forall \ r \in F$$

The numbers $\varrho_P(r), \varphi_P(r) \in [0,1]$ denote the membership degree and non-membership degree of the element $r \in F$ respectively.

We also can represent each fuzzy set as the following IFS [33] :

$$P = \{\langle r, \varrho_P(r), 1 - \varrho_P(r)\rangle : \ \ r \in F\} \tag{2.3}$$

$P = \{\langle r, \varrho_P(r), 1 - \varrho_P(r)\rangle : \ \ r \in F\}$ is easily demonstrated to be identical to equation (2.2).i.e. each fuzzy set is a specific instance of IFS.
[33] For each element $r \in F$, the degree of hesitancy of r in P is defined as:

$$\zeta_P(r) = 1 - \varrho_P(r) - \varphi_P(r)$$

It is a hesitancy degree or the degree of non-determinacy (or uncertainty) of r to P [29] and it is obvious that

$$0 \leq \zeta_P(r) \leq 1 \quad \forall \ r \in F$$

If $P \in$ Fuzzy set F then for each fuzzy set P in F, evidently

$$\zeta_P(r) = 1 - \varrho_P(r) - \varphi_P(r) - [1 - \varrho_P(r)] = 0 \quad \forall \ r \in F$$

Example 1 *Let us consider the case of an project work assigned to Shyam. Suppose he has completed t part of the work, then we define the work done as the membership function $\varrho_(t)$ and the work not done as the non-membership function $\varphi_(t)$.*
While if we consider the case of t part work of the project is done, while r part of the work is not done, and some z work of the project like for example the reading part of an experiment/thesis is completed but the observation aren't noted fairly till yet. Then $\zeta_(z)$ is called the degree of hesitance,which is $1 - (\varrho_(t) + \varphi_(r))$

Relations and Operations on IFSs

Here, we define the following relations and operations for every two IFSs L and K as follows [1], [2] :
Relations

1. Inclusion:$L \subset K \Leftrightarrow \varrho_L(t) \leq \varrho_K(t), \varphi_L(t) \geq \varphi_K(t) \forall \in T$

2. Equality:$L = K \Leftrightarrow \varrho_L(t) = \varrho_K(t), \varphi_L(t) = \varphi_K(t) \forall t \in T$

Operations

1. Union: $L \cup K = \{(t, max(\varrho_L(t), \varrho_K(t)), min(\varphi_L(t), \varphi_K(t))) \mid t \in T\}$

2. Intersection:$L \cap K = \{(t, min(\varrho_L(t), \varrho_K(t)), max(\varphi_L(t), \varphi_K(t)) \mid t \in T\}$

3. Addidtion:$L + K = \{(t, \varrho_L(t) + \varrho_K(t) - \varrho_L(t)\varrho_K(t), \varphi_L(t)\varphi_K(t)) \mid t \in T\}$

4. Multiplication:$L \times K = \{(t, \varrho_L(t)\varrho_K(t), \varphi_L(t) + \varphi_K(t) - \varphi_L(t)\varphi_K(t)) \mid t \in T\}$

5. Difference: $L - K = \{(t, min(\varrho_L(t), \varphi_K(t)), max(\varphi_L(t), \varrho_K(t)) \mid t \in T\}$

6. Cartesian product:$L \times K = \{(\varrho_L(t)\varrho_K(t)), (\varphi_L(t)\varphi_K(t)) \mid t \in T\}$

7. Complement:$L^c = \{(t, \varphi_L(t), \varrho_L(t)) \mid t \in T\}$

Axiomatic Definition of IFS

Definition 5 *Let l be a mapping $l : IFSs(T) \times IFSs(T) \to [0,1]$. Then l is said to be a similarity measure if $l(L, K)$ satisfies the following axioms as given by Mitchell (2003) [22]:*

(SA1) : $0 \leq l(L, K) \leq 1$

(SA2) : $l(L, K) = 1$ *iff* $L = K$

(SA3) : $l(L, K) = l(K, L)$

(SA4) : *If* $L \subseteq K \subseteq J, L, K, J \in IFSs(T)$, *then* $l(L, J) \leq l(L, K)$ *and* $l(L, J) \leq l(K, J)$.

2.2 Similarity Measures between Two IFSs

Similarity measures for IFSs can be classified into two categories: distance-based measures and non-distance based measures. Distance-based measures include Euclidean distance, Hamming distance, and Minkowski distance, while non-distance based measures include Jaccard similarity, cosine similarity, and correlation coefficient.

Here we would deal with only the distance-based similarity measures. Some of the existing similarity measures are as follows:

- Chen (1995)[5] similarity measure

$$S_C(L,K) = 1 - \frac{\sum_{i=1}^{n} |S_L(r_i) - S_K(r_i)|}{2n} \tag{2.4}$$

  where $S_L(r_i) = \varrho_L(r_i) - \varphi_L(r_i)$ and $S_K(r_i) = \varrho_K(r_i) - \varphi_K(r_i)$

- Hong and Kim (1999)[14] similarity measure

$$S_{HK}(L,K) = 1 - \frac{\sum_{i=1}^{n}(|\varrho_L(r_i) - \varrho_K(r_i)| + |\varphi_L(r_i) - \varphi_K(r_i)|}{2n} \tag{2.5}$$

- Mitchell (2003)[22] similarity measure

$$S_M(L,K) = \frac{\rho_\varrho(L,K) + \rho_\varphi(L,K)}{2} \tag{2.6}$$

  where $\rho_\varrho(L,K) = 1 - \sqrt{\frac{\sum_{i=1}^{n}|\varrho_L(r_i) - \varrho_K(r_i)|^p}{n}}$ and $\rho_\varphi(L,K) = 1 - \sqrt{\frac{\sum_{i=1}^{n}|\varphi_L(r_i) - \varphi_K(r_i)|^p}{n}}$

