THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 9

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Class: MATH1510G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

David 15-11-2021

Signature Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	1.5						
2	2						
3							
4	3						
					Total	6.5	/ 10

$$A+B=0$$
$$-\quad -5A+B=1$$
$$4A=-1$$

Part A

1. (a) Evaluate $\displaystyle\int \frac{1}{t^2+4t-5}\,dt.$ $\quad A=-\frac{1}{4} \quad B=\frac{1}{4}$

-1.5

(b) Using t-substitution and the result in part (a), evaluate

$$\int \frac{1}{2\sin x - 3\cos x - 2}\,dx$$

(a) $\displaystyle\int \frac{-1}{4(x+\;)} + \frac{1}{4(x-5)}\,dx$

$= +\frac{1}{4}\ln|x-1| \mp \frac{1}{4}\ln|x+5| + \text{Constant}$

(b) Let $u=\tan\frac{x}{2}$. $\qquad \tan x = \dfrac{2\tan\frac{x}{2}}{(1-\tan^2\frac{x}{2})}$

$du = \sec^2\frac{x}{2}\left(\frac{1}{2}\right)dx \qquad = \dfrac{2u}{1-u^2}$

$= (1+u^2)\left(\frac{1}{2}\right)dx$

$\sin x = \dfrac{2u}{1+u^2}$

$\cos x = \dfrac{1-u^2}{1+u^2}$

$\therefore 2\displaystyle\int \frac{1}{4u-3+3u^2-2-2u^2}\,du$

$= 2\displaystyle\int \frac{1}{u^2+4u-5}\,du$

By obtaining the result of (a), we have:

$= -\frac{1}{3}\ln|u+1| + \frac{1}{3}\ln|u-5| + \text{constant}$

$= -\frac{1}{3}\ln\left|\tan\frac{x}{2}+5\right| + \frac{1}{3}\ln\left|\tan\frac{x}{2}-5\right| + \text{Constant}$ //

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

~~cos(a+b) =~~

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

2. Evaluate the following indefinite integrals.

$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \cancel{\ldots}\right]$

(a) $\displaystyle\int x\sin(2x+1)\,dx$

(b) $\displaystyle\int x\arccos x\,dx$

(c) $\displaystyle\int \cos(\ln x)\,dx$

(a) Let $u = 2x+1$

$du = 2\,dx$ & $x = \frac{u-1}{2}$

$$\frac{1}{4}\int \underset{u}{(u-1)}\ \underset{dv}{\sin u}\,du$$

$$= \frac{1}{4}\left[-(u-1)\cos u + \int \cos u\,du\right]$$

$$= \frac{1}{4}\left(\cos u - u\cos u + \sin u\right) + \text{Constant}$$

$$= \frac{-2x\cos(2x+1) + \sin(2x+1)}{4} + \text{Constant}$$

(b) Let $x = \cos\theta$, $\arccos x = \theta$

$dx = \sin\theta\,d\theta$

$1^2 = x^2 + ?^2$

$? = \sqrt{1-x^2}$

$$\int \theta(\cos\theta)(\sin\theta)\,d\theta$$

$$= \frac{1}{2}\int \underset{u}{\theta}\,\underset{dv}{(\sin 2\theta)}\,d\theta$$

$$= \frac{1}{2}\left[\theta\left(-\frac{1}{2}\right)(\cos 2\theta) - \int\left(-\frac{1}{2}\right)(\cos 2\theta)\,d\theta\right]$$

$$= -\frac{1}{4}\theta\cos\theta + \frac{1}{8}\sin\theta + \text{Constant}$$

$= -\frac{1}{4}x(\arccos x) + \frac{1}{8}\sqrt{1-x^2} + \text{Constant}$ //

(c) $\int \cos(\ln x)\, dx$

Let $x = e^u$, $\ln x = u$.

$dx = e^u du$

$\therefore \int \underset{u}{(\cos u)} \underset{dv}{e^u du}$.

$= e^u(\cos u) + \int \underset{dv}{e^u} \underset{u}{(\sin u)}\, du$

$= e^u(\cos u) + e^u(\sin u) - \int e^u(\cos u)\, du$.

$= \dfrac{e^u(\cos u) + e^u(\sin u)}{2} + \text{Constant}$ //

$u =$

Part B

3. Define $I_n = \int \frac{x^n}{\sqrt{2x+1}}\, dx$ for all non-negative integers n.

   (a) Evaluate I_0.

   (b) Considering that

   $$\int x^{n-1}\sqrt{2x+1}\, dx = \int x^{n-1}\frac{2x+1}{\sqrt{2x+1}}\, dx = 2\int \frac{x^n}{\sqrt{2x+1}}\, dx + \int \frac{x^{n-1}}{\sqrt{2x+1}}\, dx,$$

   show that

   $$I_n = \frac{x^n\sqrt{2x+1}}{2n+1} - \frac{n}{2n+1} I_{n-1}$$

   for all integers $n \geq 1$.

   (c) Using parts (a), (b), evaluate I_3.

(a) $I_0 = \int \frac{x}{\sqrt{2x+1}}\, dx$

Let $u = 2x+1$, $x = \frac{u-1}{2}$.

