Student

Name:

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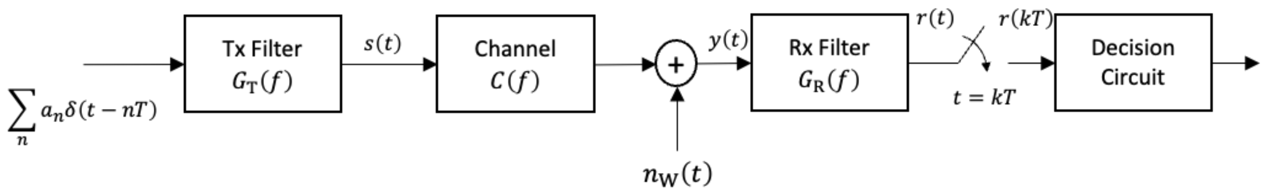
21.03.2024

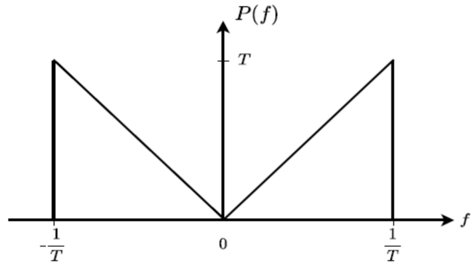
EHB 308E
COMMUNICATION II
Midterm Exam I

1. (30p) Block diagram of a binary baseband communication scheme is shown below, where $1/T$ is the symbol rate of transmission and a_n is the information symbol.



a) By defining $P(f) = G_T(f)C(f)G_R(f) = \mathcal{F}\{p(t)\}$, (i) draw an equivalent block diagram for above system. (ii) Write $r(t)$ and $r(kT)$ using the equivalent system including ISI & noise terms. (iii) Express the criterion not to have ISI on the sampled signal $r(kT)$.

b) Explain whether $P(f)$ given below has ISI or not.



c) Assume channel frequency response is flat ($C(f) = 1$), and we want to realize $P(f)$ in (b) in the system (regardless of being ISI-free or not). Determine a $G_T(f)$ and $G_R(f)$ pair that will maximize the signal-to-noise ratio (SNR) at the sampling time instants. Write the expression for $G_T(f)$ and $G_R(f)$, and draw the frequency responses.

2. (30p) Answer the following questions briefly:

a) What is the purpose of source coding in a digital communication? Consider building blocks of a PCM system. Which block/s of PCM can be put under "Source Coding"?

b) Draw the line-coded signal for bits [0 1 1 0 1 0] using alternate mark inversion (AMI). Write 2 benefits of using AMI.

c) A baseband communication signal $s(t)$ is defined below

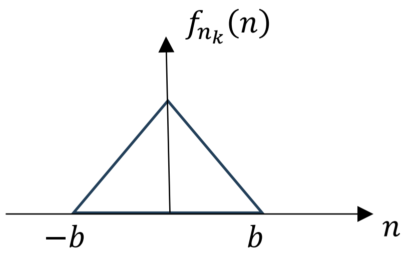
$$s(t) = \sum_{n=-\infty}^{\infty} a_n p(t - 0.025b_n - 0.1n)$$

where $p(t)$ is pulse shape, $a_n \in \{-3, -1, 1, 3\}$ and b_n are nth digital information symbols. (i) Describe the modulation scheme of $s(t)$. (ii) What is the maximum datarate of this signal can achieve (in bits per second)?

d) A digital M-PAM system will be designed using available channel bandwidth of 14 GHz. Root-raised-cosine (RRC) with excess BW parameter α is used for pulse shaping. 25 Gigabits per second is intended to be transmitted. (i) Describe possible design choices (at least 2 M and corresponding α values). (ii) If the receiver is known to be imperfect in sampling time synchronization, which design choice you would select, and why?

