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Author(s): R. C. Elston

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A WEIGHT-FREE INDEX FOR THE PURPOSE OF RANKING OR SELECTION WITH RESPECT TO SEVERAL TRAITS AT A TIME¹

R. C. ELSTON²

*Department of Biostatistics, University of North Carolina,
Chapel Hill, North Carolina, U.S.A.*

SUMMARY

The problem is considered of how to rank individuals with respect to measures on several traits jointly when nothing is to be assumed about what economic weights are appropriate. A selection or ranking index is developed on intuitive grounds and then shown to be in a certain sense weight-free. The use of this index is illustrated for two traits measured on chickens. The case where there are errors of measurements is also discussed; in this case a modification of Smith's [1936] selection index may be appropriate.

INTRODUCTION

Suppose we have a population of individuals on each of which we have measures (or scores) of p different traits, the measures being x_1, x_2, \dots, x_p . A problem that often arises in practice is that of how to select the best individuals, or how to rank the individuals, with respect to all the p traits combined; the problem becomes acute when there are negative correlations among the measures on the different traits. Consideration will be given to the selection problem in the main, but a solution to the more general problem of ranking the whole population will easily follow.

Assume it is desired to select a fraction λ of the individuals so as to maximize jointly in some sense the measures of the traits in the selected individuals. Consider the following two possibilities. Let the top fraction θ of individuals with respect to x_i belong to the set of individuals X_i , where $i = 1, 2, \dots, p$. Then we can choose θ so that the fraction of individuals is λ in either

- (1) the set $X_1 \cap X_2 \cap \dots \cap X_p$, i.e., the intersection of all p sets,

or

- (2) the set $X_1 \cup X_2 \cup \dots \cup X_p$, i.e., the union of all p sets.

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²Present address: Research Group in Biometric Medicine, Department of Statistics, University of Aberdeen, Old Aberdeen, Scotland.

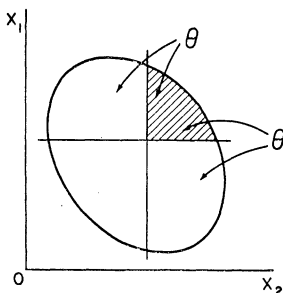


FIGURE 1.

SELECTION ON THE BASIS OF THE INTERSECTION OF TWO SETS, EACH CONTAINING THE SAME PROPORTION θ OF THE POPULATION.

This is illustrated in Figures 1 and 2 for the case $p = 2$. The whole population lies within the ellipse; for (1) we find that fraction θ such that the individuals that belong to *both* the θ best with respect to x_1 and the θ best with respect to x_2 make up a fraction λ of the whole population; for (2) we find that fraction θ such that individuals that belong to *either* the θ best with respect to x_1 or the θ best with respect to x_2 make up a fraction λ of the whole population. The selected individuals are those falling in the cross-hatched areas.

These two cases are extremes, and, from a practical viewpoint, neither is entirely satisfactory. Using method (1) we shall have to select individuals that are much lower in each trait, which is illustrated by the fact that the cross-hatched area comes much nearer to the origin; whereas using method (2) we may be led to select none of the most desirable individuals, which is illustrated by the fact that the cross-hatched area does not include that part of the ellipse furthest away from the origin.

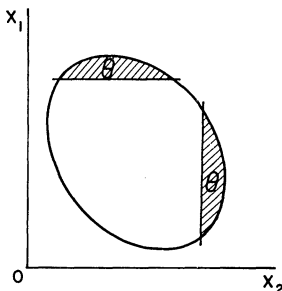


FIGURE 2.

SELECTION ON THE BASIS OF THE UNION OF TWO SETS, EACH CONTAINING THE SAME PROPORTION θ OF THE POPULATION.

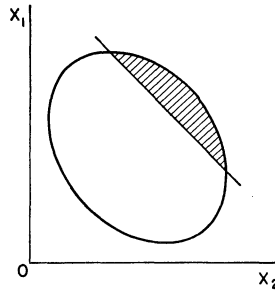


FIGURE 3.

SELECTION ON THE BASIS OF A LINEAR INDEX.

