1_motivation_big_o

June 24, 2025

1 Motivation and Big-O

1.1 Outline

- Motivation
- Time Complexity: Introduction to Big-O Notation
- Average, Best, and Worst Case
- Space Complexity

1.2 About Me

- PhD candidate in the Department of Computer Science at UofT
- Thesis is on remote patient monitoring using wearables and mobile devices
- Did BSc from Vancouver (SFU) and MSc from UofT
- Co-Founder of a remote patient monitoring startup (Tabiat)

1.3 Introducing the Learning Support Staff

- Will be available during office hours, work periods, and on Slack to answer questions

1.4 Why should a Data Scientist take this Course?

- Problem solving. This course provides you with a framework to solve coding problems you may encounter in your career.
- Efficient programs. We want to write programs that scale well with big data.
- Interview preparation. Many data science jobs require a technical interview, which involes solving algorithms problems.

1.5 Learning Objectives

- Assess options and choices around methods to solve problems and data representation methods using Big-O notation.
- Develop comfort with recursive functions.
- Decide on appropriate methods to represent data for a problem.

- Take a client-led problem and translate it into an optimization problem.
- Identify why code is running slowly and know how to improve its performance.

2 Motivating Code Demos

2.1 What are Algorithms and Data Structures

- An **algorithm** is a procedure to solve a problem
  - Sort a data observations from smallest to largest
  - Find the nearest neighbor to a data point
  - How fast is each algorithm?
- A **data structure** is a concrete method to store some data.
  - A pandas data set is a good way to store observations with many features.
  - How much space does the data sturcture need? How long does it take to access each observation?

[1]:
```
import numpy as np
import timeit
import random
```

2.2 Loop Versus Vectorized Operations

[2]:
```
size = 10**4

# Using Python lists
list_a = list(range(size))
list_b = list(range(size))

# Using NumPy arrays
array_a = np.arange(size)
array_b = np.arange(size)

# Timing for list addition
list_time = timeit.timeit(lambda:
  [a + b for a, b in zip(list_a, list_b)], number=1)

# Timing for vectorized array addition
array_time = timeit.timeit(lambda:
  array_a + array_b, number = 1)
```

2.3 Loop Versus Vectorized Operations

We will learn about what vectorized is in lecture 6.

[3]:
```python
print(f"List Addition: {list_time:.6f} seconds")
print(f"Vectorized Addition: {array_time:.6f} seconds")
```

```
List Addition: 0.000406 seconds
Vectorized Addition: 0.000333 seconds
```

- Why was the NumPy vectorized operation much faster?
- How can we describe how much faster the vectorized operation is?
- This is useful in many iterative algorithms, such as gradient descent.

2.4 Search in List Versus Set

We will learn about searching and sorting in lecture 2.

[4]:
```python
# Python list
list_time = timeit.timeit(lambda:
  -1 in list_a, number = 1)

# Python set
set_a = set(range(size))
set_time = timeit.timeit(lambda:
  -1 in set_a, number = 1)
```

2.5 Search in List Versus Set

[5]:
```python
print(f"List Search: {list_time:.6f} seconds")
print(f"Set Search: {set_time:.6f} seconds")
```

```
List Search: 0.000064 seconds
Set Search: 0.000001 seconds
```

- Why was the set search much faster?
- How can we describe how much faster the vectorized operation is?
- What are the pros and cons of choosing each data structure?

2.6 Selection Sort Versus Python Sort

For context, selection sort is a naive sorting algorithm, while Python implements Tim Sort for the default search function.

[6]:
```python
def selection_sort(arr):
  n = len(arr)

  for i in range(n):
    min_index = i
    for j in range(i+1, n):
      if arr[j] < arr[min_index]:
```

```
        min_index = j

    arr[i], arr[min_index] = arr[min_index], arr[i]
```

2.7 Selection Sort Versus Python Sort

[7]:
```
random.shuffle(list_a)
rand_list = list_a.copy()

sel_time = timeit.timeit(lambda:
  selection_sort(rand_list.copy()), number = 1)

py_time = timeit.timeit(lambda:
  sorted(rand_list.copy()), number = 1)
```

2.8 Selection Sort Versus Python Sort

[8]:
```
print(f"Selection sort: {sel_time:.6f} seconds")
print(f"Tim sort: {py_time:.6f} seconds")
```

```
Selection sort: 1.421455 seconds
Tim sort: 0.001110 seconds
```

- Why was selection sort much slower than Tim sort (not in detail)?
- If we double the size of the list, how much slower will the code be in each case?

