THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 8

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Class: MATH 1510 G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

David 8-11-2021

Signature Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	2.5						
2	4						
3	2						
4							
					Total	8.5	/ 10

$\sin 2\theta$

$$(\tan^2\theta + 1)^3 = (\sec^2\theta)^3$$

Part A

1. Evaluate $\int \frac{x^3}{(x^2+1)^3}\, dx$ by using the given substitution.

   (a) $x = \tan\theta$

   (b) $u = 1 + x^2$

$$\left(\sin\theta \cos\theta\right)^3$$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\frac{\sin\theta}{\tan\theta} = \cos\theta$

2.5

A. (a) $x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$\therefore \int \frac{\tan^3\theta}{(\sec^2\theta)^3}\, d\theta \cdot \frac{1}{\sec^2\theta}$$

$$= \int \frac{\sin^3\theta}{\cancel{\cos^3\theta}} \cdot \cos^5\theta \, d\theta$$

$$= \int \left(1 - \sin^2\theta\right)\left(\frac{1}{2}\sin 2\theta\right)^3 d\theta$$

$$=$$

(b) $u = 1 + x^2$

$du = 2x \, dx$

$$\therefore \frac{1}{2}\int \frac{u-1}{u^3}\, du$$

$$= \frac{1}{2}\int \left(u^{-2} - u^{-3}\right) du$$

$$= \frac{1}{2}\left(-u^{-1} + \frac{1}{2}u^{-2}\right) + C$$

$$= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2}$$

$+$ Constant //

$$\frac{6(x+2)+8}{(x+2)^2+4^2}$$

$$\frac{A}{(3x-2)} \quad \frac{B}{(x+1)}$$

$A+3B=9$

$A-2B=-1$

$5B=10$

$B=2$

$A=3$

2. Evaluate the following indefinite integrals.

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(a) $\displaystyle\int \frac{x^2+4x+1}{x-2}\,dx$

(b) $\displaystyle\int \frac{9x-1}{3x^2+x-2}\,dx$

(c) $\displaystyle\int \frac{6x+17}{x^2+4x+20}\,dx$

(a) By long division, we have:

$$\int\left(\frac{\cancel{(x-2)}(x+6)}{\cancel{x-2}}+\frac{13}{x-2}\right)dx$$

$$=\int (x+6)\,dx+13\int \frac{1}{x-2}\,dx$$

$$=\frac{1}{2}x^2+6x+13\ln|x-2|+\text{Constant} \;//$$

(b) $\displaystyle\int \frac{9x-1}{(3x-2)(x+1)}\,dx$

$$=\int \frac{3}{3x-2}\,dx+\int \frac{2}{x+1}\,dx$$

$$=\ln|3x-2|+2\ln|x+1|+\text{Constant}$$

$$=\ln|(x+1)^2(3x-2)|+\text{constant} \;//$$

(c) $\cancel{\int \frac{6(x+2)+5}{(x+2)^2+4^2}\,dx}$ $\qquad \frac{1}{1+x^2}$

$$= \int \frac{3(2x+8)}{x^2+8x+20}\,dx - \int \frac{7}{x^2+8x+20}\,dx$$

Let $u = x^2+8x+20$

$du = (2x+8)\,dx$

$$3\int \frac{1}{u}\,du - \int \frac{7}{(x+2)^2+16}\,dx$$

$$= 3\ln u - \frac{7}{16}\int \frac{1}{\left(\frac{x+2}{4}\right)^2+1}\,dx$$

$$= 3\ln(x^2+8x+20) - \frac{7}{16}\tan^{-1}\left(\frac{x+2}{4}\right)$$

$+$ constant //

$\cancel{\cos(a+b)} = \cancel{\cos a \cos b \cdots}$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin^2 x = \frac{1-\cos 2x}{2}$ 5

Part B $\sin(a-b) = \sin a \cos b - \cos a \sin b$

3. Evaluate the following indefinite integrals.

(a) $\int \sin^6 x \cos^3 x \, dx$ $2\cos a \sin a$.

(b) $\int \sin^4 x \cos^4 x \, dx$

(Hint: Consider the double angle formula for sine.)

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(a) $\int \sin^6 x \cos^2 x \, d(\sin x)$

$= \int \sin^6 x (1-\sin^2 x) \, d(\sin x)$

$= \int \sin^6 x - \sin^8 x \, d(\sin x)$

$= \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + \text{Constant}$.

(b) $\int (\sin x \cos x)^4 \, dx$

$= \int \left(\frac{1}{2}\sin 2x\right)^4 dx$

$= \frac{1}{16}\int \sin^4(2x) \, dx$

$$= \frac{1}{16}\int \frac{1-\cos^2(4x)}{2}\,dx$$

$$= \frac{1}{32}\int 1 - \frac{1+\sin 8x}{2}\,dx$$

$$= \frac{1}{64}\int 1 - \sin 8x\,dx$$

$$= \frac{x}{64} + \frac{\cos 8x}{512} \cancel{+}\ \text{Constant}$$

4. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{x}{\sqrt[3]{1+x^2}}\,dx$

(b) $\displaystyle\int \frac{1}{\sqrt{2x+3}-\sqrt{2x+1}}\,dx$

$$\frac{1}{1+x^2}$$

$$\cancel{u = \sqrt{2x+3} - \sqrt{2x+1}}$$

$$\cancel{du = \frac{\sqrt{2x+1}-\sqrt{2x+3}}{2\sqrt{2x+3}\,\sqrt{2x+1}}\,dx}$$

$$\int \frac{\sqrt{2x+3}+\sqrt{2x+1}}{2}\,dx$$

(a) Let $u = 1 + x^2$

$du = 2x\,dx$

$u^{-\frac{1}{3}}$

$$= \frac{1}{2}\int \frac{1}{u^{\frac{1}{3}}}\,du$$

$$= \frac{1}{2}\cdot\frac{3}{2}u^{\frac{2}{3}} + \text{Constant}$$

$$= \frac{3}{4}\left(1+x^2\right)^{\frac{2}{3}} + \text{Constant}$$

(b) $$\int \frac{\sqrt{2x+3} + \sqrt{2x+1}}{2} \, dx$$

$$= \frac{1}{4}\int \sqrt{2x+3} \; d(2x+3) + \frac{1}{4}\int \sqrt{2x+1} \; d(2x+1)$$

$$= \frac{1}{6}(2x+3)^{\frac{3}{2}} + \frac{1}{6}(2x+1)^{\frac{3}{2}} + \text{constant}$$