PROBLEM-SOLVING EXERCISE #1
DESIGN AND UNCERTAINTY SOLUTION

PART A

Roll a single standard six-sided die 36 times and tabulate your results (i.e., number of 1's, number of 2's, ..., number of 6's). Calculate the mean, variance and standard deviation of your data.

Your results will depend upon your actual data, however, your results should have been close to the following: Mean = 3.5 Variance = 2.917 Standard deviation = 1.708

PART B

Roll a pair of standard six-sided dice 36 times and tabulate your results (i.e., number of 2's, number of 3's, ..., number of 12's). Calculate the mean, variance and standard deviation of your data.

Again, your results will depend upon your actual data, however, your results should have been close to the following: Mean = 7.0 Variance = 5.833 Standard deviation = 2.415

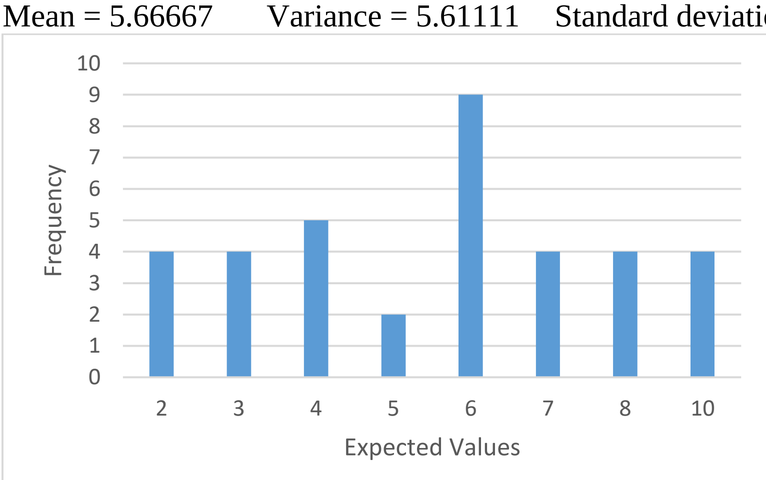
PART C

Suppose you have a pair of six-sided dice where each die contains the following sides: (1, 1, 2, 3, 5, 5). In theory, if you rolled this pair of dice 36 times, how many times would you get a total of 2, 3, 4, 5, 6, 7, 8 and 10? Using graph paper, plot the distribution of expected values. Calculate the mean, variance, and standard deviation of this distribution.

Value	***Frequency***
2	*4*
3	*4*
4	*5*
5	*2*
6	*9*
7	*4*
8	*4*
10	*4*

These results also need to be graphed,
Frequency *on the y-axis,* ***Value*** *on the x-axis*
Bar Graph (shown below) or Data Points Graph

Mean = 5.66667 Variance = 5.61111 Standard deviation = 2.3687



PART D

The number of false fire alarms in a suburb of Detroit averages 6.2 per day. Assuming that a Poisson distribution is appropriate:

What is the probability that exactly 6 false alarms will occur on a given day?

$$\mathbf{P(X = 6) = 0.1601}$$

What is the probability that less than 6 false alarms will occur on a given day?

$$\begin{aligned}\mathbf{P(X < 6)} &= \mathbf{P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)} \\ &= \mathbf{0.002029 + 0.012582 + 0.039006 + 0.080612 + 0.124948 + 0.154936} \\ &= \mathbf{0.414113}\end{aligned}$$

What is the probability that more than 6 false alarms will occur on a given day?

$$\begin{aligned}\mathbf{P(X > 6)} &= \mathbf{1 - P(X <= 6)} \\ &= \mathbf{1 - (P(X=0) + P(X = 1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6))} \\ &= \mathbf{1 - (0.002029 + 0.012582 + 0.039006 + 0.080612 + 0.124948 + 0.154936 + 0.1601\ )} \\ &= \mathbf{1 - 0.574213} \\ &= \mathbf{0.425787}\end{aligned}$$

(Hint: the probabilities of all possible cases must add to one).

PART E

College Pro Painting

You have decided that for your summer job you are going to manage a group of students who will paint houses to earn money for the summer. As Manager, your paycheck depends on your capability to efficiently schedule the team so that all the houses get painted on time with little, or ideally no, overage on scheduled man-hours.

