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香港中文大學
The Chinese University of Hong Kong

二〇一四至一五年度上學期科目考試
Course Examination 1st Term, 2014-15

科目編號及名稱 Course Code & Title : **MATH1510A/B/C/D/E/F/G Calculus for Engineers**

時間 Time allowed : 2 小時 hours 00 分鐘 minutes

學號 Student I.D. No. : ______ 座號 Seat No.: ______

Answer ALL Questions.

1. Let

$$f(r) = \begin{cases} r^2 - 4r + 4, & r < 2; \\ \sqrt{r-2}, & 2 \le r < 6; \\ |4 - r|, & r \ge 6. \end{cases}$$

(a) (2 points) Find $\lim_{r \to 2^-} f(r)$.

(b) (4 points) Is $f(r)$ continuous at $r = 2$? Justify your answer.

(c) (2 points) Use the definition of left-derivative to find the left-derivative of $f(r)$ at $r = 6$, i.e., $Lf'(6)$.

(d) (4 points) Is $f(r)$ differentiable at $r = 6$? Justify your answer.

2. Find $\dfrac{dy}{dx}$ for:

(a) (3 points) $y = \log_4 x + 7^x$;

(b) (3 points) $y = \dfrac{1}{(\sin x + e^x)^2}$;

(c) (3 points) $y = (\cos x)^x$;

(d) (3 points) $y = \displaystyle\int_{x^2}^{\sin x} (t^5 - 9t^2)\,dt$.

3. Find the following integrals:

   (a) (3 points) $\displaystyle\int \left(\frac{x - x^{-1}}{\sqrt{x}} + 3^x + \frac{\pi^2}{x} \right) dx$;

   (b) (3 points) $\displaystyle\int e^x \sin(e^x)\, dx$;

   (c) (3 points) $\displaystyle\int_0^\pi \cos^3 x\, dx$;

   (d) (3 points) $\displaystyle\int \sin(5x)\cos(3x)\, dx$;

   (e) (3 points) $\displaystyle\int_1^e \frac{\ln x}{x^2}\, dx$;

   (f) (3 points) $\displaystyle\int \sqrt{16 - 5x^2}\, dx$;

   (g) (2 points) $\displaystyle\int \frac{dx}{(x-2)(x-4)}$.

4. Solve the following problems. Justify your answers.

   (a) Let
   $$g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + 4.$$

      i. (4 points) Find the critical points and find the intervals on which the function is increasing or decreasing.

      ii. (3 points) Use the First Derivative Test to determine whether the critical point is a local min or max (or neither).

   (b) (4 points) Let
   $$h(x) = 6x^{3/2} - 4x^{1/2}, \quad x > 0.$$
   Find the critical points and apply the Second Derivative Test (or show that it fails).

5. Solve the following problems. Justify your answers.

(a) (4 points) Sketch and find the area of the plane region bounded by the given curves:

$$\begin{cases} y = x^2 - 2x; \\ y = 6x - x^2. \end{cases}$$

(b) Given:

The plane region $\mathcal{R}$ is bounded by $y = 1 + x^{3/2}$, $y = 9$ and $x = 0$.

Set up the integrals (but do not evaluate) to find the volumes of the solids obtained if the plane region $\mathcal{R}$ is rotated about

i. (2 points) the $x-$axis;

ii. (2 points) the $y-$axis.

(c) (4 points) Let

$$w(x, y) = xe^y.$$

Compute the double integral of $w(x, y)$ over the domain

$$\mathcal{D} = \{(x, y) |\ 0 \le x \le 1 \quad \text{and} \quad 0 \le y \le x\}.$$

6. Solve the following problems separately. Justify your answers.

(a) Given:

$$u(x, t) = \cos(x + \beta t) + \sin(x - \beta t),$$

where β is an arbitrary constant.

i. (3 points) Find u_x and u_t.

ii. (3 points) Find a constant A such that :

$$A\,\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}.$$

(b) (3 points) Given that

$$\begin{cases} s = p^2 - q^2 + 4t; \\ p = \phi^2 e^{3\theta}; \\ q = \cos(\phi + \theta), \end{cases}$$

where ϕ, θ, t are independent variables of real numbers. Compute s_ϕ and s_θ.

7. Solve the following problems separately. Justify your answers.

   (a) (2 points) Find the radius of convergence of the following series:
   $$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+1}}.$$

   (b) (6 points) Using the definition of the Taylor series, find the Taylor series about $x = 1$ for $d(x) = e^{x+2}$, and show that it converges to e^{x+2} for all values of x.

   (c) (6 points) Let
   $$l(x) = \begin{cases} 0, & x = 0; \\ \dfrac{\pi - x}{2}, & 0 < x < 2\pi; \\ 0, & x = 2\pi; \end{cases}$$
   and $l(x+2\pi) = l(x)$. Find the Fourier series of l on the interval $[0, 2\pi]$.

8. Solve the following problems separately. Justify your answers.

   (a) (2 points) Evaluate the following limit:
   $$\lim_{x\to 0} \left(\frac{e^x}{e^x - 1} - \frac{1}{x} \right).$$
   Show each step of your work.

   (b) (2 points) Use Taylor series about $x = 0$ to evaluate the following limit:
   $$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x^2 - x\ln(1+x)}.$$

   (c) (2 points) Let
   $$v(x) = x \ln x - x + 1.$$
   Show that $v(x) \geq 0$ for all $x \geq 1$.

   (d) (2 points) Find a point α satisfying the conclusion of Lagrange's mean value theorem for the given function and interval:
   $$p(x) = \cos x - \sin x, \quad [0,\ 2\pi].$$

   (e) (2 points) Given:
   $$q(x) = (x-3)^2.$$
   Find $\gamma \in [2, 5]$ such that $q(\gamma)$ is equal to the average value of q over $[2, 5]$.

- **End** -