The type of selection that is really desired is a modification of method (1) such that excellence with respect to one of the traits can to a certain extent compensate for, but not entirely overrule, weakness with respect to other traits. This very statement implies that there is some function of the measures on the various traits that it is desired to maximize; this function should normally be decided upon by the experimenter, and may well depend upon economic conditions. A function that is often considered appropriate is one of the form $w_1x_1 + w_2x_2 + \cdots + w_px_p$, where the w 's are appropriate economic weights; when this is the case and the w 's are known there is no problem. But such a linear function need not always be appropriate; and the experimenter may be unable, or may not wish, to assume any particular set of economic weights: it is to meet this situation only that the present paper is written.

INTUITIVE DEVELOPMENT OF AN INDEX

Consider the methods of selection illustrated in Figures 3 and 4, for $p = 2$. Figure 3 illustrates maximizing the linear function

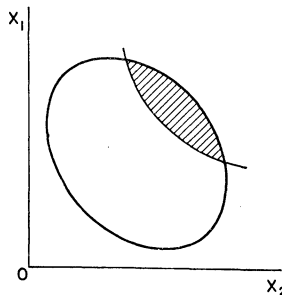


FIGURE 4.

SELECTION ON THE BASIS OF A NON-LINEAR INDEX.

$x_1 + wx_2$, where $-w$ is the slope of the boundary above and to the right of which individuals are selected. For example, giving equal weights to x_1 and x_2 is equivalent to maximizing $x_1 + x_2$, so that $w = 1$ and the boundary makes a 45 degree angle with the horizontal axis. In Figure 4 some non-linear function of x_1 and x_2 is maximized. Now notice that whatever function we decide to maximize, in practice it will be such that the boundary is everywhere convex decreasing. In fact this is a property that economists have called the *law of substitution* [Samuelson, 1955]. From this it follows that we can always change the scales on which x_1 and x_2 are measured, i.e., take some strictly monotonic functions of the observed measurements x_1 and x_2 , so as to make the boundary a straight line. Thus, provided we have measurements of the traits on appropriate scales, it is only necessary to maximize a linear function of these measurements.

As an example, consider the poultry problem of selecting female breeding stock for broiler production. Let x_1 be the number of eggs laid by a bird in one year, and x_2 the weight of the bird in pounds at 10 weeks of age. The experimenter wishes to select a fraction λ of the birds so as to maximize both x_1 and x_2 , placing equal emphasis on each trait. In this case it is clear that if we rank the birds using the index $x_1 + x_2$, and select the λ best birds on the basis of this ranking, x_2 will have little influence in determining which birds are selected—simply because on the scale on which it is measured it is much less variable than x_1 . If the measurements on x_1 are w times as variable as those on x_2 , we might consider ranking the birds on the basis of the index $x_1 + wx_2$. Then this defines what we mean by the expression “equal emphasis” when we are considering traits that are qualitatively different, and gives a method of procedure when w is known.

But a different approach may have more intuitive appeal. We have $x_1 \geq 0$, and if $x_1 = 0$, i.e., the bird lays no eggs at all, we want to be absolutely certain of not selecting it, however great x_2 is. Similarly we have $x_2 \geq k$, where k is a lower bound to the 10-week weight, and if $x_2 = k$ for some bird we want to be absolutely certain of not selecting it, however great x_1 is. We thus want as our index a function of x_1 and x_2 that satisfies the following three conditions:

- (1) it must take on its smallest value at $x_1 = 0$, whatever the value of x_2 ;
- (2) it must take on its smallest value at $x_2 = k$, whatever the value of x_1 ; and
- (3) it must take on its greatest value when both x_1 and x_2 are greatest.

The simplest mathematical function that satisfies these three conditions

is $x_1(x_2 - k)$, and it is therefore suggested that this is an appropriate index to use. More generally we can write this index as $(x_1 - k_1)(x_2 - k_2)$, where k_i is the greatest lower bound to x_i , $i = 1, 2$; and if p traits are involved the index becomes $\prod_{i=1}^p (x_i - k_i)$. In this way an individual that scores lowest in any one trait automatically has an index of zero.

The intuitive appeal of this index is illustrated in Figure 5 for the case $p = 2$. The boundary separating selected from unselected individuals belongs to the family of curves:

$$x_1 - k_1 = A/(x_2 - k_2), \quad A > 0.$$

This is a rectangular hyperbola whose asymptotes are at $x_1 = k_1$ and $x_2 = k_2$. The fraction λ of the population that is selected is a strictly decreasing function of A ; curves for three different values of λ are illustrated. We see that when λ is small this method of selection approaches the method illustrated in Figure 3; and as λ increases this

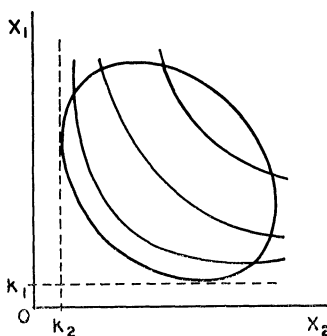


FIGURE 5.

