香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一二至一三年度上學期科目考試
Course Examination 1st Term, 2012-13

科目編號及名稱
Course Code & Title : **MATH1510A CALCULUS FOR ENGINEERS**

時間
Time allowed : 2 小時 hours 00 分鐘 minutes

學號
Student I.D. No. : ______________ 座號 Seat No.: ______________

SHOW ALL NECESSARY WORK TO GET CREDIT FOR SOLUTIONS
(NOTE: THERE ARE 100 POINTS TOTAL ON THIS EXAMINATION)

1. Solve each of the following problems separately. Justify your solution carefully.

   (a) Determine whether each of the following limits exists. If so, compute the limit.

   i. (4 points) $\displaystyle\lim_{x\to 2}\frac{2x^2+1}{x-1}$

   ii. (4 points) $\displaystyle\lim_{x\to +\infty}\frac{x^2+x+1}{(x-1)^2}$

   iii. (4 points) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

   iv. (4 points) $\displaystyle\lim_{x\to 0} x^2\cos\left(\frac{2\pi}{x}\right)$

   (b) (4 points) Let

   $$f(x)=\begin{cases} x-1, & x\le 5;\\ cx-2, & x>5,\end{cases}$$

   where c is a constant. Determine the value of c so that $f(x)$ is continuous on $(-\infty,\infty)$.

2. Find the derivative y' $\left(\text{or } \dfrac{dy}{dx}\right)$ of each function y below.

   (a) (5 points) $y = 2\sqrt[3]{x} + \dfrac{2}{\sqrt{x}} + 4\sqrt{3}$

   (b) (5 points) $y = (\sin x + \cos x)^2$

   (c) (5 points) $y = \dfrac{1 - \ln(x)}{1 + \ln(x)}$

   (d) (5 points) $y = e^{-x^2}\sin(x^2)$

3. Solve each of the following problems separately. Justify your solution carefully.

   (a) (5 points) Find all local (that is, relative) maxima and minima of the function:
   $$f(x) = 6x - x^2 + 4,$$
   if any.

   (b) Let f be the function defined as follows:
   $$f(x) = 10(x-1)e^{-2x}, \quad x \in (-\infty, \infty).$$

      i. (6 points) Determine the intervals on which f is increasing or decreasing, and find all local (that is, relative) maxima and minima, if any. Substantiate your results by determining the sign of the first derivative in the neighborhood of the stationary points (or the critical values), that is, those x at which $f'(x) = 0$.

      ii. (4 points) Determine the intervals on which f is concave up or concave down (or convex down/up), and find all inflection points, if any.

4. Evaluate the following integrals.

   (a) (5 points) $\displaystyle\int \left(2\sqrt{x} + \frac{3}{\sqrt{x}}\right) dx$

   (b) (5 points) $\displaystyle\int x^2\sqrt{2x^3+1}\, dx$

   (c) (5 points) $\displaystyle\int_{\pi/8}^{\pi/4} \sin^2(x)\, dx$

   (d) (5 points) $\displaystyle\int_1^e \ln x\, dx$

5. Solve each of the following problems separately. Justify your solution carefully.

(a) (4 points) Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 4x - x^2$.

(b) (4 points) Calculate the volume of the solid in Figure 1.

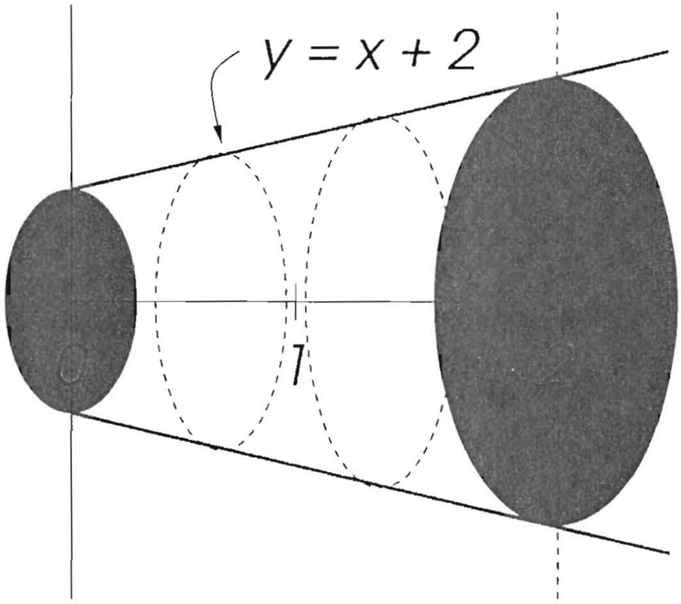

Figure 1: Question 5(b).

6. Solve each of the following problems separately. Justify your solution carefully.

(a) Consider

$$z(x, y) = e^{-3x} \cos y.$$

i. (2 points) Compute z_{xy} and z_{yx}.

ii. (1 point) What can be said about z_{xy} and z_{yx}?

(b) (3 points) Does the function

$$f(x, y) = \sqrt{x^2 + y^2}$$

satisfy the following equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0\,?$$

Justify your answer.

7. Solve each of the following problems separately. Justify your solution carefully.

   (a) Evaluate each of the following limits:

      i. (2 points) $\lim_{x\to+\infty} \frac{x^2(3+\sin^2 x)}{x+100}$

      ii. (2 points) $\lim_{x\to-\infty} xe^x$

      iii. (2 points) $\lim_{x\to+\infty} x^{1/x}$

   (b) i. (1 point) Find the Maclaurin series of the function

$$f(x) = e^{2x}.$$

      ii. (1 point) Use the Taylor series expansion to compute the following limit:

$$\lim_{x\to 0} \frac{e^{2x}-1}{x}.$$

8. Solve each of the following problems separately. Justify your solution carefully.

   (a) (1 point) Let $f(x)$ be a continuous function on $[0,1]$. Show that

$$\int_0^1 \left(\int_0^x f(t)dt\right) dx = \int_0^1 (1-x)f(x)dx.$$

   (b) (1 point) Given $w = f(x,y)$, $x = e^u \cos v$, and $y = e^u \sin v$. Show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial w}{\partial u}\right)^2 + \left(\frac{\partial w}{\partial v}\right)^2\right].$$

   (c) (1 point) Noting that for $r \neq 1$,

$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r},$$

   compute the definite integral $\int_0^1 e^x\,dx$ by finding the limit as n tends to infinity of the left Riemann sum corresponding to the partition of $[0,1]$ into n subintervals of equal length.

~~ THE END ~~