HW5

Rules & Instruction

- Compiler of programming problem.
  - C : `gcc -DONLINE_JUDGE -O2 -w -fmax-errors=3 -std=c11 main.c -lm -o main`
  - C++: `g++ -DONLINE_JUDGE -O2 -w -fmax-errors=3 -std=c++17 main.cpp -lm -o main`
  - Execution: `./main`
- Can I use theorems that haven't been mentioned in class?
  - Programming: No restriction. (無限制)
  - Handwritten: Please include its proof. (請寫上證明。)
- Can I refer to resources from the Internet or other sources that are not from textbooks or lecture slides?
  - Yes, but you must specify the references (the Internet URL you consulted with or the name and the page number of books). (可以，但必須附上參考來源，如網址或書名和頁數)
    - Handwritten: specify the references next to your solution. (手寫題請將參考附註於你的答案旁，你可以使用短網址工具)
    - Programming: specify the references at the top of your code in comments. (程式題請將參考以註解的方式附註於程式碼的最上方)
  - Although you can use external resources, it doesn't mean you can just paste the resources as your solution. **You have to answer by yourself**, and remember to specify the references; otherwise we'll view it as cheating. (雖然參考外部資源是允許的，但請用自己的話去作答，並切記要註記參考來源，否則會以作弊/抄襲處理)
- Rules of cheating.
  - Programming: We use tools to monitor code similarity. (我們用機器抓抄襲)
  - Handwritten: Though we forbid plagiarism, we still encourage you to discuss with each other. Remember that after discussion you must answer problems **in your own words**. (禁止抄襲，鼓勵大家有問題可以互相討論，**但請用自己的話去作答**。)
  - **Copycats will be scored 0.** You also need to have a cup of coffee with Professor Yeh. (**抄襲者與被抄襲者當次作業零分**，另外會跟教授喝杯咖啡☕。)
    - People who committed twice or more will be punished. (情節嚴重者，如累犯...，以校規處置)
- Rules of delay.
  NO LATE SUBMISSION IS ALLOWED.
  That is, zero toleration of late submission.

Non-Programming

1. Turnip Delivery

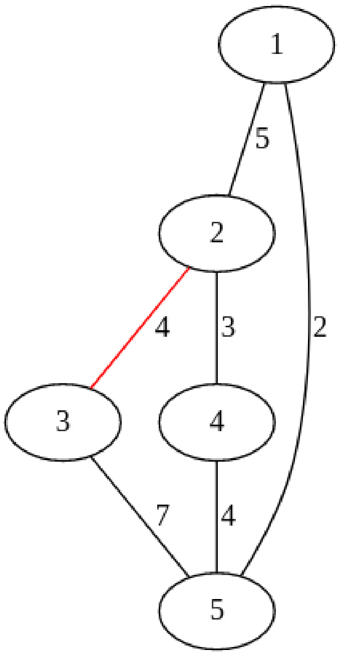
In [Animal Crossing](), there are many islands, and some of them can reach each other directly by airplane.

An airplane connects two different islands and has a weight limit of w.
You want to take some turnips to another island, but the total weight of turnips must not be higher than w, or the airplane will crash.

What is the maximum weight of turnips you can deliver from island A to island B?

It's guaranteed that all islands are connected, that is, you can reach any island from any other one by some **path**s.

For example, in $Figure\ 1$, the maximum weight of turnips we can deliver from 1 to 5 is 4.



$Figure\ 1$

There are several paths from 1 to 5:

- $1 \to 5$
  The maximum weight of turnips we can deliver through this path is 2.
- $1 \to 2 \to 4 \to 5$
  The maximum weight of turnips we can deliver through this path is 3.
- $1 \to 2 \to 3 \to 5$
  The maximum weight of turnips we can deliver through this path is 4.

So, the maximum weight of turnips you can deliver from 1 to 5 is 4.

(1). $(5\ points)$
Given a connected graph with N nodes, how many edges are in the **maximum spanning tree** of this graph?