- Liu (2005)[18] similarity measures

$$S_L(L,K) = 1 - \frac{\sqrt{\sum_{i=1}^{n}(\varrho_L(r_i) - \varrho_K(r_i))^2 + (\varphi_L(r_i) - \varphi_K(r_i))^2 + (\zeta_L(r_i) - \zeta_K(r_i))^2}}{2n} \tag{2.7}$$

- Song et al. (2014)[27] similarity measure

$$S_S(L,K) = \frac{1}{2n}\sum_{i=1}^{n}(\sqrt{\varrho_L(t_i)\varrho_K(r_i)} + 2\sqrt{\varphi_L(r_i)\varphi_K(r_i)} + \sqrt{\zeta_L(r_i)\zeta_K(r_i)} + \sqrt{(1 - \varphi_L(r_i))(1 - \varphi_K(r_i))}) \tag{2.8}$$

- Garg and Kumar (2018)[11] similarity measure

$$S_{GK}(L,K) = \frac{1}{n}\sum_{i=1}^{n}(1 - \frac{1}{3}|a_L(r_i) - a_K(r_i)| + b_L(r_i) - b_K(r_i)| + |c_L(r_i) - c_K(r_i)|) \tag{2.9}$$

  where $a(r_i) = \varrho(r_i) \times (1 - \varphi(r_i)), b(r_i) = (1 - \varrho(r_i)) \times (1 - \varphi(r_i)) - \varphi_(r_i) \times (1 - \varrho(r_i)), c_(r_i) = \varphi_(r_i) \times (1 - \varrho(r_i))$

- The normalized Hamming similarity

$$S'_{nh}(L,K)=1-(\frac{1}{2n}\sum_{i=1}^{n}[|\varrho_L(t_i)-\varrho_K(r_i)|+|\varphi_L(t_i)-\varphi_K(r_i)|+|\pi_L(t_i)-\pi_K(r_i)|]) \tag{2.10}$$

- The normalized Euclidean similarity

$$S'_{ne}(L,K)=1-(\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left[(\varrho_L(t_i)-\varrho_K(r_i))^2+(\varphi_L(t_i)-\varphi_K(r_i))^2+(\pi_L(t_i)-\pi_K(r_i))^2\right]}) \tag{2.11}$$

- Hausdorff similarity or Grzegorzewski similarity measure

$$S_{Ha}(L,K)=1-(\frac{1}{k}\sum_{i=1}^{k}max\{|\varrho_L(t_i)-\varrho_K(t_i)|,|\varphi_L(t_i)-\varphi_K(t_i)|\}) \tag{2.12}$$

- Wang-Xin similarity measure

$$\begin{aligned}S_{WX}(L,K)=&1-(\frac{1}{k}\sum_{i=1}^{k}[\frac{\{|\varrho_L(t_i)-\varrho_K(t_i)|+|\varphi_L(t_i)-\varphi_K(t_i)|\}}{4}\\&+\frac{max\{|\varrho_L(t_i)-\varrho_K(t_i)|,|\varphi_L(t_i)-\varphi_K(t_i)|\}}{2}])\end{aligned} \tag{2.13}$$

- H-max similarity measure

$$\begin{aligned}S_{H_{max}}(L,K)=&1-(\frac{1}{3k}\sum_{i=1}^{k}[|\varrho_L(t_i)-\varrho_K(t_i)|+|\varphi_L(t_i)-\varphi_K(t_i)|\\&+|max\{\varrho_L(t_i),\varphi_K(t_i)\}-max\{\varrho_K(t_i),\varphi_L(t_i)\}|])\end{aligned} \tag{2.14}$$

- WX-H-max similarity measure

$$\begin{aligned}S^{WX}_{Hmax}(L,K)=&1-(\frac{1}{k}\sum_{i=1}^{k}[\frac{\varrho_L(t_i)-\varrho_K(t_i)|+|\varphi_L(t_i)-\varphi_K(t_i)|}{4}|\\&+\frac{|max\{\varrho_L(t_i),\varphi_K(t_i)\}-max\{\varrho_K(t_i),\varphi_L(t_i)\}|}{2}])\end{aligned} \tag{2.15}$$

- Mahanta and Panda similarity measure [21] :

$$S_{JA}(L,K)=1-(\frac{1}{k}\sum_{i=1}^{k}\frac{|\varrho_L(t_i)-\varrho_K(t_i)|+|\varphi_L(t_i)-\varphi_K(t_i)|}{\varrho_L(t_i)+\varrho_K(t_i)+\varphi_L(t_i)+\varphi_K(t_i)}) \tag{2.16}$$

- Baccour and Alimi similarity measure:

$$S_{BA1}(L,K) = 1 - (\frac{1}{2k}\sum_{i=1}^{k}(\sqrt{\varrho_L(t_i)} - \sqrt{\varrho_K(t_i)})^2 + (\sqrt{\varphi_L(t_i)} - \sqrt{\varphi_K(t_i)})^2) \tag{2.17}$$

$$S_{BA2}(L,K) = 1 - (\frac{1}{4k}\sum_{i=1}^{k}(\sqrt{|\varrho_L(t_i) - \varrho_K(t_i)|} + \sqrt{|\varphi_L(t_i) - \varphi_K(t_i)|})^2) \tag{2.18}$$

- Ju et. al. similarity measure:

$$S_J(L,K) = 1 - (\frac{1}{k2ln2}\sum_{i=1}^{k}((\varrho_L(t_i) - \varrho_K(t_i))ln\frac{1+\varrho_L(t_i)}{1+\varrho_K(t_i)} + (\varphi_L(t_i) - \varphi_K(t_i))ln\frac{1+\varphi_L(t_i)}{1+\varphi_K(t_i)})) \tag{2.19}$$

