# Lecture 4

## Bond Valuation

Instructor: Prof. Chen (Alison) Yao
CUHK Business School

# Raise Capital by Issuing Bonds

- Investment in a new plant and equipment requires money
  - often a lot of money
- Government, municipalities and companies can turn to the public to raise new capital
- Two ways to raise new money:
  - borrow cash by issuing bonds
  - sell additional shares of common stock

# Bonds

- One who buys a corporate bond is a *creditor* of the company, NOT an owner (unlike stockholders who are owners).
- When companies issue bonds they promise to make a series of fixed interest payments and then to repay the debt
- As long as the company generates sufficient cash, the payments on a bond are certain

# Bond: Fixed-income Investment

- **Fixed-income** investments: you are assured a steady payout or yearly income.

- This regular income is what makes bond returns inherently less risky than stock returns.
  - Stocks are risky
    - We do not know the price for Google next week.
  - Bonds seem risk-free?
    - A 8% coupon bond of 1000-dollar face value will pay you 80 dollars each year and 1000 dollars at maturity
    - Interest rate risk
    - Default risk

# Lecture Outline

- Bond valuation
  - Coupon Bond
    - Bond Pricing
    - Bond Yield-to-Maturity
  - Zero-Coupon Bond

- Bond Risks
  - Interest rate risk
  - Default risk

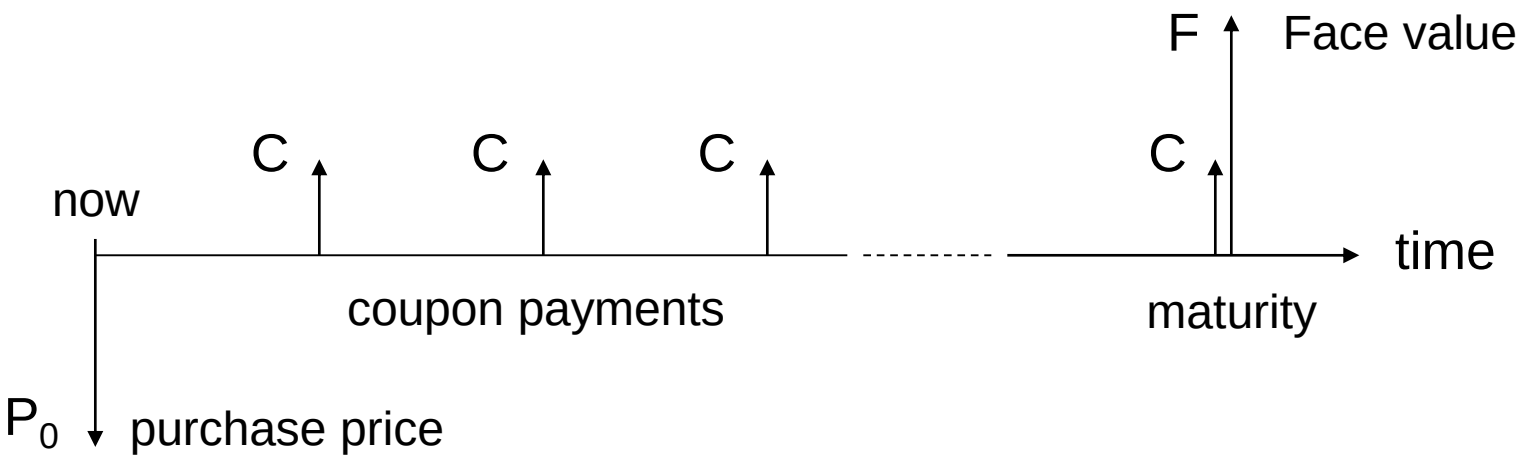
# Terminology

- **Bond**: a security issued as promise to repay a loan
  - The loan is paid off in a fixed number of payments
- **Face value (or *par value*)**: the principal amount of the loan to be repaid
  - The amount relative to which interest is computed
- **Coupon**: the stated interest payment(s) to the bond holders at (a) specified date(s)
  - Example: U.S. Treasury bonds pay coupons semi-annually
  - Zero-Coupon Bond pays no coupons
- **Maturity**: Number of years until the face value is paid
- **Yield to maturity (YTM)**: the required rate of return in the market on a bond