3. (40p) Consider a binary PAM baseband communication system where probabilities of sending bit 0 and bit 1 are P_0 and P_1, respectively. Transmitter uses rectangular pulse shape with amplitude levels $V_0 = -A$ for bit 0, and $V_1 = A$ for bit 1. Receiver samples the received signal at the middle of rectangular pulses to get sampled signal as $r_k = V_i + n_k$. PDF of the noise sample n_k is given as

$$f_{n_k}(n) = \begin{cases} \frac{b-|n|}{b^2}, & -b > n > b \\ 0, & \text{elsewhere} \end{cases}$$



a) Draw the receiver block diagram based on the description above, and write the decision rule.

b) Express the average probability of error (P_e) in terms of $P_0, P_1, A, f_{n_k}(.)$ and the decision threshold α.

c) Assuming $P_0 = 2P_1$ and $A \leq b \leq 2A$, derive P_e in terms of A, b, and α. Approximately plot P_e vs α for the range $-A \leq \alpha \leq A$.

d) By focusing on the range $\alpha \in (A-b, b-A)$, find α_{opt} that gives minimum probability of error, $P_{e_{\min}}$.

e) Find the minimum value of SNR = $\frac{A^2}{\sigma_n^2}$ in dB that will achieve error-free reception? ($\sigma_n^2 = \mathbf{E}[n_k^2]$ is the power of noise)

Useful notes:

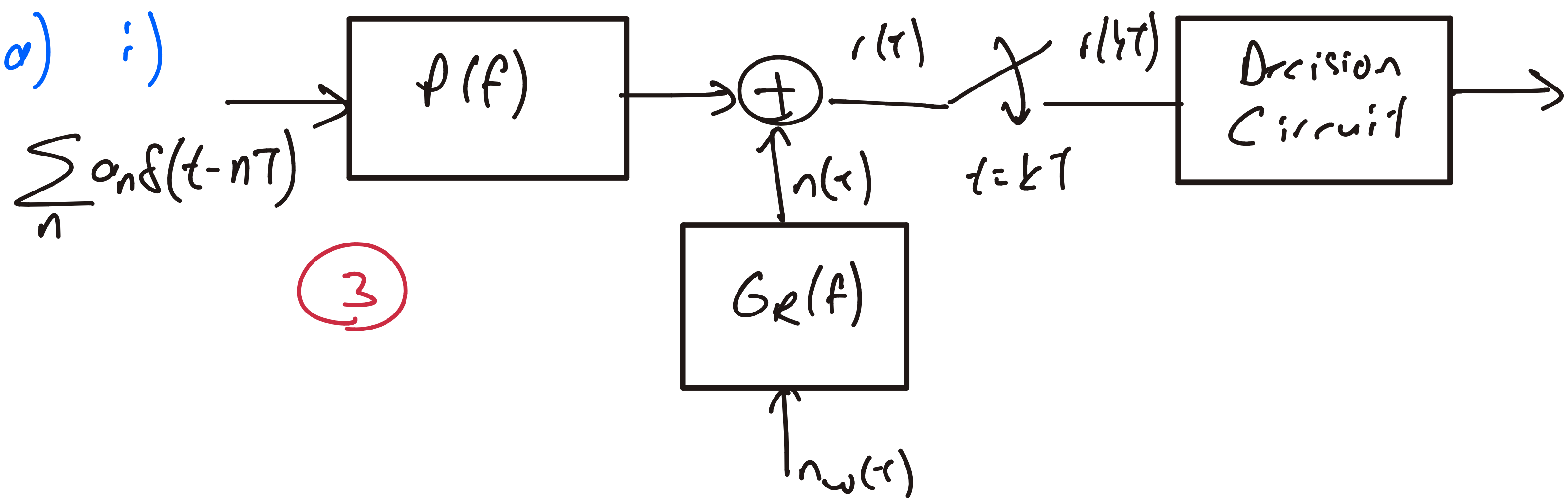
- $\text{sinc}(x) = \sin(\pi x)/(\pi x)$, $\Pi(t/\tau) \leftrightarrow \tau\text{sinc}(f\tau)$, $\text{sinc}(t\tau) \leftrightarrow (1/\tau)\Pi(f/\tau)$
- $x_{dB} = 10\log_{10} x_{linear}$

Attention:

- Please make sure your mobile phone is turned off <u>completely</u> during the exam.
- Do not leave the exam room in first 45 minutes.
- Write down you name in question <u>and</u> answer sheets and submit <u>both</u> once you are done.