(2). $(10\ points)$
Please draw down the maximum spanning tree of the graph in $Figure\ 1$.

What is the relationship between MST and this problem?
After finding out the MST, the path from A to B is unique. **Moreover, the answer (the maximum weight of turnips) is the minimum weight limit airplane on the path from A to B.**

(3). $(15\ points)$
Try to explain the reason why the answer (the essential airplane) is always in MST.
What properties of MST can help you to solve this problem?

> In pA, the question is a part of this problem: find out the minimum weight limit airplane from A to B in the given tree.
> In Bonus, the question is the same as this problem.

2. Lexicographical order AGAIN!

> In HW4, we've learned lexicographical order. If you forget the definition, make sure to review before answering the following questions.
> You can view its definition in the statement of HW4.

Someday, we forget the order of the English alphabet (the 26 letters: $a, b, c, \ldots, z$).
Luckily, some lexicographical orders of some strings had been recorded. Can we analyze the correct order of the letters?

For example:

$\mathrm{dcd} < \mathrm{dcc} < \mathrm{bddd} < \mathrm{bbbbc} < \mathrm{bbbba} < \mathrm{c}$

We can find out some information from the relationship.

- From $\mathrm{dcd} < \mathrm{bddd} < \mathrm{c}$, we can know $d < b < c$, because the first letters of them are different.
- From $\mathrm{bbbbc} < \mathrm{bbbba}$, we can know $c < a$.

So the correct order of the letters is $d < b < c < a$.

(1). $(15\ points)$
Another lexicographical order of some strings was found.

$\mathrm{zyx} < \mathrm{yzx} < \mathrm{yzy} < \mathrm{yxv} < \mathrm{wwww} < \mathrm{wwvx}$

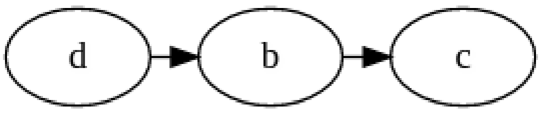
Please find out the correct order of the letters v, w, x, y, z by observation.
Show your work just like the above demonstration.

Actually, there's an algorithm that can solve this kind of problem.

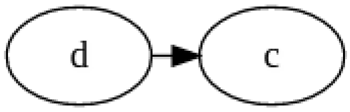
For the same example above, firstly, left align these strings.

1. dcd
2. dcc
3. $bddd$
4. $bbbbc$
5. $bbbba$
6. c

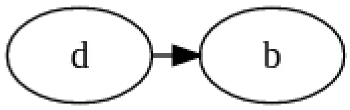
For the first letter of every string, we can create a graph to indicate their relationship. The edges point to the letter that has a higher order.



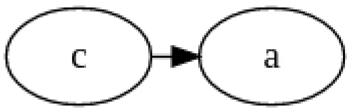
From the second letter, note that you can only create relationships with strings that **have the same prefix**.



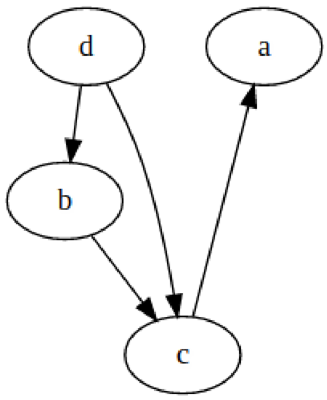
From the third letter between first and second string.



From the second letter between third and fourth string.



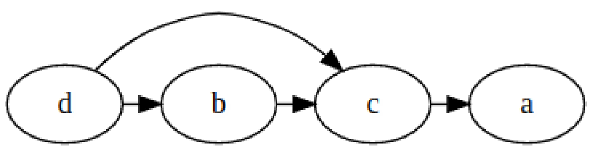
From the fifth letter between fourth string and fifth string.



Finally, combine all the relationships.

If there exists at least one solution, the combined graph should be a **DAG** (Directed Acyclic Graph).

Find a topological sorting of the DAG.



And we get the answer $d < b < c < a$.