# Coupon Bonds

**Coupon bonds:**

- Today (t=0), you pay the market price $P_0$ for the bond
- At of the end of every period, you receive coupon payment C
- At maturity, you receive the last coupon plus the face value F



# Coupon Rate

- principal ("face value")
  = final redemption amount

- coupon rate
  = stated annual interest rate

- coupon frequency:
  number of coupons per year

***"face value = F"***

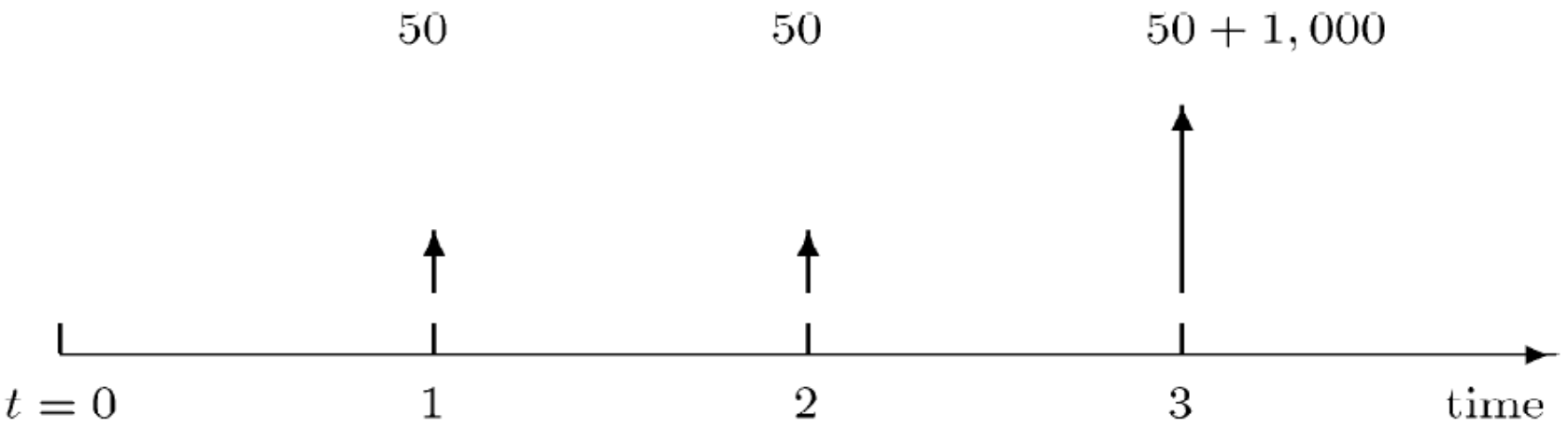
***"coupon rate = R"***

***Coupon amount:*** $C = \frac{R}{N} \times F$

***"paid N times per year"***

# Example

Example: A 3-year bond with principal of $1,000 and annual coupon payment of 5% has the following cash flow



# What Determines Bond Value?

The price of a bond depends on **the interest rates required in the market**:

- bond price should equal to the present value of all future cash flows (coupons and par value)

***Formula:***

- **bond value today = PV of coupons + PV of par value**

$$P = \sum_{i=1}^{t} \frac{C}{(1+r)^i} + \frac{F}{(1+r)^t} = \frac{1}{r}\left[1 - \frac{1}{(1+r)^t}\right] \times C + \frac{1}{(1+r)^t} \times F$$

$$= PV_A(r;t) \times C + DF(r;t) \times F$$

- bond prices are inversely related to the level interest rates required in the market

# Bond Value: Pricing

Example:

we need to price an 9% coupon (semi-annual payments), 3-year maturity bond with par value of $100, suppose the required interest rate is 4% annually, what is the value of the bond?

