香 港 中 文 大 學
The Chinese University of Hong Kong

二零一零至一一年度 上學期科目考試
Course Examination 1st Term, 2010 - 11

科目編號及名稱
Course Code & Title : MATH1510A UNIVERSITY MATHEMATICS FOR ENGINEERING

時間
Time allowed : 2 小時 hours 00 分鐘 minutes

學生編號
Student I.D. No. : _____ 座號 Seat No. : _____

Answer ALL questions. Justify all your steps.

1. (a) Sketch the function $f(x) = \dfrac{x}{(x+1)x(x-1)}$.

   (b) Let $k(x) = (\sin x)^x$. Compute $k'(x)$.

   (c) Let $f(x) = \dfrac{x}{1+x}$, $g(x) = \sqrt{1+x}$, compute $f(g(x))'$.

2. (a) Write down the Taylor's polynomial of degree 3 centered at 1 for the function

$$f(x) = \sqrt{x}.$$

   (b) Compute the value of

$$\sin 3$$

   with an error less than 0.001 using Taylor's theorem.

3. (a) Let the function $f : (a, b) \to \mathbb{R}$ satisfy $f'(x) > 0$ for each x in (a, b).
   Show that it is a strictly increasing function.

   (b) Compute the limit

$$\lim_{x \to \infty} x^{\left(\frac{1}{1+x^2}\right)}.$$

4. (a) Compute the indefinite integral

$$\int \frac{x^2+1}{x+1} dx.$$

   (b) Compute the definite integral

$$\int_0^{\pi} (\sin x) \cdot (\cosh x) dx.$$

5. (a) Find the inverse of the following 3×3 matrix.

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(b) Let $a(x), b(x), \cdots, i(x)$ be differentiable functions. Consider the function

$$f(x) = \begin{vmatrix} a(x) & b(x) & c(x) \\ d(x) & e(x) & f(x) \\ g(x) & h(x) & i(x) \end{vmatrix}.$$

Show the following formula: ($'$ means derivative with respect to x)

$$f'(x) = \begin{vmatrix} a'(x) & b(x) & c(x) \\ d'(x) & e(x) & f(x) \\ g'(x) & h(x) & i(x) \end{vmatrix} + \begin{vmatrix} a(x) & b'(x) & c(x) \\ d(x) & e'(x) & f(x) \\ g(x) & h'(x) & i(x) \end{vmatrix} + \begin{vmatrix} a(x) & b(x) & c'(x) \\ d(x) & e(x) & f'(x) \\ g(x) & h(x) & i'(x) \end{vmatrix}.$$

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