All the best..

Q1) a) i)



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ii) $r(t) = \sum_n a_n \delta(t-nT) * p(t) + n(t)$ where $n(t) = n_w(t) * g_R(t)$

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$r(t) = \sum_n a_n p(t-nT) + n(t)$

$r(kT) = \sum_n a_n p(kT-nT) + n(kT) = \underbrace{a_k p(0)}_{\text{desired symbol}} + \underbrace{\sum_{n\neq k} a_n p((k-n)T)}_{\text{ISI}} + \underbrace{n_k}_{\text{noise}}$

iii) For no ISI : $\sum_{n\neq k} p((k-n)T) = 0 \Rightarrow \sum_{m\neq 0} p(mT) = 0$ $\quad m = k-n$

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Zero ISI criterion : $p(mT) = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases}$

b) If $P_\Sigma(f) = \sum_\ell P(f - \frac{\ell}{T}) = T$, then $p(t) = \mathcal{F}^{-1}\{P(f)\}$ is zero-ISI.

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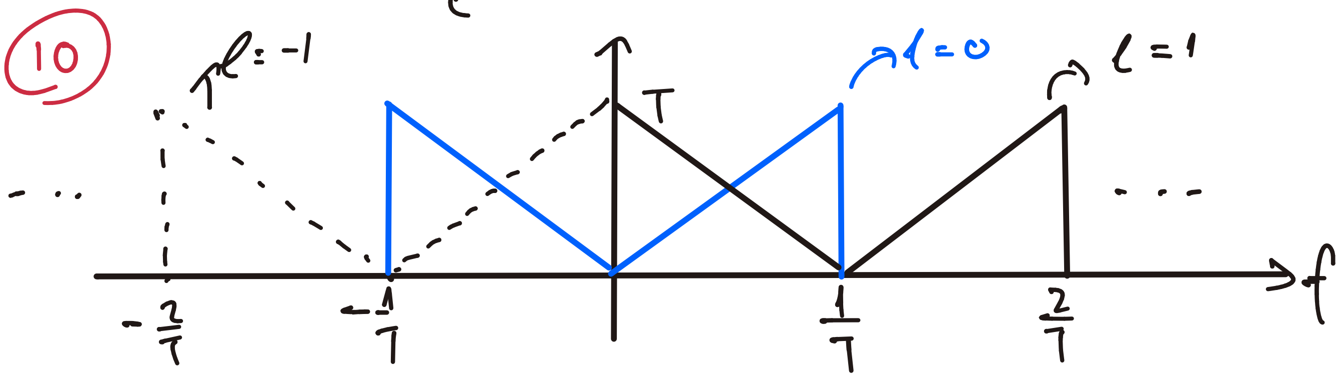
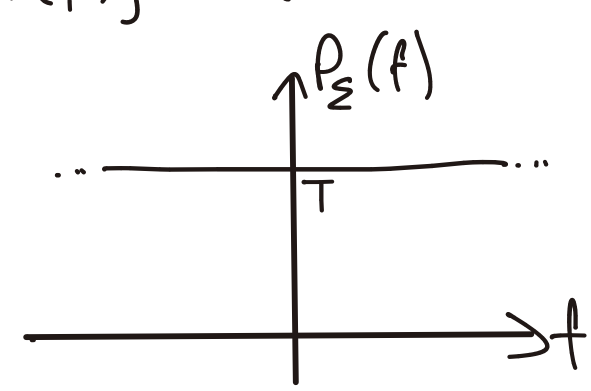


