

Part I

Single reaction

Equilibrium binding and
Cooperativity
Week 1-2,

Matlab crash course

Today's objective

Get comfortable playing with Matlab...

- Interacting with Matlab
- Enter Data
- Operations
- Some Commonly Used Functions
- Making Pretty Pictures
- M-Files and Scripts
- For, While, and If
- Solving ODEs



Useful Resources

- <http://www.greenteapress.com/matlab/>

Physical Modeling in MATLAB

by Allen B. Downey

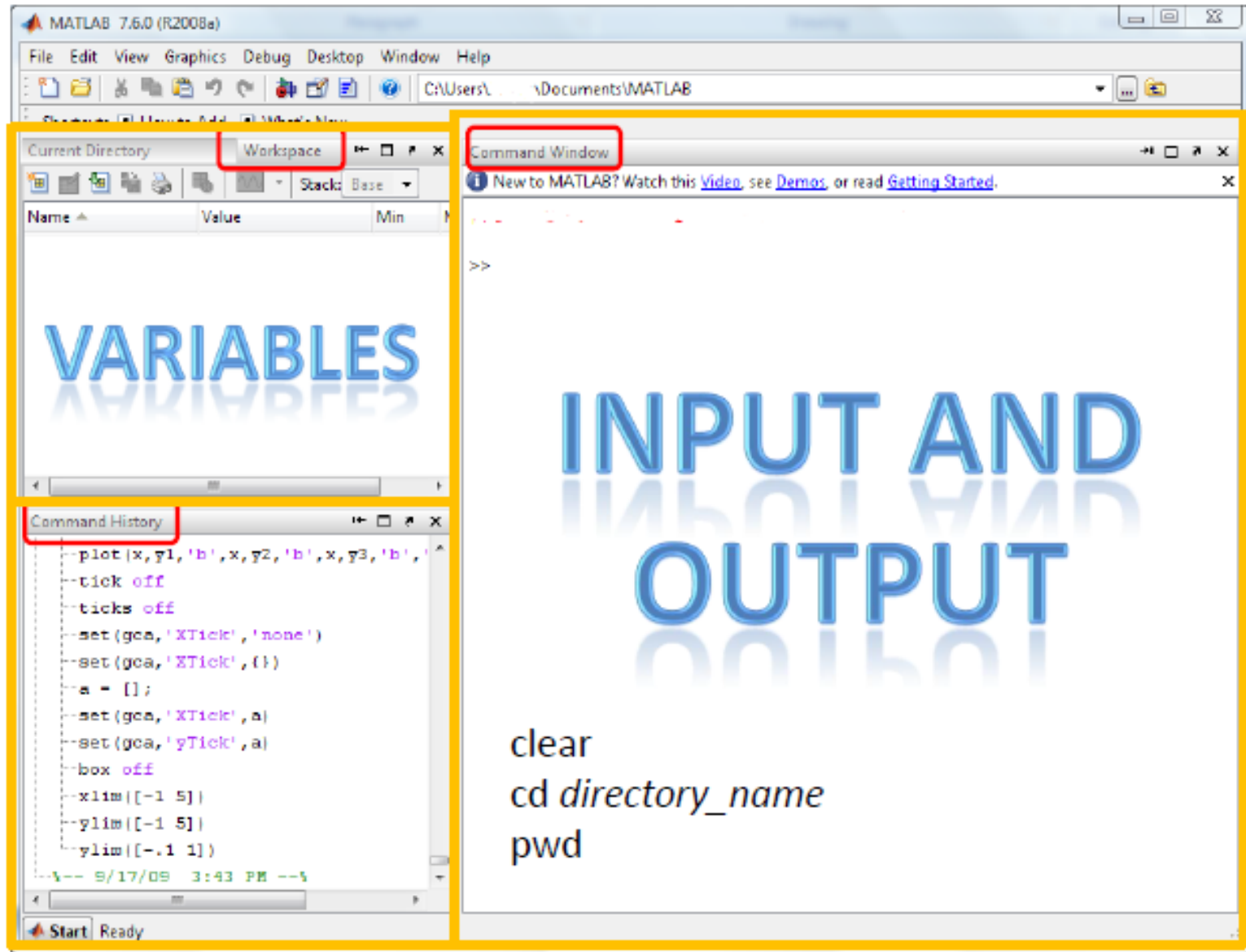
Physical Modeling in MATLAB is an introduction to programming in MATLAB and simulation of physical systems.

Download [the book in PDF](#) now, or buy the paperback edition from [Lulu.com](#) or [Amazon.com](#).



- Google
 - Search for '[matlab plot](#)'

Interacting with Matlab



Entering Data

The image shows the MATLAB 7.6.0 (R2008a) interface. The workspace window on the left lists variables A, B, ans, and t. The command window on the right shows the execution of commands to create matrices A and B, and a vector t. The command history window at the bottom left shows the sequence of commands entered. The command window also displays the output of the commands, including the matrices A and B, and the vector t. The text 'input' and 'output' are overlaid on the command window, and the text 'Semi-colon to suppress output' is overlaid on the command window. The text 'Try 'help linspace', or google 'matlab linspace'' is overlaid on the command window.

Workspace:

Name	Value	Min	Max
A	[4,1,5,6]	1	6
B	[1,2,3,4]	1	4
ans	11	11	11
t	<1x21 double>	0	10

Command Window:

```
>> S+E
ans =
    11

>> A = [4,1,5,6];
>> A
A =
     4     1     5     6

>> B = [1 2; 3 4]
B =
     1     2
     3     4

>> t = 0:0.5:10
t =
Columns 1 through 7
         0         0.5000         1.0000         1.5000         2.0000         2.5000         3.0000
Columns 8 through 14
         3.5000         4.0000         4.5000         5.0000         5.5000         6.0000         6.5000
Columns 15 through 21
         7.0000         7.5000         8.0000         8.5000         9.0000         9.5000        10.0000
```

Command History:

```
ticks off
set(gca,'XTick','none')
set(gca,'XTick',[])
a = [];
set(gca,'XTick',a)
set(gca,'YTick',a)
box off
xlim([-1 5])
ylim([-1 5])
ylim([-1 1])
9/17/09 3:43 PM --%
clc
S+E
A = [4,1,5,6];
A
B = [1 2; 3 4]
t = 0:0.5:10
```

Annotations:

- input**: Points to the command `S+E`.
- output**: Points to the result `ans = 11`.
- Semi-colon to suppress output**: Points to the semicolon in `A = [4,1,5,6];`.
- Try 'help linspace', or google 'matlab linspace'**: Points to the command `t = 0:0.5:10`.

MATLAB 7.6.0 (R2008a)

File Edit Debug Desktop Window Help

Current Directory: CAUsers\ Documents\MATLAB

Shortcuts How to Add What's New

Current Directory Workspace Stack: Base

Name	Value	Min	Max
A	[4,1,5,6]	1	6
B	[1,2;3,4]	1	4
ans	[1;3]	1	3
r	<1x400 double>	0	100
t	<1x21 double>	0	10

Command Window

New to MATLAB? Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

```
>> r = linspace(0,100,400);  
>> r(5)  
ans =  
1.0025  
  
>> r(5:10)  
ans =  
1.0025 1.2531 1.5038 1.7544 2.0050 2.2556  
  
>> B(1,2) (row, column)  
ans =  
2  
  
>> B(:,1) the 1st column of B  
ans =  
1  
3  
  
>>
```

Command History

9/17/09 3:43 PM --%
--%
--% c1c
--% S+6
--% A = [4,1,5,6];
--% A
--% B = [1 2; 3 4]
--% t = 0:0.5:10
--% c1c
--% r = linspace(0,100,400);
--% r(5)
--% r(5:10)
--% B(1,2)
--% B(:,1)

Start

OVR

MATLAB 7.6.0 (R2008a)

File Edit Debug Desktop Window Help

Current Directory: C:\Users\ Documents\MATLAB

Shortcuts How to Add What's New

Current Directory Workspace Stack: Base

Name	Value	Min	Max
a	[1,1,1,1,1]	1	1
b	[0,0,0]	0	0
c	[1,1,1,1,1,0,0,0]	0	1
d	[0,0,0,0]	0	0

Command Window

New to MATLAB? Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

```
>> a = ones(1,5)
a =
    1    1    1    1    1

>> b = zeros(1,3)
b =
    0    0    0

>> c = [a b]
c =
    1    1    1    1    1    0    0    0

>> d = zeros(2)
d =
    0    0
    0    0

>>
```

create a row vector filled with 1's

create another row vector filled with 0's

merge the two

try out: eye, rand, randn

Command History

```
clc
a = ones(1,5)
b = zeros(1,3)
c = [a b]
d = zeros(2)
```

Start OVR

Operations

The image shows the MATLAB 7.6.0 (R2008a) interface. The top menu bar includes File, Edit, Debug, Desktop, Window, and Help. The current directory is C:\Users\Documents\MATLAB. The workspace pane on the left shows variables a, ans, and b. The command window on the right displays the following code and output:

```
>> a = rand(2) % create a 2 by 2 matrix filled with random numbers from 0 to 1
a =
    0.9575    0.1576
    0.9649    0.0706

>> b = rand(2);
>> a*b % matrix multiplication
ans =
    0.9930    0.7886
    1.3997    0.9099

>> a.*b % element by element multiplication
ans =
    0.9165    0.1261
    0.4683    0.1377

>> sin(a)
ans =
    0.8178    0.1570
    0.8320    0.8252

>>
```