- Vlachos similarity measure:

$$S_V(L,K) = 1 - (\sum_{i=1}^{k}[\varrho_L(t_i)ln\frac{\varrho_L(t_i)}{\frac{1}{2}(\varrho_L(t_i)+\varrho_K(t_i))} + \varphi_L(t_i)ln\frac{\varphi_L(t_i)}{\frac{1}{2}(\varphi_L(t_i)+\varphi_K(t_i))} + \varrho_K(t_i)ln\frac{\varrho_K(t_i)}{\frac{1}{2}(\varrho_L(t_i)+\varrho_K(t_i))} + \varphi_K(t_i)ln\frac{\varphi_K(t_i)}{\frac{1}{2}(\varphi_L(t_i)+\varphi_K(t_i))}]) \tag{2.20}$$

- Ke et. al. similarity measure:

$$S_K(L,K) = 1 - (\frac{1}{k}\sum_{i=1}^{k}[(\frac{\varrho_L(t_i) - \varphi_L(t_i)}{2} - \frac{\varrho_K(t_i) - \varphi_K(t_i)}{2})^2 + \frac{1}{3}(\frac{\varrho_L(t_i) + \varphi_L(t_i)}{2} - \frac{\varrho_K(t_i) +_K (t_i)}{2})^2]^{\frac{1}{2}}) \tag{2.21}$$

Apart from these above mentioned measures, there are also many other similarity measures discussed in the theory of IFSs.

Chapter 3

An Atypic 3D Similarity Measure

Several similarity measures have been proposed for IFSs, including the intuitionistic fuzzy divergence, intuitionistic fuzzy entropy, and intuitionistic fuzzy similarity measure.

However, these measures have limitations such as being sensitive to outliers and not considering the degree of uncertainty and hesitation in the IFSs.

We have reviewed many distance and similarity measures in the literature. Several of them are unsuccessful in aiding with actual decision-making. Despite common perception , many of the similarity/distance measures do not line up with reality. In existing methods, the difference between corresponding memberships and non-membership has been taken to define the distance/similarity expression. In contrast to previous definitions, we considered the influence of individual and global factor variations on the similarity of two IFSs. A atypic similarity measure for IFSs has been introduced in the following Definition 6.

3.1 Defination of The Atypic Similarity Measure

The terminologies used throughout this section is as follows:
$T = \{t_1, t_2, t_3, ...t_n\}$ be an UOD.
$L = \{\langle t, \varrho_L(t), \varphi_L(t)\rangle\}$ and $K = \{\langle t, \varrho_K(t), \varphi_K(t)|$ are two IFSs on T.
$\Delta\varrho_i = |\varrho_L(r_i) - \varrho_K(r_i)|, \Delta\varphi_i = |\varphi_L(r_i) - \varphi_K(r_i)|, \Delta\zeta_i = |\zeta_L(r_i) - \zeta_K(r_i)|$
$\Delta\varrho_{min} = \min_i\{\Delta\varrho_i\}$, $\Delta\varphi_{min} = \min_i\{\Delta\varphi_i\}$, $\Delta\zeta_{min} = \min_i\{\Delta\zeta_i\}$
$\Delta\varrho_{max} = \max_i\{\Delta\varrho_i\}, \Delta\varphi_{max} = \max_i\{\Delta\varphi_i\}, \Delta\zeta_{max} = \max_i\{\Delta\zeta_i\}$

Definition 6 *Similarity measure between IFSs L and K is given by the following equation.*

$$l_C(L,K) = \frac{1}{3n}\sum_{i=1}^{n}\left[\frac{2-\Delta\varrho_i-\Delta\varrho_{max}}{2+\Delta\varrho_i-\Delta\varrho_{min}}(1-\Delta\varrho_i)+\frac{2-\Delta\varphi_i-\Delta\varphi_{max}}{2+\Delta\varphi_i-\Delta\varphi_{min}}(1-\Delta\varphi_i)+\frac{2-\Delta\zeta_i-\Delta\zeta_{max}}{2+\Delta\zeta_i-\Delta\zeta_{min}}(1-\Delta\zeta_i)\right] \tag{3.1}$$

Theorem 1 *$l_C(L,K)$ is a similarity measure between two IFSs L and K defined on $T=\{t_1,t_2,t_3,....t_n\}$.*

Proof To prove the axioms of Definition 6, substitute $\frac{2-\Delta\varrho_i-\Delta\varrho_{max}}{2+\Delta\varrho_i-\Delta\varrho_{min}}=\alpha_i$, $\frac{2-\Delta\varphi_i-\Delta\varphi_{max}}{2+\Delta\varphi_i-\Delta\varphi_{min}}=\beta_i$, $\frac{2-\Delta\zeta_i-\Delta\zeta_{max}}{2+\Delta\zeta_i-\Delta\zeta_{min}}=\gamma_i$ in Equation(3.1). Then Equation (3.1) becomes

$$l_C(L,K)=\frac{1}{3n}\sum_{i=1}^{n}[\alpha_i(1-\Delta\varrho_i)+\beta_i(1-\Delta\varphi_i)+\gamma_i(1-\Delta\zeta_i)] \tag{3.2}$$

(SA1):We have $0\leq\Delta\varrho_i\leq 1$ & $0\leq\Delta\varphi_i\leq 1$ & $0\leq\Delta\zeta_i\leq 1$.
$\Longrightarrow$ $0\leq\Delta\varrho_{min}\leq 1, 0\leq\Delta\varphi_{min}\leq 1,\quad 0\leq\Delta\zeta_{min}\leq 1$
$\Longrightarrow$ $0\leq\Delta\varrho_{max}\leq 1,\quad 0\leq\varphi_\varphi max\leq 1, 0\leq\Delta\zeta_{max}\leq 1$
$\Longrightarrow$ $0\leq\alpha_i\leq 1$ and $0\leq\beta_i\leq 1$ and $0\leq\varphi_i\leq 1$
$0\leq\alpha_i(1-\Delta\varrho_i)+\beta_i(1-\Delta\varphi_i)+\varphi_i(1-\Delta\zeta_i)\leq 3$ holds.
Hence $0\leq l_C(L,K)\leq 1$