(2). $(20\ points)$
Same as the previous question, for the following lexicographical order of strings:

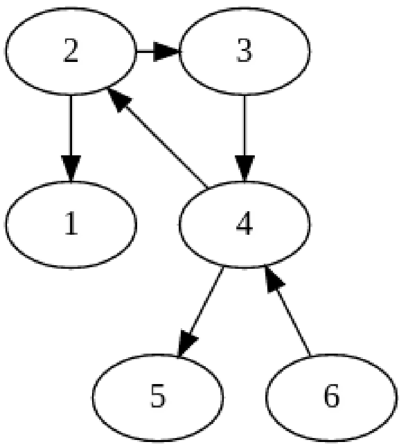
$\mathrm{zyx} < \mathrm{yzx} < \mathrm{yzy} < \mathrm{yxv} < \mathrm{wwwww} < \mathrm{wwvxy}$

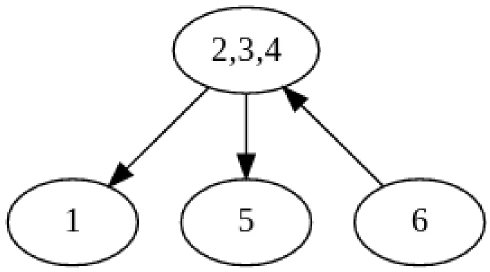
Please find out the correct order of the letters v, w, x, y, z, but this time using the topological sorting and show your work.

3. Strongly Connected Components

A directed graph is **strongly connected** if there is a path between all pairs of vertices. The **strongly connected components** of a directed graph form a partition into subgraphs that are themselves strongly connected.

Given a directed graph G, we can shrink the strongly connected component down to a single vertex. Here is an example.

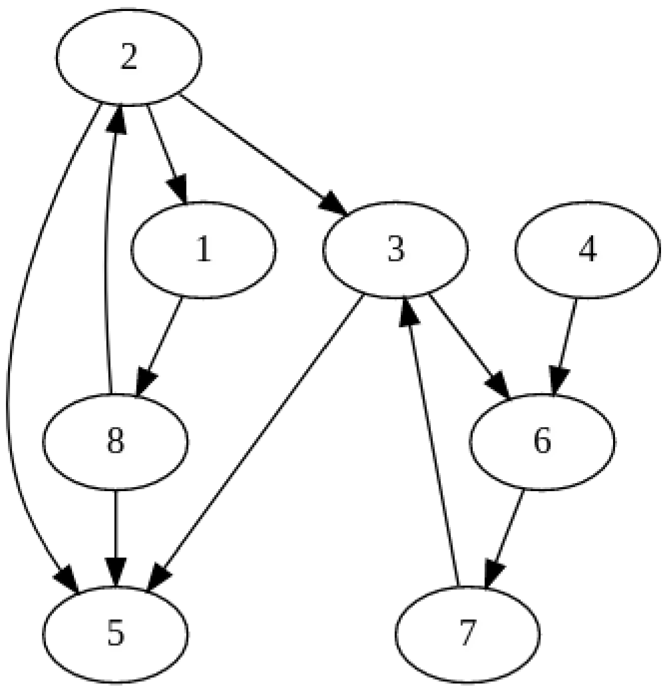




Since the vertices $2, 3, 4$ are strongly connected, they can be shrunk into a vertex (right graph).

After shrinking the graph, you will get a **DAG** (Directed Acyclic Graph).

(1). $(15\ points)$
Try to shrink all the SCCs in the graph below, and draw the result.
Note that the graph will transform into a DAG.



(2). $(15\ points)$
Denote the graph whose SCCs are all shrunk as G^{SCC}.

Prove that $(G^{SCC})^T = (G^T)^{SCC}$.

G^T is the transpose of G, G^T is obtained by reversing the directions of all edges in graph G (in lecture slide: graph.pptx, page 44).

Programming

pA: Turnip Delivery

> Time Limit: 1s
> Memory Limit: 262144 KB

Description

> Please check out the first problem of handwritten if you haven't.

