Cho Kit CHAN **MATH1510_202122T1**
Assignment Extra_exercise_1 due 01/01/2022 at 11:00pm HKT

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1. (1 point)

$$f(x) = |x^2 - a|$$

where a ia a positive real number. Find the value of:
(i) $f(0)$
(ii) $f(3\sqrt{a})$
(ii) $f(\sqrt{a}+3)$
Express your answers in term of a, without absolute value sign.

(i) _____

(ii) _____

(iii) _____

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2. (1 point) Given that
$f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 13$
Find the domain of $f \circ g$

- A. $(-\infty, -\sqrt{12}] \cup [\sqrt{12}, \infty)$
- B. $[-\sqrt{12}, \sqrt{12}]$
- C. $(-\infty, 12) \cup (12, \infty)$
- D. $(-\infty, -\sqrt{12}) \cup (\sqrt{12}, \infty)$

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3. (1 point) Given that
$f(x) = \sqrt{12x}$ and $g(x) = \sin x$
Find the range of $g \circ f$

- A. $(-\sqrt{12}, \sqrt{12})$
- B. $[-1, 1]$
- C. $\mathbb{R}$
- D. $[-\sqrt{12}, \sqrt{12}]$

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4. (1 point) For each of the following sequences, evaluate

$$\lim_{n\to\infty} a_n$$

(i) $a_n = 2 + (\frac{1}{\sqrt{7n}})$ for $n \geq 1$

(ii) $a_n = (\frac{1+n}{9n})$ for $n \geq 1$

(iii) $a_n = (\frac{\sin n}{n})$ for $n \geq 1$

(i) _____

(ii) _____

(iii) _____

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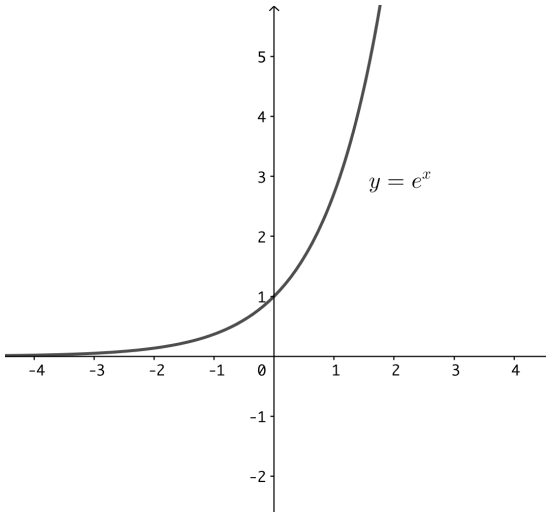
5. (1 point) The following is the graph of $f(x) = e^x$.



Without using L'Hospital's rule, evaluate the following limits. Furthermore, if the limit does not exist but diverges to $\pm\infty$, please indicate so and determine the correct sign.

(i)

$$\lim_{x\to-\infty} e^{1-6x^2-x^4}$$

(ii)

$$\lim_{x\to+\infty} \frac{9e^{4x} - e^{-2x}}{9e^{4x} - e^{2x} + 3e^{-3x}}$$

(i) _____

(ii) _____

6. (1 point) Without using L'Hospital's rule, evaluate the following limit,

$$\lim_{x\to\infty} \ln(17x^3 - 101x + 90)$$

- A. 17
- B. $-\infty$
- C. 0
- D. ∞

7. (1 point) Without using L'Hospital's rule, evaluate the following limit,

$$\lim_{x\to-\infty} \ln\left(\frac{1}{7x^4 - x}\right)$$

- A. 0
- B. $\ln\frac{1}{7}$
- C. $-\infty$
- D. ∞

8. (1 point) Without using L'Hospital's rule, evaluate the following limits.

(i)

$$\lim_{x\to\infty} \frac{\ln(2^x) + \ln(5^x)}{x}$$

(ii)

$$\lim_{x\to 4^+} \frac{x-4}{\sqrt{x^2-16}}$$

(i) _____

(ii) _____

9. (1 point) Without using L'Hospital's rule, evaluate the following limits.

(i)

$$\lim_{x\to\infty} (\sqrt{x^2+11x} - \sqrt{x^2-6})$$

(ii)

$$\lim_{x\to 6} \frac{\sqrt{x}-6}{x-36}$$

(i) _____

(ii) _____

10. (1 point) Without using L'Hospital's rule, evaluate the following limits.

(i)

$$\lim_{x\to 0} \frac{e^{8x} - e^{-8x}}{1 - e^{-16x}}$$

(ii)

$$\lim_{x\to+\infty} (1 + \frac{2}{x})^{4x}$$

(i) _____

(ii) _____

11. (1 point) Without using L'Hospital's rule, evaluate the following limits.

(i)

$$\lim_{x\to -1} \frac{x^2 + 10x + 9}{x^2 - 1}$$

(ii)

$$\lim_{x\to 2^-} \frac{|x-2|}{(x-2)(x+2)}$$

(i) _____

(ii) _____

12. (1 point) Find the following limit if it exists. If it does not exist, state whether it is $+\infty$, or $-\infty$, or neither.