3 Time Complexity: Introduction to Big-O Notation

3.1 An example

- Imagine you are writing an algorithm to search for a landing position for a rocket. You want it to be simple (to avoid bugs) and fast (since you only have 10 seconds to find a site). [1]
- It takes 1 millisecond to check each element. You decide to test a simple search and binary search on 100 elements (more on these methods later).
  - Simple search takes 100ms. Binary search takes 7ms.
- Then you test binary search with 1 billion elements and it takes 32ms.
  - Binary search is about 15 times faster than simple search, because simple search took 100 ms with 100 elements, and binary search took 7 ms. So simple search will take 30 $\times$ 15 = 450ms with 1 billion elements.
- Since that is within your threshold, you decide to go with simple search. **Is this correct?**

3.2 A practical example

- Definitely wrong!!

- The run time of different algorithms can grow at different rates.
- Big-O tells us how run time increases as the list size increases.

3.2.1 Comparing run times of simple and binary search

Elements	Simple Search	Binary Search
100	100 ms	7 ms
10,000	10 s	14 ms
1,000,000,000	11 days	32 ms

3.3 Big-O Notation

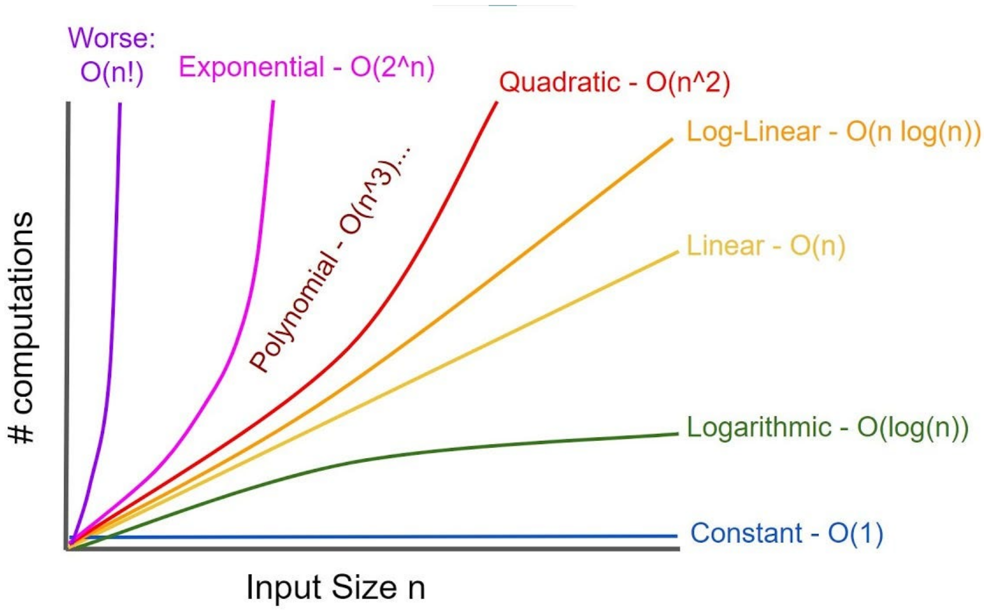
- Big-O tells you how fast an algorithm is in terms of the number of operations, n.
- Simple search needs to take each element, so it will take n operations. The run time in Big-O notation is $O(n)$.
- Binary search needs log n operations, so the run time in Big-O notation is $O(\log n)$
  - Note: log in computer science usually refers to log base 2.

3.4 Big-O is Upper Bound Run Time

- Big-O notation is about the *worst-case* scenario.
  - If you were conducting linear search through a phone book, even if you were looking for Abe Aberdeen, it is still considered $O(n)$.
- Formally, it characterizes an upper bound on the asymptotic behavior of the run time.
- For example, the function $7n^3 + 30n^2 - 200n + 9$ has highest-order term $7n^3$. The function's growth rate is n^3 because the function grows no faster than n^3. The Big-O is $O(n^3)$.

