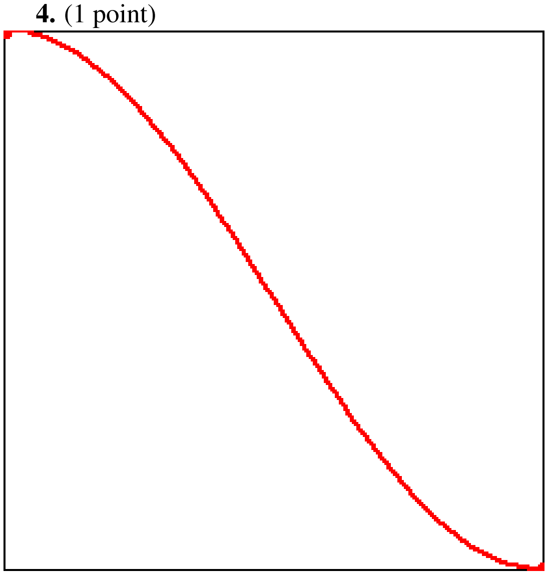
1. (1 point)
Assume that the radius r of a sphere is expanding at a rate of 9cm/min.
Determine the rate at which the volume is changing with respect to time when $r = 9$cm
The volume is changing at a rate of ________ cm^3/min.

2. (1 point)
Water is poured into a vessel in the form of a right circular cone with vertex down at a rate of $2\,\text{cm}^3$/min. The cone height and radius of the cone are 40 cm and 12 cm respectively.
Find the rate at which the water level is rising when the water is 23 cm from the vertex.
Answer: __________ cm/min
Find the rate of rise of the level when the vessel is quarterly filled.
Answer: __________ cm/min

3. (1 point) Let $f(x) = \dfrac{2x-8}{x^2-5x+4}$. Which of the following statements is/are true?

- A. $f(x)$ has a horizontal asymptote $y = 2$.
- B. $f(x)$ has a horizontal asymptote $y = 0$.
- C. $f(x)$ has a vertical asymptote $x = 4$.
- D. $f(x)$ is not well defined at $x = 1, 4$.
- E. $f(x)$ has a vertical asymptote $x = 1$.

4. (1 point)

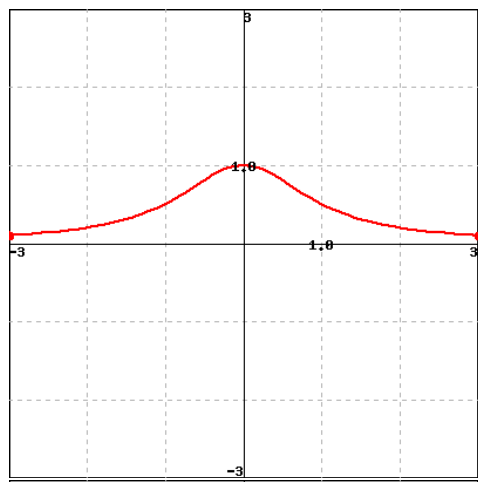


Referring to the graph above, which of the following statements is correct:

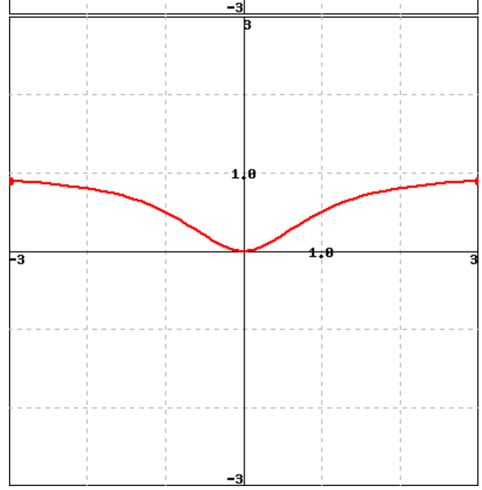
- A. $f''(x)$ changes sign from - to +
- B. $f''(x) < 0$ for all x
- C. $f''(x) > 0$ for all x
- D. $f''(x)$ changes sign from + to -

5. (1 point)
Match the functions with their graphs.

