

1.- Programar la forma estándar, directa, serie y paralelo de la siguiente función de transferencia

$$G(z) = \frac{uz^{-3} + (2-u)z^{-2} + (u-3)z^{-1} + (1-\frac{u}{4})}{5uz^{-3} + (6-u)z^{-2} + (7-u)z^{-1} + (1-\frac{u}{8})}$$

u: último número del numero de cuenta del alumno

EQUIPO 2

u=2

Solución:

$$G(z) = \frac{(2)z^{-3} + (2-2)z^{-2} + (2-3)z^{-1} + (1-\frac{2}{4})}{5(2)z^{-3} + (6-2)z^{-2} + (7-2)z^{-1} + (1-\frac{2}{8})}$$

$$G(z) = \frac{2z^{-3} - 1z^{-1} + (\frac{4}{4} - \frac{2}{4})}{10z^{-3} + (4)z^{-2} + 5z^{-1} + (\frac{8}{8} - \frac{2}{8})}$$

$$G(z) = \frac{2z^{-3} - z^{-1} + (\frac{2}{4})}{10z^{-3} + 4z^{-2} + 5z^{-1} + (\frac{6}{8})}$$

$$G(z) = \frac{2z^{-3} - z^{-1} + \frac{1}{2}}{10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}} \text{ (simplificada)}$$

PROGRAMACIÓN FORMA DIRECTA

$$G(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-3} - z^{-1} + \frac{1}{2}}{10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}}$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-3} - z^{-1} + \frac{1}{2}}{10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}}$$

$$\frac{Y(z)\left(10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}\right)}{1} = \frac{X(z)\left(2z^{-3} - z^{-1} + \frac{1}{2}\right)}{1}$$

$$Y(z)\left(10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}\right) = X(z)\left(2z^{-3} - z^{-1} + \frac{1}{2}\right)$$

$$10Y(z)z^{-3} + 4Y(z)z^{-2} + 5Y(z)z^{-1} + \frac{3}{4}Y(z) =$$

$$2X(z)z^{-3} - X(z)z^{-1} + \frac{1}{2}X(z)$$

$$+ \frac{3}{4}Y(z) =$$

$$2X(z)z^{-3} - X(z)z^{-1} + \frac{1}{2}X(z) - \left(10Y(z)z^{-3} + 4Y(z)z^{-2} + \frac{3}{4}Y(z)\right)$$

$$+ Y(z) =$$

$$\frac{4}{3}\left[2X(z)z^{-3} - X(z)z^{-1} + \frac{1}{2}X(z) - \left(10Y(z)z^{-3} + 4Y(z)z^{-2} + \frac{3}{4}Y(z)\right)\right]$$

$$+ Y(z) =$$

$$\frac{4}{3}\left[2X(z)z^{-3} - X(z)z^{-1} + \frac{1}{2}X(z) - 10Y(z)z^{-3} - 4Y(z)z^{-2} - \frac{3}{4}Y(z)\right]$$

$$Y(z) =$$

$$\frac{8}{3}X(z)z^{-3} - \frac{4}{3}X(z)z^{-1} + \frac{4}{6}X(z) - \frac{40}{3}Y(z)z^{-3} - \frac{16}{3}Y(z)z^{-2} - \frac{3}{4}Y(z)$$

$$Y(z) =$$

$$\frac{8}{3}X(z)z^{-3} - \frac{4}{3}X(z)z^{-1} + \frac{2}{3}X(z) - \frac{40}{3}Y(z)z^{-3} - \frac{16}{3}Y(z)z^{-2} - \frac{3}{4}Y(z)$$

**DIAGRAMA DE BLOQUES
PROGRAMACION DIRECTA**

PROGRAMAR FORMA ESTANDAR

$$G(z) = \frac{2z^{-3} - z^{-1} + \frac{1}{2}}{10z^{-3} + 4z^{-2} + 5z^{-1} + \frac{3}{4}} \quad (\text{simplificada})$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}}{30z^{-3} + z^{-1} + \frac{1}{4}}$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{V(z)} \frac{V(z)}{X(z)} = \frac{6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}}{30z^{-3} + z^{-1} + \frac{1}{4}}$$

$$\frac{Y(z)}{V(z)} \frac{V(z)}{X(z)} = \left(\frac{6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}}{1} \right) \left(\frac{1}{30z^{-3} + z^{-1} + \frac{1}{4}} \right)$$

