1 Question 1

For the data sets in Problems 1–4, construct a divided difference table. Is smoothing with a low-order polynomial appropriate? If so, choose an appropriate polynomial and fit using the least-squares criterion of best fit. Analyze the goodness of fit by examining appropriate indicators and graphing the model, the data points, and the deviations.

1.

x	0	1	2	3	4	5	6	7
y	2	8	24	56	110	192	308	464

Code Pasting Area

```
x = [0, 1, 2, 3, 4, 5, 6, 7];
y = [2, 8, 24, 56, 110, 192, 308, 464];
```

2.

x	0	1	2	3	4	5	6	7
y	23	48	73	98	123	148	173	198

Code Pasting Area

```
x = [0, 1, 2, 3, 4, 5, 6, 7];
y = [23, 48, 73, 98, 123, 148, 173, 198];
```

3.

x	0	1	2	3	4	5	6	7
y	7	15	33	61	99	147	205	273

Code Pasting Area

```
x = [0, 1, 2, 3, 4, 5, 6, 7];
y = [7, 15, 33, 61, 99, 147, 205, 273];
```

4.

x	0	1	2	3	4	5	6	7
y	1	4.5	20	90	403	1808	8103	36316

Code Pasting Area

```
x = [0, 1, 2, 3, 4, 5, 6, 7];
y = [1, 4.5, 20, 90, 403, 1808, 8103, 36316];
```

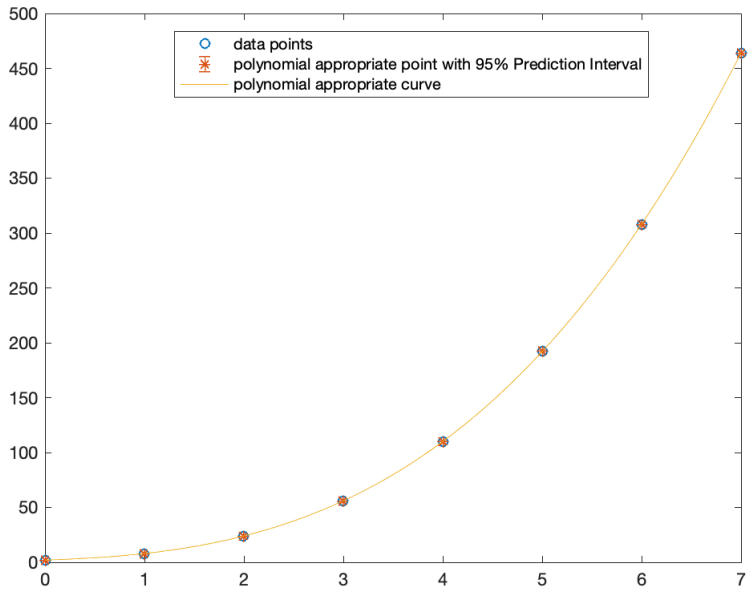
1.1 Problem 1

x	0	1	2	3	4	5	6	7
y	2	8	24	56	110	192	308	464

The divided difference table for data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	2				
		6			
1	8		5		
		16		1	
2	24		8		0
		32		1	
3	56		11		0
		54		1	
4	110		14		0
		82		1	
5	192		17		0
		116		1	
6	308		20		
		156			
7	464				

So I chose the 3-order polynomial to fit this data set:



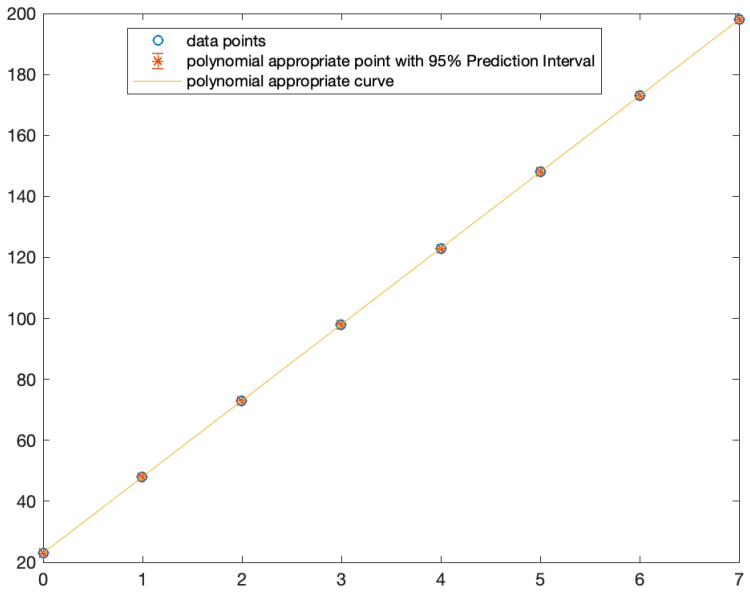
1.2 Problem 2

x	0	1	2	3	4	5	6	7
y	23	48	73	98	123	148	173	198

The divided difference table for data:

x	y	Δy	$\Delta^2 y$
0	23		
		25	
1	48		0
		25	
2	73		0
		25	
3	98		0
		25	
4	123		0
		25	
5	148		0
		25	
6	173		0
		25	
7	198		

So I chose the 1-order polynomial to fit this data set :



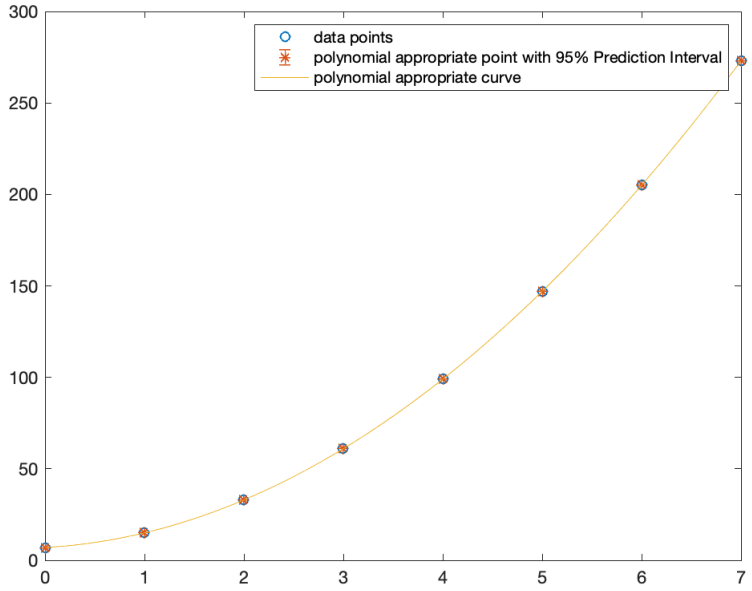
1.3 Problem 3

x	0	1	2	3	4	5	6	7
y	7	15	33	61	99	147	205	273

The divided difference table for data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	7			
		8		
1	15		5	
		18		0
2	33		5	
		28		0
3	61		5	
		38		0
4	99		5	
		48		0
5	147		5	
		58		0
6	205		5	
		68		
7	273			

So I chose the 2-order polynomial to fit this data set :



