THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 1

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I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

David 13-9-2021
Signature Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	4	5					
2	1						
3	3						
4a	2						
4b					Total	10	/ 10

$$\frac{\cancel{(x-1)}(x+1)}{\cancel{1-x}}$$

$x^2-1 = (x-1)(x+1)$

$|x-1|$

Part A

1. (a) Given that

$$f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = \sqrt{x-2},$$

write down the function $f \circ g$ explicitly. Find the domain of $f \circ g$ and express your answer in interval notation. $\Rightarrow f(g(x))$

(b) Suppose

$$f(x) = \frac{x^2-1}{|x-1|}.$$

i. Rewrite the function $f(x)$ as a piecewise function in terms of polynomials in the following form.

$$f(x) = \begin{cases} x+1 & \text{if } x > 1, \\ \text{undefined} & \text{if } x = 1, \\ -(x+1) & \text{if } x < 1. \end{cases}$$

ii. Find $f(-100000)$ and $f(100000)$.

(a) $$f \circ g = \frac{1}{\sqrt{x-2}}$$

Domain of $f \circ g(x)$: $x-2 > 0$

$x > 2$.

Interval notation : $(2, \infty)$

(b) ii) $f(-100000) = -(-100000+1)$

$= 99999$ //

$f(100000) = 100000 + 1$

$= 100001$ //

2. For the sequence

$$a_n = \left\{\sqrt{n^2+n} - \sqrt{n^2+(-1)^n}\right\}, \quad \text{for } n \geq 1$$

fill the following table (correct to 4 decimal places) and guess the value of a_n when n gets very large (approaches ∞).

n	100	1000	10000
a_n	0.4938 (4d.p.)	0.4993 (4d.p.)	0.4999 (4d.p.)

When n gets very large (i.e. $n = \infty$),

a_n will approach 0.5. //

Part B

3. Let $f(x) = x^2 + 2x + 2$ and $g(x) = \ln x$.

   (a) By completing square, find the minimum value of $f(x)$.

   (b) Find the range of $g \circ f$. Express your answer in interval notation.

(a) $f(x) = x^2 + 2x + 2$

$= x^2 + 2x + 1 - 1 + 2$

$= (x+1)^2 - 1 + 2$

$= (x+1)^2 + 1$

$\because (x+1)^2 \geq 0$ for all real number of x

∴ The minimum value of $f(x)$ $= 0+1 = 1$ //

(b) $g \circ f(x) = g(f(x))$

$= \ln(x^2+2x+2)$

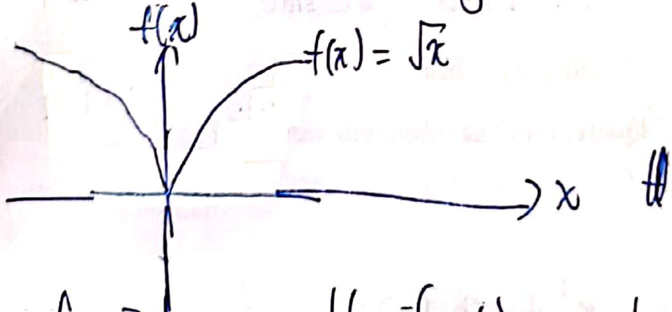
∵ $x^2+2x+2 \geq 1$ for all real number of x.

∴ Interval notation: $[0,\infty)$

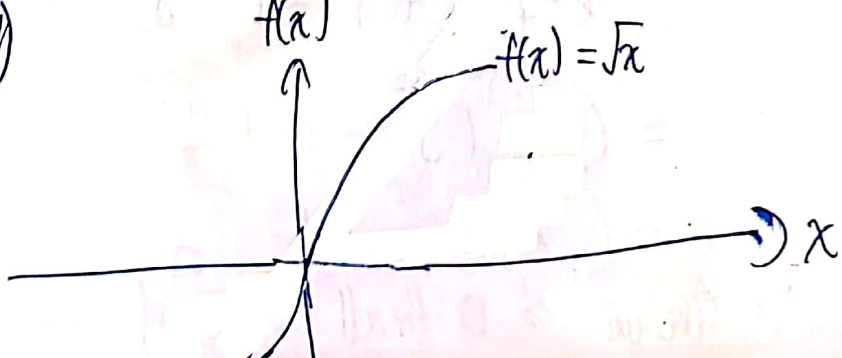
4. (a) Let $f:\mathbb{R}\to\mathbb{R}$ be a function. Given that $f(x)=\sqrt{x}$ for $x\geq 0$, sketch the graph of $f(x)$ if
   i. f is an even function;
   ii. f is an odd function.

   (b) Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $f(x)=0$ when $-1\leq x\leq 0$ and $f(x)=x$ when $0<x<1$.
   Suppose that f is a periodic function with period 2. Sketch the graph of $f(x)$.