Solution:

- coupon rate R = 9%
- Interest rate r = 4%
- maturity T = 3 (years)
- semi-annual coupons (N = 2)

- $C = \frac{R}{N} \times F = \frac{9\%}{2} \times 100 = 4.5$
- $t = N \times T = 2 * 3 = 6$
- $\frac{r}{N} = \frac{4\%}{2} = 2\%$

# Bond Value: Pricing (method 1)

Example:

we need to price an 9% coupon (semi-annual payments), 3-year maturity bond with par value of $100, suppose the required interest rate is 4% annually, what is the value of the bond?

| | ❶ | ❷ | ❸ | ❹ | ❺ | ❻ |
|---|---|---|---|---|---|---|
| | period | coupon payments | principal redemption | total cash flows | discount factor | discounted cash flows |
| | *i* | $(R/N)\times F$ | $F$ | $=❷+❸$ | $1/(1+r/N)^i$ | $=❹\times❺$ |
| ***1 year*** → | 1 | 4.50 | | 4.50 | 0.9804 | 4.41 |
| | 2 | 4.50 | | 4.50 | 0.9612 | 4.33 |
| ***2 years*** → | 3 | 4.50 | | 4.50 | 0.9423 | 4.24 |
| | 4 | 4.50 | | 4.50 | 0.9238 | 4.16 |
| ***3 years*** → | 5 | 4.50 | | 4.50 | 0.9057 | 4.08 |
| | 6 | 4.50 | 100.00 | 104.50 | 0.8879 | 92.79 |
| | | | | | | **$P_0$ = 114.01** |

# Bond Value: Pricing (method 2)

Example:

we need to price an 9% coupon (semi-annual payments), 3-year maturity bond with par value of $100, suppose the required interest rate is 4% annually, what is the value of the bond?

$$P = PV_A(r;t) \times C + DF(r;t) \times F = \frac{C}{r}\left[1 - \frac{1}{(1+r)^t}\right] + \frac{F}{(1+r)^t}$$

Since coupon is paid semi-annually, we replace *r* with *r/N*, and replace *t* with $N \times T$

$$P = \frac{C}{r/N}\left[1 - \frac{1}{(1+r/N)^{T\times N}}\right] + \frac{F}{(1+r/N)^{T\times N}}$$

$$= \frac{4.5}{0.04/2}\left[1 - \frac{1}{(1+0.04/2)^{3\times 2}}\right] + \frac{100}{(1+0.04/2)^{3\times 2}} = \$114.01$$

# Yield to Maturity (YTM)

- *Yield to maturity* (also known as *yield*), *market interest rate*, and *interest rate required in the market* are used interchangeably.

- YTM is the discount rate that equates the bond's price to the present value of all its promised cash flows

- Given the bond price, time to maturity, coupon rate and coupon frequency, we can we calculate its implicit discount rate (or yield-to-maturity)

- It is a measure of the average rate of return you will earn over the bond's life if you hold it to maturity

# Calculating YTM

**Example:** Suppose an 8% coupon (semi-annual payments), 30-year maturity bond sells at $1,276.76, what is the yield?

$$1276.76 = \sum_{i=1}^{60} \frac{40}{(1+y/2)^i} + \frac{1{,}000}{(1+y/2)^{60}} = \mathrm{PV_A}(y/2; 60) \times 40 + \mathrm{DF}(y/2; 60) \times 1000$$

**Solution:**

Method 1). Trial and error gives y = 6%

Method 2). **YIELD( )** function in Excel

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | Semiannual coupons | | Annual coupons | |
| 2 | | | | | |
| 3 | Settlement date | 1/1/2000 | | 1/1/2000 | |
| 4 | Maturity date | 1/1/2030 | | 1/1/2030 | |
| 5 | Annual coupon rate | 0.08 | | 0.08 | |
| 6 | Bond price (flat) | 127.676 | | 127.676 | |
| 7 | Redemption value (% of face value) | 100 | | 100 | |
| 8 | Coupon payments per year | 2 | | 1 | |
| 9 | | | | | |
| 10 | **Yield to maturity (decimal)** | **0.0600** | | **0.0599** | |
| 11 | | | | | |
| 12 | The formula entered here is: =YIELD(B3,B4,B5,B6,B7,B8) | | | | |

**note:**

- settlement date is the purchase date
- redemption = face value

# Valuation of a Bond: Example

Suppose the Star Co. were to issue a bond with 10 years to maturity. The Star bond has an annual coupon of $80. Similar bonds have a yield to maturity of 8 percent. The Star bond will pay $80 per year for the next 10 years in coupon interest. In 10 years, Star will pay $1000 to the owner of the bond. What would this bond sell for?