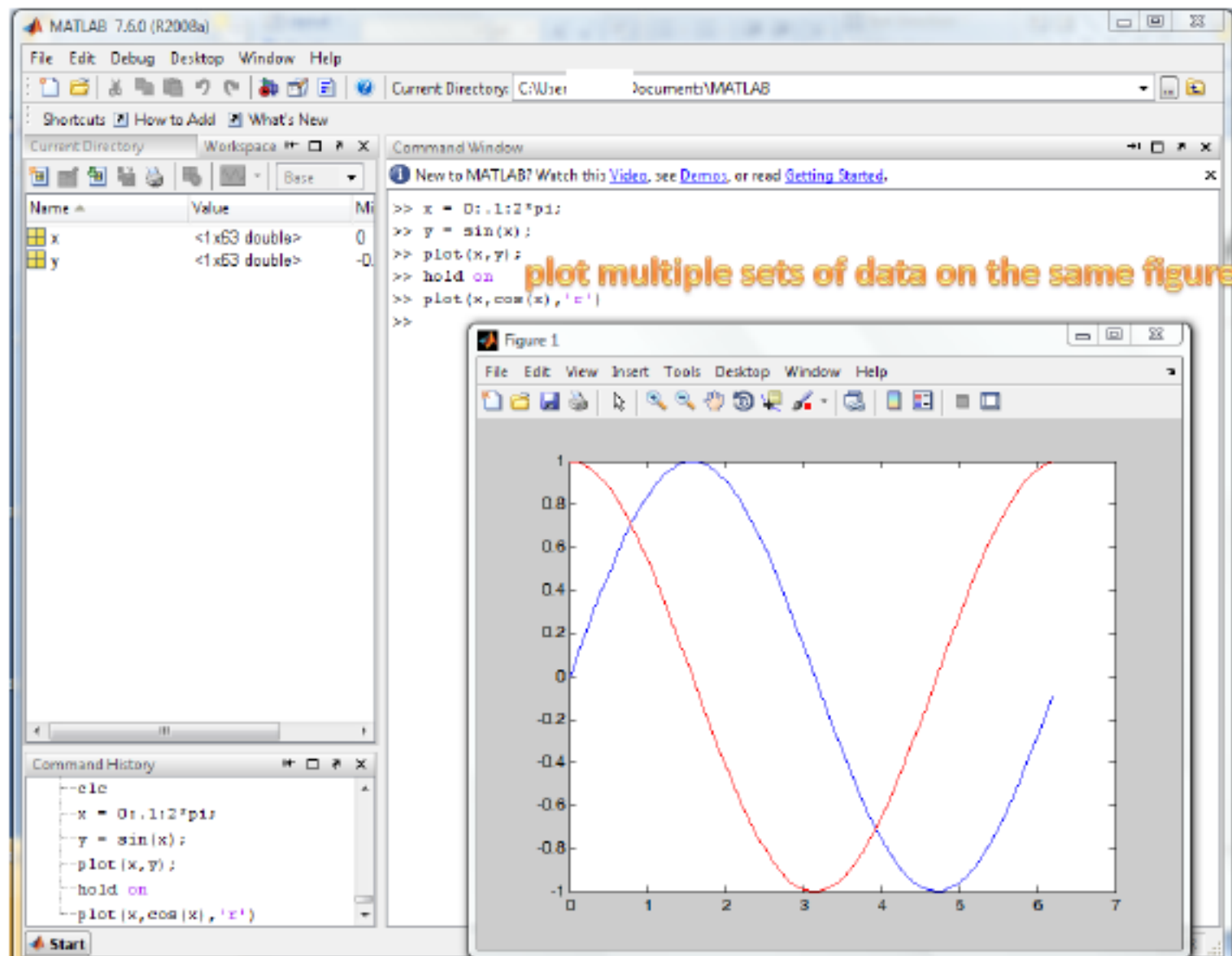
The command history pane at the bottom left shows the following commands:

```
a = rand(2) % create a 2 by
b = rand(2);
a*b % matrix multiplication
a.*b % element by element m
sin(a)
```

On the right side of the image, the following text is displayed:

You can also do:
log(a), log10(a), exp(a),
sum(a), max(a), etc.

Plotting Data / Making Pretty Pictures



Shortcuts How to Add What's New

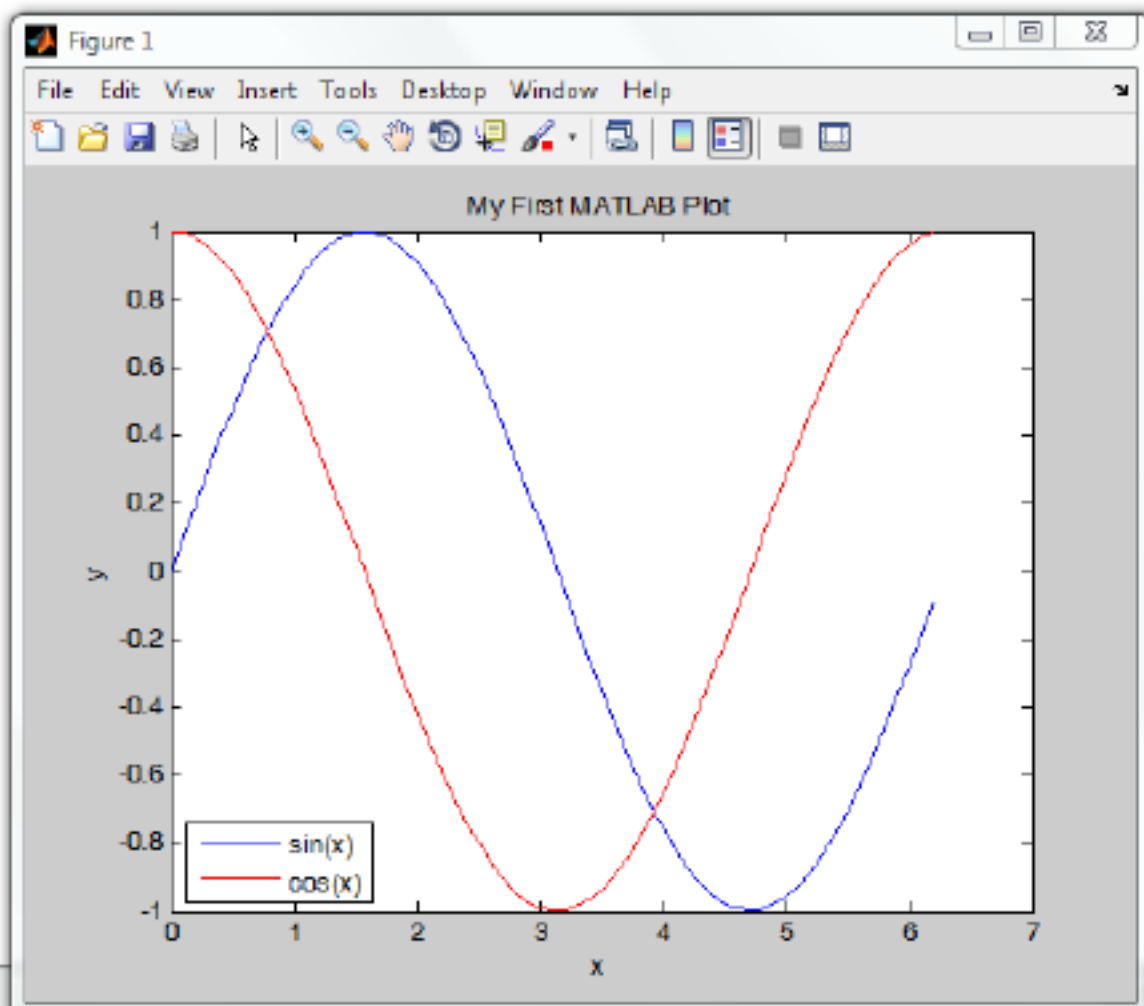
Current Directory Workspace Base

Name	Value	Memory
x	<1x63 double>	0
y	<1x63 double>	-0

Command Window

New to MATLAB? Watch this [Video](#), see [Demos](#), or read [Getting Started](#).

```
>> xlabel('x')
>> ylabel('y')
>> title('My First MATLAB Plot')
>> legend('sin(x)', 'cos(x)', 'Location', 'SouthWest')
>>
```



Try 'help plot',
'help title',
'help legend',
and 'help axis'
for more info

Command History

```
clear
xlabel('x')
ylabel('y')
title('My First MATLAB Plot')
legend('sin(x)', 'cos(x)', 'Location', 'SouthWest')
```

More About Plotting

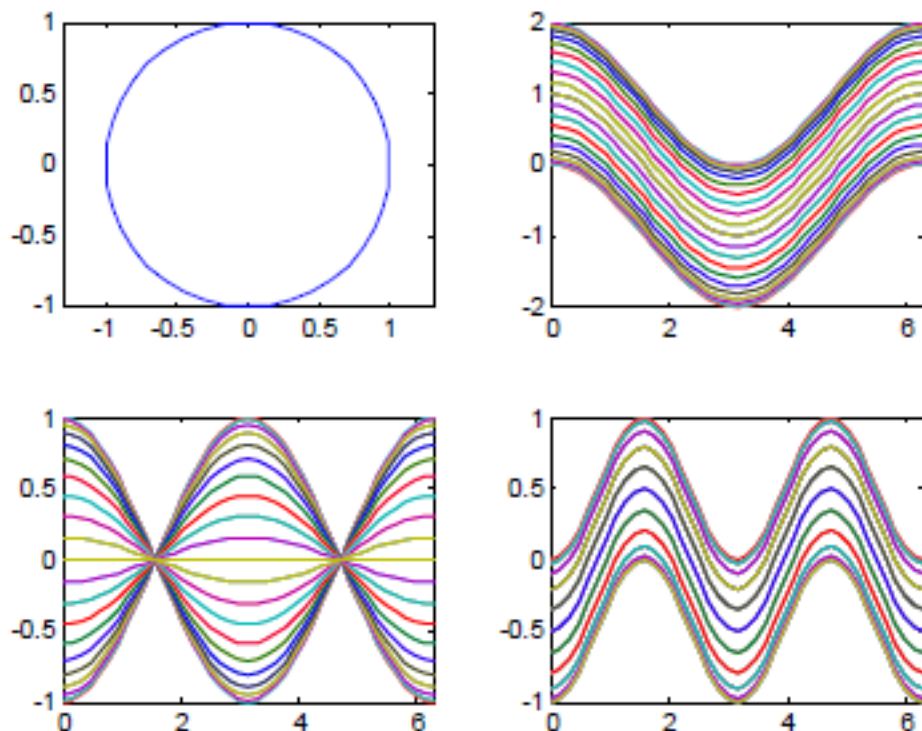
```
t = 0:pi/20:2*pi;  
[x,y] = meshgrid(t); % look up meshgrid
```

```
subplot(2,2,1) % creates a 2x2 array of plots, and plot in the first subplot  
plot(sin(t),cos(t))  
axis equal % this is a parametric plot
```

```
subplot(2,2,2)  
z = sin(x)+cos(y); % z is a matrix  
plot(t,z)  
axis([0 2*pi -2 2]) % plotting each column of z  
% versus t
```

```
subplot(2,2,3)  
z = sin(x).*cos(y);  
plot(t,z)  
axis([0 2*pi -1 1])
```

```
subplot(2,2,4)  
z = (sin(x).^2)-(cos(y).^2);  
plot(t,z);  
axis([0 2*pi -1 1])
```



% for 3-D plotting, try mesh, surf, surfl, waterfall, etc

M-Files and Functions

- Let's make our own functions
- To start the editor, type 'edit'

The screenshot displays the MATLAB environment. The main window shows a script with the following code:

```

1 function y = myfactorial(x)
2     % function y = myfactorial(x)
3
4     if x==1      % really inefficient
5         y = 1;
6     else
7         y = x*myfactorial(x-1);
8     end
9
10    % this file should be saved with the same name, i.e. 'myfactorial.m'

```

The Command Window at the bottom shows the execution of the function:

```

>> myfactorial(5)

ans =

    120