(SA2): let $l_C(L,K)=1$
$\Longleftrightarrow$ $\frac{1}{3n}\sum\limits_{i=1}^{n}[\alpha_i(1-\Delta\varrho_i)+\beta(1-\Delta\varphi_i)+\gamma_i(1-\Delta\zeta_i)]=1$
$\Longleftrightarrow$ $\alpha_i(1-\Delta\varrho_i)+\beta_i(1-\Delta\varphi_i)+\varphi_i(1-\Delta\zeta_i)=3$ for any i=1,2,....n.
$\Longleftrightarrow$ $\alpha_i(1-\Delta\varrho_i)=1$ & $\beta_i(1-\Delta\varphi_i)=1$ & $\gamma_i(1-\Delta\zeta_i)=1$
$\Longleftrightarrow$ $\alpha_i=1, 1-\Delta\varrho_i=1, \beta_i=1, 1-\Delta\varphi_i=1, \gamma_i=1, 1-\Delta\zeta_i=1$
$\Longleftrightarrow$ $\Delta\varrho_i=0$ & $\Delta\varphi_i=0$ & $\Delta\zeta_i=0$ $\forall i$
$\Longleftrightarrow$ $\varrho_L(r_i)=\varrho_K(r_i)$ & $\varphi_L(r_i)=\varphi_K(r_i)$ & $\zeta_L(r_i)=\zeta_K(r_i)$
$\Longleftrightarrow$ $L=K$

(SA3): Proof of this part is trivial.

(SA4): Suppose $L\subseteq K\subseteq J$,
$L,K,J\in$ IFSs(T)
$\Rightarrow \varrho_L(r_i)\leq\varrho_K(r_i)\leq\varrho_J(r_i)$ *and*
$\varphi_L(r_i)\geq\varphi_K(r_i)\geq\varphi_J(r_i)$
$\zeta_L=1-\varrho_L-\varphi_L$
$\zeta_K=1-\varrho_K-\varphi_K$
$\zeta_J=1-\varrho_J-\varphi_J$
$|\zeta_L(r_i)-\zeta_J(r_i)|\geq|\zeta_L(r_i)-\zeta_K(r_i)|$
$\Delta\varrho_i^{LJ}\geq\Delta\varrho_i^{LK}\implies\Delta\varrho_{min}^{LK}\geq\Delta\varrho_{min}^{LK}$ and $\Delta\varrho_{max}^{LJ}\geq\Delta\varrho_{max}^{LK}$ $\forall i=1,2,...,n$
$\Delta\varphi_i^{LJ}\geq\Delta\varphi_i^{LK}\implies\Delta\varphi_{min}^{LK}\geq\Delta\varphi_{min}^{LK}$ and $\Delta\varphi_{max}^{LJ}\geq\Delta\varphi_{max}^{LK}$ $\forall i=1,2,...,n$

$\Delta\zeta_i^{LJ} \geq \Delta\zeta_i^{LK} \implies \Delta\zeta_{min}^{LK} \geq \Delta\zeta_{min}^{LK}$ and $\Delta\zeta_{max}^{LC} \geq \Delta\zeta_{max}^{LK}$ $\forall i = 1, 2, ..., n$

let $f(a, b, c) = \frac{2-a-b}{2+c-b}$
Differentiate f partially w.r.t a, b, c. $\frac{\partial f}{\partial a} = \frac{-1}{2+c-b} < 0, \frac{\partial f}{\partial b} = \frac{-c-a}{(2+c-b)^2} \leq 0, \frac{\partial f}{\partial c} = \frac{-2+a+b}{(2+c-b)^2} < 0$
Since all the partial derivatives of f are negative for a,b and c, so f is monotonically decreasing.
Let $t = (\Delta\varrho_i^{LJ}, \Delta\varrho_{min}^{LJ}, \Delta\varrho_{max}^{LJ})$ and $r = (\Delta\varphi_i^{LJ}, \Delta\varphi_{min}^{LJ}, \Delta\varphi_{max}^{LJ})$, Then $t \geq r$, $\implies f_{(at\ \ t)} \leq f_{(at\ \ r)}$
$\alpha_i^{LJ} \leq \alpha_i^{LK}and\beta_i^{LJ} \leq \beta_i^{LK}$ and$\gamma_i^{LJ} \leq \gamma_i^{LK}$
Then $l_C(L, J) \leq l_C(L, K)$
Similarly $l_C(L, J) \leq l_C(K, J)$ can also be proved.

3.2 Mathematical Properties

Preposition 1
Let $l_C(L, K)$ is the similarity measure between two IFSs L and K. Then, it satisfies the following properties:

1. $l_C(L, K) = l_C(L^c, K^c)$
2. $l_C(L, K^c) = l_C(L^c, K)$

Proof 1:

$$l_C(L, K) = \frac{1}{3n} \sum_{i=1}^{n} [\alpha_i(1 - \Delta\varrho_i) + \beta_i(1 - \Delta\varphi_i) + \gamma_i(1 - \varphi\zeta_i)] \tag{3.3}$$

Here $L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ and $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$\Delta\varrho_i = |\varrho_L - \varrho_K|$ & $\Delta\varphi_i = |\varphi_L - \varphi_K|$ & $\Delta\zeta_i = |\zeta_L - \zeta_K|$

$$l_C(L^c, K^c) = \frac{1}{3n} \sum_{i=1}^{n} [\alpha_i(1 - \Delta\varrho_i) + \beta_i(1 - \Delta\varphi_i) + \gamma_i(1 - \varphi\zeta_i)] \tag{3.4}$$