In Animal Crossing, there are many islands, and some of them can reach each other directly by airplane.

An airplane connects two different islands and has a weight limit of w.
You want to take some turnips to another island, but the total weight of turnips must not be higher than w, or the airplane will crash.

We wonder what is the maximum weight of turnips you can deliver from island A to island B.

It's guaranteed that all islands are connected, and they form a **Tree**. So, there is no cycle in the given graph, and the path between any two islands is unique.
Note that all the airplanes are bidirected.

Input Format

The first line contains an integer N, indicating the number of islands.
For the next $N-1$ lines, each line contains three space-separated integers u_i, v_i, and w_i, indicating there is an airplane between the island with the number u_i and the island with v_i and the airplane has a weight limit of w_i.
Last line contains two space-separated integers s and t, indicating we want to deliver turnips from island s to t.

For all test data, it is guaranteed:

- $1 \leq N \leq 3 \times 10^3$
- $1 \leq u_i, v_i \leq N,\ u_i \neq v_i$
- $1 \leq w_i \leq 10^4$
- $1 \leq s, t \leq N,\ s \neq t$

Subtask1 (25%)

- $1 \leq N \leq 3$

Subtask2 (75%)

- No other restrictions.

Output Format

Output a number, indicating the maximum weight of turnips that can deliver from island s to t.

Sample Input

```
4
1 2 5
2 3 4
3 4 7
1 4
```

Sample Output

```
4
```

Sample Input

```
6
1 2 5
6 2 5
2 3 6
3 5 7
2 4 3
6 5
```

Sample Output

```
5
```

Hint

You can use this online tool to draw graph.

pB: Lava

> Time Limit: 1s
> Memory Limit: 262144 KB

Description

A-Hua usually stays up late playing Minecraft. In Minecraft worlds, there are many dangerous things, such as high mountains, creepers, spiders, etc.

Today, A-Hua encounters troubles because of lava, but he finds the rule of how lava flows. Now, he tries to determine when will every single land be flowed on in the Minecraft world.

There are three types of land in the Minecraft world, sizing $N \times M$. Moreover, A-Hua only wants the map before K seconds.

- type L: Lava, and there is exactly one lava land in the world. Lava will flow out 1 unit per second (the available directions are up, right, down, and left).
- type O: Obstacle, where lava cannot flow.
- type C: Common land, where lava can flow.

Please draw the map for A-Hua, which contains the status of whether lava flowing on after K seconds or not for every land so that he can safely pass it (maybe :P).

Input Format

The first line is N, M, K separated by space indicates the height, the width of the world, and the duration.
For the next $N \times M$ consisting of type L, O, C land indicates the initial status of the world.

Note that there is exactly one type L land in the world.

For all test data, it is guaranteed:

- $1 \leq N \leq 10^3$
- $1 \leq M \leq 10^3$
- $1 \leq K \leq 2^{31} - 1$
- There is exactly one type L land in the world.

Subtask1 (10%)

- $K = 1$
- $1 \leq N \leq 10$
- $1 \leq M \leq 10$
- There is no type O land in the world.

Subtask2 (25%)

- There is no type O land in the world.

Subtask3 (65%)

- No other restrictions.

Output Format

Output the map; if the land has been flowed on, please output 'X.' There is more information in the Hint part.

If the land has not flowed or cannot be flowed on by lava or it is lava originally, please output the land's initial status.