```

M-Files and Functions

- Local workspace and Scoping
- To make variables global: `global variable_name`

The screenshot displays the MATLAB environment. The main editor window contains the following code:

```

1 function y = myfactorial(x)
2     % function y = myfactorial(x)
3
4     if x==1      % really inefficient
5         y = 1;
6     else
7         y = x*myfactorial(x-1);
8     end
9
10    % this file should be saved with the same name, i.e. 'myfactorial.m'

```

The Command Window at the bottom shows the execution of the function:

```

>> myfactorial(5)

ans =

    120

```

For, While, and If

```
for m = 1:100
    num = 1/(m+1)
end
```

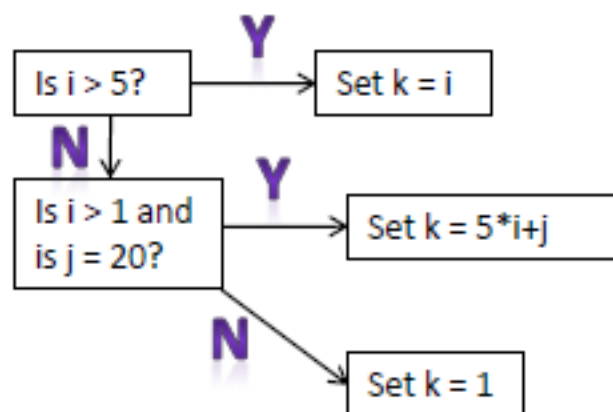
A for loop

```
% find all the powers
% of 2 below 10000
while num < 10000
    num = 2^i;
    v = [v; num];
    i = i+1;
end
```

A while loop

```
i = 6; j = 21;
if i > 5
    k = i;
elseif (i > 1) & (j == 20)
    k = 5*i+j;
else
    k = 1;
end
```

- And: $a \& b$
- Or: $a | b$
- Not-equal: $a \sim = b$
- Equal: $a == b$



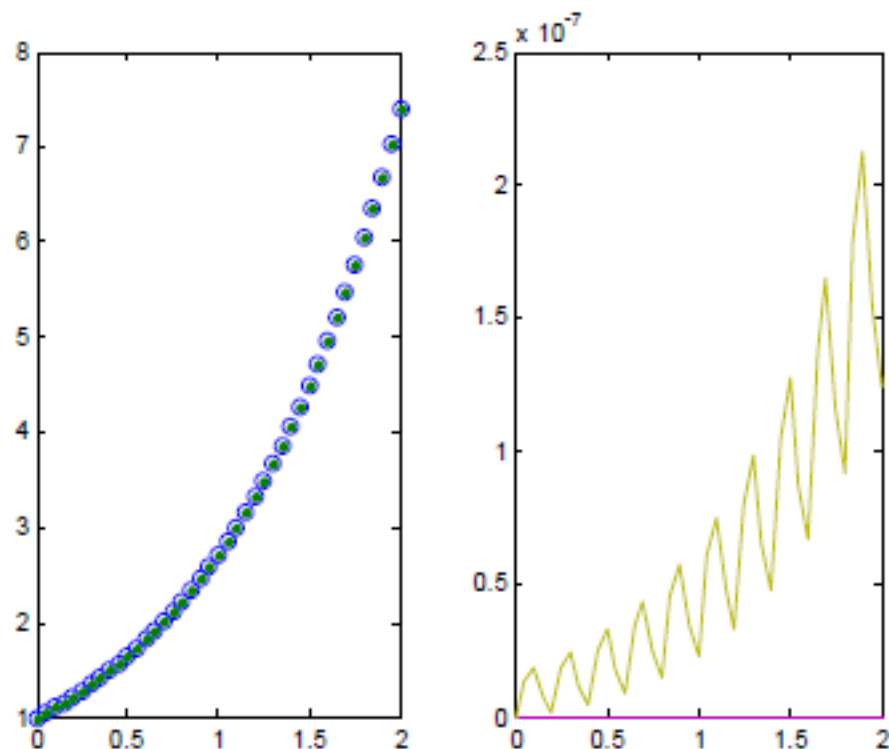
Solving ODEs

- A very simple case: $\frac{dy}{dt} = y(t) \quad 0 \leq t \leq 2 \quad y(0) = 1$

```
function dy = simpleode(t,y)
dy = y; % save as simpleode.m
```

- Type in command line:

```
[t y] = ode45(@simpleode, [0, 2], [1]);
subplot(1,2,1), plot(t,y,'o',t,exp(t),'.')
subplot(1,2,2), plot(t,(y-exp(t))/exp(t))
```



Solving ODEs

Lorenz equations $\frac{dx}{dt} = c(y - x), \frac{dy}{dt} = x(r - z) - y, \frac{dz}{dt} = xy - bz$

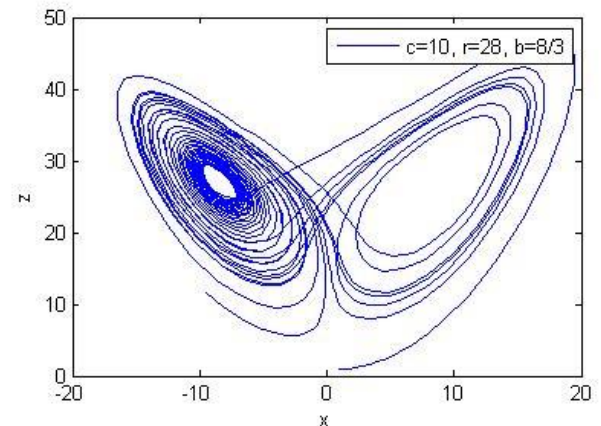
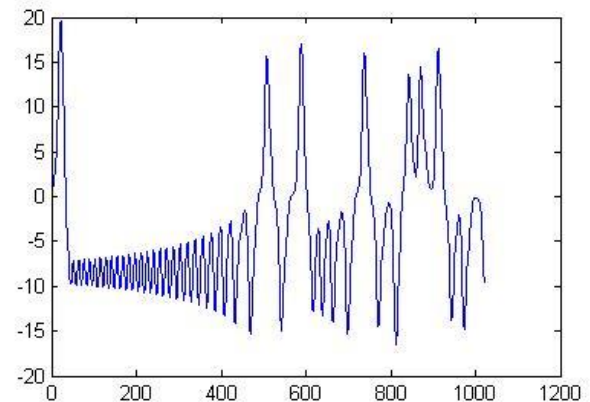
$c = 10, r = 28, b = 8 / 3, x(0) = y(0) = z(0) = 1, 0 < t < 30$

Edit and save as lorenzfunc.m

```
function dydt=f(t,y,flag,c,r,b)
% x=y(1), y=y(2),z=y(3)
dydt=[c*(y(2)-y(1)); y(1)*(r-y(3))-y(2);
      y(1)*y(2)-b*y(3)];
```

Edit and save as lorenz.m

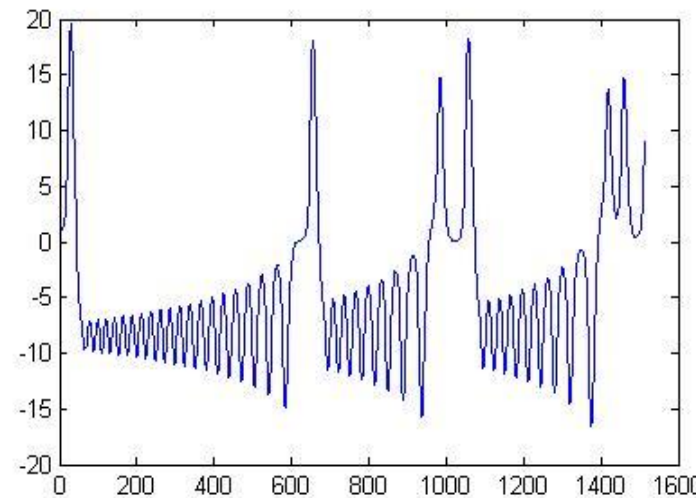
```
clear;
c=10; r=28; b=8/3;
options=[];
x0=[1 1 1];
[t y]=ode45('lorenzfunc',[0 30], x0, options, c,r,b);
subplot(2,1,1);plot(y(:,1));
subplot(2,1,2);plot(y(:,1),y(:,3));
xlabel('x'); ylabel('z');
legend('c=10, r=28, b=8/3');
```



Butterfly effect



**Exponential
amplification of
small errors!**



Add to the end of lorenz.m

hold on;

plot(y(end,1),y(end,3),'b+');

x1=x0+[0 0 1e-6];

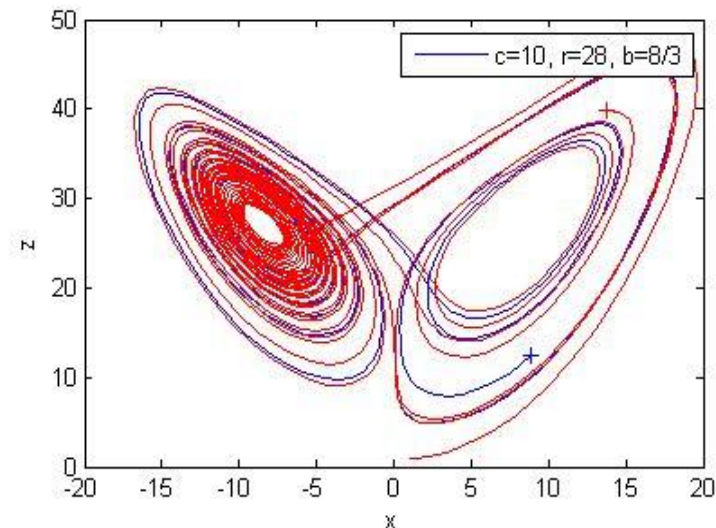
[t1 y1]=ode45('lorenzfunc',[0 30],x1,options,c,r,b);

subplot(2,1,2); hold on

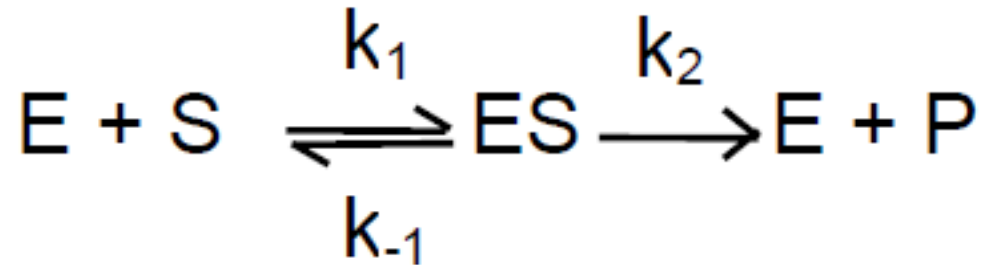
plot(y1(:,1),y1(:,3),'r-');

plot(y1(end,1),y1(end,3),'r+');

hold off



Review of Michaelis-Menten Kinetics



$$\frac{d[S]}{dt} = -k_1[E][S] + k_{-1}[ES]$$

$$\frac{d[E]}{dt} = -k_1[E][S] + (k_{-1} + k_2)[ES]$$

$$\frac{d[ES]}{dt} = k_1[E][S] - (k_{-1} + k_2)[ES]$$

$$\frac{d[P]}{dt} = k_2[ES] \equiv v$$

Simplified Equations

$$E_o = [E] + [ES]$$

$$\frac{d[S]}{dt} = -k_1 E_o [S] + (k_1 [S] + k_{-1}) [ES]$$

$$\frac{d[ES]}{dt} = k_1 E_o [S] - (k_1 [S] + k_{-1} + k_2) [ES]$$

Initial conditions:

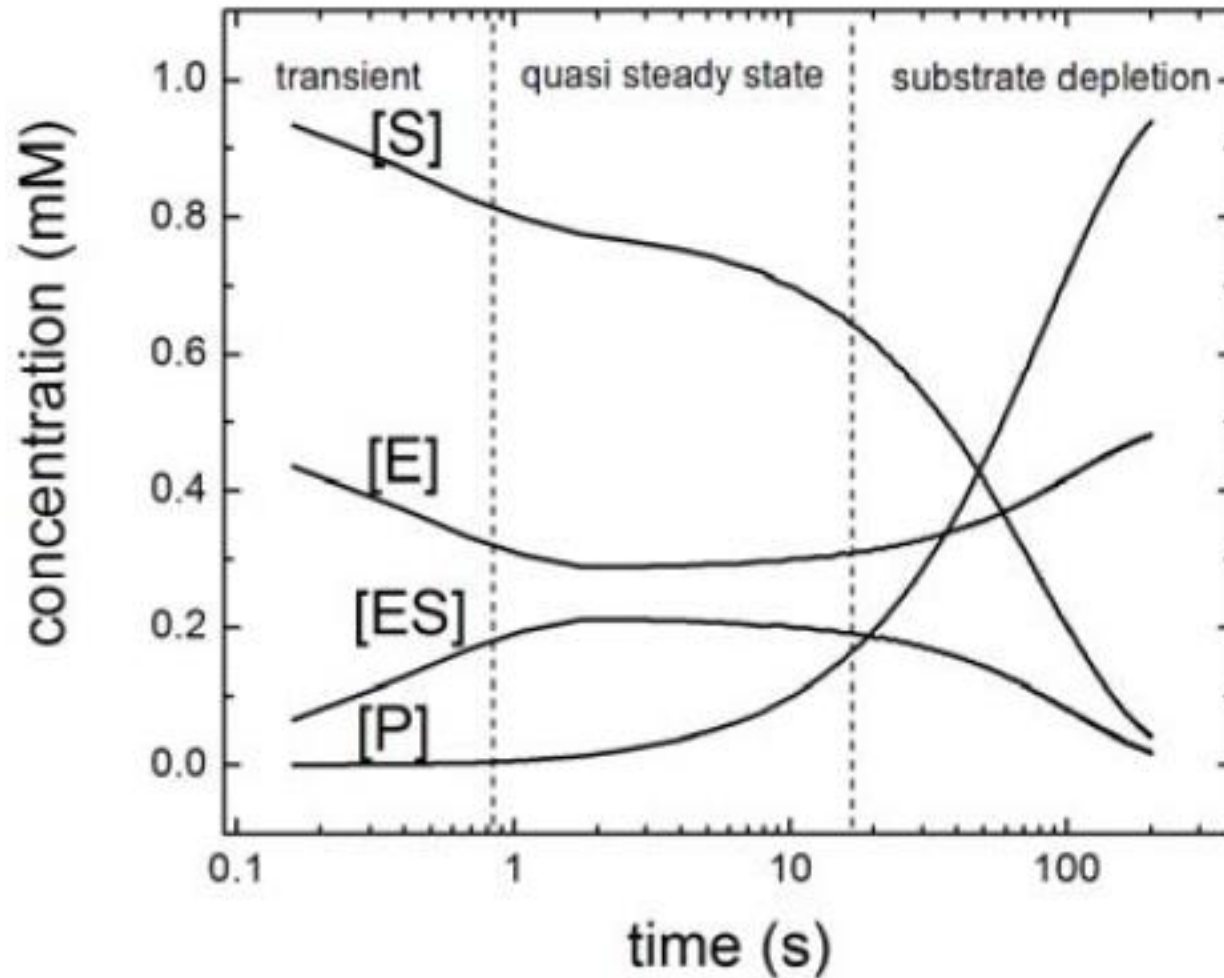
$$[S]_{t=0} = S_o$$

$$[E]_{t=0} = E_o$$

$$[ES]_{t=0} = 0$$

$$[P]_{t=0} = 0$$

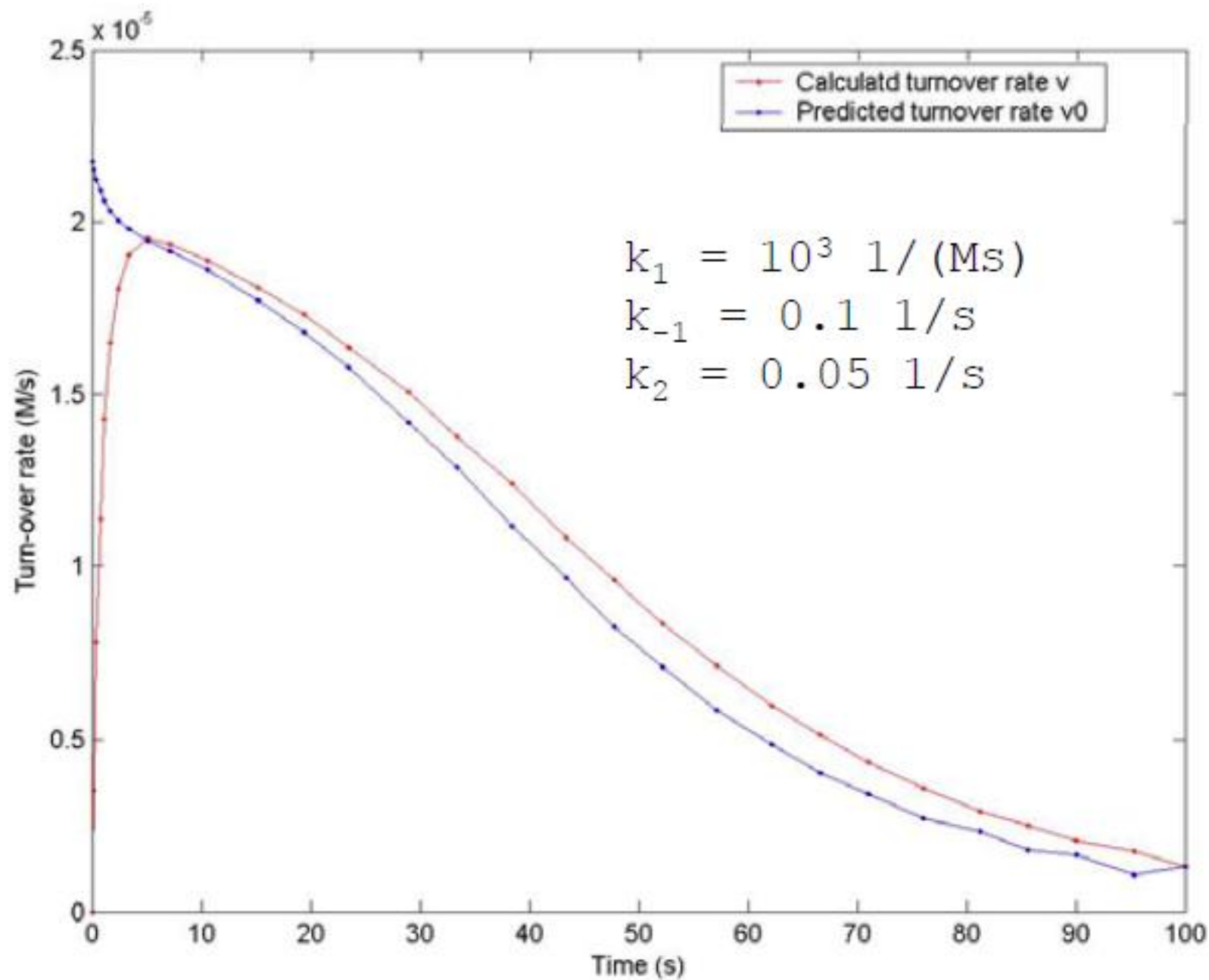
By solve the simplified equation we derive this description



Further simplification to Michaelis-Menten equation

$$V = \frac{v_{\max} S}{K_m + S}$$

Good approximation if
in quasi-steady state



Practice:

change the kinetic parameters from

$k_1=1e3$, $k_{-1}=0.1$, $k_2=0.05$

to

$k_1=1e3$, $k_{-1}=1e-4$, $k_2=0.05$

And compare the difference between simulation and quasi-steady state approximation again using `mm.m` and `mmfunc.m`.

And explain why.

Equilibrium binding

- Previously it was assumed that one substrate molecule binds to one enzyme
- In biological reactions proteins often bind multiple substrates
- Assume protein P has n binding sites, P_j denotes protein bound to j substrates S



where $j = 1, 2, \dots, n$.