(i)

$$\lim_{x\to -1} \frac{x^2 + 7x + 6}{x^2 - 1}$$

(ii)

$$\lim_{x\to 10^-} \frac{|x-10|}{(x-10)(x+2)}$$

(i) _____

(ii) _____

13. (1 point) Evaluate the following limit,

$$\lim_{h\to 3}\frac{h^2-2h-3}{|h-3|}$$

- A. 0
- B. The limit does not exist.
- C. $+\infty$
- D. $-\infty$

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14. (1 point) Find the following limit if it exists. If it does not exist, state whether it is $+\infty$, or $-\infty$, or neither.
(i)

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{7-\sqrt{6x+31}}$$

(ii)

$$\lim_{x\to -8}\left(\frac{-15x+72}{x^3+512}-\frac{1}{x+8}\right)$$

(i) _____

(ii) _____

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15. (1 point) Find the following limit if it exists. If it does not exist, state whether it is $+\infty$, or $-\infty$, or neither.
(i)

$$\lim_{x\to \frac{\pi}{4}}\frac{8\tan^2 x-7\tan x-1}{5\tan^2 x-4\tan x-1}$$

(ii)

$$\lim_{x\to +\infty}\left(\sqrt{x^2+6x+1}-\sqrt{x^2-6x+1}\right)$$

(i) _____

(ii) _____

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16. (1 point) Evaluate the following limit if it exists. If it does not exist, state whether it is $+\infty$, or $-\infty$, or neither.
(i)

$$\lim_{\theta\to 0}\theta\sin\frac{6\pi}{\theta}$$

(ii)

$$\lim_{t\to -1}\frac{(t+1)^9\cos\left(\frac{1}{t+1}\right)+\sin(t+1)}{t}$$

(i) _____

(ii) _____

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17. (1 point) Let

$$f(x)=\frac{x^2}{(x+5)(10-x)}$$

(i) What is the domain of $f(x)$?

- A. $\mathbb{R}$
- B. $(-5,10)$
- C. $(-\infty,-5]\cup[10,\infty)$
- D. $(-\infty,-5)\cup(-5,10)\cup(10,\infty)$

(ii) What is the subset of the domain where $f(x)\geq 0$?

- A. $\mathbb{R}$
- B. $(-5,10)$
- C. $(-\infty,-5]$
- D. $[10,\infty)$

(iii) What is the x- and y-intercept(s)?

x-intercept(s) _____

y-intercept(s) _____

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18. (1 point) What are the natural domains of the following functions?

(i)

$$F(r)=\ln\left(-\frac{r^2}{4}+1\right)$$

- A. $(-2,2)$
- B. $(-2,0)\cup(0,2)$
- C. $[-2,2]$
- D. $\mathbb{R}$

(ii)

$$H(t)=\sqrt{\frac{t}{(9-t)(t+5)}}$$

- A. $(-\infty,-1)$
- B. $(-\infty,-1)\cup[0,\infty)$

- C. $[0,9)$
- D. $\mathbb{R}$

Notice:
When no domain is specified for a function, we assume that it is the largest set of real numbers for which the rule for the function makes sense, i.e. gives real values. This is called the natural domain.

Numbers that you should exclude from the natural domain are those values that would cause division by zero, the square root of a negative number, and so on.

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19. (1 point) Evaluate the following limit,

$$\lim_{t\to 1^+} \frac{e^t-1}{t^2+9t-10}$$

- A. ∞
- B. 0
- C. Limit does not exist.
- D. $-\infty$

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20. (1 point) Given that

$$f(x)=\sqrt{x}$$

$$g(x)=2\tan x$$

What if the range of $g\circ f$?

- A. $(-2,2)$
- B. $(-1,1)$
- C. $[-1,1]$
- D. $\mathbb{R}$

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21. (1 point) Let $f(x)=\dfrac{2x^2+10x-209}{x^2-13x+40}$.
(a) Evaluate the following limits.
$\lim_{x\to-\infty} f(x)=$_____, $\lim_{x\to+\infty} f(x)=$_____.
(c) It is known that the vertical asymptotes of $f(x)$ are $x=a$ and $x=b$ respectively, with $a\le b$.
$a=$_____, $b=$_____
(d) $\lim_{x\to b^-} f(x)=$
- -infinity
- +infinity
- Neither of above

, $\lim_{x\to b^+} f(x)=$
- -infinity
- +infinity
- Neither of above

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22. (1 point) Let $f(x)=\dfrac{9e^x+6e^{-x}}{3e^x-5e^{-x}}$.
$\lim_{x\to-\infty} f(x)=$_____
$\lim_{x\to+\infty} f(x)=$_____

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23. (1 point) Given that $f(x)=\sqrt{x-10}$ and $g(x)=x^2-4x+4$.
Find the domain of $f\circ g$.
- A. $[2-\sqrt{10},2+\sqrt{10}]$
- B. $(2-\sqrt{10},2+\sqrt{10})$
- C. $[10,+\infty)$
- D. $(-\infty,2-\sqrt{10}]\cup[2+\sqrt{10},+\infty)$