SELECTION ON THE BASIS OF THE INDEX $(x_1 - k_1)(x_2 - k_2)$.

method more and more approaches method (1). In practice it should not be difficult for the experimenter to determine k_1 and k_2 , and then the exact form of the boundary depends only on λ .

PROPERTIES OF THE INDEX

Firstly it should be noticed that if we change the scales on which our measurements are made, letting

$$x'_i = \log (x_i - k_i), \quad i = 1, \dots, p,$$

then, so far as concerns ranking and selection, the index that is proposed here is equivalent to the linear function $\sum_{i=1}^p x'_i$; and on this

scale an individual that scores lowest in any one trait has an index of minus infinity.

Secondly the index is, in a certain sense, weight-free: the order of ranking is invariant under magnification of any of the scales used to measure the x_i . Thus assume $x_i \geq 0$ and $k_i = 0$, $i = 1, \dots, p$; such situations are common in practice. Suppose we believe our index should be based on the weighted measurements w_1x_1, \dots, w_px_p , but do not know what weights w_i to use. Then, rather than choose an arbitrary set of weights and use the index $\sum_{i=1}^p w_ix_i$, it would seem preferable to use an index that is invariant under choice of the w_i , i.e. under magnification of the scales on which x_i was measured. That the proposed index has this property is seen from the fact that, for any set of w_i , the index is equivalent to

$$\sum_{i=1}^p \log w_ix_i = \sum_{i=1}^p \log x_i + \sum_{i=1}^p \log w_i;$$

the weights only affect the second term, which is a constant for all individuals. When $k_i \neq 0$ this invariance is a property of the measure $x_i - k_i$, i.e. of the measures adjusted so that each scale "starts" at 0; we shall now show that this is not an unreasonable property.

Consider the case where we wish to select individuals so as to maximize just the two traits x_1 and x_2 , placing equal emphasis on each. Consider x_1 and x_2 as random variables, each with its own frequency distribution. If the frequency distributions are the same apart from a location parameter, then the linear index $x_1 + x_2$ meets the intuitive requirement indicated above, namely that we take the sum of two equally variable measurements. But, suppose that the frequency distributions of x_1 and x_2 differ only by an unknown scale factor $w > 0$, i.e. the distribution of x_1 is the same as the distribution of wx_2 . That is, denoting the frequency distributions of x_1 and x_2 by $f_1(x)$ and $f_2(x)$ respectively, $f_1(x) = wf_2(wx)$. Let the distribution of $\log x_1$ be $g_1(\log x)$ and that of $\log x_2$ be $g_2(\log x)$. Then, provided $x \geq 0$, $w > 0$,

$$g_1(\log x) = xf_1(x) = wxf_2(wx)$$

and

$$g_2(\log wx) = wxf_2(wx).$$

Thus $g_1(\log x) = g_2(\log w + \log x)$, which shows that the distributions of $\log x_1$ and $\log x_2$ differ by only a location parameter; it is therefore reasonable to use the index $(\log x_1 + \log x_2)$ if the distributions of x_1 and x_2 differ only by a scale factor.

Now in practical situations we shall have

$$f_i(x) > 0, \quad k_i \leq x \leq m_i$$

and

$$f_i(x) = 0, \quad \text{otherwise.}$$

Notice that for x_1 to have the same distribution as $w x_2$ we must have $k_1 = w k_2$, i.e., the distributions must be appropriately located. It is therefore advisable to ensure at least this condition is satisfied, by considering the variables $x_i - k_i$ instead of x_i . For we then have

$$f_i(x - k_i) > 0, \quad 0 \leq x \leq m_i - k_i$$

and

$$f_i(x - k_i) = 0, \quad \text{otherwise,}$$

and the corresponding condition for the variables $(x_i - k_i)$ is $0 = w 0$, which is clearly always satisfied. This justifies our using the general index $\prod_{i=1}^p (x_i - k_i)$, rather than $\prod_{i=1}^p x_i$, even when $x_i > 0$ for all i .