- For unbound protein P_0

$$\frac{d[P_0]}{dt} = -k_{+1}[P_0][S] + k_{-1}[P_1]$$

Equilibrium binding

- In steady state, $d[P_0]/dt=0$, the association constant K_a ($1/K_d$)

$$K_a = \frac{k_{+1}}{k_{-1}}$$

$$K_a = \frac{[P_1]}{[P_0][S]}$$

- To characterize all n reactions: we introduce the n association constants K_j

$$K_j = \frac{[P_j]}{[P_{j-1}][S]}$$

- For average number of substrates bound r

$$r = \frac{[P_1] + 2[P_2] + 3[P_3] + \dots + n[P_n]}{[P_0] + [P_1] + [P_2] + \dots + [P_n]}$$

$$r = \frac{K_1[S] + 2K_1K_2[S]^2 + 3K_1K_2K_3[S]^3 + \dots + nK_1K_2\dots K_n[S]^n}{1 + K_1[S] + K_1K_2[S]^2 + \dots + K_1K_2\dots K_n[S]^n}$$

Adair's equation

Four scenarios of relations between binding sites

Identical and independent binding sites

- Assuming binding to each site is independent of the states of other binding sites.
- In steady state, there are n possible binding sites available for binding the first substrate, only 1 possibility to loose a substrate going from P_1 to P_0 .

$$0 = -nk_+[P_0][S] + k_-[P_1]$$

- Similarly we get

$$0 = -(n-1)k_+[P_1][S] + 2k_-[P_2]$$

- If intrinsic association constant K is defined as: $K \equiv \frac{k_+}{k_-}$ we have

$$K_j = \frac{(n-j+1)K}{j}$$

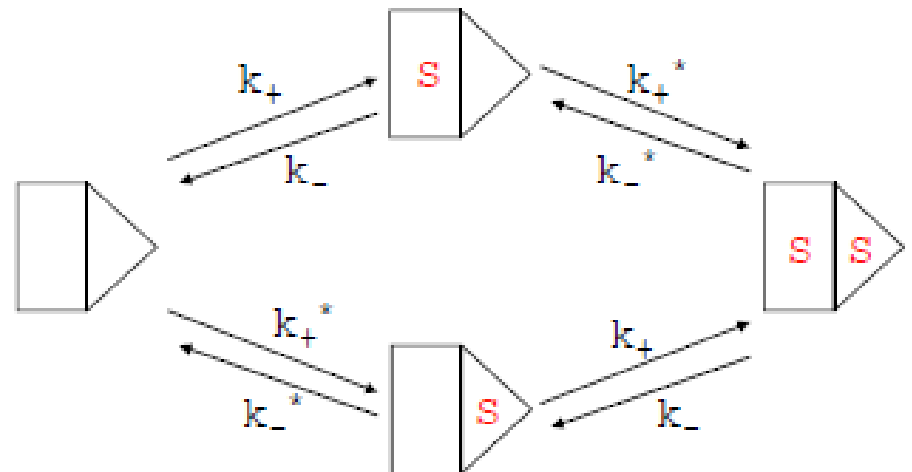
And the final result is:
$$r = \frac{nK[S]}{1+K[S]}$$

Non-identical and independent binding sites

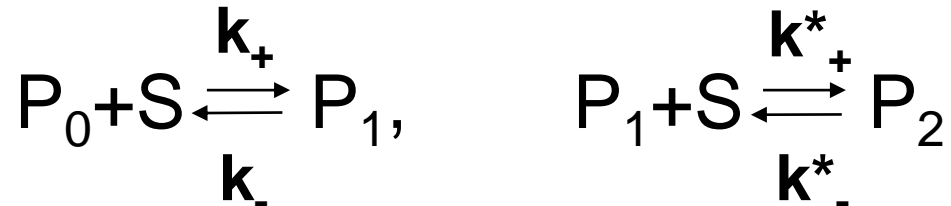
- Each binding site family (n_j) has its own association constant K_j .
- At lower concentration the high affinity binding sites will be occupied, the lower affinity site will be occupied at larger $[S]$

$$r = \frac{n_1 K_1 [S]}{1 + K_1 [S]} + \frac{n_2 K_2 [S]}{1 + K_2 [S]} + \dots + \frac{n_m K_m [S]}{1 + K_m [S]}$$

$$\sum n_j = n$$



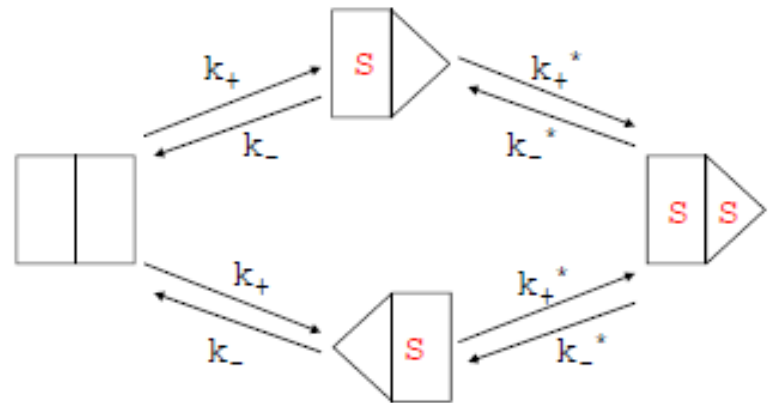
Two identical and interacting binding sites



- Two intrinsic association constants $K = k_+/k_-$, $K^* = k_+^*/k_-^*$, we find:

$$K_1 = 2K$$

$$K_2 = \frac{1}{2}K^*$$



- Together with Adair's equation, the saturation function $Y = r/n$ is:

$$Y = \frac{K[S] + KK^*[S]^2}{1 + 2K[S] + KK^*[S]^2}$$

Two identical and interacting binding sites

- For $K=K^*$ we have the Michaelis-Menten like equation:

$$\tilde{Y} = \frac{K[S]}{1+K[S]}$$

- The difference between the two is:

$$Y - \tilde{Y} = \frac{(K^* - K)K[S]^2}{(1+K[S])(1+2K[S]+KK^*[S]^2)}$$

- Positive cooperativity is often defined as $Y - \tilde{Y} > 0$
 - The affinity of binding the second ligand is higher than the first
- Negative cooperativity is often defined as $Y - \tilde{Y} < 0$
 - The affinity of binding the second ligand is lower than the first

Two identical and interacting binding sites

- Further considering the limit for which intermediate states can be neglected, meaning single-bound states are very unlikely, the effective reaction would be:



- The saturation function is now:

$$Y = \frac{K[S]^2}{1 + K[S]^2}$$

- Where $K = [P_2]/([P_0][S]^2)$ is the association constant of the reaction. This limit was first considered by Hill. It is called hill function and 2 in this function is called hill coefficient.

Hill coefficient and number of binding sites

Hill coefficient is often used as an estimation of the number of binding sites of a protein.

Just be careful about the assumption of no intermediate states.

Non-identical and interacting binding sites

- Beyond the scope of this class
- If possible, experimental determination is better.
- Further reading on enzyme kinetics and cooperativity
 - D. Fell, Understanding the control of metabolism (1997)
 - J.D. Murray, Mathematical Biology (2002)
 - H. Bisswanger, Enzyme kinetics (2002)

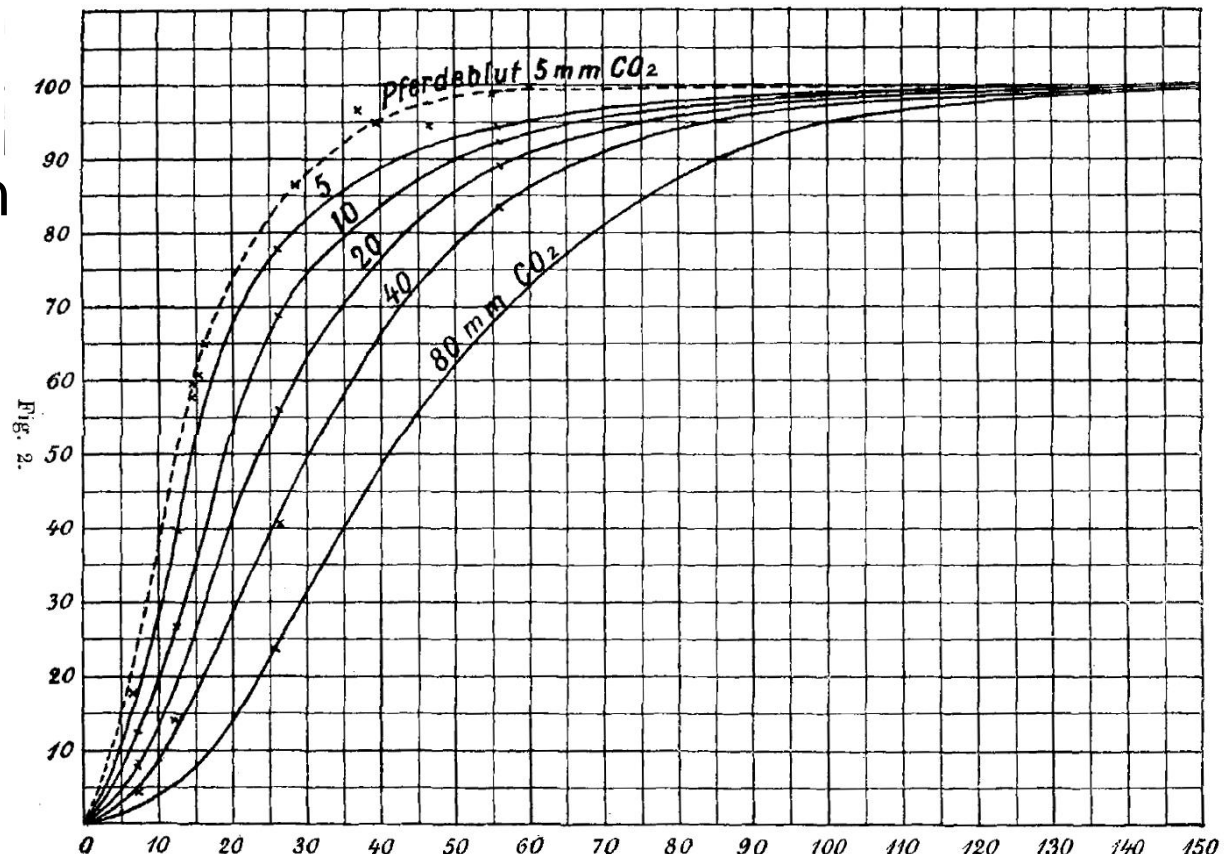
Examples of cooperativity binding in biology

Hemoglobin

1. In 1904, Christian Bohr studied hemoglobin binding to oxygen under different condition.

Sigmoidal curve:

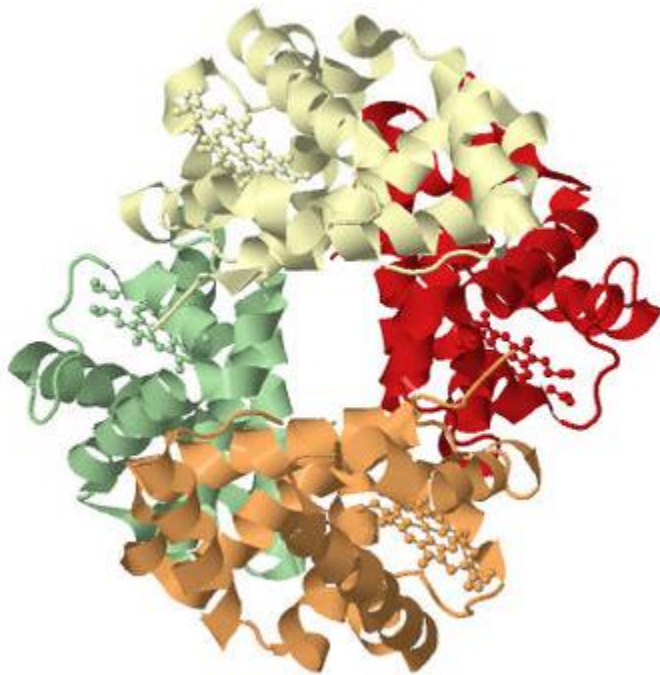
The more oxygen is bound to hemoglobin, the easier it is for more oxygen binding until all binding sites are saturated.



