THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 4

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Class: MATH1510G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/
Signature: Daniel Date: 4-10-2021

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	5						
2	4						
3							
4	1						
					Total	10	/ 10

Part A

$x^{-\frac{1}{3}}$

1. Find $f'(x)$ if

(a) $f(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

(b) $f(x) = (2x+1)(3x^2 - 5x + 3)$

(c) $f(x) = (x+1)(x+2)(x+3)(x+4)$

(d) $f(x) = \dfrac{2x-1}{x+3}$

(e) $f(x) = \dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

(a) $f'(x) = \frac{1}{3}(x^{-\frac{2}{3}}) + (-\frac{1}{3})(x^{-\frac{4}{3}})$

$= \dfrac{1}{3 \cdot \sqrt[3]{x^2}} - \dfrac{1}{3 \cdot \sqrt[3]{x^4}}$

(b) $f'(x) = 2(3x^2 - 5x + 3) + (2x+1)(6x - 5)$

$= 6x^2 - 10x + 6 + 12x^2 - 4x - 5$

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$= 18x^2 - 14x + 1$ //

(c) Let $y = f(x)$

$\ln y = \ln(x+1) + \ln(x+2) + \ln(x+3) + \ln(x+4)$

$\frac{dy}{dx}\left(\frac{1}{y}\right) = \frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} + \frac{1}{x+4}$

$\frac{dy}{dx} = \left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} + \frac{1}{x+4}\right)(x+1)(x+2)(x+3)(x+4)$ //

(d) $f'(x) = \dfrac{2(x+3) - (2x-1)}{(x+3)^2}$

$= \dfrac{2x+6-2x+1}{(x+3)^2}$

$= \dfrac{7}{(x+3)^2}$ //

(e) $f'(x) = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(\sqrt{x}-1) - \frac{1}{2}x^{-\frac{1}{2}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$

$= \dfrac{\sqrt{x}-1-\sqrt{x}-1}{2\sqrt{x}\,(\sqrt{x}-1)^2}$

$= -\dfrac{1}{\sqrt{x}(\sqrt{x}-1)^2}$ //

Part B

2. Suppose that

$$f(x) = \begin{cases} ax + b & \text{if } x < 0; \\ \sin x + 3 & \text{if } x \geq 0, \end{cases}$$

where a and b are real numbers.

Given that f is differentiable at $x = 0$, find the values of a and b.

Given that f is differentiable at $x=0$,

f is continuous at $x=0$.

$\therefore$ We have $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$:

$a(0) + b = 0+3 = 0+3$

$b = 3$ //

Also, we have $\frac{d}{dx}\Big|_{x=0} (ax+3) = \frac{d}{dx}\Big|_{x=0} (\sin x + 3)$.

$a = \cos(0)$.

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$a = 1$ //

$$\frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)$$

$$= \frac{x}{\sqrt{x^2-1}}$$

3. Let

$$f(x) = \sqrt{x^2-1} \quad \text{with domain } D_f = (-\infty,-1] \cup [1,\infty).$$

(a) By the first principle, find the derivative of $f(x)$ for any $x \in (-\infty,-1) \cup (1,\infty)$.

(b) Show that f is not differentiable at $x = \pm 1$.

(Hint: Show that

$$Rf'(1) = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} \text{ and } Lf'(-1) = \lim_{h\to 0^-} \frac{f(-1+h)-f(-1)}{h}$$

do not exist.)

(a) By the first principle, we have $f'(x) =$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{(x+\Delta x)^2-1}-\sqrt{x^2-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x^2+2x\Delta x+(\Delta x)^2-1}-\sqrt{x^2-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2\cancel{-1}\cancel{-x^2}\cancel{+1}}{\Delta x\left(\sqrt{x^2+2x\Delta x+(\Delta x)^2-1}+\sqrt{x^2-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{2x+\Delta x}{\sqrt{x^2+2x\Delta x+(\Delta x)^2-1}+\sqrt{x^2-1}}$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}$$

$$= \frac{x}{\sqrt{x^2-1}} \quad //$$

(b) $f'(x) = \dfrac{x}{\sqrt{x^2-1}}$

$= \dfrac{x}{\sqrt{(x-1)(x+1)}}$

when $x = \pm 1$, the $f'(-1)$ and $f'(1)$ is undefined.

Also, we have:

$$Rf'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{(1+\Delta x)^2 - 1} - \sqrt{1-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{2\Delta x + (\Delta x)^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \sqrt{\frac{2}{\Delta x} + 1}$$

$\Rightarrow$ DNE.

$$Lf'(-1) = \lim_{\Delta x \to 0} \frac{f(-1+\Delta x) - f(-1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{-2\Delta x + (\Delta x)^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} -\sqrt{1 - \frac{2}{\Delta x}}$$

$\Rightarrow$ DNE

$\therefore f$ is not differentiable at $x = \pm 1$ //

4. Let $T > 0$ and let $f : \mathbb{R} \to \mathbb{R}$ be a function such that

$$f(x+T) = f(x)$$

for all $x \in \mathbb{R}$.

Show by the first principle that, if f is differentiable, then

$$f'(x+T) = f'(x)$$

for all $x \in \mathbb{R}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x+T) = \lim_{\Delta x \to 0} \frac{f(x+T+\Delta x) - f(x+T)}{\Delta x} \quad \ldots (1)$$

$\because f(x) = f(x+T)$, ... (2)

$\therefore$ sub (2) into (1): $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= f'(x)$.

$\therefore$ We have $f'(x+T) = f'(x)$. //