The above considerations suggest that we can go one step further. Suppose the distributions of $\log(x_i - k_i) = x'_i$ differ by only scale factors. Then we can use the index $\prod_{i=1}^p x'_i$, provided $x'_i \geq 0$ for all i ; but, by the same argument that has just been given, it is more reasonable to use the index $\prod_{i=1}^p (x'_i - k'_i)$, where k'_i is the lower bound to x'_i .

Letting

$$x''_i = \log(x'_i - k'_i), \quad i = 1, \dots, p,$$

we see that this index is equivalent to the linear function $\sum_{i=1}^p x''_i$, so far as concerns ranking and selection. Furthermore, on the assumption that $k_i = k'_i = 0$, all i , this index is invariant under both multiplication and raising to a power. Thus if we convert the original x_i to $w_i x_i^{v_i}$, using any set of weights w_i and powers v_i , the index becomes equivalent to

$$\sum_{i=1}^p \log \log w_i x_i^{v_i} = \sum_{i=1}^p \log \log x_i + \sum_{i=1}^p \log \log w_i + \sum_{i=1}^p \log v_i,$$

and the terms that depend upon the choice of w_i and v_i are the same for all individuals. Theoretically we can take as many steps as we like, gaining more invariance at each step. In practice, however, it is doubtful whether much would be gained by going any further than this second step.

PRACTICAL PROCEDURE

Using the above principles, we can give the following practical procedure for selecting a small fraction of individuals on the basis of several traits that are to be "equally emphasized":

(1) Arrange for each trait to be measured on a scale on which it is desired to choose individuals with larger measurements. Thus if there is a trait for which a smaller measurement is desirable (an example of this occurs below), we should change the direction of the scale on which it is measured; this can be conveniently done by taking either the reciprocals or the negatives of the observed values.

(2) If the distribution of the measurements on one of the traits is of an entirely different type from those of the others, then those measurements should, if possible, be transformed to lessen this difference. Thus we should at least require that when histograms are drawn up for the various traits they should all approximate to distributions with the same number of modes; usually these will be unimodal distributions.

(3) From each measurement on a particular trait subtract the smallest sample measurement that occurs for that trait, i.e., obtain $(x_i - k_i)$.

(4) If the histograms obtained for the various traits are similarly shaped, e.g., if they are all approximately symmetric, use the index $\prod_{i=1}^p (x_i - k_i)$.

(5) If the histograms are not similarly shaped, subtract from each value of $x'_i = \log (x_i - k_i)$ the smallest sample value of x'_i that occurs (ignoring the value $x'_i = -\infty$), i.e., obtain $(x'_i - k'_i)$, and use the index $\prod_{i=1}^p (x'_i - k'_i)$.

Provided we have a large number of individuals, and we only wish to select a small fraction of them, it should be quite adequate to use the above procedure in which k_i and k'_i are chosen to be the smallest values of x_i and x'_i occurring among the individuals on which we have measurements. However, if we want our index to rank all the individuals, this procedure is inadequate, for it necessitates that at least p individuals (those with lowest score for any one trait) tie for lowest rank. This can be avoided by letting

$$k_i = (n \min x_i - \max x_i)/(n - 1)$$

where n is the number of individuals. This can be justified on the assumption that the n values of x_i that we have form a random sample from a rectangular distribution; for then the above value of k_i is the minimum variance unbiased estimate of the point at which the distribution starts. Of course the x_i need not be from a rectangular distribution, and for any unimodal distribution we may expect the above value of k_i to be on the large side; however, since it is small enough to prevent zero values of $(x_i - k_i)$ from occurring, it seems reasonable enough to use it. Similarly we can let

$$k'_i = (n \min x'_i - \max x'_i)/(n - 1), \text{ if required.}$$

EXAMPLE

The application of the above principles will be demonstrated for the following example. The experimenter has 20 chickens on each of which he has measurements of the weight at 9 weeks of age, x , and of the weight at 21 weeks of age, y . He wishes to select those birds that are best suited for the breeding of other animals that will be killed for meat at 9 weeks of age. He wishes to select those birds that have largest values of x , since their offspring will tend to have larger values of x ; at the same time he wishes to select those birds that have smallest values of y , since their offspring will tend to have smaller values of y , and so in general eat less and thus be less expensive to keep. It is desired to give equal emphasis to each trait.

