THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1510 Calculus for Engineers (2020-2021)
Supplementary Exercise 7

Partial Fractions

1. Resolve the following expressions into partial fractions.

   (a) $\dfrac{5}{x^2+x-6}$ (Hint: $\dfrac{5}{x^2+x-6} \equiv \dfrac{A}{x+3} + \dfrac{B}{x-2}$)

   (b) $\dfrac{1}{x(x^2+1)}$ (Hint: $\dfrac{1}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$)

   (c) $\dfrac{5x^2-3x+4}{(x+1)(x^2-2x+6)}$

2. Resolve the following expressions into partial fractions.

   (a) $\dfrac{x^2+3x}{x^2+3x+2}$

   (b) $\dfrac{x^4+2x+4}{(2x^2+3)(x-2)}$

   (c) $\dfrac{2x^5}{(x^2-1)(x^2-4)}$

3. Resolve the following expressions into partial fractions.

   (a) $\dfrac{x^3+1}{(x-2)^4}$ (Hint: $\dfrac{x^3+1}{(x-2)^4} \equiv \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-2)^3} + \dfrac{D}{(x-2)^4}$)

   (b) $\dfrac{2x^2+1}{x^2(x^2+1)^2}$ (Hint: $\dfrac{2x^2+1}{x^2(x^2+1)^2} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$)

Indefinite Integration

4. (a) Resolve $\dfrac{x^5+3x^2+1}{(x-1)(x^2+4)}$ into partial fractions.

   (b) Hence, evaluate $\displaystyle\int \frac{x^5+3x^2+1}{(x-1)(x^2+4)}\,dx$

5. **(Integration by substitutions)**
   Evaluate the following integrals.

   (a) $\displaystyle\int (2x-1)^{10}\,dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{5x+7}}\,dx$

   (c) $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\,dx$

   (d) $\displaystyle\int x^2\sqrt{x^3+2}\,dx$

(e) $\displaystyle\int e^x \sin e^x \, dx$

(f) $\displaystyle\int \frac{(\ln x)^4}{x} \, dx$

(g) $\displaystyle\int \frac{\cos x}{\sqrt{\sin^3 x}} \, dx$

(h) $\displaystyle\int \frac{1}{\sqrt{x} + \sqrt{x+1}} \, dx$

(i) $\displaystyle\int \frac{e^{3x} + 1}{e^x + 1} \, dx$

(j) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} \, dx$

(k) $\displaystyle\int \sec 2x \tan 2x \, dx$

(l) $\displaystyle\int \left(1 - \cos\frac{x}{2}\right)^2 \sin\frac{x}{2} \, dx$

(m) $\displaystyle\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) \, dx$

6. **(Integration by parts)**
Evaluate the following integrals by using integration by parts.

(a) $\displaystyle\int x \sin\frac{x}{2} \, dx$

(b) $\displaystyle\int x \ln x \, dx$

(c) $\displaystyle\int xe^{3x} \, dx$

(d) $\displaystyle\int \tan^{-1} x \, dx$

(e) $\displaystyle\int \sin^{-1} x \, dx$

(f) $\displaystyle\int x \sec^2 x \, dx$

(g) $\displaystyle\int x^3 e^x \, dx$

(h) $\displaystyle\int e^x \sin x \, dx$

(i) $\displaystyle\int e^{-x} \cos x \, dx$

7. **(Powers of trigonometric functions)**
Evaluate the following integrals.

(a) $\displaystyle\int \cos^3 x \sin x \, dx$

(b) $\displaystyle\int \sin^4 x \cos x \, dx$

(c) $\displaystyle\int \sin^3 x \, dx$

(d) $\displaystyle\int \cos^3 x \, dx$

(e) $\displaystyle\int \cos^4 x \sin^2 x \, dx$

(f) $\displaystyle\int \sec^2 x \tan x \, dx$

(g) $\displaystyle\int \sec^3 x \tan x \, dx$

(h) $\displaystyle\int \sec^4 x \tan^2 x \, dx$

8. **(Products of sines and cosines)**
Evaluate the following integrals.

(a) $\int \cos 3x \sin 2x \, dx$

(b) $\int \sin^3 x \sin 3x \, dx$

(c) $\int \cos x \cos 7x \, dx$

(d) $\int \sin^2 x \cos 3x \, dx$

(e) $\int \cos^3 x \sin 2x \, dx$

(f) $\int \sin x \sin 2x \sin 3x \, dx$

9. **(Trigonometric substitutions)**
Evaluate the following integrals.

(a) $\int \sqrt{25 - x^2} \, dx$

(b) $\int \frac{1}{8 + 2x^2} \, dx$

(c) $\int \frac{1}{\sqrt{4 + x^2}} \, dx$

(d) $\int \frac{x^2}{\sqrt{9 - x^2}} \, dx$

(e) $\int \frac{1}{x^2\sqrt{x^2 - 1}} \, dx$

(f) $\int \frac{x^2}{4 + x^2} \, dx$

(g) $\int \frac{2}{x^3\sqrt{x^2 - 1}} \, dx$

10. **(Integration of rational functions by partial fractions)**
Evaluate the following integrals.

(a) $\int \frac{x + 4}{x^2 + 5x - 6} \, dx$

(b) $\int \frac{x + 3}{2x^3 - 8x} \, dx$

(c) $\int \frac{x^3}{x^2 + 2x + 1} \, dx$

(d) $\int \frac{x^2}{(x - 1)(x + 1)^2} \, dx$

(e) $\displaystyle\int \frac{1}{(x+1)(x^2+1)}\,dx$

(f) $\displaystyle\int \frac{x^2}{x^4-1}\,dx$

(g) $\displaystyle\int \frac{x^4}{x^2-4}\,dx$

11. **(Integration by t-substitution)**

(a) Let $t = \tan\dfrac{x}{2}$, show that $\dfrac{dt}{dx} = \dfrac{1}{2}(1+t^2)$.

(b) Express $\sin x$ and $\cos x$ in terms of t.

(c) By considering the substitution $t = \tan\dfrac{x}{2}$, evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{2+\sin x}\,dx$

(ii) $\displaystyle\int \frac{1}{3-2\cos x}\,dx$

(iii) $\displaystyle\int \frac{1}{2+\sin x+\cos x}\,dx$

(iv) $\displaystyle\int \frac{1}{(2+\cos x)\sin x}\,dx$