Sample Input 1

```
3 3 1
CCC
CLC
CCC
```

Sample Output 1

```
CXC
XLX
CXC
```

Sample Input 2

```
3 3 2
CCC
CLC
CCC
```

Sample Output 2

```
XXX
XLX
XXX
```

Sample Input 3

```
3 3 3
CCC
CLC
CCC
```

Sample Output 3

```
XXX
XLX
XXX
```

Sample Input 4

```
3 5 3
CCCLO
OCOCC
CCCCC
```

Sample Output 4

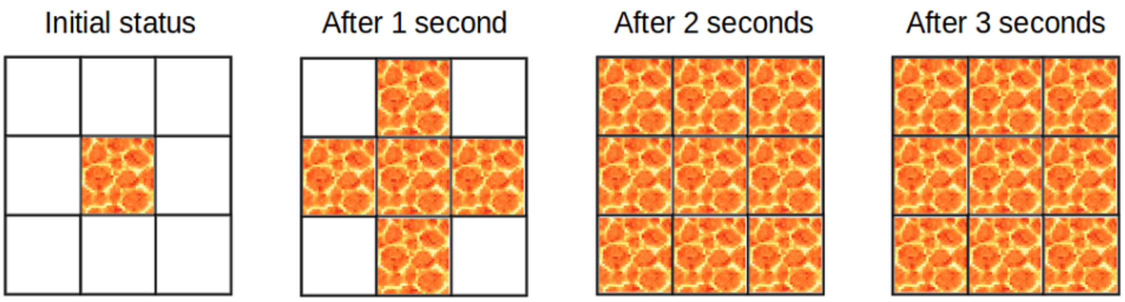
```
XXXLO
OXOXX
CCXXX
```

Hint

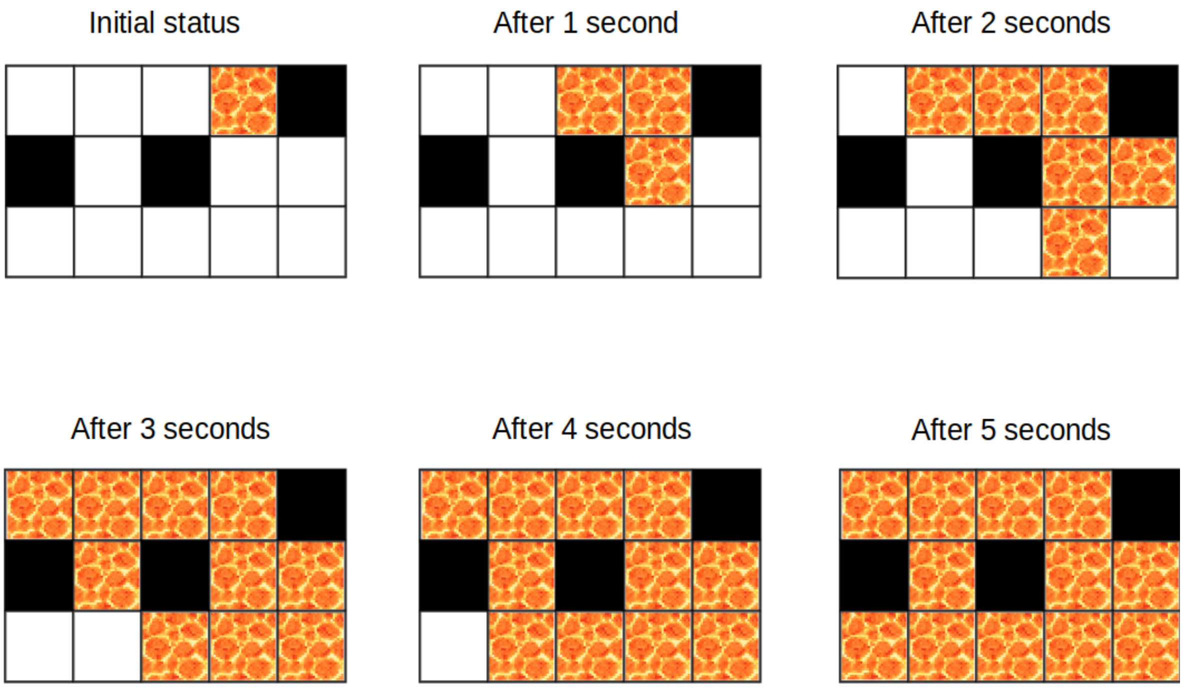
> It is called flood fill.

White land means common land.
Black land means obstacle land.

For sample input and output 1~3.



For sample input and output 4.



> Credit:
> Lava icon from [SummerFields]