Problem 3

Part1: (a) I got the meaning of the data from the readme.txt x1: intercept term x2: number of cylinders x3: displacement x4: horsepower x5: weight x6: acceleration x7: model year from the program I got

Wls =

23.4282

-0.6668

0.9898

0.0974

-5.9046

0.3229

2.7529

In that case,

The number of cylinders and weight of a car draw a negative correlation to the result; i.e. with the number of cylinders go higher and weight got higher, the miles per gallon will go down. Other indexes draw positive correlation to the result, i.e. if they goes larger, the result of miles per gallon go higher.

Part1:

The result of Mean and standard deviation of MAE are listed below:

MAE Mean =

8.5719

MAE StDeviation =

P = 4:

```
Part2:
(a)
Here I attached 2 examples of the result of the program
P = 1:
RMSE_Mean =
   10.4892
RMSE_StDeviation =
    1.3803
P = 2:
RMSE_Mean =
   10.6616
RMSE_StDeviation =
    0.3453
P = 3:
RMSE_Mean =
   10.5077
RMSE_StDeviation =
    1.1251
```

```
RMSE_Mean =
   10.6404
P = 1:
RMSE_Mean =
   10.4082
RMSE_StDeviation =
    1.3495
P = 2:
RMSE_Mean =
   10.6810
RMSE_StDeviation =
    1.6637
P = 3:
RMSE_Mean =
   10.6607
RMSE_StDeviation =
    1.5144
P = 4:
RMSE_Mean =
   10.6652
```

RMSE_StDeviation =

0.5380

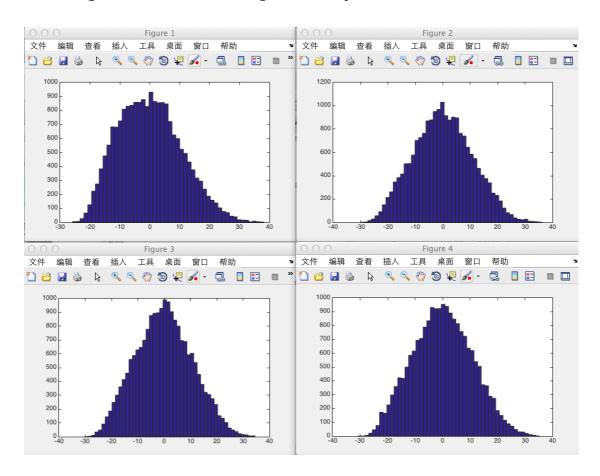
RMSE_StDeviation =

0.4450

From the examples above and my other observations, for p=1,2,3,4 the mean of RMSE is roughly the same. However, the standard deviation varies significantly with respect to p. We can see from the examples, when p=3 or p=4, the standard deviations are significantly smaller than the others. With some more examples when I run programs, p=3 is better than p=4 on most occation. So p=3 maybe the best choice.

(b)

The histograms are listed below, figure k is for p=k;



(c)

The Maximum Likelihood obey the following equation

ML:
$$\arg \max_{w} -\frac{1}{2\sigma^{2}} ||y - Xw||^{2}$$

So with this criterion, and the deriving method of in lecture, here is the result of the programming results :

P = 1:

mean =

0.0906

variance =

108.1178

P = 2:

mean =

-0.1221

variance =

113.9527

P = 3:

mean =

-0.0444

variance =

114.7182

P = 4:

```
mean =
0.0768

variance =
114.8160
```

For log likelihood function, the mean and variance still fit the former Guassian distribution. So the expected means and variance remains the same.

From the result above, we can see that when p=3, the means and variance is the smallest, so it correspond the conclusion we have in (a), i.e. p=3 may be the best choice.

Source Code Attachment

Problem 3, part 1, (a)

```
originX=importdata('X.txt');
originY=importdata('Y.txt');
% X1=transpose(originX)
% X2=inv(originX)
% randint(372,1, [1 ,392])
% originY(1,:)
All(:,1)=originY;
All(:,2:8)=originX;

nRows=size(All,1);
randRows=randperm(nRows);%# generate random ordering of row indices
AllTrain=All(randRows(1:372),:);
AllTest=All(randRows(373:end),:);
YTrain=AllTrain(:,1);
XTrain=AllTrain(:,2:8);
YTest=AllTest(:,1);
```

```
XTest=AllTrain(:,2:8);
Wls=inv(transpose(XTrain)*XTrain)*transpose(XTrain)*YTrain
```