Donde:

$$\frac{Y(z)}{V(z)} = 6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}$$

$$\frac{V(z)}{X(z)} = \frac{1}{30z^{-3} + z^{-1} + \frac{1}{4}}$$

De aquí, hacemos un cambio de variable para Y(z)
y X(z), (despeje)

$$Y(z) = V(z) \left(6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2} \right)$$

$$Y(z) = 6V(z)z^{-3} - 4V(z)z^{-2} + 3V(z)z^{-1} - \frac{1}{2}V(z)$$

$$V(z) \left(30z^{-3} + z^{-1} + \frac{1}{4} \right) = X(z)$$

$$\left(30V(z)z^{-3} + V(z)z^{-1} + \frac{1}{4}V(z) \right) = X(z)$$

$$\left(+ \frac{1}{4}V(z) \right) = X(z) - \left(30V(z)z^{-3} + V(z)z^{-1} \right)$$

$$\left(+ \frac{1}{4}V(z) \right) = X(z) - 30V(z)z^{-3} - V(z)z^{-1}$$

$$V(z) = 4 \left[X(z) - 30V(z)z^{-3} - V(z)z^{-1} \right]$$

$$V(z) = 4X(z) - 120V(z)z^{-3} - 4V(z)z^{-1}$$

**DIAGRAMA DE BLOQUES
PROGRAMACION STANDAR**

**PROGRAMACIÓN FORMA SERIE O
CASCADA**

$$H(z) = \prod_{k=1}^K H_k(z)$$

$$K = \frac{N+1}{2}$$

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2} + b_{k3}z^{-3}}{1 + a_{k0} + a_{k1}z^{-1} + a_{k2}z^{-2} + a_{k3}z^{-3}}$$

Tenemos G(Z):

$$G(z) = \frac{6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}}{30z^{-3} + z^{-1} + \frac{1}{4}} \text{ (simplificada)}$$

Obteniendo polos y ceros de la función de transferencia:

$$G(z) = \frac{(z^{-1} - 0.205666)(z^{-1} - [0.2305 + j0.5933])(z^{-1} - [0.2305 - j0.5933])}{(z^{-1} + 0.1496)(z^{-1} - [0.07479 + j0.2239])(z^{-1} - [0.07479 - j0.2239])}$$

Multiplicando y dividiendo por sus factores respectivos tenemos:

$$G(z) = \frac{(0.205666z^{-1} - 1)(z^{-1} - [0.2305 + j0.5933])(z^{-1} - [0.2305 - j0.5933])}{(z^{-1} + 0.1496)(z^{-1} - [0.07479 + j0.2239])(z^{-1} - [0.07479 - j0.2239])}$$

MEJOR:

$$G(z) = \frac{6 - 4z + 3z^2 - \frac{1}{2}z^3}{30 + z^2 + \frac{1}{4}z^3}$$

$$G(z) = \frac{-\frac{1}{2}z^3 + 3z^2 - 4z + 6}{\frac{1}{4}z^3 + z^2 + 30}$$

$$G(z) = \frac{(z - [0.5689 + j1.4644])(z - [0.5689 - j1.4644])(z - 4.86)}{(z - [1.3425 + j4.0184])(z - [1.3425 + j4.0184])(z - 6.68)}$$

Dividiendo entre Z

$$G(z) = \frac{(1 - [0.5689 + j1.4644]z^{-1})(1 - [0.5689 - j1.4644]z^{-1})(1 - 4.86z^{-1})}{(1 - [1.3425 + j4.0184]z^{-1})(1 - [1.3425 + j4.0184]z^{-1})(1 - 6.68z^{-1})}$$

Y un posible empaquetamiento de los polos y ceros sería:

$$(1 - 0.5689z^{-1} - j1.4644z^{-1})(1 - 0.5689z^{-1} + j1.4644z^{-1})$$

$$1 - 0.5689z^{-1} - j1.4644z^{-1} - 0.5689z^{-1} + (0.5689)^2 z^{-2} + (0.5689z^{-1})(j1.4644z^{-1}) + j1.4644z^{-1} - (0.5689z^{-1})(j1.4644z^{-1}) - (j1.4644z^{-1})^2$$

$$1 - 0.5689z^{-1} - 0.5689z^{-1} + (0.5689)^2 z^{-2} - (j1.4644z^{-1})^2$$

$$1 - 2(0.5689z^{-1}) + (0.5689)^2 z^{-2} + 2.1445z^{-2}$$

$$1 - 1.1378z^{-1} + 2.4681z^{-2}$$

$$G(z) =$$

$$\frac{(1 - [0.5689 + j1.4644]z^{-1})(1 - [0.5689 - j1.4644]z^{-1})}{(1 - [1.3425 + j4.0184]z^{-1})(1 - [1.3425 - j4.0184]z^{-1})}$$

