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MSRI

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Dima Arinkin

Talk Title: The Geometric Langlands Correspondence

Date: 9/3/14 **Time:** 10:30 am / pm (circle one)

List 6-12 key words for the talk: properties of the Geometric Langlands correspondence, geometric Satake correspondence.

Please summarize the lecture in 5 or fewer sentences: Arinkin defines rk N vector bundles on a Riemann surface and he also defines rk N local systems on X. He describes ~~other~~ Hecke eigensheaves, Hecke category and geometric Satake equivalence and describes when the category of quasicoherent sheaves on local systems of G dual is equivalent to D-modules on G vector bundles.

CHECK LIST

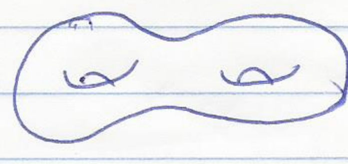
(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- ☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- ☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
  - Computer Presentations: Obtain a copy of their presentation
  - Overhead: Obtain a copy or use the originals and scan them
  - Blackboard: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
  - Handouts: Obtain copies of and scan all handouts
- ☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- ☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- ☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

The Geometric Langlands Correspondence

Dima Arinkin

Wed, Sept 3, 2014. 10:30–11:30 am

$X =$  global unramified Langlands correspondence (function field) for $GL(N)$.

$\mathrm{Bun} = \{ \text{rk } N \text{ v. bdles on } X \}$

$LS = \{ \text{rk } N \text{ l. systems on } X \}$

$$\begin{array}{ccc} \mathcal{L} & \longmapsto & \mathrm{Aut}_{\mathcal{L}} \\ \cap & & \cap \\ LS & & D\text{-mod}(\mathrm{Bun}) \end{array}$$

s.t.

- Hecke eigen property
- $\{\mathrm{Aut}_{\mathcal{L}}\}$ form a basis.

(- Parabolic induction)

$\{\mathrm{Aut}_{\mathcal{L}}\}_{\mathcal{L} \in LS}$ should form a LS-family of D-modules Aut on $LS \times \mathrm{Bun}$ is an $\mathcal{O}$-D_{Bun}-module.

Get a functor

$$\mathrm{QCoh}(\mathcal{LS}) \longrightarrow \text{D-mod}(\mathrm{Bun})$$
$$p_{2,*}(\mathrm{Aut} \otimes p_1^*(-))$$

$$\begin{tikzcd} & \mathcal{LS} \times \mathrm{Bun} \arrow[dl, "p_1"'] \arrow[dr, "p_2"] & \\ \mathcal{LS} & & \mathrm{Bun} \end{tikzcd}$$

Derive everything!

(Naive) GLC: This is an equivalence.

Other groups

G = reductive group / $\mathbb{C}$
$G^\vee$ = Langlands dual of G.
$\mathrm{Bun}_G = \{ G\text{-bundles on } X \}$
$\mathcal{LS}_{G^\vee} = \{ G^\vee\text{-local systems on } X \}$

Aut is $\mathcal{LS}_{G^\vee}$-family of D_{Bun_G}-mods.

giving an equivalence
$$\mathrm{QCoh}(\mathcal{LS}_{G^\vee}) \longrightarrow \text{D-modules}(\mathrm{Bun}_G)$$

How to modify G-bundles at $x \in X$?

<u>Modification</u> : $\mathcal{F}_1, \mathcal{F}_2 \in Bun_G$, and

$$\mathcal{F}_1 |_{X \setminus \{x\}} \cong \mathcal{F}_2 |_{X \setminus \{x\}}$$

<u>Classified by</u>

$$G(\mathcal{O}) \backslash G(K) / G(\mathcal{O}) \qquad \begin{array}{l} K = \mathbb{C}((z)) \\ \mathcal{O} = \mathbb{C}[[z]], \end{array}$$

z is a word at x.

<u>EX</u>. $G = GL(N)$, minimal modification.

$$\begin{pmatrix} z & & 0 \\ & \ddots & \\ 0 & & \end{pmatrix}$$

<u>Classically</u> : $K = \mathbb{F}_q((z))$.

<u>Hecke alg</u> $\{$ Functions : $G(\mathcal{O}) \backslash G(K) / G(\mathcal{O}) \to \mathbb{C} \}$

algebra under convolution.

Hecke category:

$$Gr = G(\mathcal{K}) / G(\mathcal{O}) \circlearrowleft G(\mathcal{K}) \supseteq G(\mathcal{O})$$

affine Grassmannian

$$\mathcal{H} := D\text{-mod}(Gr)^{G(\mathcal{O})}$$

$$= D\text{-mod}\left(G(\mathcal{O}) \backslash G(\mathcal{K}) / G(\mathcal{O})\right)$$

- $\mathcal{H}$ is a monoidal category under convolution $* : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$

$\mathcal{H}$ acts on $D\text{-mod}(Bun_G)$

Informally:

$$\begin{tikzcd}
& \left\{ \left(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_1|_{X \setminus \{x\}} \cong \mathcal{F}_2|_{X \setminus \{x\}} \right) \right\} \arrow[dl, "p_1"'] \arrow[d, "p_2"] \arrow[dr, "\tau"] & \\
\mathcal{F}_1 \in Bun_G & \mathcal{F}_2 \in Bun_G & \text{type of modification}
\end{tikzcd}$$

$$G(\mathcal{O}) \backslash G(\mathcal{K}) / G(\mathcal{O})$$

$\mathcal{M} \in \mathcal{H}$ acts as

$$p_{1,*}\left(p_2^*(-) \otimes \tau^*(\mathcal{M})\right)$$

Geometric Satake equivalence

Thm $\mathcal{H} \simeq Rep(G^\vee)$

representations of $G^\vee$.

as monoidal cats.

Rem. For abelian categories.

This is ~~an~~ used in Hecke eigenproperty.

$\mathcal{L} \in \mathcal{LS}_{G^\vee} \longmapsto Aut_\mathcal{L} \in D\text{-mod}(Bun_G)$.

Given $M \in \mathcal{H}$,

$M * Aut_\mathcal{L} = \lambda_M \otimes Aut_\mathcal{L}$ for some

v. space λ_M (eigenvalue of M on $Aut_\mathcal{L}$)

As x varies, λ_M becomes a l. system (of v. spaces) on X.

$$\begin{array}{l} M \in \mathcal{H} \\ \downarrow \\ \rho \in Rep(G^\vee) \end{array} \quad \text{so } \boxed{\lambda_M = \rho(\mathcal{L})}$$

<u>Example</u> : $G = GL(N) = G^\vee$

minimal modification $\in {}_{G(\mathcal{O})}\backslash G(K) / G(\mathcal{O})$

$$\delta_{m,m} \in \mathcal{H}$$

$$\updownarrow$$

standard rep $\in Rep(G^\vee)$

- $QCoh(LS_{G^\vee}) \simeq D\text{-mod}(Bun_G)$