Designing the Steggie

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***Abstract*—This report will detail the necessary background information of boat making through mathematical functions, explain specific design decisions and reasoning behind these choices, elaborate on the various calculations involved in this process, and discuss the real world performance of the boat. The performance qualifications of this project were to design a boat that floats in water, has a parallel deck to the surface of the water. The angle of vanishing stability is between 120°and 140°, has a maximum computer righting moment of at least 0.2 Newton-meters, and is as fast as possible under a constant force of .1 Newtons. The boat will be made out of hardboard and vinyl, and will contain a mast and eyelet. The project required a understanding of the background and terminology involved to make proper calculations using Mathematica. This use of mathematica was only possible with a understanding of the basics of multi-variable calculus and the physics ideas of center of mass, center of buoyancy, and torque. The design was started as simple and was adjusted to the needs of the Mathematica calculations and utilized to find equations that define the boat. These equations were then exported to Solidworks. The parts of the boat were laser cut out of hardboard and then glued together and wrapped and vinyl. The boat was then tested for the above requirements.**

I. Background and Terminology

To understand the boat, a basic foray into the terminology and ideas behind boats and boat building was necessary. One such area needing exploration was the parts of the boat. The hull is the main body of the ship of the vessel including the bottom, sides, and deck. The bow it the forward part of the hull of the ship or boat it is a necessary proponent to drag. The stern in contrast is the after most part of the hull. The deck is an important part of the design and is the topmost portion of the boat. Where the hull meets the water is the waterline, which is essential to calculations. The buttock is the lines formed at a point of intercepts between the centerline plane and the sheer plane. A section is a division of a boat on a plane. The heel angle is the amount the vessel is tilted from upright. Center of mass (COM) is a point representing the mean position of matter. Physics calculations can be focused on that one point. The center of buoyancy(COB) is the centroid of the submerged part of the boat. This can thought of as the center of mass of the displaced water for simplicity. One facet of this project is the angle of vanishing stability (AVS). The AVS is the heel angle at which the the boat is between capsizing and self righting. It can barely balance due to the center of mass and center of buoyancy being at a equilibrium. The righting moment is the is the restoring force of the torque which is created by the center of mass and center of buoyancy to self right when the heel angle is above the AVS.

II. Design Considerations

A. Units and Conventions

All of our units were in inches and grams, and we used a right-hand, 3D coordinate system with the origin at the center of the bottom of the hull. The x-axis ran port to starboard, with starboard being positive, and divided the boat in half. The y-axis ran lengthwise, also dividing the boat in half, with the bow in the positive direction. The positive z-axis extended skyward. (See Figure 1.)

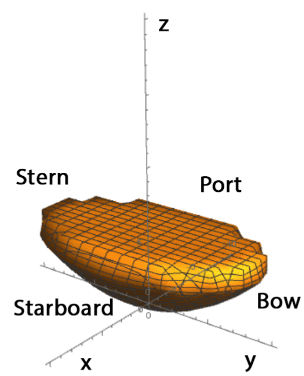

Fig. 1: Our coordinate system was defined with the x-axis running from port to starboard, the y-axis running lengthwise, and the z-axis extending vertically.

In order to simplify our calculations, we decided to model a rotating waterline rather than a rotating boat. As such, the forces of gravity and buoyancy rotated with the water. θ represented the angle of rotation of the waterline, with positive being a counter-clockwise rotation about the y-axis.

B. Making a Floating Boat

The first step in designing a boat is to make sure it's going to float. Generally speaking, when an object floats in water, the buoyant force pushing back up on that object is equal and opposite to the force pushing the object down in the water. With a boat, the downward force is the force due to gravity (Figure 2). Therefore, for a buoyant boat:

$$\vec{F}_B = -\vec{F}_{G_{boat}} \tag{1}$$

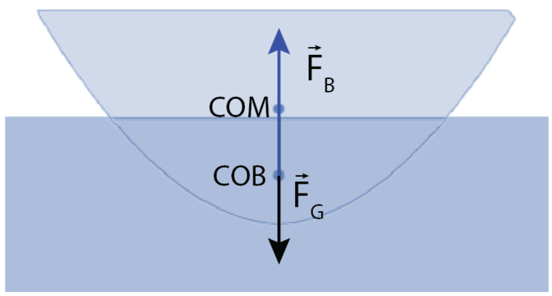

Fig. 2: When a boat is floating, the buoyant force is equal and opposite the force due to gravity acting on the boat.