$$(1 - 1.3425z^{-1} - j4.0184z^{-1})$$

$$(1 - 1.3425z^{-1} + j4.0184z^{-1})$$

$$1 - 1.3425z^{-1} - 1.3425z^{-1}$$

$$+ (1.3425)^2 z^{-2}$$

$$- (j4.0184z^{-1})^2$$

$$1 - 2.685z^{-1}$$

$$+ 1.8023z^{-2}$$

$$+ (16.1475z^{-2})$$

$$1 - 2.685z^{-1} + 17.9498z^{-2}$$

$$1 - 1.1378z^{-1} + 2.4681z^{-2}$$

$$G(z) =$$

$$\frac{(1 - 1.1378z^{-1} + 2.4681z^{-2})(1 - 4.8622z^{-1})}{(1 - 2.685z^{-1} + 17.9498z^{-2})(1 + 6.6851z^{-1})}$$

$$(1 - 2.685z^{-1} + 17.9498z^{-2})(1 + 6.6851z^{-1})$$

$$G_1(z) = \frac{(1 - 1.1378z^{-1} + 2.4681z^{-2})}{(1 - 2.685z^{-1} + 17.9498z^{-2})}$$

$$G_2(z) = \frac{(1 - 4.8622z^{-1})}{(1 + 6.6851z^{-1})}$$

$$G(z) = G_1(z)G_2(z)$$

DIAGRAMA DE BLOQUES
PROGRAMACION STANDAR

PROGRAMACIÓN FORMA PARALELO

$$G(z) = G_1(z) + G_2(z) + G_3(z) + \dots$$

$$G(z) = \frac{(1 - 1.1378z^{-1} + 2.4681z^{-2})(1 - 4.8622z^{-1})}{(1 - 2.685z^{-1} + 17.9498z^{-2})(1 + 6.6851z^{-1})}$$

$$G(z) = \frac{(z^2 - 1.1378z + 2.4681)(z - 4.8622)}{(z^2 - 2.685z + 17.9498)(z + 6.6851)}$$

$$\frac{(z^2 - 1.1378z + 2.4681)(z - 4.8622)}{(z + 6.6851)(z^2 - 2.685z + 17.9498)} = \frac{A}{(z + 6.6851)} + \frac{Bz + C}{(z^2 - 2.685z + 17.9498)}$$

Multiplicando por el minimo común múltiplo:

$$(z^2 - 1.1378z + 2.4681)(z - 4.8622) = A(z^2 - 2.685z + 17.9498) + (Bz + C)(z + 6.6851)$$

$$\begin{aligned} (z^3 - 6z^2 + 8z - 12) &= A(z^2 - 2.685z + 17.9498) + (Bz + C)(z + 6.6851) \\ (z^3 - 6z^2 + 8z - 12) &= Az^2 - 2.685Az + 17.9498A + (Bz + C)(z + 6.6851) \end{aligned}$$

$$\begin{aligned} (z^3 - 6z^2 + 8z - 12) &= Az^2 - 2.685Az + 17.9498A + Bz^2 + 6.6851Bz + Cz + 6 \\ (z^3 - 6z^2 + 8z - 12) &= Az^2 + Bz^2 + 6.6851Bz + Cz - 2.685Az + 17.9498A + 6 \\ (z^3 - 6z^2 + 8z - 12) &= (A + B)z^2 + (6.6851B + C - 2.685A)z + (17.9498A + 6) \end{aligned}$$

igualando terminos:

$$1 = 0 \text{ } \underline{\hspace{1cm}} \text{ } ERROR$$

$$(A + B) = -6$$

$$(6.6851B + C - 2.685A) = 8$$

$$(17.9498A + 6.6851C) = -12$$

$$A = -4.1398$$

$$B = -1.8602$$

$$C = 9.3204$$

$$G(z) = \frac{-4.1398}{(z + 6.6851)} + \frac{-1.8602z + 9.3204}{(z^2 - 2.685z + 17.9498)}$$

2.- Determinar la estabilidad de $G(z)$ por la prueba de Jury

$$G(z) = \frac{6z^{-3} - 4z^{-2} + 3z^{-1} - \frac{1}{2}}{30z^{-3} + z^{-1} + \frac{1}{4}}$$

$$D(z) = 30z^{-3} + z^{-1} + \frac{1}{4}$$

$$D(z) = 30 + z^2 + \frac{1}{4}z^3$$

Renglón	Z^0	z	Z^2	z^3
1	30	0	1	0.25
2	0.25	1	0	30
3	899.9375	-0.25	30	