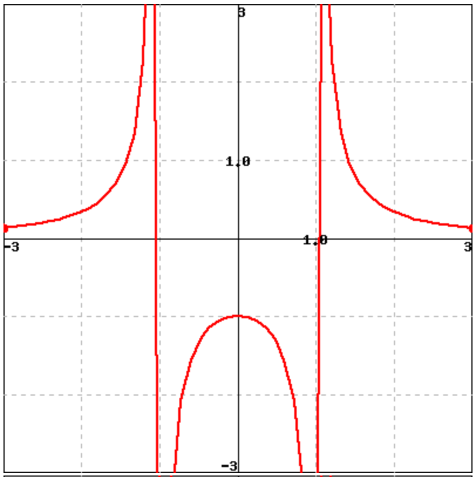
___1. $\frac{1}{x^2+1}$
___2. $\frac{x}{x^2-1}$
___3. $\frac{1}{x^2-1}$
___4. $\frac{x^2}{x^2+1}$



A.



B.

C. 

D. 

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6. (1 point)

$$f(x) = \frac{2}{x^2+3}$$

a) Find the first and second derivatives.

$f'(x) =$ __________

$f''(x) =$ __________

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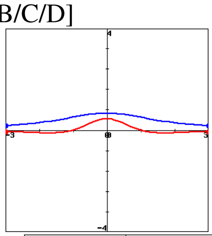
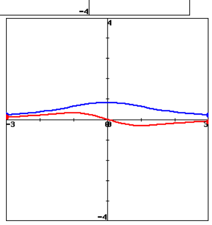
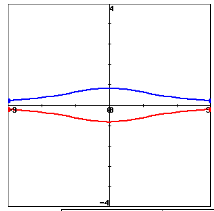
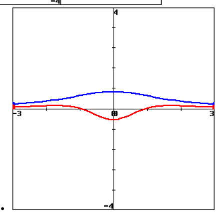
b) Identify the graph that displays f in blue and f'' in red. [?/A/B/C/D]

A. 

B. 

C. 

D. 

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c) Using the graphs of f and f'', indicate where f is concave up and concave down. Give your answer in the form of an interval.

NOTE: When using interval notation in WeBWorK, remember that:
You use 'INF' for ∞ and '-INF' for $-\infty$.
Separate multiple intervals with commas ",".
Enter **DNE** if an answer does not exist.

f is concave up on __________

f is concave down on __________

---

7. (1 point) Consider the function $f(x) = \cos(x) + \frac{\sqrt{2}}{2}x$.
This function has two critical points at $x = x_1, x_2$ in $[0, 2\pi]$, with $x_1 < x_2$. Find x_1 and x_2.

$x_1 =$ _____

$x_2 =$ _____

Also, find $f''(x_1)$ and $f''(x_2)$.

$f''(x_1) =$ __________

$f''(x_2) =$ __________

Thus $f(x)$ has a local $\boxed{?}$ at $x = x_1$

and a local $\boxed{?}$ at $x = x_2$.

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8. (1 point) Suppose that

$$f(x) = \frac{7e^x}{7e^x+6}.$$

(A) Find all critical points of f. If there are no critical points, enter *None* . If there are more than one, enter them separated by commas, e.g. 0,1.

Critical point(s) at x = _____

(B) Use **interval notation** to indicate the interval(s) where $f(x)$ is concave up. (Use commas "," to separate multiple intervals.)

Concave up: ______

(C) Use **interval notation** to indicate the interval(s) where $f(x)$ is concave down. (Use commas "," to separate multiple intervals.)

Concave down: ______

(D) Find all inflection points of f. If there are no inflection points, enter *None* . If there are more than one, enter them separated by commas.
Inflection point(s) at $x =$ ______

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9. (1 point)
Suppose that

$$f(x) = 4x^2 \ln(x), \quad x > 0.$$

(A) List all the critical points of $f(x)$. Note: If there are no critical points, enter 'NONE'.

