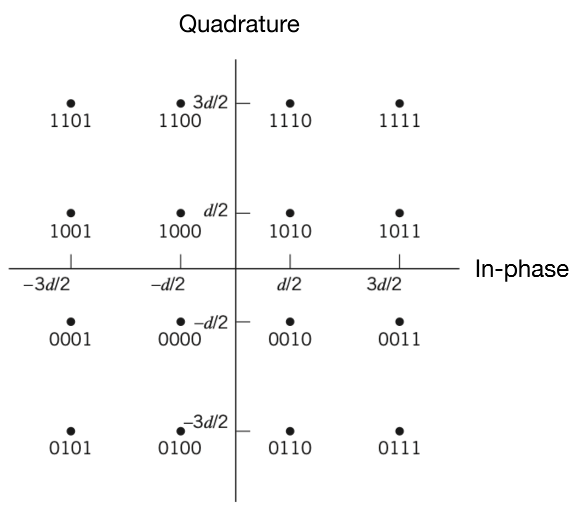
Homework 2

1. Consider a binary baseband communication system with polar NRZ line coding and rectangular transmitter pulse shaping with T seconds of pulse duration. The transmitted signal can be written as $s(t) = \sum_n a_n g_T(t - nT)$ where $a_n \in \{-1,1\}$ is nth transmitted symbol and $g_T(t) = \frac{1}{\sqrt{T}} \Pi\left(\frac{t - T/2}{T}\right)$.
   After being impacted by AWGN noise with two-sided power spectral density of $N_0/2$, the received signal passes through matched filter, sampler and decision circuit at the receiver. Due to a timing issue, the sampling at matched filter output occurs Δt seconds later than the ideal position. That is, $r_k = r(kT + \Delta t)$. Assume $0 \leq \Delta t < T$, and equal probability of bit 0 being sent vs bit 1.
   a. Derive the expression for the sampled received signal r_k as function of Δt.
   b. Obtain an expression for the average probability of error P_e.
   c. If BER target is $< 10^{-8}$ for E_b/N_0 = 12 dB, what is maximum allowable timing error Δt relative to T? (You can use MATLAB qfunc() and increase Δt value until reaching target error rate)

2. Consider an M-PSK modulation with E_s denoting the symbol energy.
   a. Show that approximate probably of symbol error in M-PSK is $P_e \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$.
   b. What is minimum bit error probability P_b in terms of E_b/N_0. Explain the condition minimizing P_b.
   c. Draw constellation diagram for $M = 16$, by labeling each point with corresponding quad-bits.

3. By using MATLAB program, do the following: Generate a 16-QAM signal with rectangular pulse shaping. Use 10 samples per symbol (e.g., represent $-1 \times g_T(t)$ as [-1 -1 -1 -1 -1 -1 -1 -1 -1 -1]). Use the given constellation for bits-to-symbol mapping. Bits to be transmitted are [0 0 1 0 1 1 1 0 0 1 0 1 0 0 0 1]. Carrier frequency $f_c = 4/T$.



   a. Plot in-phase component of 16-QAM signal.
   b. Plot phasor diagram of signal with x-axis showing in-phase part and y-axis showing quadrature part. Hint: You can use polarplot() function in MATLAB and use phase&magnitude of the signal as function arguments, or you can use plot3(X,Y,Z), with Z being time axis, and X&Y for I&Q components.
   c. Repeat steps 1 and 2 by using sinc pulse shape instead of rectangular pulse shape. Inspect the difference in curves you plotted when you use raised cosine.