Here $L^c = \langle \varphi_L, \varrho_L, \zeta_L \rangle$ and $K^c = \langle \varphi_K, \varrho_K, \zeta_K \rangle$
Llso, $\Delta\varrho_i = |\varphi_L - \varphi_K|$ & $\Delta\varphi_i = |\varrho_L - \varrho_K|$ & $\Delta\zeta_i = |\zeta_L - \zeta_K|$
The $\Delta\varrho_i$ and $\Delta\varphi_i$ of Eq(3.3) and Eq(3.4) are interchanged.
Similarly the value of $\Delta\varrho_{max}, \Delta\varrho_{min}$, and $\Delta\varphi_{max}, \Delta\varphi_{min}$ are also interchanged in Eq(3.3) and Eq(3.4). Therefore, $l_C(L, K) = l_C(L^c, K^c)$

Proof 2: $l_C(L, K^c) = l_C(L^c, K)$
Here $L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ and $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L^c = \langle \varphi_L, \varrho_L, \zeta_L \rangle$ and $K^c = \langle \varphi_K, \varrho_K, \zeta_K \rangle$
In L.H.S., $\Delta\varrho_i = |\varrho_L - \varphi_K|$ & $\Delta\varphi_i = |\varphi_L - \varrho_K|$ & $\Delta\zeta_i = |\zeta_L - \zeta_K|$

In R.H.S., $\Delta\varrho_i = |\varphi_L - \varrho_K|$ & $\Delta\varphi_i = |\varrho_L - \varphi_K|$ & $\Delta\zeta_i = |\zeta_L - \zeta_K|$

$\Delta\varrho_i$ and $\Delta\varphi_i$ are interchanged.
Hence $l_C(L, K^c) = l_C(L^c, K)$ is trivial.

Preposition 2
Let $l_C(L, K)$ is the similarity measure between two IFSs L and K. Then, it satisfies the following properties:

1. $l_C(L, K) = l_C(L \cap K, L \cup K)$
2. $l_C(L, L \cap K) = l_C(K, L \cup K)$
3. $l_C(L, L \cup K) = l_C(K, L \cap K)$

Proof 1:
$$l_C(L, K) = l_C(L \cap K, L \cup K) \tag{3.5}$$
Case-I Let $L \subseteq K$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle \;\; and \; K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_K, \varphi_K \rangle = K$
$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$
$$\therefore l_C(L, K) = l_C(L \cap K, L \cup K)$$
Case-II Let $K \subseteq L$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle \;\; and \; K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$
$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_K, \varphi_K \rangle = K$
$$\therefore l_C(L, K) = l_C(L \cap K, L \cup K)$$
Case-III Let L $=$ K
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle \;\; and \; K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$\varrho_L = \varrho_K \& \varphi_L = \varphi_K$
$L \cup K = L$
$L \cap K = L = K$
$$\therefore l_C(L, K) = l_C(L \cap K, L \cup K)$$

Proof 2:

$$l_C(L, L \cap K) = l_C(K, L \cup K) \tag{3.6}$$

Case-I Let $L \subseteq K$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_K, \varphi_K \rangle = K$
$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$
$l_C(L, L \cap K) = l_C(L, L) = 1$ and $l_C(K, L \cup K) = l_C(K, K) = 1$

$$\therefore l_C(L, L \cap K) = l_C(K, L \cup K)$$

Case-II Let $K \subseteq L$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$
$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_K, \varphi_K \rangle = K$

$l_C(L, L \cap K) = l_C(L, K)$ and $l_C(K, L \cup K) = l_C(K, L)$

$$\therefore l_C(L, L \cap K) = l_C(K, L \cup K)$$

Case-III Let L = K
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$\varrho_L = \varrho_K \& \varphi_L = \varphi_K$
$L \cup K = L = K$
$L \cap K = L = K$
$l_C(L, L \cap K) = l_C(L, L) = 1$ and $l_C(K, L \cup K) = l_C(K, K) = 1$

$$\therefore l_C(L, L \cap K) = l_C(K, L \cup K)$$

Proof 3:

$$l_C(L, L \cup K) = l_C(K, L \cap K) \tag{3.7}$$

Case-I Let $L \subseteq K$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_K, \varphi_K \rangle = K$
$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$
$l_C(L, L \cup K) = l_C(L, K)$ and $l_C(K, L \cap K) = l_C(K, L)$

$$\therefore l_C(L, L \cup K) = l_C(K, L \cap K)$$

Case-II Let $K \subseteq L$
$L = \langle \varrho_L, \varphi_L, \zeta_L \rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K \rangle$
$L \cup K = \{\langle t, max(\varrho_L, \varrho_K), min(\varphi_L, \varphi_K) \rangle\} = \langle t, \varrho_L, \varphi_L \rangle = L$

$L \cap K = \{\langle t, min(\varrho_L, \varrho_K), max(\varphi_L, \varphi_K)\rangle\} = \langle t, \varrho_K, \varphi_K\rangle = K$

$$l_C(L, L \cup K) = l_C(L, L) = 1 \text{ and } l_C(K, L \cap K) = l_C(K, K) = 1$$

$$\therefore l_C(L, L \cup K) = l_C(K, L \cap K)$$

Case-III Let L = K
$L = \langle \varrho_L, \varphi_L, \zeta_L\rangle$ *and* $K = \langle \varrho_K, \varphi_K, \zeta_K\rangle$
$\varrho_L = \varrho_K \quad \& \quad \varphi_L = \varphi_K, \quad L \cup K = L = K, \quad L \cap K = L = K$
$l_C(L, L \cup K) = l_C(L, L) = 1$ and $l_C(K, L \cap K) = l_C(K, K) = 1$

$$\therefore l_C(L, L \cup K) = l_C(K, L \cap K)$$

Example 2 *Let us consider an IFSs* $A = \{\langle \lambda, 1 - \lambda\rangle\}$ *where* $\lambda \epsilon [0, 1]$ *in* $T = \{t\}$. *The similarity measure of this IFS from the three IFSs* $A_1 = \{\langle 1, 0\rangle\}, A_2 = \{\langle 0, 1\rangle\}$, *and* $A_3 = \{\langle 0.5, 0.5\rangle\}$ *is shown in Figure 3.1.*