In this case we will select for large values of either x and $-y$ or x and $1/y$. It is clearly preferable to use $-y$, rather than $1/y$, if we wish the distributions of the two chosen variables to be as similar as possible. Before going any further, however, consider what happens if it is assumed that the distributions of x and y are the same apart only from a location parameter. In this case the index $x - y$ gives "equal emphasis" to the two traits, each in the appropriate direction. But notice that this index depends only on the gain in weight between 9 and 21 weeks of age, and so any selection that is based on it will not depend on x *per se*. This suggests that what the experimenter really wishes to do is not so much to select for large values of x and small values of y , as to select for large values of x and small values of $y - x$ (the gain in weight after 9 weeks of age). We shall assume this to be the case and so consider the problem of selection on the basis of $x_1 = x$ and $x_2 = x - y$ jointly.

The columns at the left of Table 1 list the twenty birds, identified by a letter, with their values of x_1 and x_2 in decagrams, and their ranks on the basis of each of these traits separately. Suppose we wish to select the four best birds. By method (1), in which the intersection of sets is selected, we should take birds B , C , M and S ; for if we let $\theta = 9/20$ we find these are just the birds that are among both the θ best with respect to x_1 and the θ best with respect to x_2 . By method (2), in which the union of sets is selected, we should take birds C , H , K and N ; for if we let $\theta = 2/20$ we find these are the birds that are among either the θ best with respect to x_1 or the θ best with respect to x_2 . Notice that there is just one bird, C , that would be selected by both of these methods; we might therefore reasonably want any selection index to rank this bird the highest.

As previous experience suggests that x_1 and x_2 have similar distributions, we shall only consider the simple index $(x_1 - k_1)(x_2 - k_2)$.

If we take $k_1 = 86$ and $k_2 = -135$, the minimum values of x_1 and x_2 respectively that occur in the sample, we find that the four best birds are C , M , O and S , with C in fact ranking the highest; this is seen in the middle column of Table 1, where it is also clear that two birds, H and K , tie for last place with an index of zero.

TABLE 1
DATA ON 20 CHICKENS

Bird	x_1	Rank	x_2	Rank	$(x_1-86) \times$ (x_2+135) Rank	$(x_1-84) \times$ (x_2+137) Rank	$(x_1-79) \times$ (x_2+153) Rank	
A	110	11	-103	8	768	7	1550	7
B	113	6	-105	9	810	6	1632	5
C	116	4	- 88	1	1410	1	2405	1
D	112	7	-107	11	728	8	1518	8
E	120	3	-122	16	442	14	1271	11
F	107	16	-108	12	567	10	1260	12
G	91	18	-105	9	150	17	576	18
H	86	20	- 90	2	0	19	441	19
I	89	19	-125	18	30	18	280	20
J	108	15	- 97	5	836	5	1624	6
K	123	2	-135	20	0	19	792	17
L	110	11	-124	17	264	16	899	16
M	112	7	-102	7	858	4	1683	4
N	126	1	-121	15	560	11	1504	9
O	110	11	- 95	4	960	3	1798	3
P	99	17	- 92	3	559	12	1220	13
Q	110	11	-109	13	624	9	1364	10
R	111	10	-117	14	450	13	1152	14
S	112	7	- 98	6	962	2	1815	2
T	114	5	-125	18	280	15	980	15

If we wish to rank all 20 birds we can, as suggested in the previous section, take $k_1 = [20(86) - 126]/19 = 84$ to the nearest integer, and similarly $k_2 = -137$. The result of doing this is also given in Table 1, and it is seen that the same four birds rank highest. If values of k_1 and k_2 are known from other data, then these values should be used. Thus from a large body of similar data it is found that the smallest value of x_1 is 79 and the smallest value of x_2 is -153. The result of using these values for k_1 and k_2 is shown in the last column of Table 1; the same four birds are ranked highest, but the ranking of all 20 birds is slightly altered.

EXTENSION TO THE CASE WHERE THERE ARE
ERRORS OF MEASUREMENT

So far we have only considered selecting for the best *measurements*. Often, however, we wish to select for some best values which our measurements approximate. The difference may not be important when our errors of measurement are small, or are all of the same order of magnitude. But when these errors vary in magnitude, or when they are correlated, it may be preferable to take this fact into account. This is essentially the problem that was solved by Smith [1936]; in his case the measurements were phenotypic values, and he wished to select for the best genetic values. It is here proposed that Smith's method be followed, with the modification that x'_i or x''_i be taken as the observed values and not x_i . In this way the resulting index is invariant, to the same extent as before, under changes in the scales on which the x_i are measured. In what follows x'_i can be replaced by x''_i , if the more invariant index is considered appropriate.