*Cash flows*

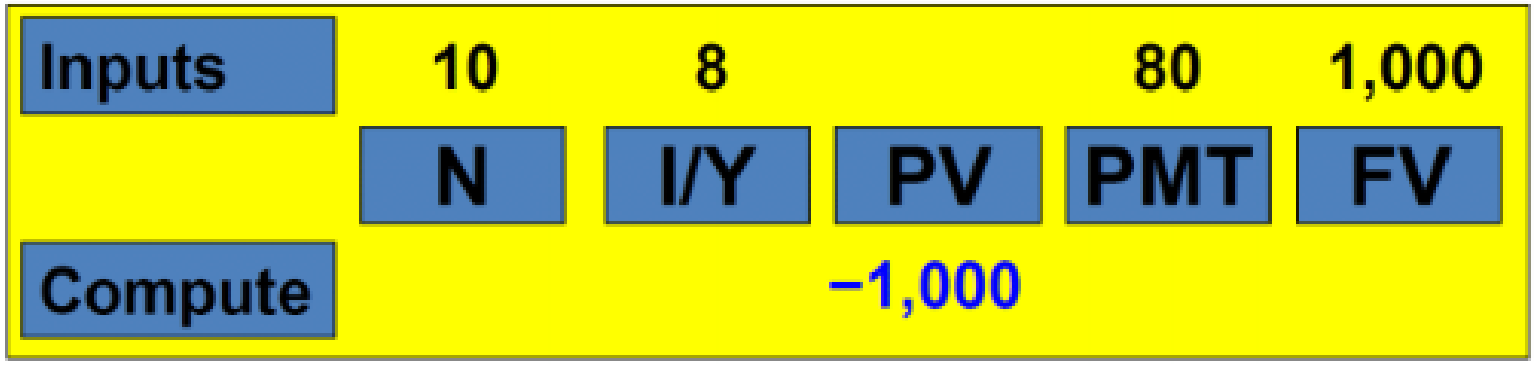

As shown, the Star bond has an annual coupon of $80 and a face, or par, value of $1,000 paid at maturity in 10 years.

# Valuation of a Bond: Example

- Bond value $= C \times \left\{ \frac{1-[1/(1+r)^t}{r} \right\} + \frac{F}{(1+r)^t}$

$$= 80 \times \left\{ \frac{1-1/1.08^{10}}{0.08} \right\} + \frac{1000}{1.08^{10}} = \$1000$$

- The bond sells at its face value, as coupon rate=YTM



# Valuation of a Bond: Example

- Suppose one year later, the interest rate in the market has risen to 10 percent.
- Bond value $= C \times \left\{ \frac{1-[1/(1+r)^t}{r} \right\} + \frac{F}{(1+r)^t}$

$$= 80 \times \left\{ \frac{1-1/1.1^9}{0.1} \right\} + \frac{1000}{1.1^9} = \$884.82$$

| Inputs | 9 | 10 | | 80 | 1,000 |
|---|---|---|---|---|---|
| | N | I/Y | PV | PMT | FV |
| Compute | | | −884.82 | | |

- The bond sells for less than its face value (discount bond), as coupon rate < YTM.

# Valuation of a Bond: Example

- Suppose one year later, the interest rate in the market has fallen to 6 percent.
- Bond value $= C \times \left\{ \frac{1-[1/(1+r)^t}{r} \right\} + \frac{F}{(1+r)^t}$

$$= 80 \times \left\{ \frac{1-1/1.06^9}{0.06} \right\} + \frac{1000}{1.06^9} = \$1{,}136.03$$

| Inputs | 9 | 6 | | 80 | 1,000 |
|---|---|---|---|---|---|
| | N | I/Y | PV | PMT | FV |
| Compute | | | −1,136.03 | | |

- The bond sells for more than its face value (premium bond), as coupon rate > YTM.