Additionally, the buoyant force is equal to the mass of the water displaced multiplied by the acceleration due to gravity.

$$\vec{F}_B = m_{water} g \tag{2}$$

Since the mass of water is equal to its volume times its density, we can also write:

$$\vec{F}_B = \rho_{water} V_{water} \vec{g} \tag{3}$$

That means that in order for our boat to float, the boat must have suitable density and displace enough water such that the buoyant force can counteract the force of gravity.

C. Finding the Waterline

The buoyancy concept can be extended to find the waterline on the submerged vessel. If the floating boat has a mass, m_{boat}, then the water it displaces must also have the same mass:

$$m_{boat} = m_{waterdisplaced} \tag{4}$$

The mass of the boat can be calculated by integrating all of the point masses in the region defined by the boat. Similarly, the mass of the displaced water can be found by taking the volume of the submerged region of the boat (found by integration) and multiplying it by the density of water. (One must remember to take into account the fact that the waterline is rotated when defining the submerged region.) Combining these two concepts allows us to rewrite the above equation as follows:

$$\iint\limits_{\vec{r} \in Boat} \rho_{boat} d\vec{r} = \iint\limits_{\vec{r} \in Submerged} \rho_{water} d\vec{r} \tag{5}$$

Solving for the upper bound on the submerged region will give you the waterline.

D. Making it Sit Flat

In order to meet the second requirement, it is necessary to design the boat such that the COM and COB are aligned along the x- and y-axes when the plane of the deck is parallel to the plane of the water. Otherwise, a torque will result about the COM, as illustrated for the x-axis in Figure 3.

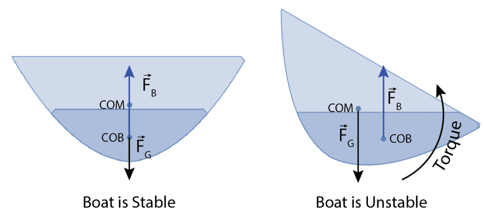

Fig. 3: Unless the COM and COB are aligned vertically, there will be a torque about the COM and the boat will rotate.

E. Determining the Angle of Vanishing Stability

In a well designed boat, heeling the boat ($\theta \neq 0$) will result in the COB moving out from under the COM in the direction the boat is leaning. This will provide a stabilizing torque, which will right the boat (Figure 4 center). In a poorly designed boat, such as one that is too narrow (Figure 4 right), the COB moves in the opposite direction and provides a destabilizing torque, causing the boat to capsize.

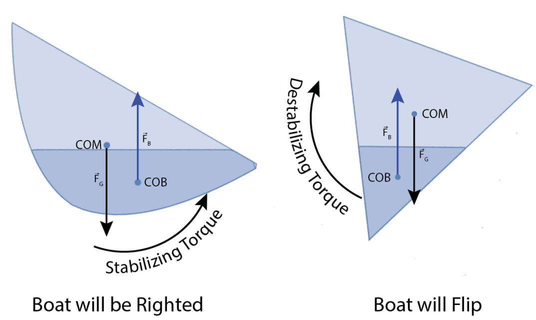

Fig. 4: The stability of a boat depends on the relative locations of the COM and COB. In order for a boat to sit flat (left), the COM and COB must be aligned in the horizontal axis. In a well designed boat (center), the COB moves in the direction the boat is leaning, and the buoyant force rights the boat. In a poorly designed boat (right), the COB moves in the opposite direction, and the buoyant force flips the boat.

To achieve the desired angle of vanishing stability (AVS), the boat should be designed such that the relation of the COM and COB shifts from the middle scenario to the right scenario at that value for θ.

1) Finding the Center of Mass: To determine the torque acting on the boat at a given heel angle, we first need to calculate the COM and COB. To calculate the COM in $\mathbb{R}3$, one can use the double integral

$$\vec{r}_{COM} = \frac{1}{M_{boat}} \iint\limits_{\vec{r} \in Boat} \rho \vec{r} \, d\vec{r} \tag{6}$$

Since M_{boat} can be found by integrating all of the point masses in the region, the above equation can be rewritten as follows:

$$\vec{r}_{COM} = \frac{\iint\limits_{\vec{r} \in Boat} \rho_{boat} \vec{r} \, d\vec{r}}{\iint\limits_{\vec{r} \in Boat} \rho_{boat} \, d\vec{r}} \tag{7}$$

2) Finding the Center of Buoyancy: In order to find the COB, which is where we assume the buoyant force to be acting, one needs to find the centroid of the submerged region of the boat. If $\vec{r} \in Submerged$ is the region bound by the water (as determined in Section II-C) and the boat, then the centroid can be computed using the following integral:

$$\vec{r}_{COB} = \frac{1}{V_{boat}} \iint\limits_{\vec{r} \in Submerged} \vec{r} \, d\vec{r} \tag{8}$$