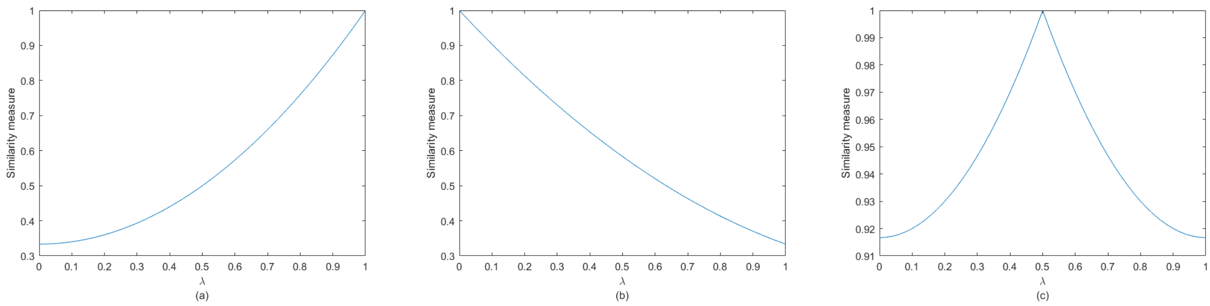

Figure 3.1: Behaviour of the purposed similarity measure $l_C(A, A_i)$ with variation in λ, where (a), (b), (c) represent the similarity $l_C(A, A_1), l_C(A, A_2), l_C(A, A_3)$ respectively.

Chapter 4

Applications

4.1 Pattern Recognition

Similarity measures for IFSs are being continously used in pattern classification problems (Wang and Xin,2005[33]; Mitchell, 2003[22]; Liang and Shi, 2003[17]; Hatzimichailidis et al.,2012[13]; Luo and Ren, 2016[19]; Xiao, 2019[35]; Park et al.,2007[25]; Song et al.,2015[28]). Consequently, the preceding example demonstrates the suggested similarity measure's relevance to pattern recognition issues. Let $T = \{t_1, t_2, ...t_n\}$ is an universe of discourse (UOD) and there are m numbers of patterns $\Gamma_1, \Gamma_2, \Gamma_3, ...\Gamma_m$ given as IFS and a test sample Ω. The following steps are followed to identify the unknown pattern.
(Step 1): Compare the similarity between each pattern Γ_i and Ω by using Equation (1).
(Step 2): Find maximum k^* of the similarity measure value by the following equation:

$$k^* = arg\max_i\{Sim(\Gamma_i, \Omega)\}$$

(Step 3): The pattern (Γ_k) for which the similarity value k^* is maximum will be selected pattern.

Example 3 *Consider three given patterns $A_1, A_2 and A_3$ denoted by IFSs in $T = t_1, t_2, t_3$, and they were used in Chen et al. (2016a,b)[6], Dengfeng and Chuntian (2002)[9], Hung and Yang (2008)[15], Nguyen (2016a,b)[23] and Ye (2011)[36] . The three patterns are given as follows:*
$A_1 = \{\langle t_1, 1.0, 0.0\rangle, \langle t_2, 0.8, 0.0\rangle, \langle t_3, 0.7, 0.1\rangle\}$
$A_2 = \{\langle t_1, 0.8, 0.1\rangle, \langle t_2, 1.0, 0.0\rangle, \langle t_3, 0.9, 0.0\rangle\}$
$A_3 = \{\langle t_1, 0.6, 0.2\rangle, \langle t_2, 0.8, 0.0\rangle, \langle t_3, 1.0, 0.0\rangle\}$

IFS B is the sample as:
$B = \{\langle t_1, 0.5, 0.3\rangle, \langle t_2, 0.6, 0.2\rangle, \langle t_3, 0.8, 0.1\rangle\}$

Table 4.1: **Relative comparison of similarity between IFSs**

Measures	$S(A_1, B)$	$S(A_2, B)$	$S(A_3, B)$	*Results*
S_C	**0.7833**	**0.7833**	0.8500	A_3
S_{HK}	**0.7833**	**0.7833**	0.8500	A_3
S_M	**0.7833**	**0.7833**	0.8500	A_3
S_L	0.7547	0.7630	0.8423	A_3
S_S	0.8868	0.9134	0.9361	A_3
S_{GK}	0.7644	0.7622	0.8378	A_3
l	0.7326	0.7185	0.8099	A_3

4.2 Software Quality Evaluation

(Step 1): Create a decision matrix using the IFSs of attributes and criteria. The intuitionistic decision matrix is given by $D = [a_{ij}]m \times n$, where $aij = \langle \zeta_{ij}, \eta_{ij} \rangle$.
(Step 2): Find the Positive Ideal Solution $(PIS)S^+$ and Negative Ideal Solution $(NIS)S^-$ using the following formulas.

$$S^+ = \bigcup_{j=1}^{m} S_i = \{\langle max\{\zeta_{S_i}(t_j)\}, min\{\eta_{S_i}(t_j)\}\rangle | t_j \in T\} \tag{4.1}$$

$$S^- = \bigcap_{j=1}^{m} S_i = \{\langle min\{\zeta_{S_i}(t_j)\}, max\{\eta_{S_i}(t_j)\}\rangle | t_j \in T\} \tag{4.2}$$

(Step 3): Calculate the similarity between each alternative S_i and $PIS(S^+)$ and $NIS(S^-)$ by using Equation (3.1).
(Step 4): Calculate the closeness coefficient by the given equation.

$$\Phi_{l_C} = \frac{l_C(S_i, S^+)}{l_C(S_i, S^+) + l_C(S_i, S^-)} \tag{4.3}$$

(Step 5): A larger closeness coefficient indicates that an option is closer to the PIS while being further from the NIS. The descending order of closeness is used to decide the rank.