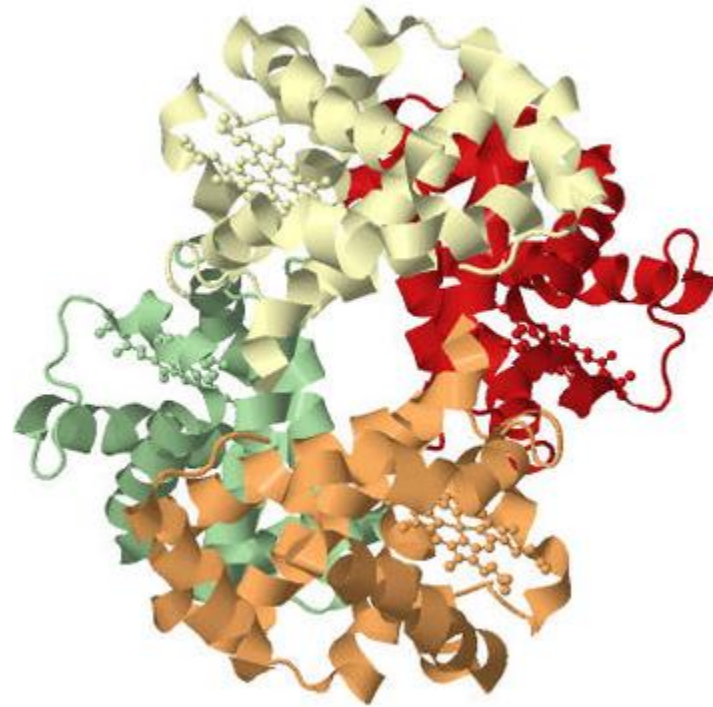
Examples of cooperativity binding in biology

Hemoglobin

2. The molecular evidence for cooperativity: In 1960, Max Perutz solved the **tetrameric** hemoglobin structure contain **four** hemes for oxygen binding.



Free

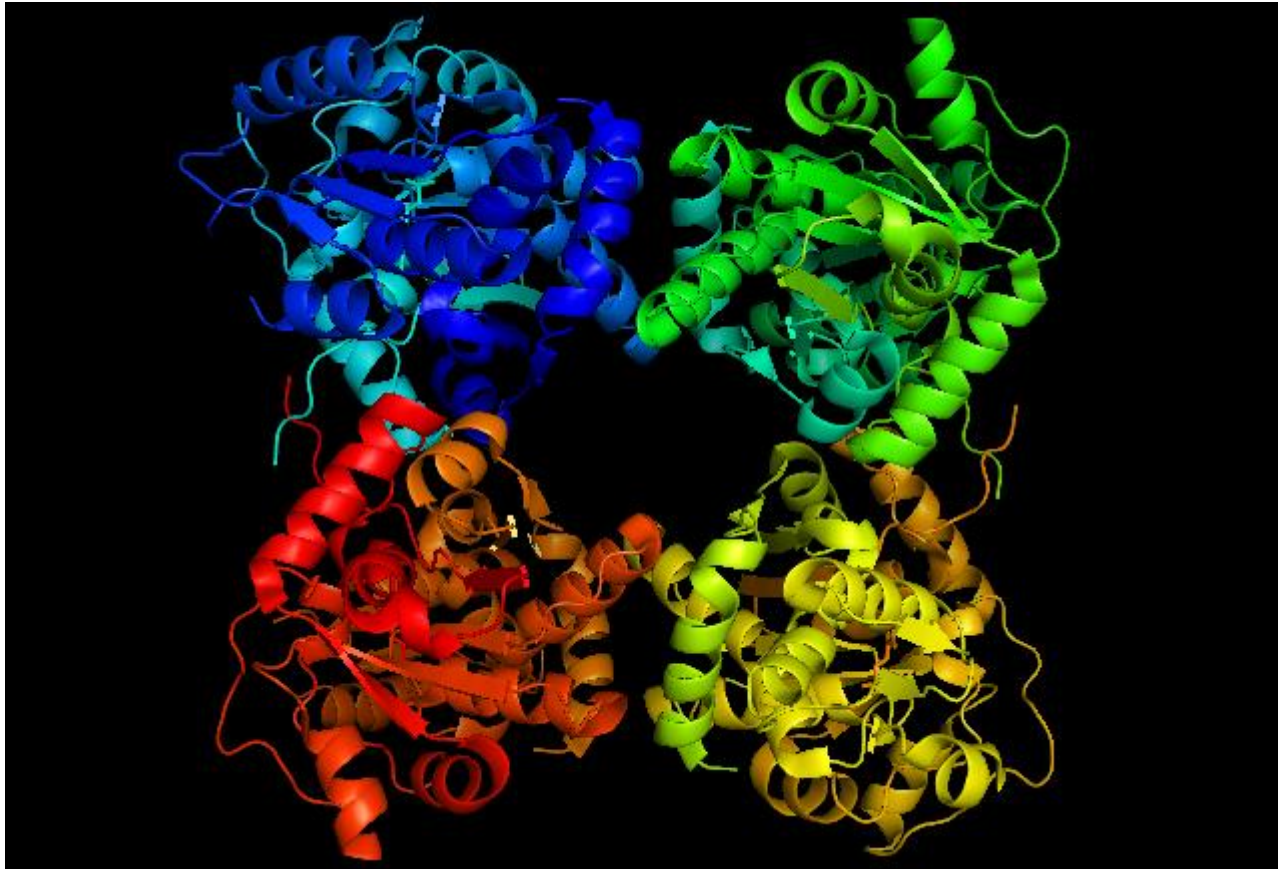


Oxygen-bound

Examples of cooperativity binding in biology

Multimeric enzymes carry several binding sites for the regulator

Threonine deaminase



Examples of cooperativity binding in biology

Ion Channels: are formed of several identical monomers, arranged symmetrically in membrane. Several classes of such channel whose opening is regulated by ligands exhibit cooperative binding.

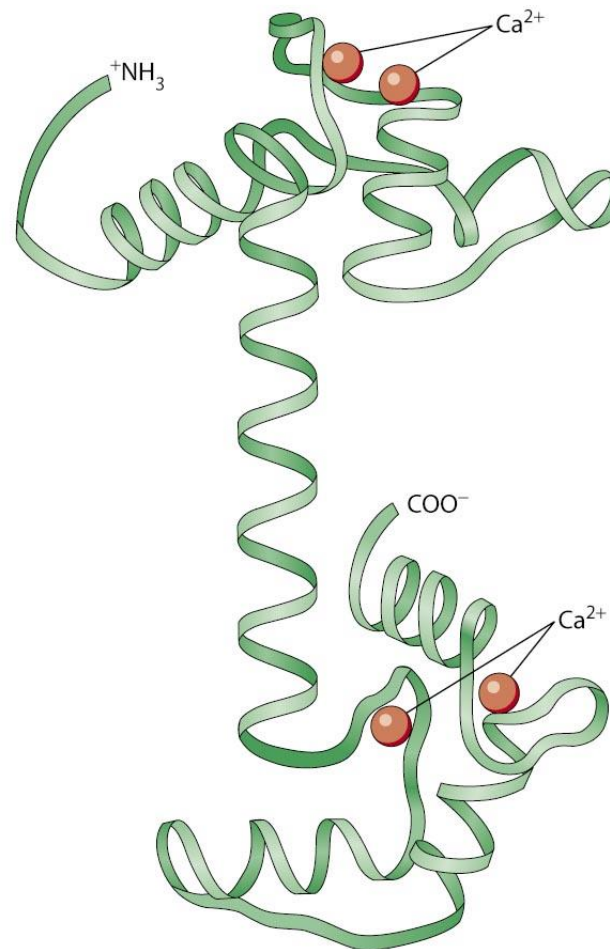
Nicotinic acetylcholine receptors



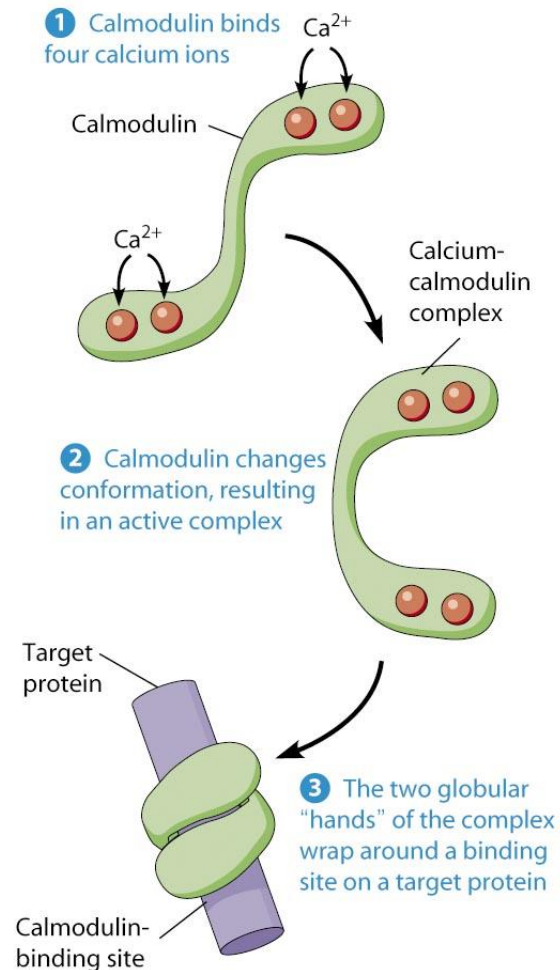
Examples of cooperativity binding in biology

