Design and Uncertainty

Discussion Points

- Mean
- Variance
- Standard Deviation
- Probabilities
- Uniform Distribution
- Normal Distribution
- Computational Form for Variance and Standard Deviation
- Poisson Distribution
- Manufacturing Scheduling and Uncertainty

<u>**Mean (μ)**</u>

Given a set of N numbers, X_1 , X_2 , ... , X_N , the ***mean***, or average, is calculated by summing the numbers and dividing by N.

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

For example, for the numbers **3, 4, 5**

$$\mu = (3 + 4 + 5) / 3$$

$$= 12 / 3$$

$$= 4$$

Variance (σ^2)

The ***variance*** is a measure of the spread of a distribution of numbers. It is computed as the average squared deviation of each number from the mean of the distribution.

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

For the numbers **3, 4, 5**

$$\sigma^2 = ((3 - 4)^2 + (4 - 4)^2 + (5 - 4)^2) / 3$$

$$= (1 + 0 + 1) / 3$$

$$= 0.667$$

Standard Deviation (σ)

The ***standard deviation*** of a distribution of numbers is the square root of the variance.

For the numbers **3, 4, 5**

$$\sigma^2 = 0.667$$

$$\sigma = 0.816$$

Another Example

For the numbers **2, 4, 6**

$$\mu = (2 + 4 + 6) / 3$$

$$= 12 / 3$$

$$= 4$$

$$\sigma^2 = ((2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2) / 3$$

$$= (4 + 0 + 4) / 3$$

$$= 2.667$$

$$\sigma = 1.633$$

Coin Toss

Possible Outcomes

(Head)   (Tail)

Each outcome has a probability of 1/2.

Single Standard Die

Possible Outcomes

(1)   (2)   (3)   (4)   (5)   (6)

Each outcome has a probability of 1/6.

These are both ***Uniform*** Distributions.

Pair of Standard Dice

Possible Outcomes

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Frequency Distribution			*Probability*
2	-	1	1/36
3	-	2	2/36
4	-	3	3/36
5	-	4	4/36
6	-	5	5/36
7	-	6	6/36
8	-	5	5/36
9	-	4	4/36
10	-	3	3/36
11	-	2	2/36
12	-	1	1/36

The 68-95-99.7 Rule

All normal density curves satisfy the following property, which is often referred to as the ***Empirical Rule***.

68%

of the observations fall within 1 ***standard deviation*** of the ***mean***, that is between $\mu - \sigma$ and $\mu + \sigma$.

95%

of the observations fall within 2 ***standard deviations*** of the ***mean***, that is between $\mu - 2\sigma$ and $\mu + 2\sigma$.

99.7%

of the observations fall within 3 ***standard deviations*** of the ***mean***, that is between $\mu - 3\sigma$ and $\mu + 3\sigma$.

For a pair of Standard 6-Sided Dice:

$\mu = 7$ $\quad \sigma^2 = 5.833$ $\quad \sigma = 2.415$

$\mu - \sigma$ to $\mu + \sigma$ = 4.585 - 9.415 → 24 / 36 = 66.666%

$\mu - 2\sigma$ to $\mu + 2\sigma$ = 2.17 - 11.83 → 34 / 36 = 94.444%

$\mu - 3\sigma$ to $\mu + 3\sigma$ = -0.245 - 14.245 → 36 / 36 = 100.0%

<u>Computation Form for Variance</u>

Computational form for the variance in a population.
All summations are assumed to be from i equals 1 to N.

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

$$= \frac{\sum (X^2 - 2 X \mu + \mu^2)}{N}$$

$$= \frac{\sum X^2}{N} - \frac{2 \mu \sum X}{N} + \frac{\sum \mu^2}{N}$$

$$= \frac{\sum X^2}{N} - (2 \mu) \mu + \mu^2$$

$$\sigma^2 = \frac{\sum X^2}{N} - \mu^2$$