4.2.1 Implementation of the purposed algorithm

Several models (Thao and Chou,2021[32]; Li et al.,2014[16]; Chang et al.,2008[4]) have been purposed using the different theories of the fuzzy set to evaluate the quality of the software. Increasing demand for high-quality software necessities for evaluation of software quality in today's digital world. In this part, we provide a framework for assessing software quality based on the suggested similarity measure. For quality assessment, we have five different types of software available, and 13 attributes are used for quality assessment. A new standard for evaluating system and software quality identifier was established by ISO 25010 in 2011. The following important quality identifiers are verified and included in this standard. The set Q = Financial Sustainability (Q_1), Functional Correctness (Q_2), Testablility (Q_3), Performance efficiency (Q_4), Compatibility (Q_5), Usability (Q_6), Appropriateness Recognizability (Q_7), User interface Aesthetics (Q_8), Reliability (Q_9), Security (Q_{10}), Maintainability (Q_{11}), Modifiability (Q_{12}), Portability (Q_{13}) is collection of attributes and the set S = S_1, S_2, S_3, S_4, S_5 is collection of alternatives. The intuitionistic fuzzy decision matrix is given in the Table 9. Calculate

PIS S^+ and NIS S^- by using Equation (4.1) and (4.2).

$S^+ = \langle$ 0.81,0.04$\rangle$, $\langle$ 0.81,0.01$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 1,0$\rangle$, $\langle$ 0.81,0.04$\rangle$, $\langle$ 1,0$\rangle$. $\langle$ 1,0$\rangle$

$S^- = \langle$0.25,0.25$\rangle$, $\langle$0.6,0.16$\rangle$, $\langle$0.49,0.19$\rangle$, $\langle$0.8,0.1$\rangle$, $\langle$0.25,0.3$\rangle$, $\langle$0.25,0.4$\rangle$, $\langle$0.16,0.4$\rangle$, $\langle$0.49,0.19$\rangle$, $\langle$0.25,0.3$\rangle$,$\langle$0.16,0.4$\rangle$,$\langle$0.25,0.25$\rangle$, $\langle$0.25,0.4$\rangle$, $\langle$0.25,0.4$\rangle$

Table 4.2: **Software Vs Attributes of as IFSs**

Attributes	S_1	S_2	S_3	S_4	S_5
Financial Sustainability (Q_1)	$\langle 0.49, 0.1\rangle$	$\langle 0.6, 0.04\rangle$	$\langle 0.36, 0.04\rangle$	$\langle 0.81, 0.05\rangle$	$\langle 0.25, 0.25\rangle$
Functional Correctness (Q_2)	$\langle 0.7, 0.16\rangle$	$\langle 0.8, 0.01\rangle$	$\langle 0.73, 0.03\rangle$	$\langle 0.6, 0.11\rangle$	$\langle 0.81, 0.05\rangle$
Testablility (Q_3)	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.01\rangle$	$\langle 1, 0\rangle$	$\langle 0.49, 0.19\rangle$	$\langle 0.64, 0.1\rangle$
Performance efficiency (Q_4)	$\langle 0.81, 0.05\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 1, 0\rangle$	$\langle 0.8, 0.01\rangle$	$\langle 0.81, 0.05\rangle$
Compatibility (Q_5)	$\langle 1, 0\rangle$	$\langle 0.25, 0.1\rangle$	$\langle 1, 0\rangle$	$\langle 0.25, 0.3\rangle$	$\langle 0.25, 0.3\rangle$
Usability (Q_6)	$\langle 0.25, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 0.25, 0.16\rangle$	$\langle 0.25, 0.4\rangle$
Appropriateness Recognizability (Q_7)	$\langle 0.25, 0.4\rangle$	$\langle 0.6, 0.04\rangle$	$\langle 1, 0\rangle$	$\langle 0.16, 0.18\rangle$	$\langle 0.81, 0.05\rangle$
User interface Aesthetics (Q_8)	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0\rangle$	$\langle 1, 0\rangle$	$\langle 0.49, 0.19\rangle$	$\langle 0.64, 0.1\rangle$
Reliability (Q_9)	$\langle 1, 0\rangle$	$\langle 0.25, 0.1\rangle$	$\langle 1, 0\rangle$	$\langle 0.25, 0.3\rangle$	$\langle 0.25, 0.3\rangle$
Security (Q_{10})	$\langle 1, 0\rangle$	$\langle 0.6, 0.04\rangle$	$\langle 1, 0\rangle$	$\langle 0.16, 0.176\rangle$	$\langle 0.81, 0.05\rangle$
Maintainability (Q_{11})	$\langle 0.49, 0.1\rangle$	$\langle 0.6, 0.04\rangle$	$\langle 0.36, 0.04\rangle$	$\langle 0.81, 0.05\rangle$	$\langle 0.25, 0.25\rangle$
Modifiability (Q_{12})	$\langle 0.25, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 0.25, 0.16\rangle$	$\langle 0.25, 0.4\rangle$
Portability (Q_{13})	$\langle 0.3, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 1, 0\rangle$	$\langle 0.25, 0.16\rangle$	$\langle 0.25, 0.4\rangle$

Table 4.3: **Ranking based on correlation coefficient.**

Software	$l_C(S_i, S^+)$	$l_C(S_i, S^-)$	Φ_{l_C}	**Rank**
S_1	0.6438	0.7813	0.4518	3
S_2	0.7667	0.6203	0.5528	2
S_3	0.9310	0.4830	0.6584	1
S_4	0.5307	0.8242	0.3917	4
S_5	0.5536	0.8659	0.3900	5

4.3 Medical Diagnosis

Predicting the disease that a patient is suffering from is the most common problem that can be solved implementing the similarity measures in Intuitionistic fuzzy sets. We will use the proposed similarity measures to calculate similarity and obtain a similar result to the other existing similarity(distance).
Let patients are represented by set P={Riya, Suresh, Kasus, Gautam} and symptoms are collected in set S={Temperature, Headache, Stomach Pain, Cough, Chest pain} and D={Viral fever, Malaria, Typhoid, Stomach problem, Chest problem} is the diagnosis set as represented in Table 4.4 and Table 4.6. The numerical values related to this medical diagnosis problem have been collected from [21], [10].