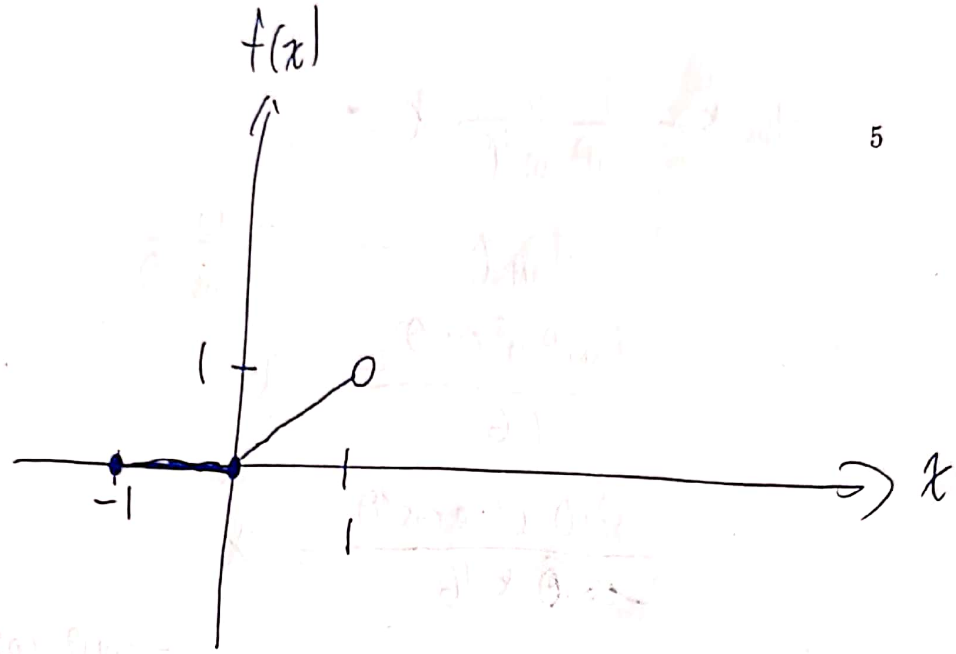
(a) i) f is an even function when $f(-x) = f(x)$ for all $\mathbb{R}$ and symmetric about y-axis



ii) f is an odd function when $f(x) = -f(x)$.



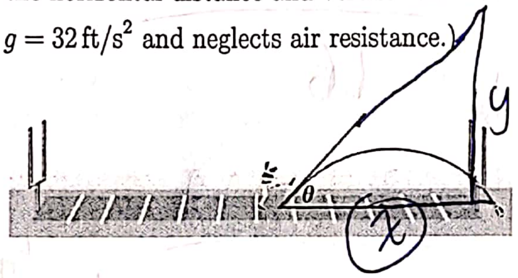
(b)



5. The path traveled by an object that is projected at an initial height of h_0 feet, an initial speed of v feet per second, and an initial angle θ is given by

$$y = -\frac{16}{v^2 \cos^2 \theta} x^2 + (\tan \theta) x + h_0$$

where x and y are the horizontal distance and vertical distance respectively. (This model takes $g = 32\,\mathrm{ft/s^2}$ and neglects air resistance.)



If a football is kicked from the ground level with speed v,

(a) Show that the total horizontal distance traveled is $\dfrac{v^2 \sin\theta \cos\theta}{16}$.

(b) With what angle θ will the total horizontal distance traveled be maximized? (Hint: Consider the double angle formula: $\sin 2x = 2 \sin x \cos x$)

$h_0 =$

(a) Total horizontal distance $= x$ when $y = 0$.

Sub $y = 0$:

$$0 = -\frac{16}{v^2 \cos^2\theta} x^2 + (\tan\theta) x$$

$$x\left(\tan\theta - \frac{16}{v^2 \cos^2\theta} x\right) = 0$$

$x = 0$ (rejected.) or

$$\tan\theta - \frac{16}{u^2\cos^2\theta}x = 0$$

$$\tan\theta = \frac{16}{u^2\cos^2\theta}x$$

$$\frac{\tan\theta\, u^2\cos^2\theta}{16} = x$$

$$\frac{\sin\theta\, u^2 \cos^2\theta}{\cancel{\cos\theta} \times 16} = x$$

$$x = \frac{u^2\sin\theta\cos\theta}{16} \quad //$$

(b) $$x = \frac{u^2\sin\theta\cos\theta}{16}$$

$$= \frac{u^2\sin 2\theta}{32},$$ ~~by considering~~

For $\sin 2\theta$, $0^\circ < 2\theta < \frac{\pi}{2}$,

& $0 \leq \sin 2\theta \leq 1$.

When $2\theta = \frac{\pi}{2}$

$$\theta = \frac{\pi}{4} \quad //$$

(45°)