Problem 3, part 1, (b)

```
originX=importdata('X.txt');
originY=importdata('Y.txt');
% X1=transpose(originX)
% X2=inv(originX)
% randint(372,1, [1 ,392])
% originY(1,:)
All(:,1)=originY;
All(:,2:8)=originX;
MAE=1:1000;
for i=1:1000
   nRows=size(All,1);
   randRows=randperm(nRows);%# generate random ordering of row indices
   AllTrain=All(randRows(1:372),:);
   AllTest=All(randRows(373:end),:);
   YTrain=AllTrain(:,1);
   XTrain=AllTrain(:,2:8);
   YTest=AllTest(:,1);
   XTest=AllTrain(:,2:8);
   Wls=inv(transpose(XTrain)*XTrain)*transpose(XTrain)*YTrain;
   sum1=0;
   for j=1:20
      sum1=sum1+abs(YTest(j,:)-XTest(j,:)*Wls);
   MAE(1,i) = sum1/20;
end
sum2=0;
for k=1:1000
   sum2=sum2+MAE(1,k);
end
MAE_Mean=sum2/1000;
```

Problem 3, part 2, (a)

```
originX=importdata('X.txt');
```

```
originY=importdata('Y.txt');
% X1=transpose(originX)
% X2=inv(originX)
% randint(372,1, [1 ,392])
% originY(1,:)
originX2=originX.*originX;
originX3=originX2.*originX;
originX4=originX3.*originX;
All(:,1)=originY;
All(:,2:8)=originX;
All(:,9:15)=originX2;
All(:,16:22)=originX3;
All(:,23:29)=originX4;
RMSE=1:1000;
for p=1:4
disp(['P = ',num2str(p),':'])
for i=1:1000
   nRows=size(All,1);
   randRows=randperm(nRows);
   AllTrain=All(randRows(1:372),:);
   AllTest=All(randRows(373:end),:);
   YTrain=AllTrain(:,1);
   XTrain=AllTrain(:,2:7*p+1);
   YTest=AllTest(:,1);
   XTest=AllTrain(:,2:7*p+1);
   Wls=pinv(transpose(XTrain)*XTrain)*transpose(XTrain)*YTrain;
   sum1=0;
   for j=1:20
       sum1=sum1+(YTest(j,:)-XTest(j,:)*Wls)^2;
   end
   RMSE(1,i) = sqrt(sum1/20);
end
sum2=0;
for k=1:1000
   sum2=sum2+RMSE(1,k);
end
RMSE Mean=sum2/1000;
sum3=0;
for 1=1:1000
   sum3=sum3+(RMSE(1,i)-RMSE Mean)^2;
```

```
end
RMSE_StDeviation=sqrt(sum3/1000);
RMSE_Mean
RMSE_StDeviation
end
```

Problem 3, part 2, (b)

```
originX=importdata('X.txt');
originY=importdata('Y.txt');
% X1=transpose(originX)
% X2=inv(originX)
% randint(372,1, [1 ,392])
% originY(1,:)
originX2=originX.*originX;
originX3=originX2.*originX;
originX4=originX3.*originX;
All(:,1)=originY;
All(:,2:8)=originX;
All(:,9:15)=originX2;
All(:,16:22)=originX3;
All(:,23:29)=originX4;
Error=20:1000;
for p=1:4
disp(['P = ',num2str(p),':'])
for i=1:1000
   nRows=size(All,1);
   randRows=randperm(nRows);
   AllTrain=All(randRows(1:372),:);
   AllTest=All(randRows(373:end),:);
   YTrain=AllTrain(:,1);
   XTrain=AllTrain(:,2:7*p+1);
   YTest=AllTest(:,1);
   XTest=AllTrain(:,2:7*p+1);
   Wls=pinv(transpose(XTrain)*XTrain)*transpose(XTrain)*YTrain;
   for j=1:20
      Error(j,i)=YTest(j,:)-XTest(j,:)*Wls;
   end
end
```

```
figure; hist(Error(:),50);
end
```

Problem 3, part 2, (c)

```
originX=importdata('X.txt');
originY=importdata('Y.txt');
% X1=transpose(originX)
% X2=inv(originX)
% randint(372,1, [1 ,392])
% originY(1,:)
originX2=originX.*originX;
originX3=originX2.*originX;
originX4=originX3.*originX;
All(:,1)=originY;
All(:,2:8)=originX;
All(:,9:15)=originX2;
All(:,16:22)=originX3;
All(:,23:29)=originX4;
Error=20:1000;
for p=1:4
disp(['P = ',num2str(p),':'])
for i=1:1000
   nRows=size(All,1);
   randRows=randperm(nRows);
   AllTrain=All(randRows(1:372),:);
   AllTest=All(randRows(373:end),:);
   YTrain=AllTrain(:,1);
   XTrain=AllTrain(:,2:7*p+1);
   YTest=AllTest(:,1);
   XTest=AllTrain(:,2:7*p+1);
   Wls=pinv(transpose(XTrain)*XTrain)*transpose(XTrain)*YTrain;
   for j=1:20
      Error(j,i)=YTest(j,:)-XTest(j,:)*Wls;
   end
end
% figure; hist(Error(:),50);
sum1=0;
```

```
sum2=0;

for j=1:1000
    for i=1:20
        sum1=sum1+Error(i,j);
    end
end
mean=sum1/20000

for j=1:1000
    for i=1:20
        sum1=sum1+(Error(i,j)-mean)^2;
    end
end
variance=sum1/20000
```