Table 4.4: **Symptoms Vs patients**

Patient	**Temperature**	**Headache**	**Stomach pain**	**Cough**	**Chest pain**
Riya	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.1, 0.6\rangle$
Suresh	$\langle 0.0, 0.8\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.1, 0.8\rangle$
Kasus	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.0, 0.6\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.0, 0.5\rangle$
Gautam	$\langle 0.6, 0.1\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.3, 0.4\rangle$

Table 4.5: **Symptoms Vs Diseases**

Disease	**Temperature**	**Headache**	**Stomach pain**	**Cough**	**Chest pain**
Viral fever (Vf)	$\langle 0.4, 0.0\rangle$	$\langle 0.3, 0.5\rangle$	$\langle 0.1, 0.7\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.1, 0.7\rangle$
Malaria (Ma)	$\langle 0.7, 0.0\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.0, 0.9\rangle$	$\langle 0.7, 0.0\rangle$	$\langle 0.1, 0.8\rangle$
Typhoid (Ty)	$\langle 0.3, 0.3\rangle$	$\langle 0.6, 0.1\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0.1, 0.9\rangle$
Stomach problem (Sp)	$\langle 0.1, 0.7\rangle$	$\langle 0.2, 0.4\rangle$	$\langle 0.8, 0.0\rangle$	$\langle 0.2, 0.7\rangle$	$\langle 0.2, 0.7\rangle$
Chest problem (Cp)	$\langle 0.1, 0.8\rangle$	$\langle 0.0, 0.8\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.8, 0.1\rangle$

In the following bar diagram, in each of the four cases, the bar with highest height denotes the proper detection of disease for a that person, and Table 4.7

Table 4.6: **Similarity measure of Disease of Patients**

Disease	Riya	Suresh	Kasus	Gautam
Viral fever (Vf)	0.7156	0.6195	0.5979	0.6761
Malaria (Ma)	0.7523	0.5722	0.5652	0.7001
Typhoid (Ty)	0.7365	0.6848	0.6798	0.5882
Stomach problem (Sp)	0.4787	0.8488	0.5542	0.5837
Chest problem (Cp)	0.5173	0.6064	0.5037	0.4610

shows a result comparison between the efficiency of our proposed similarity with the existing. Diagnosis demonstration is consolidated by the similarities of result in the postulated measure of similarity to the existing ones, per patient wise.

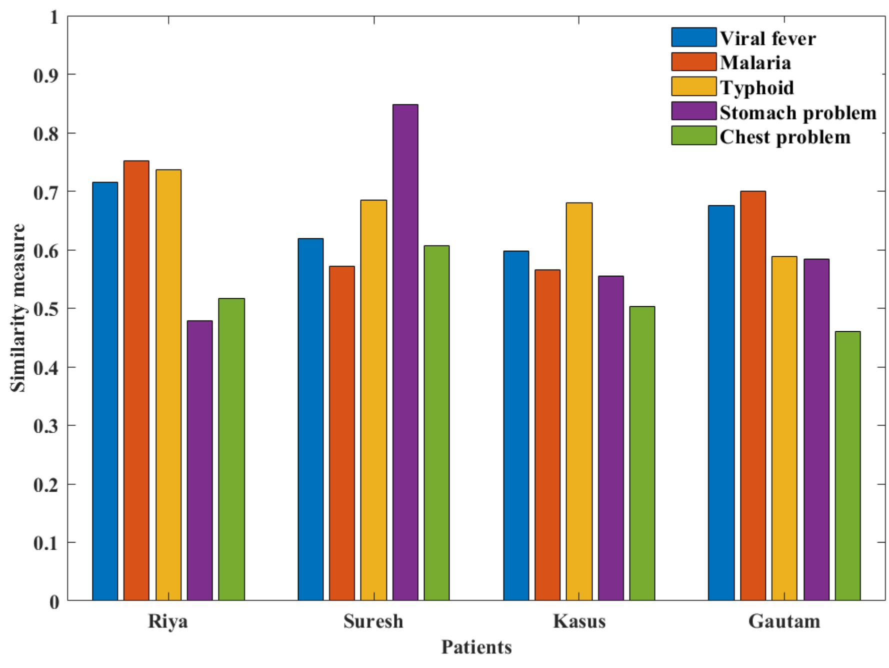

Figure 4.1: Relative Comparison of Medical Diagnosis

The maximum similarity gives us the proper detection of malaria in Riya. Suresh has pain in the stomach. Kasus is identified with Typhoid. Gautam suffers from viral fever which is compared with others prediction and thus obtaining similar result.

Table 4.7: **Relative comparison of prediction of diseases**

Patient	**Our Prediction**	**Others' prediction**
Riya	Malaria	Malaria[20],[30],[8],[34],[26] and viral fever[31]
Suresh	Stomach problem	Stomach problem[20],[30]−[26]
Kasus	Typhoid	Typhoid[7, 8, 20, 26, 30] and malaria[8]
Gautam	Malaria	Viral fever[30],[34]−[26] and malaria[20],[31]−[24]

Chapter 5

Conclusions and Future Scopes

This thesis summarizes the basic concepts of similarity between two IFSs and some basic operations on IFSs. We have concluded that IFSs are suitable for providing a flexible model to explain uncertain facts involving uncertain boundary. The notion of IFS also helps to solve various applications. In this thesis, a new similarity measure based on membership, non-membership, and hesitancy degrees has been proposed. It has been shown that this new similarity measure fulfills all the properties mentioned in the axiomatic definition of similarity measure. It is observed that this proposed similarity measure can detect a slight change either in membership degree or in non-membership degree or hesitancy degree. We have also applied this similarity measure in pattern recognition, career determination, and medical diagnosis.

This measure can further be applied in face recognition and various other emerging field like natural language processing (NLP) in machine learning.

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