Lecture 10

Modeling with ODEs

Modeling with differential equations

- Modeling strategy:
  - We model the change in the values of all variables:
  - We describe the change in the values of all variables:
  - We describe the change as a function of the current values of all variables:
  - If the process takes place continuously in time, it leads to differential equations

Modeling with differential equations

- Says that we have information about the change of our system in the form $\frac{dx_1}{dt} = f_1(x_1, \dots, x_n)$, $\frac{dx_2}{dt} = f_2(x_1, \dots, x_n)$, $\dots$, $\frac{dx_n}{dt} = f_n(x_1, \dots, x_n)$
  - Discrete time leads to difference equations (iterated maps)
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- If the time is continuous: 3 series
  - Continuous time leads to:
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  - Euler approximation: average rate of change of f: $\frac{f(x+h)-f(x)}{h}$
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  - Allow dt to approach 0: $df/dx=df$
    - instantaneous rate of change
    - instantaneous rate of change

Figure 1: Rate of change in a discrete time system

Function of a 1-D input is the rate of change of the function output with respect to time

$P_{n+1} = rP_n$
$\text{Rate}_n^{\text{dec}} = P_{n+1} - P_n$
$P_{n+1} = P_n + \text{Rate}_n^{\text{dec}}$

Modeling with differential equations

- The use of differential equations
  - instantaneous rate of change in continuous problems
  - approximate an average rate of change in discrete problems
    - with discrete time we may be around a functional relationship even in seemingly discrete system

$\dot{x}$ $\dot{y}$

The derivative as a rate of change

- Development of calculus was triggered by the study of motion (17th and 17th century)
  - Galileo studied how x_0 at time t_0 is x_1 at time t_1
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  - Galileo: $x_1 = x_0 + v(t_1 - t_0)$
  - Galileo: $x_1 = x_0 + v(t_1 - t_0)$
- If $x(t)$ is a differentiable function, then the derivative $\dot{x}(t)$ at any point can be interpreted as the instantaneous rate of change of x with respect to t at that point
  - $\dot{x}(t) = \lim_{h \to 0} \frac{x(t+h)-x(t)}{h}$

Figure 2: Rate of change in a continuous time system

Function of a 1-D input is the rate of change of the function output with respect to time

The derivative as the slope of the tangent line

- Since $\dot{x}$ is understood the motion of interest
  - is constructing tangent lines
  - $\dot{x}$ is understood the motion of interest
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