$du = 2dx$

$\frac{1}{4}\int \frac{u-1}{\sqrt{u}}\, du$

$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

$= \frac{1}{4}\left(\frac{2}{3}\right)u^{\frac{3}{2}} - \frac{1}{4}(2)u^{\frac{1}{2}} + \text{Constant}$

$= \frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}\sqrt{2x+1} + \text{Constant}$ //

dv u

(b) $\because \int x^{n-1}\sqrt{2x+1}\, dx = 2I_n + I_{n-1}$

$\therefore 2I_n - I_{n-1} = \frac{x^n\sqrt{2x+1}}{n} - \int \frac{x^n}{n} \cdot \frac{1}{\sqrt{2x+1}}\, dx$

$\frac{(2x-3)}{15 \; 30}$

$$2I_n - I_{n-1} = \frac{x^n\sqrt{2x+1}}{n} - \frac{1}{n} I_n .$$

$$I_n \left(\frac{2n+1}{n}\right) = \frac{x^n\sqrt{2x+1}}{n} + I_{n-1}$$

$$I_n = \frac{x^n\sqrt{2x+1}}{2n+1} + \left(\frac{n}{2n+1}\right) I_{n-1}$$

(c) $I_1 = \frac{x\sqrt{2x+1}}{3} + \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{2}\sqrt{2x+1}$ + Constant

$I_2 = \frac{x^2\sqrt{2x+1}}{5} + \frac{2}{5}\left[\left(\frac{x}{3} - \frac{1}{2}\right)\sqrt{2x+1} - \frac{1}{18}(2x+3)^{\frac{3}{2}}\right]$ + Constant

$I_3 = \frac{x^3\sqrt{2x+1}}{7} + \frac{3}{7}\left(\frac{x^2\sqrt{2x+1}}{5} + \frac{2x-3}{15}\sqrt{2x+1} - \frac{1}{45}(2x+3)^{\frac{3}{2}}\right)$

$= \frac{x^3\sqrt{2x+1}}{7} + \frac{3x^2\sqrt{2x+1}}{35} + \frac{(2x-3)\sqrt{2x+1}}{35} - \frac{(2x+3)^{\frac{3}{2}}}{105}$ + Constant

4. Define $I_n = \int \sec^n x \, dx$ for all non-negative integers n.

(a) By applying integration by parts to

$$\int \sec^n x \, dx = \int \sec^{n-2} x \sec^2 x \, dx,$$

show that

$$I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

for all integers $n \geq 2$.

(b) Using (a), find I_4 and I_5.

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(a) $\int \underbrace{\sec^{n-2} x}_{u} \cdot \underbrace{\sec^2 x \; dx}_{dv}$

$$I_n = \tan x \cdot \sec^{n-2} x - \int \tan x \, (n-2) \sec^{n-3} x \cdot \sec x \tan x \, dx$$

$$= \tan x \cdot \sec^{n-2} x - (n-2) \int \tan^2 x \cdot \sec^{n-2} x \, dx$$

$$= \tan x \cdot \sec^{n-2} x - (n-2) \int \sec^n x - \sec^{n-2} x \, dx$$

$$= \tan x \cdot \sec^{n-2} x - (n-2) I_n + (n-2) I_{n-2}$$

$$I_n(n-1) = \tan x \cdot \sec^{n-2} x + (n-2) I_{n-2}$$

$$I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2} \quad //$$

(b) $I_2 = \tan x + 0 \times \cancel{\tan x}\, I_0$

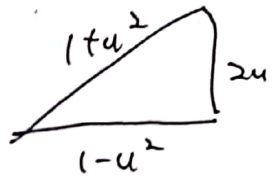
$= \tan x$

$$I_4 = \frac{\sec^2 x \tan x}{3} + \frac{2}{3}(\tan x)$$

$$= \frac{(\sec^2 x + 2)\tan x}{3} + C \quad //$$

$$I_1 = \int \sec x \, dx$$

$$= \int \frac{1}{\cos x} dx$$



Let $u = \tan\frac{x}{2}$, $\tan x = \frac{2u}{1-u^2}$

$du = (1+u^2)\,dx$ $\quad \cos x = \frac{1-u^2}{1+u^2}$

$$= \int \frac{1}{1-u^2} du$$

$$\frac{(1+t)^2}{1-t^2}$$

$$= \frac{1}{2}\int \frac{1}{1+u} + \frac{1}{1-u}\, du$$

$$= \frac{1}{2} \ln|\tan x + \sec x| + \text{constant}$$

$$I_3 = \frac{\sec x \tan x}{2} + \frac{1}{4} \ln|\tan x + \sec x|) + \text{constant}$$

$$I_5 = \frac{\sec^3 x \tan x}{4} + \frac{3}{4}\left(\frac{\sec x \tan x}{2} + \frac{1}{4}\ln\left|\tan x + \sec x\right|\right) + \text{constant}$$

$$= \frac{\sec^3 x \tan x}{4} + \frac{3}{8}(\sec x \tan x) + \frac{3}{8}\ln\left|\tan x + \sec x\right|$$

$$+ \text{constant} \; //$$