We assume that for each individual

$$x'_i = g_i + e_i, \quad i = 1, 2, \dots, p,$$

where e_i is in the nature of an error, and the index we should like to use is $\sum_{i=1}^p g_i$. Since the g_i are unknown, we take instead a linear function $\sum_{i=1}^p b_i x_i$ that is most highly correlated with this index (this reasoning follows Kempthorne [1957]); and such a linear function is obtained when we choose the b_i to be the solution to the p equations

$$b_1 \sigma_{1i} + b_2 \sigma_{2i} + \dots + b_p \sigma_{pi} = \tau_{1i} + \tau_{2i} + \dots + \tau_{pi}, \quad i = 1, 2, \dots, p,$$

where σ_{ki} is the covariance between x'_k and x'_i (variance if $i = k$) and τ_{ki} is the covariance between g_k and g_i (variance if $i = k$).

This method assumes (1) there is no covariance between g_i and e_k ($i, k = 1, 2, \dots, p$) and (2) the τ_{ki} and σ_{ki} are known. In practice only estimates of these are available, and so the merit of this procedure depends in part on how good these estimates are. Often there are insufficient data even to estimate the τ_{ki} ; in such cases it may be best to assume that the τ_{ii} are all equal, since this is implied in the desire to use an index that approximates $\sum_{i=1}^p g_i$.

In conclusion the example of the previous section will be reconsidered assuming there may be errors of measurement. For simplicity we shall assume that it is desired to select the birds on the basis of their phenotypes, and so here there is no question of considering any genetic values. In this example it is reasonable to assume that the covariance between x'_1 and x'_2 is the same for both the true and the measured values,

i.e. $\tau_{12} = \sigma_{12}$; it is then found that

$$\frac{b_2}{b_1} = \frac{\sigma_{11}\tau_{22} + \sigma_{12}(\sigma_{11} - \sigma_{12} - \tau_{11})}{\sigma_{22}\tau_{11} + \sigma_{12}(\sigma_{22} - \sigma_{12} - \tau_{22})}.$$

Thus $b_1/b_2 = 1$ if either

$$(1) \quad \sigma_{11} = \tau_{11} \quad \text{and} \quad \sigma_{22} = \tau_{22}$$

or

$$(2) \quad \sigma_{11} = \sigma_{22} \quad \text{and} \quad \tau_{11} = \tau_{22} ,$$

and so there is no need to calculate b_1 and b_2 if either there are no errors of measurement or if the true and observed variances remain constant for the two different traits (as measured on the logarithmic scales used). We have no information about τ_{11} and τ_{22} , but we can estimate σ_{11} and σ_{22} from the data in the sample. If we take the 20 sample values of $\log_{10}(x_1 - 84)$ and $\log_{10}(x_2 + 137)$ the estimated variances are 0.104 and 0.100 respectively, and the difference between these two is clearly insignificant. If the values of $\log_{10}(x_1 - 79)$ and $\log_{10}(x_2 + 153)$ are taken, the estimated variances are 0.0470 and 0.0223 respectively. These would not be judged significantly different on the basis of a two-tailed F -test, but they will be used to illustrate how a modified index may be obtained; we shall assume $\tau_{11} = \tau_{22}$, so that the difference between these two variances is due solely to a difference in the magnitude of measurement errors, and obtain an index that allows for this.

The sample covariance between $\log_{10}(x_1 - 79)$ and $\log_{10}(x_2 + 153)$ is -0.0086 . Therefore, on the assumption that $\tau_{11} = \tau_{22}$, $b_2/b_1 = (\sigma_{11} - \sigma_{12})/(\sigma_{22} - \sigma_{12})$ is estimated by $0.0556/0.0309 = 1.80$. Thus we can arbitrarily let $b_1 = 1$ and take as our index $\log(x_1 - 79) + 1.8 \log(x_2 + 153)$. If this is used to rank the 20 birds it is found that the ranks are mostly the same as for the simple index $(x_1 - 84)(x_2 + 137)$; some ranks differ from these by one place, and one rank differs by two places. It must be emphasized that the estimates of σ_{11} , σ_{12} and σ_{22} , based as they are on a sample of 20 individuals, are very poor, and so there is little justification for using the modified index in this case. Smith [1936] indicates the properties of this type of index when the variances and covariances are known, but the effect of substituting estimates of these quantities for the true parameters is unknown.

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