3.5 Common Big-O Run Times

Here are seven Big-O run times that you'll encounter frequently, sorted from fasted to slowest.



- $O(1)$, known as *constant time*. Ex: addition, division
- $O(\log n)$, known as *logarithmic time*. Ex: binary search
- $O(n)$, known as *linear time*. Ex: Linear search
- $O(n \log n)$. Ex: Tim Sort
- $O(n^2)$, known as *quadratic time*. Ex: Selection sort.
- $O(2^n)$, known as *exponential time*. Ex: Naive recursive solution for nth Fibonacci number
- $O(n!)$, known as *factorial time*. Ex. Traveling salesperson

3.6 Determining Time Complexity

Consider the following code. How can you determine the Big-O?

[1] Example from Grokking Algorithms

[9]:
```
def f(n):
  for i in range(n):
    for j in range(n):
      print(i, j)
```

3.7 Determining Time Complexity

- With "raw" Python code, you can usually count the number of nested `for` loops to determine the Big-O
  - A loop gives $O(n)$
  - A nested loop gives $O(n^2)$
- It's usually not so simple in Data Science because of packages we use.

- There are many factors affecting the constants in your run time
  - How complicated is each step? Is it n or $2000n$?
  - How are your algorithms implemented? Is your programming language fast? Are your libraries fast?
- Implementation issues will be covered later in the course.

3.8 Big-O with Two Variables

Consider the following code. What is it's Big-O?

[10]:
```
def fun(n,m):
  for i in range(n):
    for j in range(m):
      print("Hello")
```

3.9 Big-O with Two Variables

- The time complexity here is $O(nm)$.
  - If $n = m$, then $O(n^2)$.
- All terms should be combined into one Big-O
  - $O(nm)$ is correct and $O(n)O(m)$ is incorrect.
  - $O(n + m)$ is correct and $O(n) + O(m)$ is incorrect.
  - $O(n^2 + mn + m)$ is written as $O(n^2 + nm)$. We can't throw away either term because we don't know which term will dominate.
- Important to think about this when working with datasets.
  - They have n rows and p columns.
  - Can you reason how long it will take to fit a decision tree?

4 Best, Average, and Worst Case

4.1 Best, Average, and Worst Case

- Big-O deals with worst case.
- If we can develop a notion of an "average input," then we can devise the average case of an algorithm.
- Best case is useful to think about the constants in your algorithm.
  - $O(\log n)$ is always faster than $O(n)$ expect with very small n.

5 Space Complexity

5.1 What is Space Complexity

- Aside from our algorithm taking too long to run, its also an issue if you run out of memory.
  - Note, memory (RAM), is not the same as disk space.
  - The computer will load data into memory from the disk
- It will be problematic if you need to load 2 billion observations all at once.
- We can also analyze space complexity with Big-O notation
- Notice that time complexity is usually about the *algorithm*, while space complexity is about the *data structure*.

5.2 Examples

- Code that prints `hello {your name}` will have $O(1)$ space.
- Code that sums a list of size n has $O(n)$ space.
- You have users on Instagram, and you want to store who follows who. The answer depends (why?). The worst case space is $O(n^2)$

6 Recommended Problems and References

6.1 Recommended Problems

- Cormen: Chapter 1 exercises
  - 1.2-1, 1.2-2, 1.2-3
- Bhargava: Chapter 1 exercises
  - 1.3 to 1.5
- Additional (for the mathematically inclined)
  - In CS, log is usually base 2, but a strong distinction is not made because *logs of different bases only differ by a constant factor* and constants are dropped in Big-O. Show this is true
  - Show that exponents of different bases **do not** differ by a constant factor

6.2 References

- Bhargava, A. Y. (2016). *Grokking algorithms: An illustrated guide for programmers and other curious people.* Manning. Chapter 1.
- Cormen, T. H. (Ed.). (2009). *Introduction to algorithms* (3rd ed). MIT Press. Chapter 1 and 3.