Multi-site molecules: Calmodulin

Transcriptional
factors as well



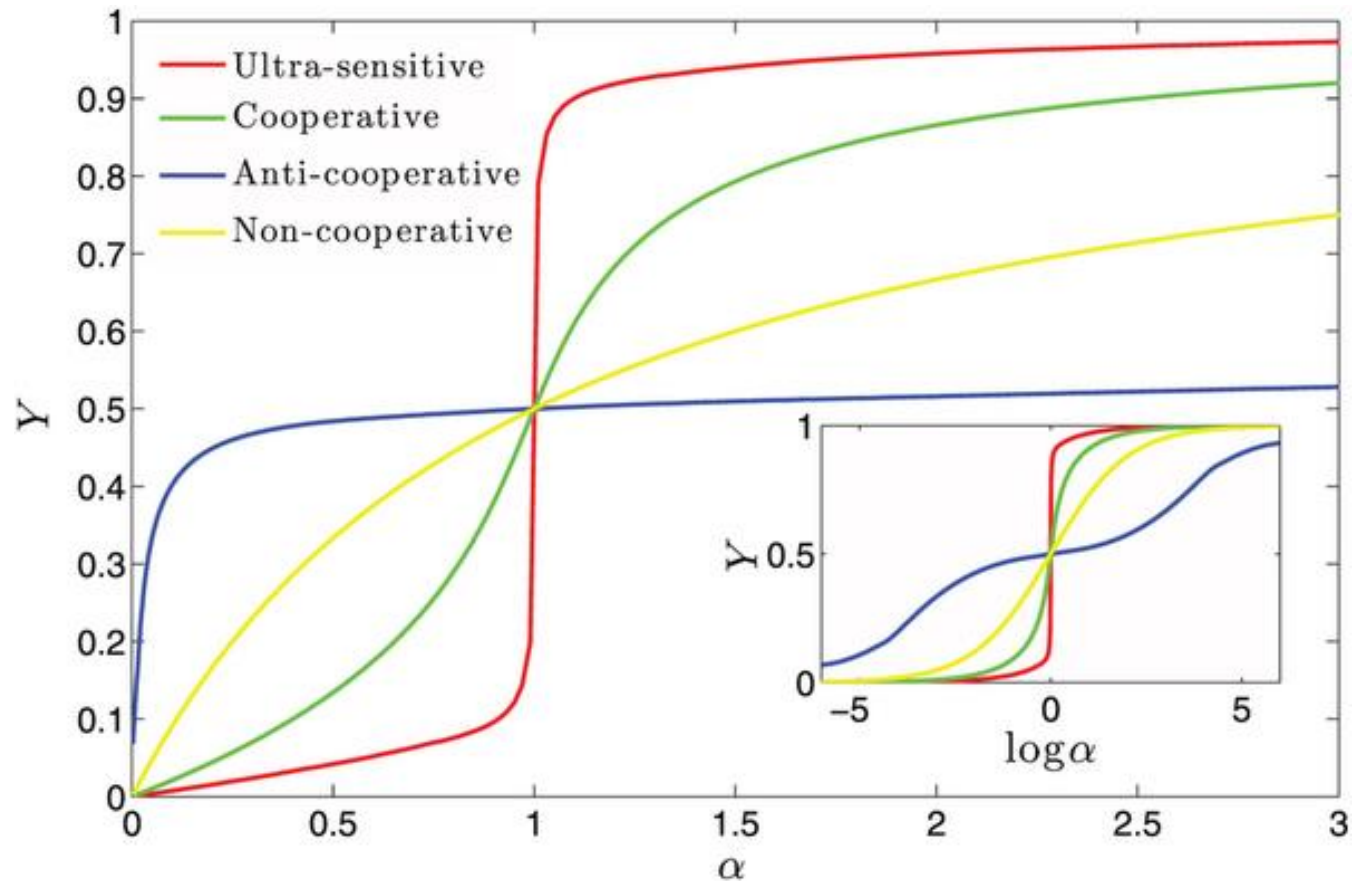
(a) Structure of Ca^{2+} -calmodulin complex



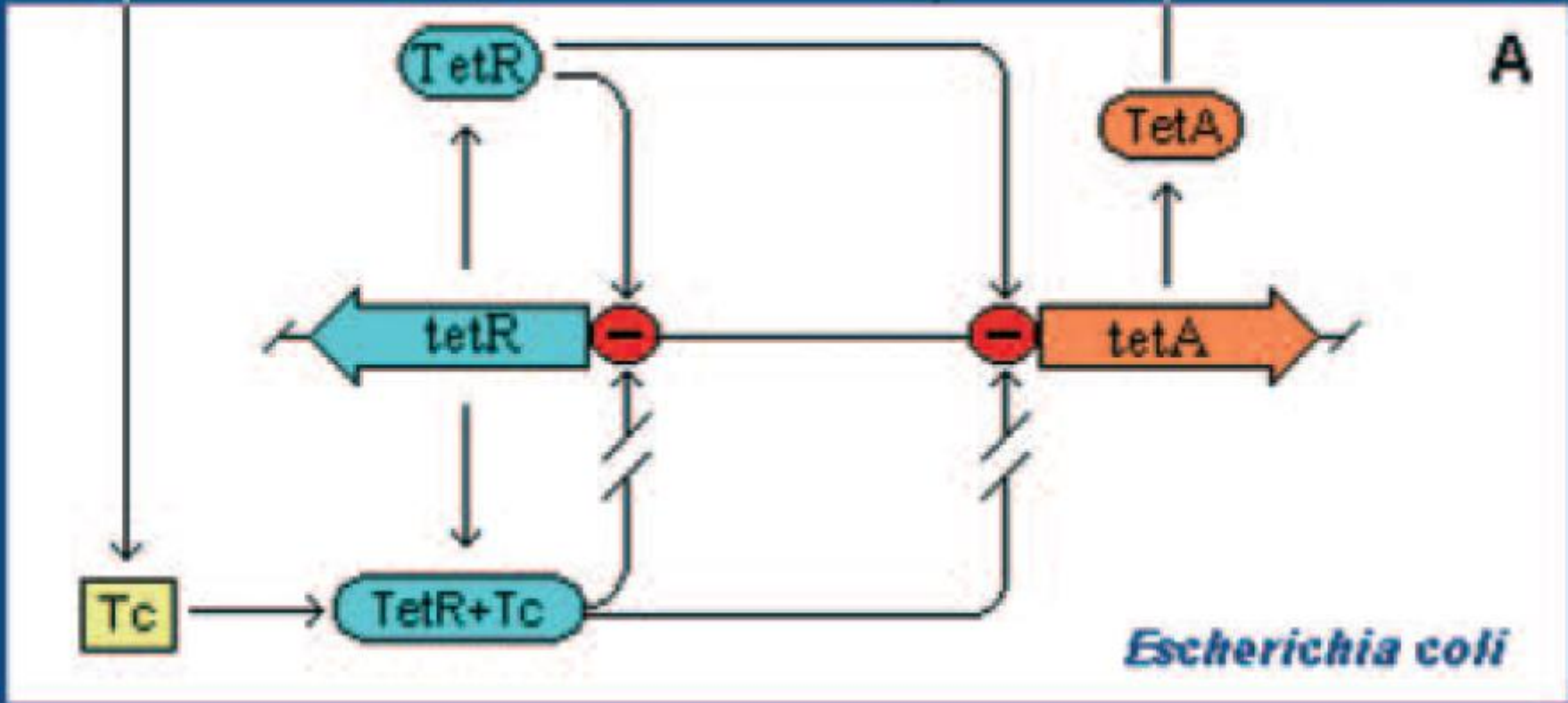
(b) Function of Ca^{2+} -calmodulin complex

What is cooperativity for (function)?

Sensitivity



Mechanisms for tetracycline resistant



TetR

Crystal structure

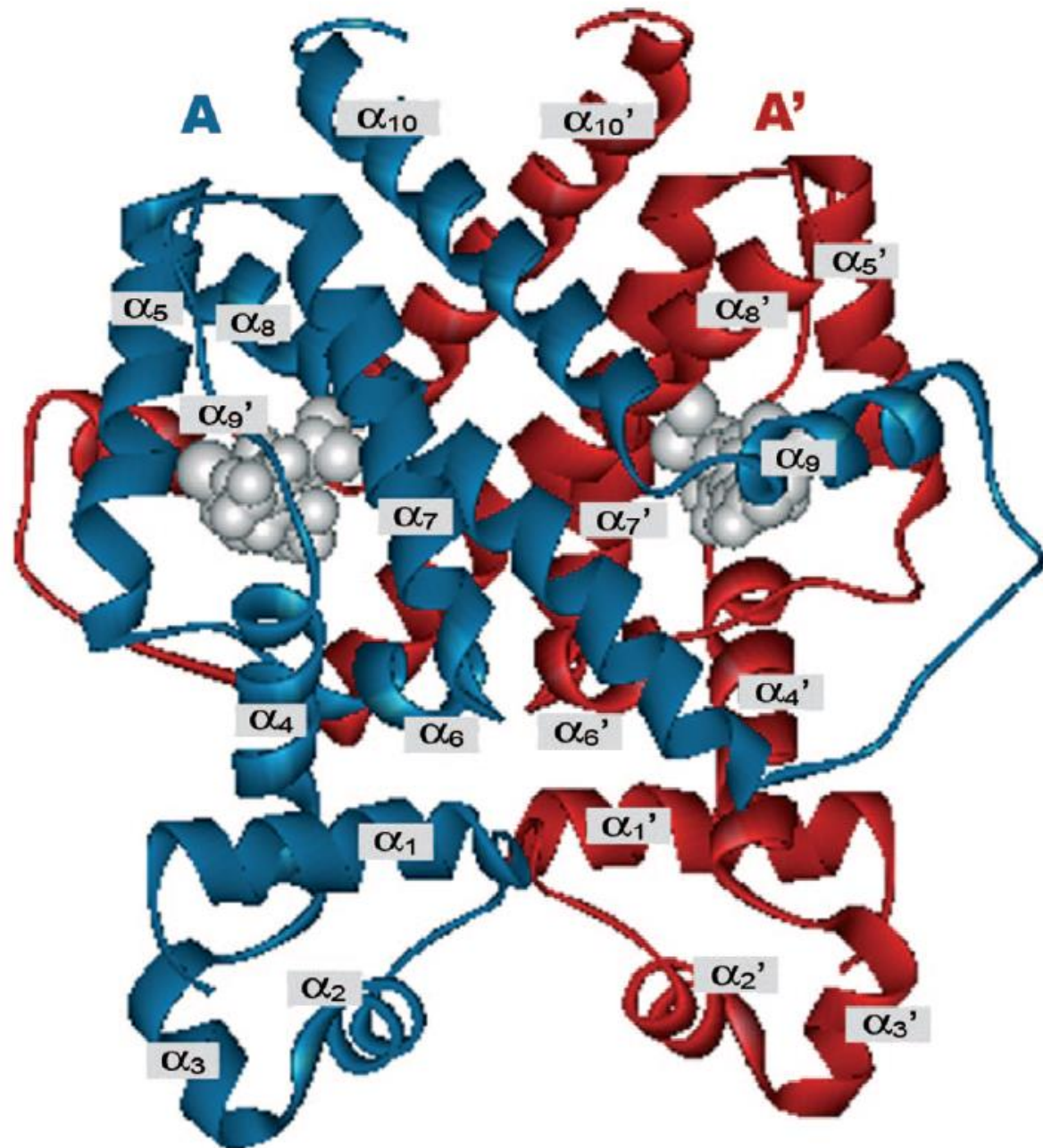


FIG. 2. Ribbon diagram of a TetR homodimer. Monomers are shown in blue or red. Two tetracycline molecules, each bound to a monomer, are shown in grey. α -Helices 2 and 3 in the blue monomer and α_2' and α_3' in the red monomer constitute the shared HTH DNA binding domain. α -Helix 1 and part of helix α -4, together with α -helices 2 and 3, comprise the sequence that best defines the TetR family profile. (Adapted from Hinrichs et al. [150] with permission of the publisher.)