Given that a house requires H man-hours to paint, your team has N houses to paint and S student-painters on the team:

i. Write the expression that relates the number of students that you will need to schedule to finish painting the houses in 12 weeks assuming 40-hour work weeks.

ii. If it takes 120 man-hours to paint a house and your team has 24 houses to paint, how many students do you need to hire to paint the houses in 12 weeks assuming 40-hour workweeks?

iii. A friend of yours, Bionka, managed a College Pro Painting team last summer and tells you that since every student is not as efficient as you may expect combined with the fact that the neighborhood you're covering has both small and large houses, the number of man-hours to paint a house, H, is not exactly 128. Instead, she tells you that H is equally likely to take anywhere from 100 man-hours to 140 man-hours. Additionally, Bionka hints that not all of your workers show up as expected. In fact, during a given week a man-day (8 hours) of work is lost on the average of 1.1 times. Fill in the following probability tables:

H	100	110	120	130	140
P(H)	0.2	0.2	0.2	0.2	0.2

Let R = Number of times 8 man-hours (a man-day) will be lost in a week:

R	0	1	2	3	4	5
P(R)	0.33287	0.36615	0.20138	0.07384	0.02031	0.00446

Recall that you have 12 weeks to paint these houses. Considering the two 'risks of uncertainty' characterized by the above tables that contain more realistic information regarding the number of man-hours that will *probably* be required to paint the 24 houses and the number of man-days of work that will *probably* be lost per week for each of the 12 weeks, how many students should you recruit to be most efficient? Briefly explain your answer.

Part E – Solution

Let: H = # man-hours to paint house

N = # Houses

S = # of Student-painters

Given : 12 weeks to paint 40 man-hours/week/student-painter

i.) N * H = # man-hours to finish all houses

S* 40 * 12 = # man-hours available in 12 weeks

Want these two to equal, so

S = (N * H) / 480

ii.) Given H = 120 man-hours/house

N = 24 houses

S = (24 * 120)/ 480 = 6, Therefore need 6 student-painters

iii) Assume a uniform distribution

H	P(H)
100	.20
110	.20
120	.20
130	.20
140	.20

Now calculate the number of man-hours to finish

(.20)(24)(100) + (.20)(24)(110) + (.20)(24)(120) + (.20)(24)(130) + (.20)(24) (140) =

480 + 528+ 576+ 624 + 672 = **2880 man-hours**

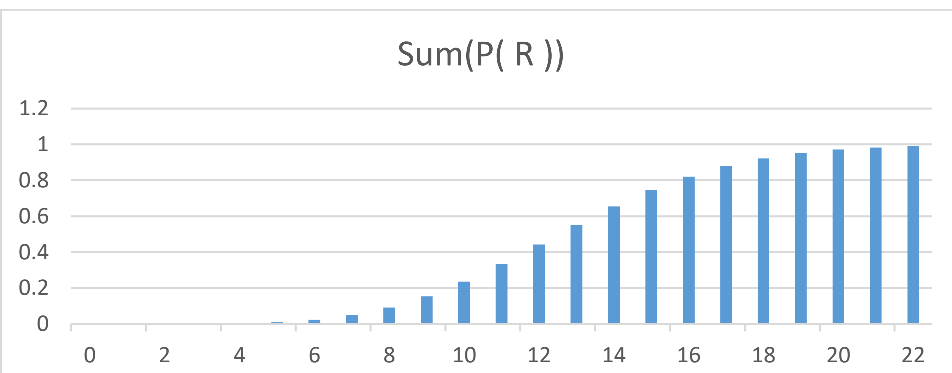
Apply Poisson Distribution for number of times 8 man-hours lost per week

R	P(R)
0	0.33287
1	0.36615
2	0.20138
3	0.07384
4	0.02031
5	0.00446

The above probabilities are for an interval of one week, which is a subinterval. The time interval we need to consider is 12 weeks. So the λ for that interval is: 1.1 * 12 = 13.2

Probabilities and chart are on the next two pages:

R	P(R)	Sum(P(R))
0	1.8506E-06	1.8506E-06
1	2.44279E-05	2.62785E-05
2	0.000161224	0.000187503
3	0.000709387	0.00089689
4	0.002340978	0.003237868
5	0.006180182	0.00941805
6	0.0135964	0.02301445
7	0.025638926	0.048653375
8	0.042304227	0.090957602
9	0.0620462	0.153003802
10	0.081900984	0.234904786
11	0.09828118	0.333185966
12	0.108109298	0.441295265
13	0.109772518	0.551067783
14	0.103499803	0.654567586
15	0.091079827	0.745647413
16	0.075140857	0.82078827
17	0.058344665	0.879132936
18	0.042786088	0.921919024
19	0.029725072	0.951644095
20	0.019618547	0.971262643
21	0.012331658	0.983594301
22	0.007398995	0.990993296



From the above probabilities, there is nearly 100% that 22 or less man-days will be lost over the interval of 12 weeks.

Now add this to the man-hours to finish houses, first convert man-days to man-hours: 22 * 8 = 176 man-hours:

Add to the previous total: 2880 + 176 = **3056 man-hours**

Substitute into formula for student painters:

S = 3056 / 480 = 6.366667 **We will need to hire 7 students.**