______

(B) Find all intervals (separated by commas if more than one) where $f(x)$ is increasing. Pay attention to endpoints!
Note: Use 'INF' for ∞, '-INF' for $-\infty$, and use 'U' for the union symbol. If there is no interval, enter 'NONE'.
Increasing: ______

(C) Find all intervals (separated by commas if more than one) where $f(x)$ is decreasing. Pay attention to endpoints!
Decreasing: ______

(D) List the x values of all local maxima of $f(x)$. If there are no local maxima, enter 'NONE'.
x values of local maxima = ______

(E) List the x values of all local minima of $f(x)$. If there are no local minima, enter 'NONE'.
x values of local minima = ______

(F) Find all intervals where $f(x)$ is concave up.
Concave up: ______

(G) Find all intervals where $f(x)$ is concave down.
Concave down: ______

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10. (1 point) The rate of transmission in a telegraph cable is observed to be proportional to

$$x^2 \ln(1/x)$$

where x is the ratio of the radius of the core to the thickness of the insulation $(0 < x < 1)$. What value of x gives the maximum rate of transmission?

= ______

Hint: Think about what does 'proportional' means in this case. Essentially, this means that the rate of transmission is a constant multiple of the expression given. Write the rate, R, this way, differentiate and apply your knowledge of rates of change and maxima and minima to solve for x.

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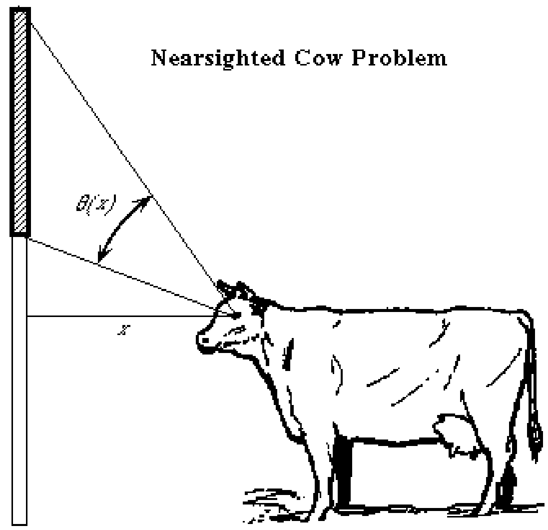
11. (1 point)
The Nearsighted Cow Problem: A Calculus Classic.
A rectangular billboard 6 feet in height stands in a field so that its bottom is 13 feet above the ground. A nearsighted cow with eye level at 4 feet above the ground stands x feet from the billboard. Express θ, the vertical angle subtended by the billboard at her eye, in terms of x. Then find the distance x_0 the cow must stand from the billboard to maximize θ.
$\theta(x) =$ ______
θ is maximized when $x_0 =$ ______



(**Click on image for a larger view**)

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12. (1 point)
Demonstrate how to:

Find the point on the curve
$y = \sqrt{x}$ closest to the point $(8,0)$.

by dragging all relevant statements below into the **right column** in an appropriate order.

(Leave all irrelevant or incorrect statements in the left column.)

---

0. Hence, $f(x)$ is strictly decreasing on $(7.5,+\infty)$ and strictly increasing on $(7.5,+\infty)$.
1. To find the critical points of $f(x)$, we first find $f'(x) = 2x - 15$, which is defined everywhere.
2. $f'(x)$ is positive on the interval $(-\infty, 7.5)$ and negative on $(7.5,+\infty)$.
3. $f''(x) = 2$

4. Hence, $f(x)$ is strictly increasing on $(7.5, +\infty)$ and strictly decreasing on $(7.5, +\infty)$.
5. Minimizing the distance between $(8,0)$ and the curve $y = \sqrt{x}$ is equivalent to minimizing the square of the distance.
6. Then, solve $f'(c) = 0$ to obtain $c = 7.5$.
7. $f'(x)$ is negative on the interval $(-\infty, 7.5)$ and positive on $(7.5, +\infty)$.
8. It follows that the point on $y = \sqrt{x}$ closest to $(8,0)$ is $(7.5, \sqrt{7.5})$.
9. So, $f(7.5)$ must be the minimum value of $f(x)$.
10. The square of the distance between a point (x,y) on the curve and the point $(8,0)$ is $f(x) = (x-8)^2 + y^2 = x^2 - 15x + 8^2$.