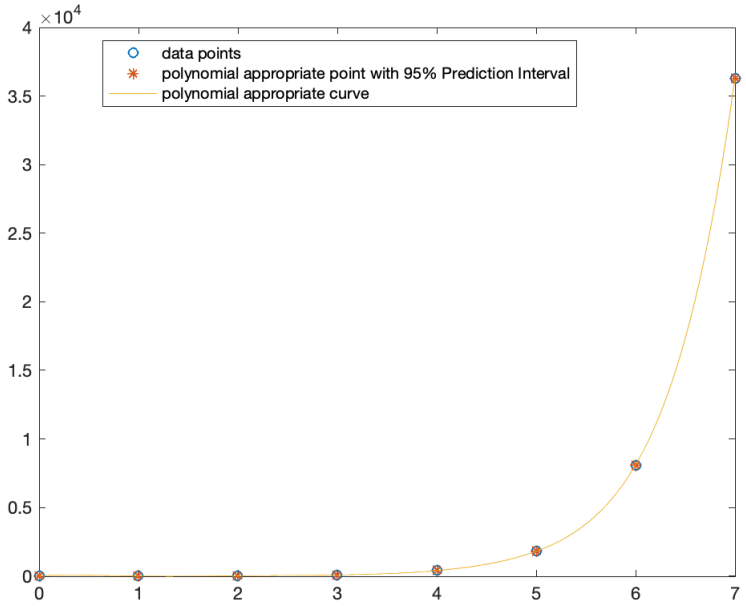
1.4 Problem 4

x	0	1	2	3	4	5	6	7
y	1	4.5	20	90	403	1808	8103	36316

The divided difference table for data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$
0	1							
		3.5						
1	4.5		6					
		15.5		7.0833				
2	20		27.25		6.0833			
		70		31.417		4.2875		
3	90		121.5		27.521		2.4639	
		313		141.5		19.071		1.2338
4	403		546		122.87		11.101	
		1405		633		85.675		
5	1808		2445		551.25			
		6295		2838				
6	8103		10959					
		28213						
7	36316							

So I chose the 7-order polynomial to fit this data set :



2 Question 2

In the following data, X represents the diameter of a ponderosa pine measured at breast height, and Y is a measure of volume—number of board feet divided by 10. Construct a scatterplot of the given data. Is there a trend in the data? Are any of the data points outliers? Construct a divided difference table. Is smoothing with a low-order polynomial appropriate? If so, choose an appropriate polynomial and fit using the least-squares criterion of best fit. Analyze the goodness of fitt by examining appropriate indicators and graphing the model, the data points, and the deviations.

X	17	19	20	22	23	25	31	32	33	36	37	38	39	41
Y	19	25	32	51	57	71	141	123	187	192	205	252	248	294

Remove the outliers form the data set:

