1 Question 1

Solve the ODE system as follows

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -x + y, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -x - y$$

$$x = -\frac{\mathrm{d}y}{\mathrm{d}t} - y$$
$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}y}{\mathrm{d}^2t} - \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t} + 2y$$

$$\frac{\mathrm{d}y}{\mathrm{d}^2t} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 2y = 0$$
$$r^2 + 2r + 2 = 0$$
$$r_1 = -1 + i, \quad r_2 = -1 - i$$
$$\alpha = -1, \quad \beta = 1$$
$$y = e^{\alpha t}(C_1 \cos(\beta t) + C_2 \sin(\beta t))$$
$$y = e^{-t}(C_1 \cos(t) + C_2 \sin(t))$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = e^{-t}(C_1(-\sin(t) - \cos(t)) + C_2(\cos(t) - \sin(t)))$$
$$x = -\frac{\mathrm{d}y}{\mathrm{d}t} - y = e^{-t}(C_1 \sin(t) - C_2 \cos(t))$$

$$C_1 = 1$$
$$C_2 = 0$$
$$x(t) = e^{-t} \sin(t)$$
$$y(t) = e^{-t} \cos(t)$$

2 Question 2

Try the MATLAB ODE solver by implementing the three numerical examples in the lecture note.

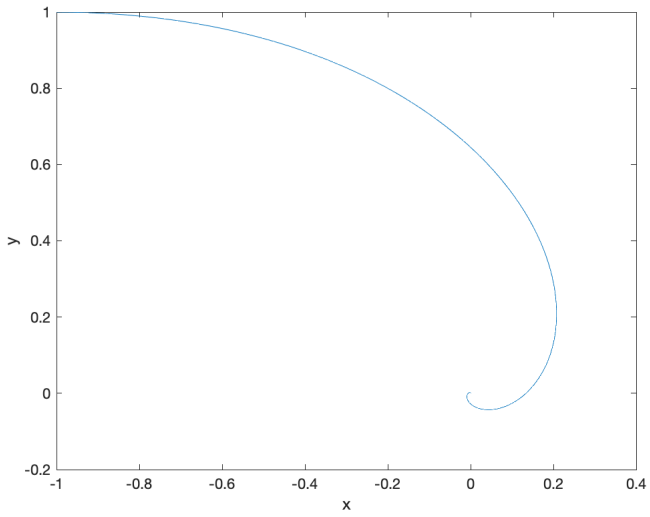
2.1 Example model 1

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -x + y, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -x - y$$

Let $x_0 = 0, y_0 = 1, t_0 = 0, t_e = 1000$:

```
function dydt = m1func(t,Y)
    x = Y(1);y = Y(2);
    dydt = [- x + y;
            - x - y];

[t xy]=ode45(@m1func,[0:0.01:1000],[0,1]);
x=xy(:,1);y=xy(:,2);
figure(1); plot(x,y);xlabel("x");ylabel("y");
```



2.2 Example model 2

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax - bxy, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = my - nxy$$

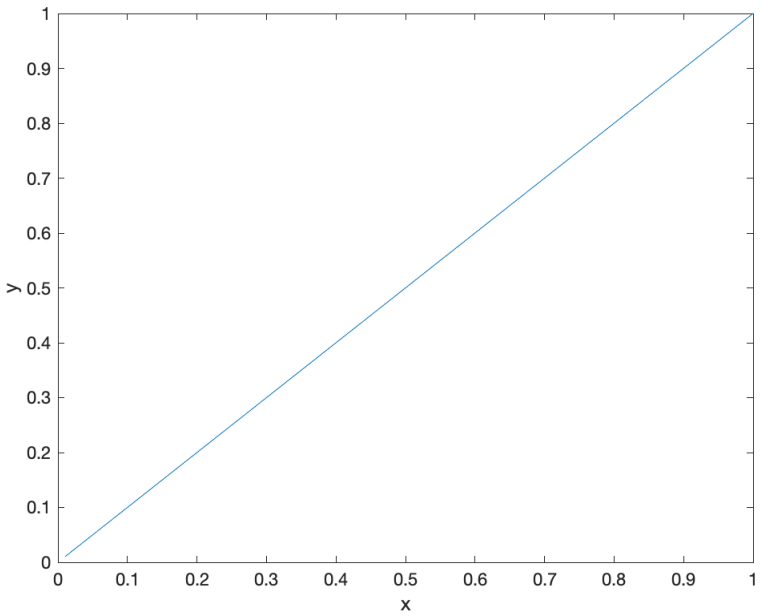
Let $a = 1, b = 100, m = 1, n = 100,$

$x_0 = 1, y_0 = 1, t_0 = 0, t_e = 10000$:

```
function dydt = m2func(t,Y)
    a = 1;b = 100;m = 1;n = 100;
    x = Y(1);y = Y(2);
    dydt = [a * x - b * x * y;
            m * y - n * x * y];

[t xy]=ode45(@m2func,[0:10000],[1,1]);

x=xy(:,1);y=xy(:,2);
figure(1); plot(x,y);xlabel("x");ylabel("y");
```



2.3 Example model 3

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -ax + by + c, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = mx - ny + p$$

Let $a = 1, b = 1, c = 1, m = 1, n = 1, p = 1$

$x_0 = 1, y_0 = 1, t_0 = 0, t_e = 10000$:

```
function dydt = m3func(t,Y)
    a = 1;b = 1;c = 1;m = 1;n = 1;p = 1;
    x = Y(1);y = Y(2);
    dydt = [- a * x + b * y + c;
            m * x - n * y + p];

[t xy]=ode45(@m3func,[0:10000],[1,1]);

x=xy(:,1);y=xy(:,2);
figure(1); plot(x,y);xlabel("x");ylabel("y");
```