# YTM vs. Coupon Rate

Relationship between YTM and coupon rate :

- If $y = R$ (YTM equals coupon rate), then bond price *P* equals par *F*
- if $y < R$, then $P > F$ (bond sells at a premium)
- If $y > R$, then $P < F$ (bond sells at a discount)

# Zero-Coupon Bonds

- **Zero-coupon bonds** (discount bonds):
  - Today (t=0), you pay the market price $P_0$ for the bond
  - no coupon payments are made during the bond's lifetime
  - at maturity, you receive the face value F
  - interest is implicit in the difference between $PV_0$ and F (the "discount")


- Even though no interest payments are made on the bond, the bond calculation uses semiannual periods to be consistent with coupon bond calculation

$$P_0 = \frac{F}{(1 + y/2)^{T \times 2}}$$

# Zero-Coupon Bonds: Example

The Eight-Inch Nails (EIN) Company issues a $1,000 face value, five-year zero-coupon bond. The initial price is $508.35. What is the bond's yield to maturity and the total interest paid over the life of the bond?

**Solution**

The yield to maturity is the discount rate that equates the bond's price to the present value of all its promised cash flows

$$P_0 = \frac{F}{(1+y/2)^{T\times 2}} \qquad 508.35 = \frac{1000}{(1+y/2)^{5\times 2}} \qquad y = 14\%$$

The total interest paid over the life of the bond is $1,000 - $508.35 = $491.65

# Lecture Outline

- Bond valuation
  - Coupon Bond
    - Bond Pricing
    - Bond Yield-to-Maturity
  - Zero-Coupon Bond

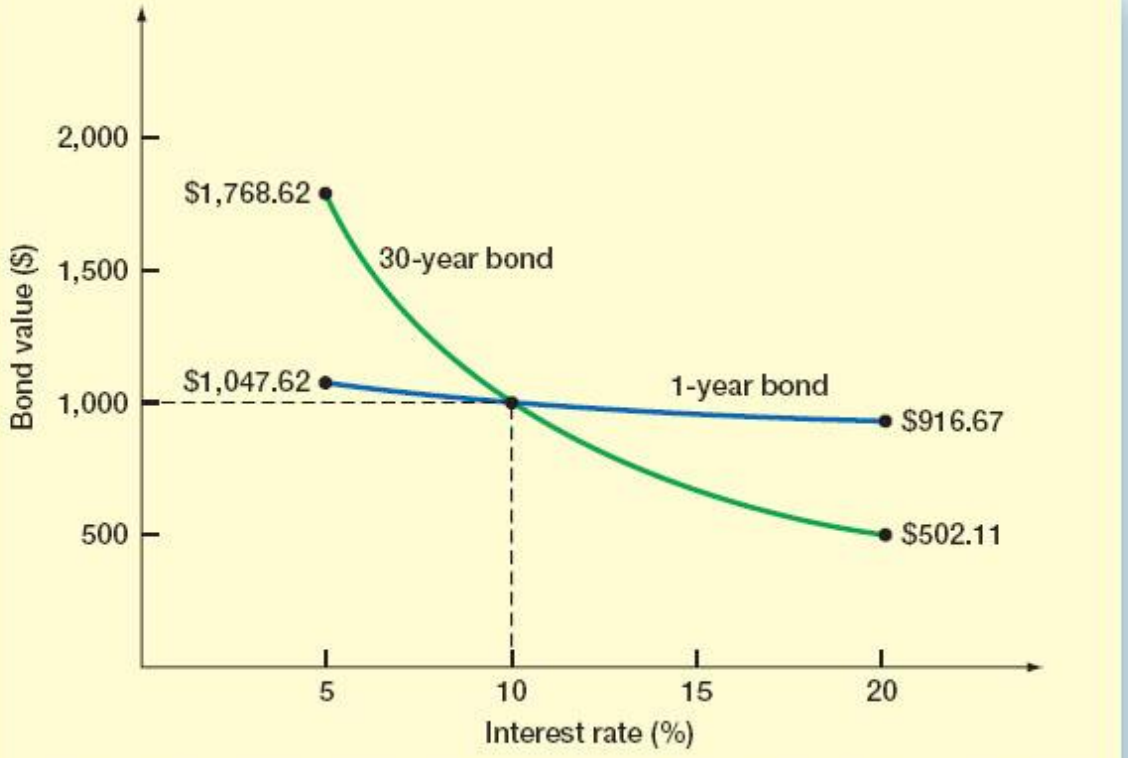
- Bond Risks
  - Interest rate risk
  - Default risk