X	17	19	20	22	23	25	31	36	37	39	41
Y	19	25	32	51	57	71	141	192	205	248	294

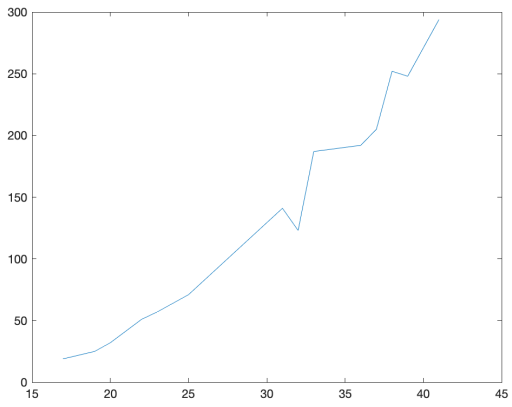


Figure 1: Before

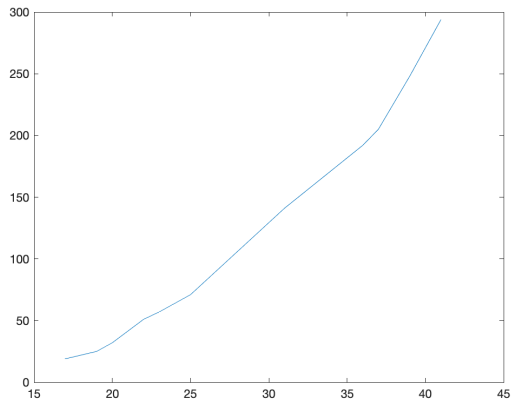


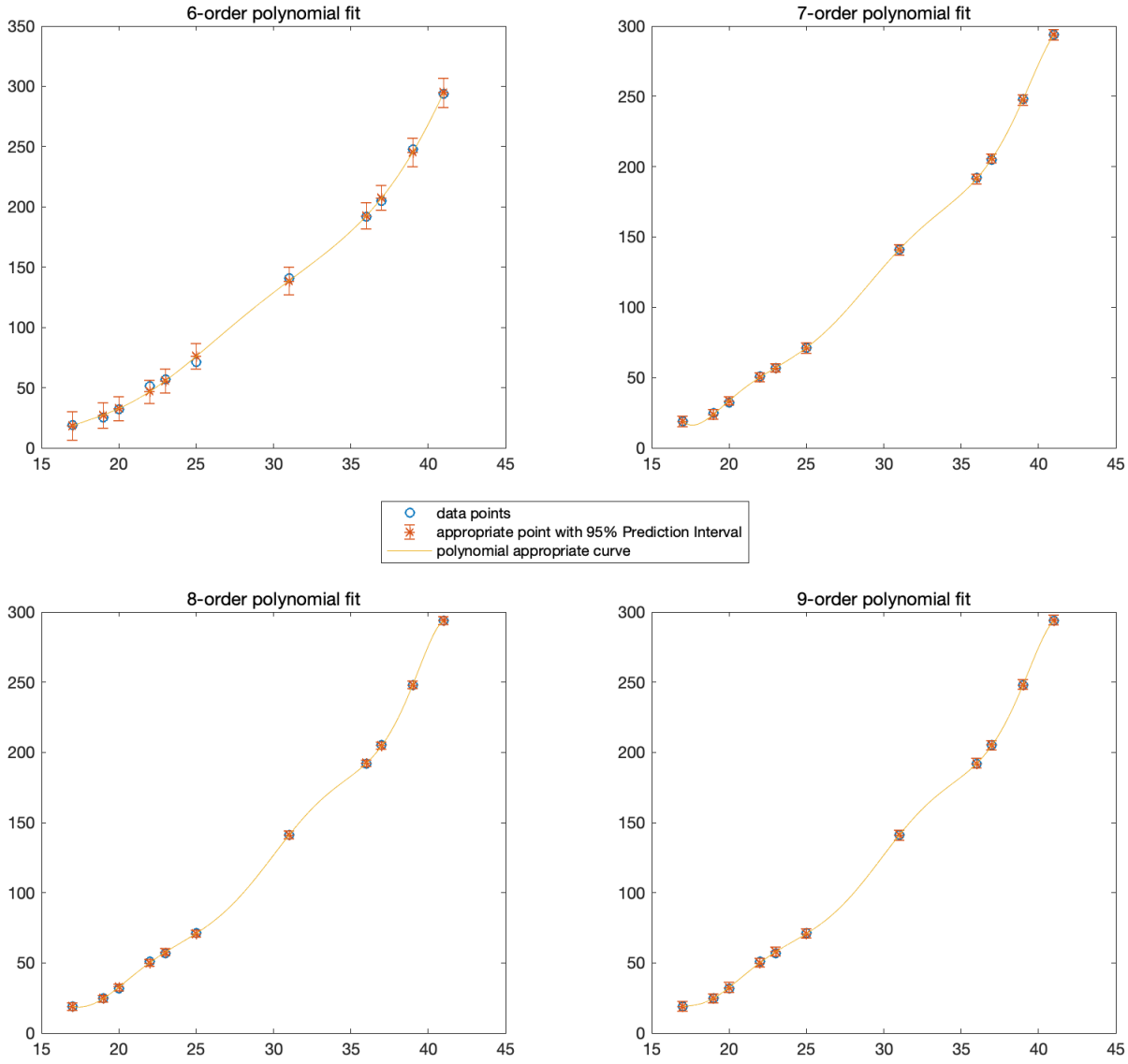
Figure 2: After

The divided difference table for data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$
17	19							
		3						
19	25		1.3333					
		7		-0.1				
20	32		0.8333		-0.0667			
		9.5		-0.5		0.025		
22	51		-1.1667		0.1333		-0.0027	
		6		0.3		-0.0132		$0.1879*10^{-3}$
23	57		0.3333		-0.0247		0.0008	
		7		0.0278		0.0012		$-0.0478*10^{-3}$
25	71		0.5833		-0.0059		-0	
		11.667		-0.0551		0.0009		0
31	141		-0.1333		0.0075		-0	
		10.2		0.05		0.0006		$-0.0186*10^{-3}$
36	192		0.4667		0.0176		-0.0004	
		13		0.2958		-0.006		
37	205		2.8333		-0.0788			
		21.5		-0.4917				
39	248		0.375					
		23						
41	294							

$\Delta^8 y$	$\Delta^9 y$	$\Delta^{10} y$
$-0.1179*10^{-4}$		
	$0.6446*10^{-6}$	
$0.0239*10^{-4}$		$-3.3071*10^{-8}$
	$-0.1491*10^{-6}$	
$-0.0089*10^{-4}$		

So I chose the 6-order, 7-order, 8-order and 9-order polynomial to fit this data set :



It is found that the 7-order polynomial is enough to fit this data set.