$\Rightarrow$



$\Rightarrow$ Given $P(f)$ is a zero-ISI pulse. ✓

c) Rx filter $G_R(f)$ that maximizes SNR at sampling time T is the one that is matched to Tx filter $G_T(f)$. 5

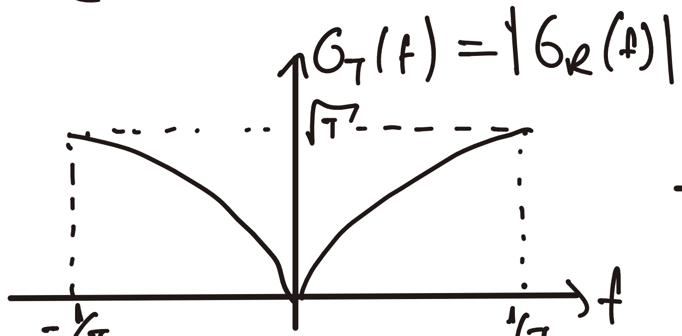
$\Rightarrow G_R(f) = G_T^*(f) e^{-j2\pi f T}$

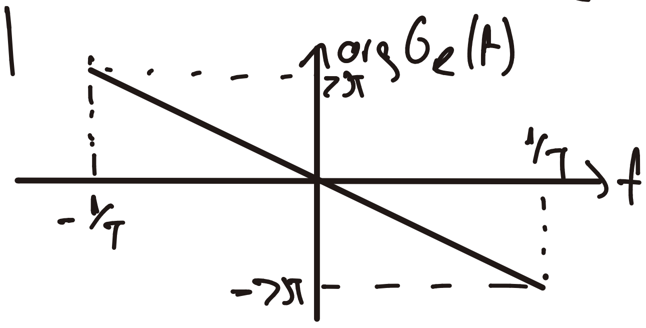
$|G_R(f)| = |G_T(f)| = \sqrt{P(f)}$

$P(f) = \begin{cases} T^2|f|, & -\frac{1}{T} \leq f \leq \frac{1}{T} \\ 0 & \text{ow} \end{cases}$

Let $G_T(f) = \sqrt{P(f)} = \begin{cases} T\sqrt{f} & 0 \leq f \leq \frac{1}{T} \\ T\sqrt{-f} & -\frac{1}{T} \leq f < 0 \\ 0 & \text{ow} \end{cases}$

Then, $G_R(f) = \begin{cases} T\sqrt{f}\, e^{-j2\pi f T}, & 0 \leq f \leq \frac{1}{T} \\ T\sqrt{-f}\, e^{-j2\pi f T}, & -1/T \leq f < 0 \\ 0, & \text{ew} \end{cases}$





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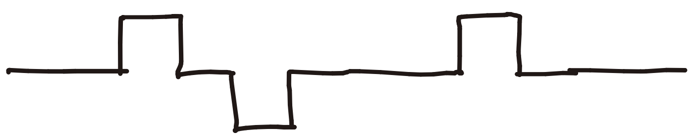
Q2) a) Source coding converts raw information into series of bits

6 it also represents data as smallest number of bits as possible by compression (reducing redundancy).

"Samples", "Quantizes" and "Coder" are parts of "Source Coding" in PCM.

b) 0 1 1 0 1 0



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- AMI can detect 1 bit error in channel.
- It has very low DC component.

c) $s(t)$ has both PAM and PPM modulation.

Each symbol has total duration of 0,1 seconds $\Rightarrow$ 10 symbols/s.

b_n modulates the "position" of pulse with 0,025 seconds steps

$\Rightarrow$ 4 possible positions $\Rightarrow$ $\underline{b_n}$ carries $\log_2 4 = 2$ bits per symbol.

& $\underline{a_n}$ has 4 possible levels $\Rightarrow$ $\log_2 4 = 2$ bits per symbol.