# Interest Rate Risk

- Interest Rate Risk: change in investment's value due to change in the level of market interest rate

- Rules of thumb:
  - The longer the time to maturity, the greater the interest rate risk
  - The greater the coupon rate, the lower the interest rate risk

# Bonds with Longer Maturity Are More Risky




Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

| | Time to Maturity | |
|---|---|---|
| Interest Rate | 1 Year | 30 Years |
| 5% | $1,047.62 | $1,768.62 |
| 10 | 1,000.00 | 1,000.00 |
| 15 | 956.52 | 671.70 |
| 20 | 916.67 | 502.11 |

***Q: What are the percentage changes in bond prices when interest rate goes from 10% to 15%?***

# Bonds with Larger Coupons are Less Risky

- Suppose we have two bonds with the same maturity
  - Zero coupon bond
  - 8% coupon bond
- Which one is more risky?

# Example

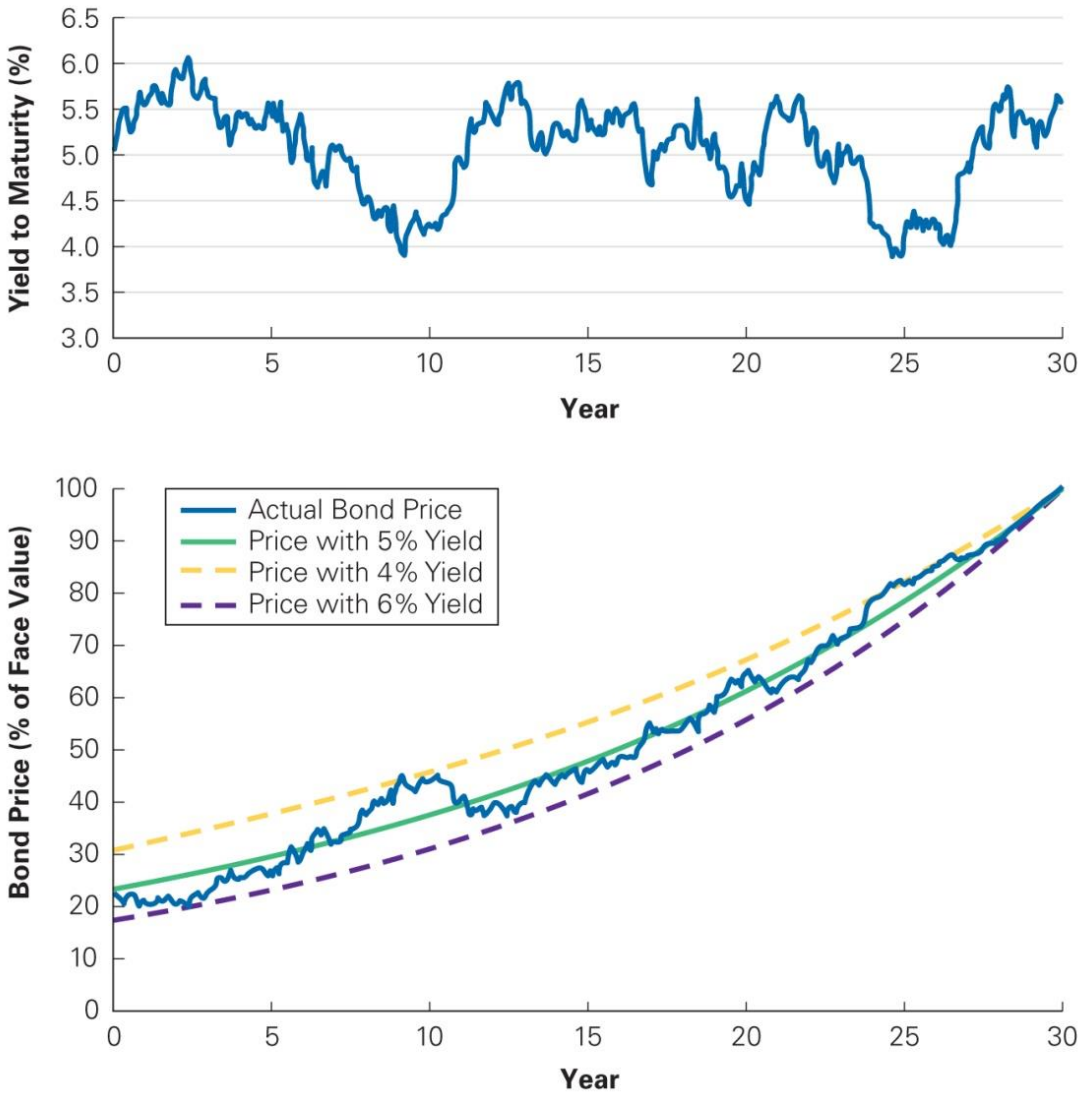
- Suppose midterm worth 99% of the score
- What will you do after the midterm?