Since V_{boat} can be found by integrating the submerged region, the above equation can be rewritten as follows:

$$\vec{r}_{COB} = \frac{\iint\limits_{\vec{r} \in Submerged} \vec{r} \, d\vec{r}}{\iint\limits_{\vec{r} \in Submerged} d\vec{r}} \tag{9}$$

Once the COB and COM have been determined, one needs to calculate the moment arm, which is the position vector of the COB relative to the COM. This can be calculated like so:

$$\vec{r}_{momentarm} = \vec{r}_{COB} - \vec{r}_{COM} \tag{10}$$

3) Calculating the Buoyant Force: When calculating the buoyant force, one must remember to account for the rotated waterline (as illustrated in Figure 5). Given that the buoyant force is equal and opposite the force of gravity and that θ represents the angle of rotation of the waterline, the components of the buoyant force acting on the heeled boat (Figure 5) can be found like so:

$$\vec{F}_{B_x} = -|\vec{F}_G| sin(\theta)$$
$$\vec{F}_{B_y} = |\vec{F}_G| cos(\theta)$$

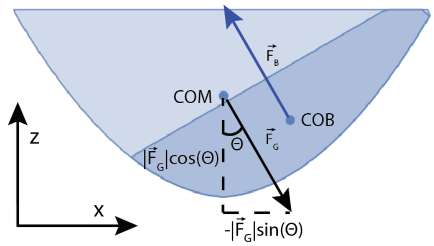

Fig. 5: When considering the buoyant force, it is important to remember it does not point directly in the positive z direction, due to the rotated waterline.

4) Determining the Moment and the AVS: With the moment arm and buoyant force vectors calculated, it's a simple enough task of calculating the resulting torque. All one has to do is cross the two vectors:

$$\vec{\tau} = \vec{r}_{momentarm} \times \vec{F}_B \tag{11}$$

Performing the calculation for a variety of heel angles, plotting the results, and checking where the moment is zero will reveal the AVS. (The result for our boat can be seen in Figure 9.)

III. Proposed Design and Justification

We initially toyed with the idea of a hull shape with a concave region, like that shown in Figure 6. However, we ran into stability problems due to the COB shifting in the wrong direction at low heel angles.

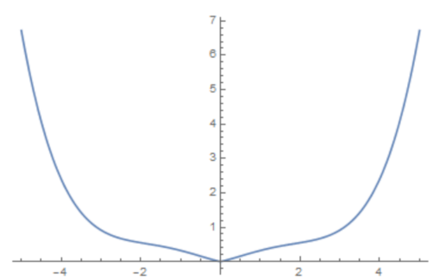

Fig. 6: Our original hull shape with concavity. We were forced to abandon this design due to stability issues.

In an attempt to fix our stability problems, we switched to a parabolic hull shape. We tweaked the coefficients on $|x|$ and x^2 with the goal of designing a stable boat with the narrowest hull possible, but we were unable to fully optimize the shape due to time constraints and issues with our Mathematica code. We ended up settling on a ”good-enough” hull definition and chose a coefficient for y^2 that would make our boat about 20 inches long. In the end, our hull declaration could be described by $z = 0.14(0.53|x| + x^2) + 0.0528y^2$. Bounding this on the top with our deck equation of

$z = 5.2$, we ended up with the following inequality to define our boat ($\mathbb{B}$) in $\mathbb{R}3$:

$$\mathbb{B} = 0.14(0.53|x| + x^2) + 0.0528y^2 \leq z \leq 5.2 \quad (12)$$

A section, buttock, and deck view can be seen in Figure 7:

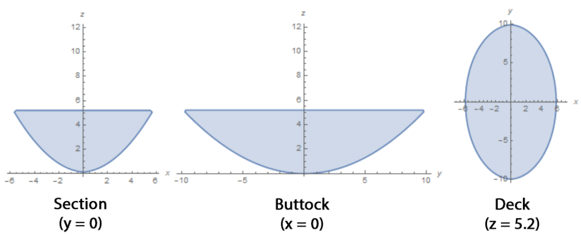

Fig. 7: Views of the hull and deck from various angles.

