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MSRI

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Mee Seong Im **Email/Phone:** mim2@illinois.edu

Speaker's Name: Dima Arinkin

Talk Title: The Geometric Langlands Correspondence

Date: 9/2/14 **Time:** 10:30 am / pm (circle one)

List 6-12 key words for the talk: Compact Riemann surface, general linear group setting of the Geometric Langlands correspondence

Please summarize the lecture in 5 or fewer sentences: There are two sides to the geometric Langlands conjecture: the automorphic side and the Galois side, where the conjecture is precisely the correspondence between two completely different objects. We will explore this correspondence in the setting for rk N vector bundles on X; i.e., $Bun := Bun_{GL(N)} X$.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

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The Geometric Langlands Correspondence
Dima Arinkin
Tuesday Sept 2, 2014, 10:30–11:30 am

Introduction to Geometric Langlands Conjecture

<u>Day 1.</u> $GL(N)$
<u>Day 2.</u> Properties
<u>Day 3.</u> Toward proof

1. X = Compact Riemann surface
   X/k smooth projective curve $/k$
   $\mathrm{char}(k) = 0$

<u>Automorphic</u> [redacted] <u>Galois</u>

$Bun = Bun_{GL(N)} X$
$= \left\{ \begin{array}{c} \text{rk } N \text{ v. bundles} \\ \text{on } X \end{array} \right\}$

<u>Classically</u>: $X/\mathbb{F}_q$, $\mathbb{F}_q(X)$ function field of X

Functions: $Bun(\mathbb{F}_q) \to \mathbb{C}$

<u>Geometrically</u>:

Cat. of $\boxed{\text{D-modules on } Bun}$

D-modules $\parallel$ alg systems of PDEs
= quasicoh. sheaves with flat connections

Bun has to be viewed as a stack.

indexed by local systems on X

Rough goal:
Find a family of "special" D-modules on Bun that forms a "basis."

Defn Galois side
Rank N local systems on X,
is a (rk N) v. bdle on X
together with a flat connection.
Classically, $\pi_1(X) \to GL(N)$.

$$LS = \{\text{loc. sys}\}$$

(2) Example (Geometric class field theory)
$N=1$ (rk 1 v. bdle on X)

Bun $= J =$ Jacobian of X
Starting from a rk 1 loc system ℓ
on X $\longleftrightarrow$ Aut_ℓ : D-module on J.

ℓ is given by $Mon(\ell) : \pi_1(X) \to \mathbb{C}^*$

$$\pi_1(J) = H_1(X) = \pi_1(X)^{ab}$$

Define Aut_ℓ to be
rk 1 loc. system on J s.t. $Mon(Aut_\ell)$

③ How are these $Aut_{\mathcal{L}}$ special?

$\mathcal{L} \in \mathcal{LS}$

$\downarrow$

$Aut_{\mathcal{L}}$ is a D-module on Bun.

Hecke eigenproperty.

<u>Ex</u> $N=1$.

Fix $x \in X$; it gives $t_x : Bun \to Bun$

$E \mapsto E(x)$

Classically,

$H := Bun(\mathbb{F}_q)$, operators $\circlearrowright t_x$

t_x^* act on $\{\text{functions } H \to \mathbb{C}\}$,

$(x \in X)$

find eigenvectors.

Geometrically:

$t : X \times Bun \to Bun$

$(x, E) \longmapsto E(x)$

Hecke eigenproperty:

$$\boxed{\ell \boxtimes Aut_\ell = t^*(Aut_\ell)}$$ canonical isom.

$$\ell_x \otimes Aut_\ell = t_x^*(Aut_\ell)$$

What happens if $N > 1$?

Universal Hecke correspondence

$$\mathrm{Hecke} = \{ (\underset{\in \mathrm{Bun}}{E \to E_1} \text{ s.t. } 0 \to E \to E_1 \to E_1/E) \}$$

E_1/E = length 1 sheaf at $x \in X$

$$\begin{tikzcd} & \mathrm{Hecke} \arrow[dl, "p"'] \arrow[dr, "p_1"] & \\ X \times \mathrm{Bun} & & \mathrm{Bun} \end{tikzcd}$$

$(x, E) \mapsfrom (E \to E_1) \mapsto E_1$

Hecke eigenproperty $\quad \mathcal{L} \in \mathcal{LS}$

$$\mathcal{L} \boxtimes \mathrm{Aut}_{\mathcal{L}} = p_*(p_1^!(\mathrm{Aut}_{\mathcal{L}}))$$

$\mathcal{L}$ ← eigenvalues

$\mathrm{Bun}(\mathbb{F}_q)$