# Thank God, Alison's Course is Over!

# Bonds with Larger Coupons are Less Risky

- Alison's course is one semester
- If midterm counts 99%, you are almost sure that you will get a good course grade if you do well in the mid-term
- If midterm counts 30%, you still need to work hard after the mid-term
- For a bond, if larger cash flows come in earlier, the bond becomes less riskier

# YTM and Bond Price Fluctuations Over Time



# Bond Rate of Return

- When you invest in a bond, you receive a regular coupon payment
- As bond prices change, you may also make a capital gain or loss

Example

Suppose you buy the 5.5% coupon Treasury bond today for a price of $1,056.03 and sell it next year at a price of $1,080, what is the return on your investment?

# Bond Rate of Return

The **rate of return** on your investment of $1,056.03 is

$$\textbf{Rate of Return} = \frac{\textbf{Coupon Income} + \textbf{Price Change}}{\textbf{Investment}}$$

$$= \frac{\$55 + \$23.97}{\$1{,}056.03} = 7.48\%$$

- Bond prices
  - fall when market interest rates rise
  - rise when market interest rates fall
- The rate of return fluctuates with market interest rates

# Bond Rate of Return

Example:

- Consider a 5.5% coupon bond that has 3 years left until maturity and sells today for $1,056.03
- Its yield to maturity is 3.5%
- Suppose that by the end of the year, interest rates have fallen and the bond's yield to maturity is now only 2.0%.
- What will be the bond's rate of return?

# Bond Rate of Return

- At the end of the year, the bond will have only 2 years to maturity, and the value of the bond will be

$$P_1 = \frac{\$55}{1.02} + \frac{\$1{,}055}{(1.02)^2} = \$1{,}067.95$$

- You invested \$1,056.03 and at the end of the year you receive a coupon payment of \$55 and have a bond worth \$1,067.95

- Your rate of return is therefore

$$\frac{\$55 + (\$1{,}067.95 - \$1{,}056.03)}{1{,}056.03} = 6.34\%$$

# Lecture Outline

- Bond valuation
  - Coupon Bond
    - Bond Pricing
    - Bond Yield-to-Maturity
  - Zero-Coupon Bond

- Bond Risks
  - Interest rate risk
  - Default risk

# Corporate Bonds

**Corporate bonds** are used by large corporations for long-term borrowing

- similar in structure to Treasury bonds, except corporate bonds are subject to **credit risk:**
  - there is some chance that the company will fall on hard times and will not be able to repay its debts
  - the owner bears the risk that the issuing firm may default

- Investors take this default risk into account when they price bonds and demand a higher interest rate to compensate