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Total bitrate : $\boxed{R_b = \frac{1}{T_{sym}} \times (2+2) = 40 \text{ bits per second}}$

d) $B_{ch} \geq \frac{1+\alpha}{2T}$

i) $R_b = \frac{\log_2 M}{T}$

$\rightarrow B_{ch} \geq \frac{1+\alpha}{2\log_2 M} R_b$

$14.10^9 \geq \frac{1+\alpha}{\log_2 M} 12{,}5.10^9$

6 $0 \leq \alpha \leq 1$

$\log_2 M \geq (1+\alpha)\frac{25}{28}$

Option 1 : $\boxed{M = 2 \quad \& \quad 0 \leq \alpha \leq \frac{3}{25}}$

Option 2 : $\boxed{M = 4 \quad 0 \leq \alpha \leq \frac{31}{25} \Rightarrow 0 \leq \alpha \leq 1}$

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ii) Larger α in RC

$\downarrow$

Smaller sidelobes in time beyond symbol duration

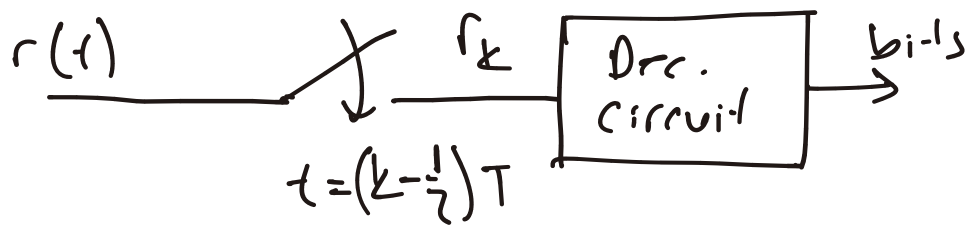
$\downarrow$

Less ISI in case of imperfect sampling -time

choice is : $\downarrow$

$\boxed{M = 4 \text{ and } \alpha = 1}$

Q3) a)



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$$\boxed{\begin{array}{l}\text{Decision rule:}\\ \text{If } r_k \geq \alpha \text{, decide "1"}\\ \text{elseif } r_k < \alpha \text{, " "0"}\end{array}}$$

b) $P_e = P_0 \times P(e \mid \text{bit 0 is sent}) + P_1 \times P(e \mid \text{bit 1 is sent})$

$= P_0 \times P(r_k \geq \alpha \mid \text{bit 0 is sent}) + P_1 \times P(r_k < \alpha \mid \text{bit 1 is sent})$

6 $\quad = P_0 \times \int_{\alpha}^{\infty} f_{r|v_0}(r)\,dr + P_1 \times \int_{-\infty}^{\alpha} f_{r|v_1}(r)\,dr$

$$\boxed{P_e = P_0 \times \int_{\alpha}^{\infty} f_n(r+A)\,dr + P_1 \times \int_{-\infty}^{\alpha} f_n(r-A)\,dr} \qquad \boxed{f_{r|v_i}(r) = f_n(r-v_i)}$$

c)