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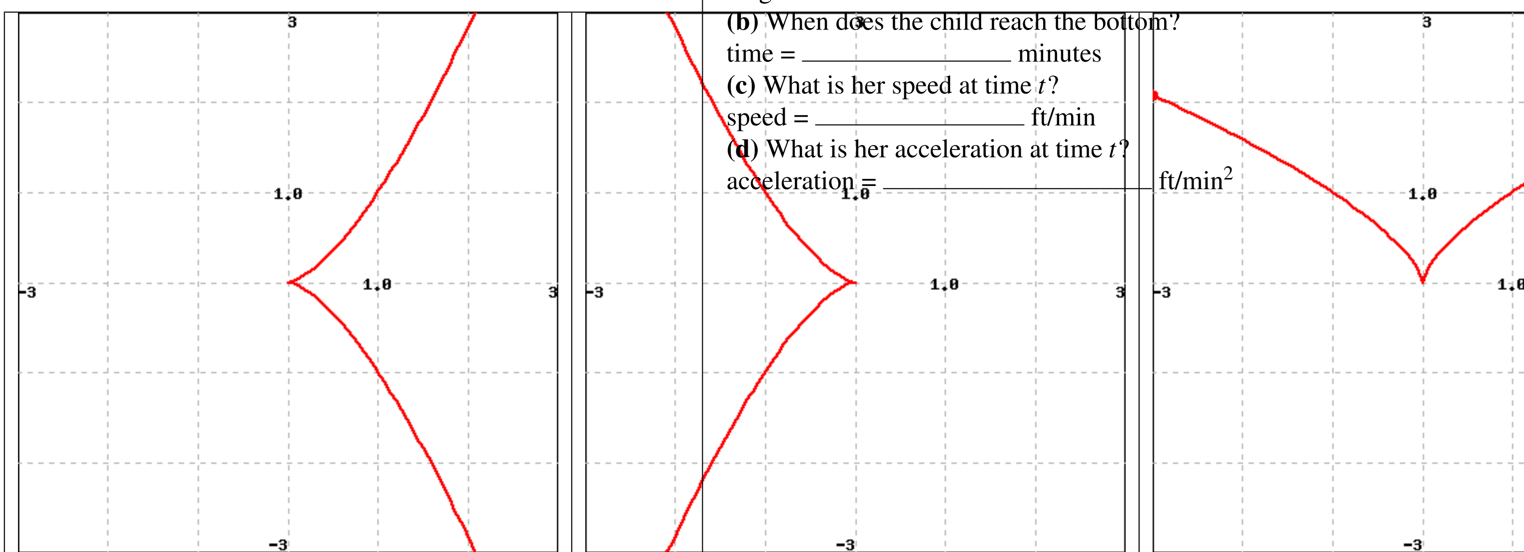
13. (1 point)

Consider the parametric curve

$$\mathbf{r}(t) = \left\langle t^2, -t^3 \right\rangle.$$

Determine which of the following graphs is the correct image of this curve:



A B C

Also, compute the velocity and acceleration vectors at $t = 1$, and use this information to determine the direction of motion for the curve.

The correct graph is $\boxed{?}$.

$\mathbf{v}(1) =$ ________.

$\mathbf{a}(1) =$ ________.

The direction of motion is $\boxed{?}$.

Usage: To enter a vector, for example $\langle 1, -3, 2 \rangle$, type ”¡ 1, -3, 2 ¿”.

---

14. (1 point)

Determine the speed $s(t)$ of a particle with a given trajectory at a time t_0 (in units of meters and seconds).

$$c(t) = (\ln(t^2+1), t^3),\ t_0 = 12$$

____

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15. (1 point) A child wanders slowly down a circular staircase from the top of a tower. With x, y, z in feet and the origin at the base of the tower, her position t minutes from the start is given by

$$x = 30\cos t, \quad y = 30\sin t, \quad z = 120 - 5t.$$

(a) How tall is the tower?
height = ____________ ft

(b) When does the child reach the bottom?
time = ____________ minutes

(c) What is her speed at time t?
speed = ____________ ft/min

(d) What is her acceleration at time t?
acceleration = ____________ ft/min^2