This inequality made our boat symmetric about the x- and y-axes, which meant our COM along those axes was zero:

$$\vec{r}_{COM_x} = 0$$
$$\vec{r}_{COM_y} = 0 \quad (13)$$

In order to find the COM along the z-axis, we needed to calculate the COM for the mast, ballast, hull sections, deck, and structural keel separately and combining the result. First we had to find the weighted sum of the mass of the hull sections, where each hull section was at one of the y positions in the set H:

$$H = \{-8, -6.75, -4.5, -2.25, 2.25, 4.5, 6.75, 8\} \quad (14)$$

To do that, we summed up the points in each region multiplied by their displacement (z) and the density of fiberboard (ρ_f):

$$W_{sections} = \sum_{y \in H} \iint_{\mathbb{B}} \rho_f z \; dz dx \quad (15)$$

Once we had the mass of the hull sections, we added that to the weighted mass sum of the structural keel, ballast, and deck, which was computed using a similar method. The resulting sum can be called $S_{weighted}$. Furthermore, if M_{boat} is the mass of the entire boat, then we can find the COM by doing the following:

$$\vec{r}_{COM_z} = \frac{S_{weighted}}{M_{boat}} \quad (16)$$

After running a few simulations, shifting our ballast up and down, and looking at our stability curve (Figure 9), we realized we would need to make the COM as low as possible to get an AVS above 120°. (A different hull equation may have fared us better, but we didn't have time to redesign the hull entirely.) Thus, we used eight solid hull sections, cut out a significant portion of the deck, and placed our 990g of ballast at the lowest point possible in our hull. Figure 8 shows a 3D CAD rendering of our boat skeleton.

Fig. 8: A rendering of our CAD boat model. Solid hull sections and a cut-out deck, combined with situating the ballast at the bottom of the hull, gave us the lowest COM.

Using a Mathematica model, we were able to predict the stability curve and AVS shown in Figure 9.

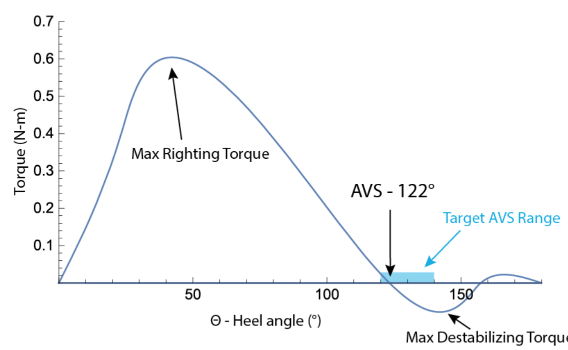

Fig. 9: Plotting the resulting torque on the boat at various heel angles revealed our AVS to be about 122°.

IV. Performance

During the test the boat was successfully able to float flat. It was completely airtight and had no leakage or issues. In addition, our AVS was 122°, which was exactly as our Mathematica model predicted (within measurement error). The final portion of this project was the drag test. Under 0.1 Newtons of force the vessel's speed was 0.401 m/s. Overall the boat performed exceedingly well.

V. Conclusion

The boat had a variety of successes and failures over the course of its development. In the end, she met all the design requirements and performed according to our calculations. She also looked beautiful on snow (Figure 10). Going forward with this subsequent improvements would be to increase the speed of the boat. Although the speed of 0.401 m/s is respectable, there are many different boat designs and shapes that outclass Steggie in this regard. Further research would need to be done towards speed based boat designs to augment the current design while continuing to meet the other requirements. As shown below the final project was assembled with lots of care as to fall in line with the calculations.

Fig. 10: Steggie showcased in the fresh snow.

Alexander Core & Kyle Combes
Quantitative Engineering Analysis Students
QEA Module II
February 23, 2017

Rebecca Christianson
Quantitative Engineering Analysis Instructor

Dear Editor,

We have addressed your concerns regarding our report. Here is a summary, broken up according to affected section:

In our abstract, we changed it so it became much more descriptive and actually detailed the entire project. Another big change was the boat requirements no longer being bullet pointed.

In our section titled “Background and Terminology,” we defined center of mass and center of buoyancy so they could be utilized to explain the angle of vanishing stability.

In our section titled “Design Considerations,” we corrected our equations so that every vector is correctly denoted by an arrow. All annotated vectors in figures had arrow hats added as well. We also reworded the sentence in Section II-C which previously stated incorrectly that gravity was always acting in the negative *z* direction. Now it correctly states it rotates with the waterline. We also added a reminder to remember the rotated waterline in Section II-C and two further mentions of the angle of rotation, ϴ, in Section II-E.

In our section titled “Proposed Design and Justification,” we added discussion on the process of settling on our particular hull shape. We also added details, including equations, on our center of mass calculation for our boat design.

In our section titled “Conclusion” we added a short sentence to tie in our picture.

We look forward to hearing your feedback on our revised edition.

Thank you for your time,

Alex Core and Kyle Combes