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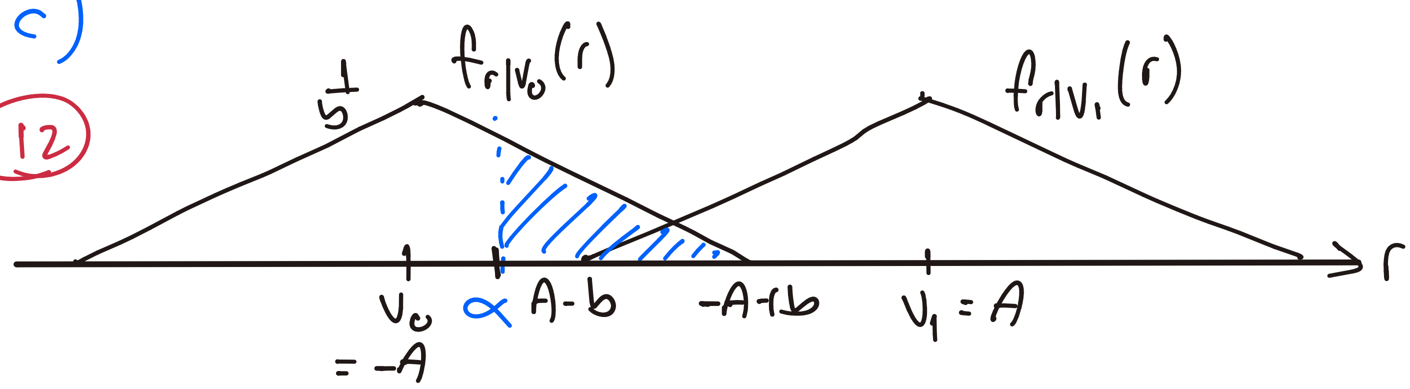


$P_0 => P_1$

We can split $(-A, A)$ range into 3 for easier calculations.

<u>for $-A < \alpha < A-b$</u>

$$P_e = P_0 \times \int_{\alpha}^{\infty} f_n(r+A)\,dr + P_1 \times \underbrace{\int_{-\infty}^{\alpha} f_n(r-A)\,dr}_{=0 \text{ for } \alpha < A-b \text{ region}} = P_0 \int_{\alpha}^{\infty} f_n(r+A)\,dr$$

$$= P_0 \int_{\alpha}^{-A+b} \frac{b-(r+A)}{b^2}\,dr$$

$$= P_0 \left(\frac{b-A}{b^2} r - \frac{1}{2b^2} r^2 \right) \Big|_{\alpha}^{-A+b} = P_0 \left[\left(\frac{(b-A)^2}{b^2} - \frac{(b-A)^2}{2b^2} \right) - \left(\frac{(b-A)\alpha}{b^2} - \frac{\alpha^2}{2b^2} \right) \right]$$

$$= P_0 \left(\frac{(b-A)^2}{2b^2} - \frac{2(b-A)\alpha}{2b^2} + \frac{\alpha^2}{2b^2} \right) = P_0 \underbrace{\frac{(b-A-\alpha)^2}{2b^2}}$$

Area of dashed region above.
It can also be found using geometry.

for $A-b \leq \alpha < b-A$:

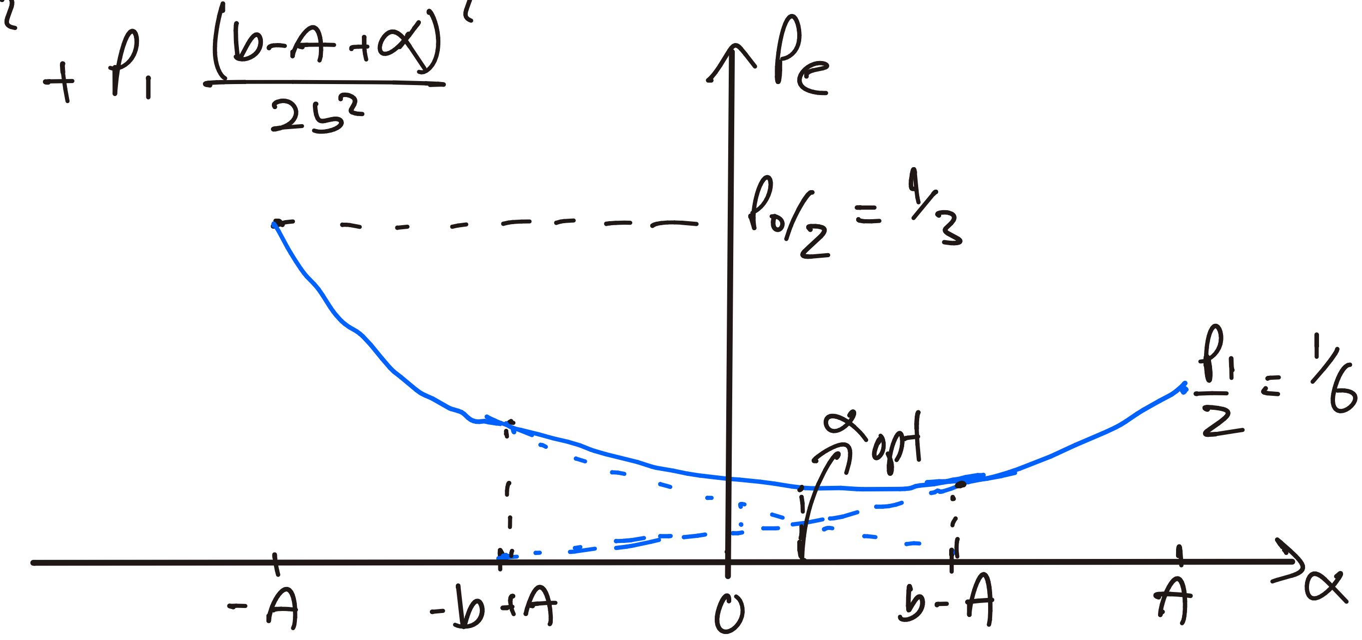
$$P_e = P_0 \times \int_{\alpha}^{\infty} f_n(r+A)\,dr + P_1 \times \int_{-\infty}^{\alpha} f_n(r-A)\,dr$$