TetR dimer Bind to Palindromic DNA sequence- TetO

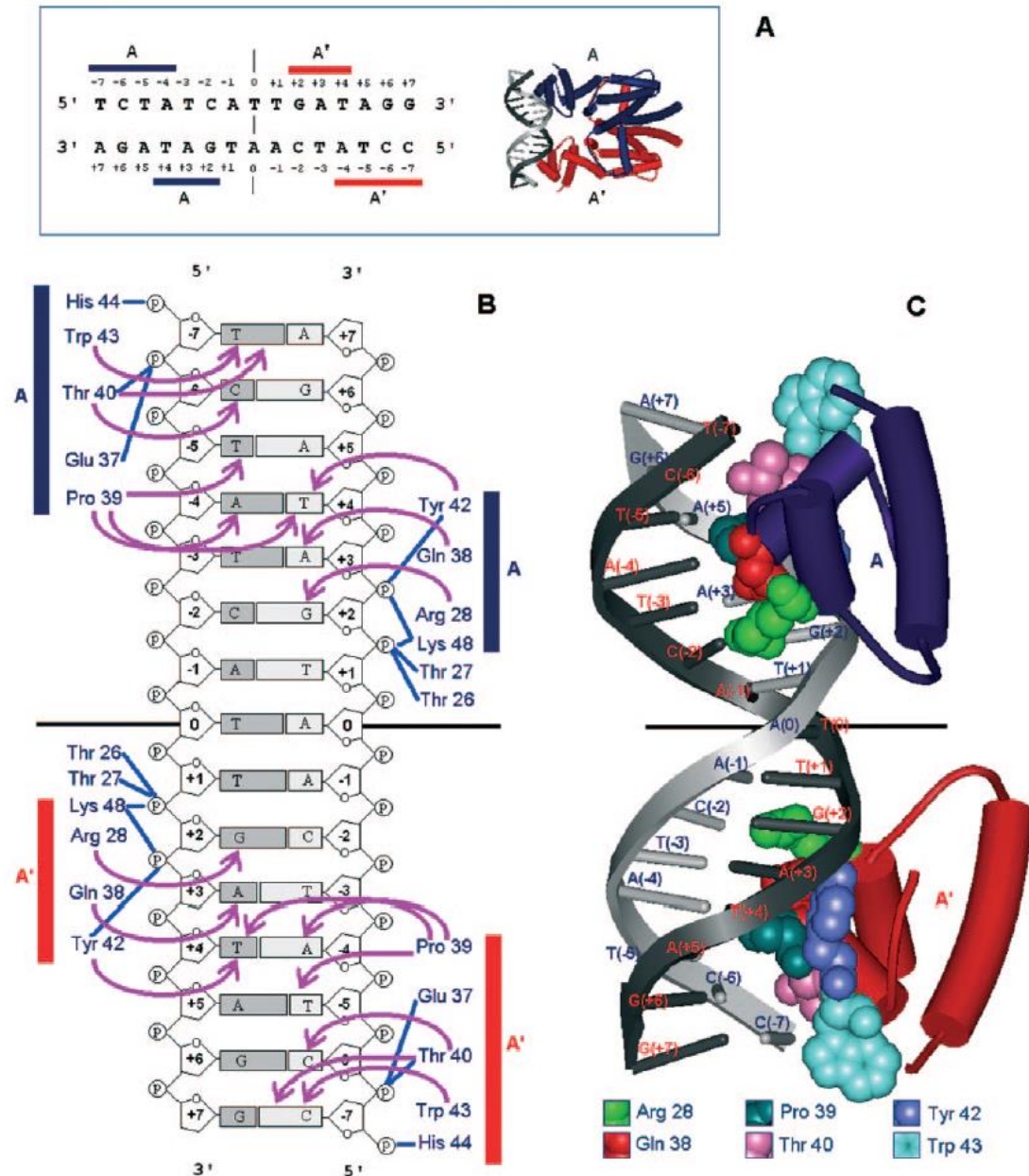
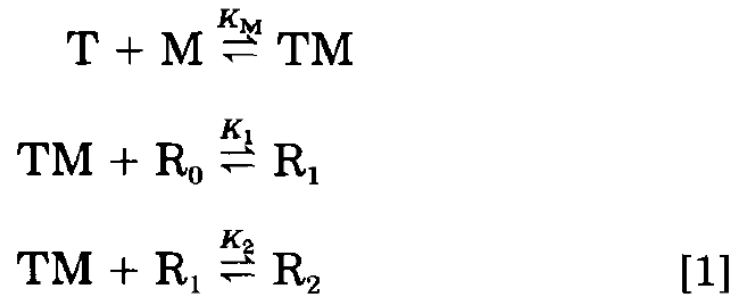


FIG. 3. Binding of TetR to its operator site. A) *tetR* operator and contact regions. The *tetR* operator is a palindromic sequence. Horizontal bars show nucleotides contacted by each monomer of the TetR dimer. B) Interaction of TetR residues with specific nucleotides (arrows) and phosphate backbone (blue lines) in the operator region. The amino acids involved in DNA binding extend from residues 27 to 48. Contacts established with

Cooperative binding of tetracycline to TetR



where T, M and TM represent free tetracycline, Mg^{2+} , and tetracycline complexed by Mg^{2+} , respectively. R_0 , R_1 , and R_2 are the free repressor dimer, repressor dimer with one TM, and repressor dimer with two TM, respectively. K_M , K_1 , and K_2 are the equilibrium association constants of the respective reactions. Since the repressor dimer consists of two chemically identical subunits (14), the two binding sites can be considered intrinsically identical, having an association constant K for tetracycline. The binding of the first tetracycline may modify the binding affinity of the second tetracycline to the repressor to $K \times \alpha$. The macroscopic binding constants K_1 and K_2 are then related to K and α by the equations

$$\begin{aligned}K_1 &= 2 \times K \\K_2 &= K \times \alpha/2.\end{aligned}$$

Cooperative binding of tetracycline to TetR

TABLE 1

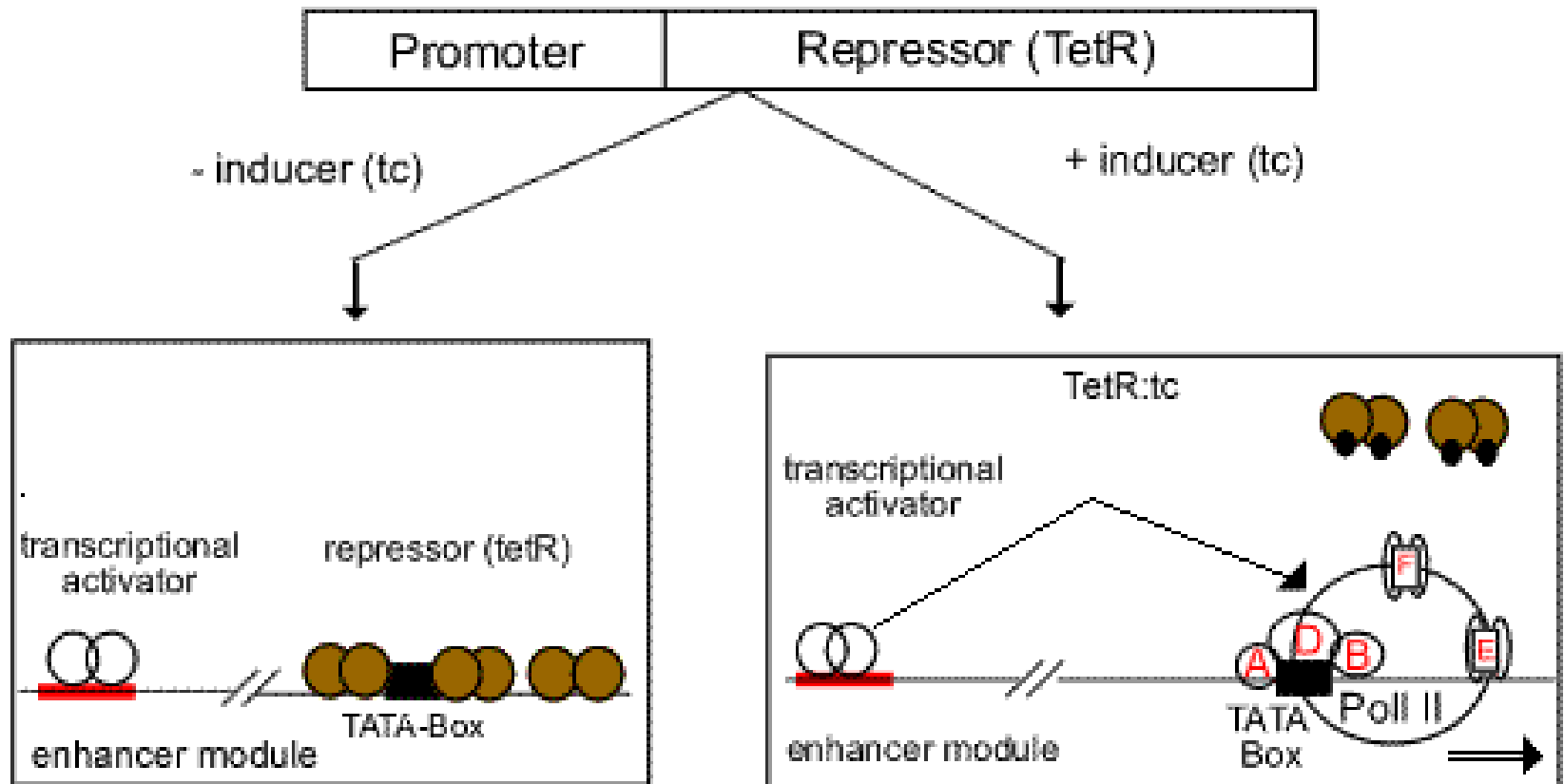
Effect of Repressor Concentration on the Binding Parameters of Tetracycline to the Repressor

[Repressor] (μM)	α	K ($\times 10^9 \text{ M}^{-1}$)	K for $\alpha = 1$ ($\times 10^9 \text{ M}^{-1}$)
0.11	0.9	2.3	2.4
0.33	0.7	2.1	2.0
1.0	1.8	2.0	2.7
1.1	2.0	2.6	3.1
1.1	1.5	1.1	1.2
1.1	1.5	3.6	3.8
5.3	2.0	4.4	6.0

Note. K and α were determined as described in the text. K was also determined considering that the binding is noncooperative ($\alpha = 1$).

Induced expression system with WT tetR

Tet as a promoter repressing system



Induced expression system with engineered tetR

The Tet regulatory system