Find the domain of $g\circ f$.
- A. $[2-\sqrt{10},2+\sqrt{10}]$
- B. $(-\infty,2-\sqrt{10}]\cup[2+\sqrt{10},+\infty)$
- C. $[10,+\infty)$
- D. $(2-\sqrt{10},2+\sqrt{10})$

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24. (1 point) For the sequence

$$a_n=\{\sqrt{n^2+1n}-\sqrt{n^2-3}\},\qquad \text{for } n\ge 1$$

find the value of a_n when $n\to\infty$.
$\lim_{n\to\infty} a_n=$_____

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25. (1 point) Without using L'Hôpital's rule, evaluate the following limit of sequence.
(You may enter "±inf" to indicate $\pm\infty$)

$$\lim_{n\to\infty}(\sqrt{n^2+6n-8}-\sqrt{n^2+10n+6})$$

_____

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26. (1 point) Without using L'Hôpital's rule, evaluate the following limit of sequence.
(You may enter "±inf" to indicate $\pm\infty$)

$$\lim_{n\to\infty}\frac{\sin(3^n)+\cos(8/n)}{n}$$

_____

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27. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate $\pm\infty$)

$$\lim_{x\to 6}\frac{\sqrt{2x+7}-\sqrt{19}}{\sqrt{x+6}-\sqrt{12}}$$

_____

28. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate ±∞)

$$\lim_{x\to 0}\frac{\sin(8+x)-\sin(8-x)}{x}$$

_____

29. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate ±∞)

$$\lim_{x\to -\infty}\frac{5|x|-10x}{4x-4}$$

_____

30. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate ±∞)

$$\lim_{x\to +\infty}\frac{\ln(e^{7x}+8)}{\ln(e^{9x}+7)}$$

_____

$$\lim_{x\to -\infty}\frac{\ln(e^{7x}+8)}{\ln(e^{9x}+7)}$$

_____

31. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate ±∞)

$$\lim_{x\to 0}\frac{\tan(10x)}{\sin(8x)}$$

_____

32. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "±inf" to indicate ±∞)

$$\lim_{x\to +\infty}\left(\frac{9x+10}{9x-9}\right)^{x}$$

_____

33. (1 point) Let a be a real number and let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f=\begin{cases} e^{\frac{1}{3x}} & \text{if } x<0;\\ 0 & \text{if } x=0;\\ \cos(x)-a & \text{if } x>0.\end{cases}$$

Assume $\lim_{x\to 0} f(x)$ exists.
(a) Find the value of a. _____
(b) Is f continuous at $x=0$? [Yes/No]

34. (1 point) Let $\gamma(t)=(x(t),y(t))=(8\cos 2t-7, 8\sin 2t+8), t\in\mathbb{R}$ be a curve.
Which of the following equation describes the curve?

- A. $(x+7)^2+(y-8)^2=8$
- B. $(x+7)^2+(y-8)^2=64$
- C. $(x-7)^2+(y+8)^2=64$
- D. $(x-7)^2+(y+8)^2=4$

35. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "± inf" to indicate ±∞)

$$\lim_{x\to \pi/6}\frac{2\sin^2 x+5\sin x-3}{2\sin^2 x-15\sin x+7}$$

_____

36. (1 point) Consider $f(x)=e^x, g(x)=\dfrac{1}{x-2}$, which of the following is the range of $g\circ f$?

- A. $(-\infty,1)\cup(1,+\infty)$
- B. $(-\infty,2)\cup(2,+\infty)$
- C. $(-\infty,0)\cup(0,+\infty)$
- D. $\left(-\infty,-\dfrac{1}{2}\right)\cup(0,+\infty)$

Which of the following is the range of $f\circ g$?

- A. $(0,+\infty)$
- B. $(0,1)\cup(1,+\infty)$
- C. $(1,+\infty)$
- D. $\left(-\infty,-\dfrac{1}{2}\right)\cup(0,+\infty)$

37. (1 point) Consider $f(x)=\begin{cases}\dfrac{\sin(6x)+2x}{2\cos(2x)+\cos(2/x)} & \text{if } x<0;\\ 0 & \text{if } x=0;\\ \dfrac{\sqrt{1+2x}-1}{x} & \text{if } x>0\end{cases}$

Evaluate the following limits.
(You may enter "± inf" to indicate ±∞)
$\lim_{x\to 0^-} f(x)$_____
$\lim_{x\to 0^+} f(x)$_____
Is f continuous at $x=0$? [Yes/No]

38. (1 point) Without using L'Hôpital's rule, evaluate the following limit.
(You may enter "± inf" to indicate ±∞)

$$\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36}$$

_____

39. (1 point) Let $f(x) = 3\sin^2 x + 24\sin x + 51$, find the range of f.

- A. $[30, +\infty)$
- B. $[30, 78]$
- C. $(-\infty, +\infty)$
- D. $[3, +\infty)$

40. (1 point) Let $f(x) = \tan(\sqrt{9-x})$, find the range of f.

- A. $(-\infty, +\infty)$
- B. $[0, \tan(9)]$
- C. $(-\infty, \tan(9)]$
- D. $[0, +\infty)$