$$= P_0 \frac{(b-A-\alpha)^2}{2b^2} + P_1 \frac{(b-A+\alpha)^2}{2b^2}$$

$P_0 = \frac{2}{3}$, $P_1 = \frac{1}{3}$

for $b-A \leq \alpha < A$:

$$P_e = P_1 \frac{(b-A+\alpha)^2}{2b^2}$$



d)

$$\frac{\partial P_e}{\partial \alpha} = \frac{\partial}{\partial \alpha}\left(\frac{P_0(\alpha^2 - 2\alpha(b-A) + (b-A)^2) + P_1(\alpha^2 + 2\alpha(b-A) + (b-A)^2)}{2b^2}\right)$$

$$= \frac{1}{2b^2}\left(2\alpha P_0 - 2P_0(b-A) + 2\alpha P_1 + 2P_1(b-A)\right) = 0$$

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$$\alpha_{opt}(P_0 + P_1) = (b-A)(P_0 - P_1)$$

$$\Rightarrow \boxed{\alpha_{opt} = \frac{P_0 - P_1}{P_0 + P_1} \cdot (b-A) = \frac{b-A}{3}}$$

e) For error-free reception, $b < A$ should be satisfied.

Power of the noise with given pdf :

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$$N = \sigma_n^2 = E[n_t^2] = \int_{-\infty}^{\infty} f_n(n_t)\cdot n_t^2\,dn_t = \int_{-b}^{0} \frac{b+n_t}{b^2}\cdot n_t^2\,dn_t + \int_{0}^{b} \frac{b-n_t}{b^2}\,n_t^2\,dn_t$$

$$= \int_{-b}^{0} \frac{n_t^2}{b} + \frac{n_t^3}{b^2}\,dn_t + \int_{0}^{b} \frac{n_t^2}{b} - \frac{n_t^3}{b^2}\,dn_t = \left(\frac{n_t^3}{3b} + \frac{n_t^4}{4b^2}\right)\Bigg|_{-b}^{0} + \left(\frac{n_t^3}{3b} - \frac{n_t^4}{4b^2}\right)\Bigg|_{0}^{b}$$

$$= \frac{b^2}{3} - \frac{b^2}{4} + \frac{b^2}{3} - \frac{b^2}{4} = \frac{b^2}{6}$$

$$\boxed{SNR = \frac{A^2}{\sigma_n^2} = \frac{A^2}{b^2/6} > 6 \Rightarrow 7{,}78\ [dB]}$$