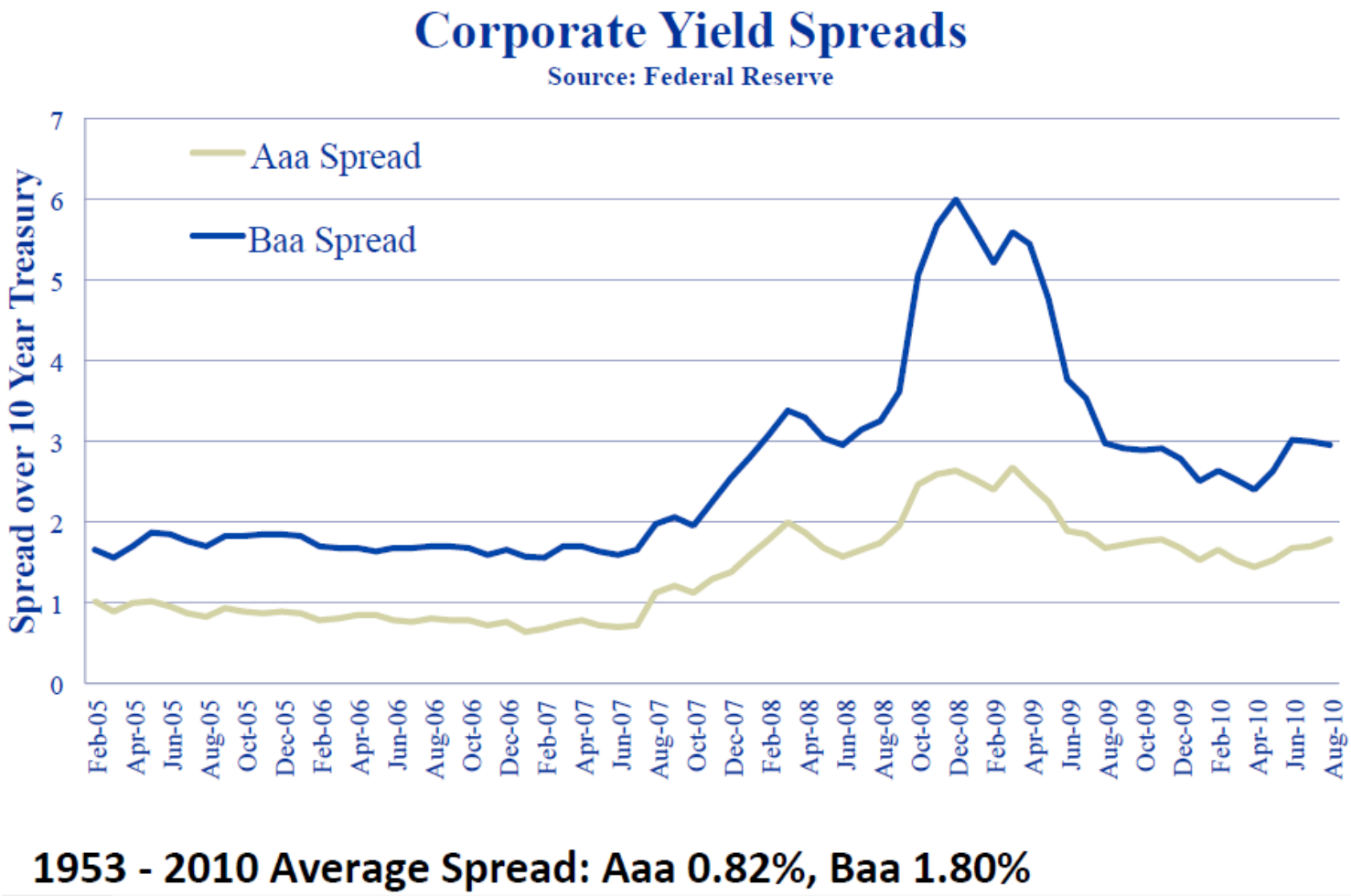
# Bond Ratings

Corporate bonds are rated by rating agencies based on the creditworthiness of the corporate issuers.

| | Moody | S&P | Safety |
|---|---|---|---|
| Investment Grade Bonds | Aaa | AAA | Extremely strong capacity to pay interest and principal |
| | Aa | AA | Very strong capacity to pay interest and repay principal |
| | A | A | Strong capacity to pay interest and repay principal |
| | Baa | BBB | Adequate capacity to pay interest and repay principal |
| Speculative/Junk Bonds | Ba; B | BB; B | Predominantly speculative w.r.t. capacity to pay interest and repay principal. Although such debt is likely to have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions. Issues rated C by Moody's are typically in default. |
| | Caa | CCC | |
| | Ca | CC | |
| | C | C | |
| | | D | Default |

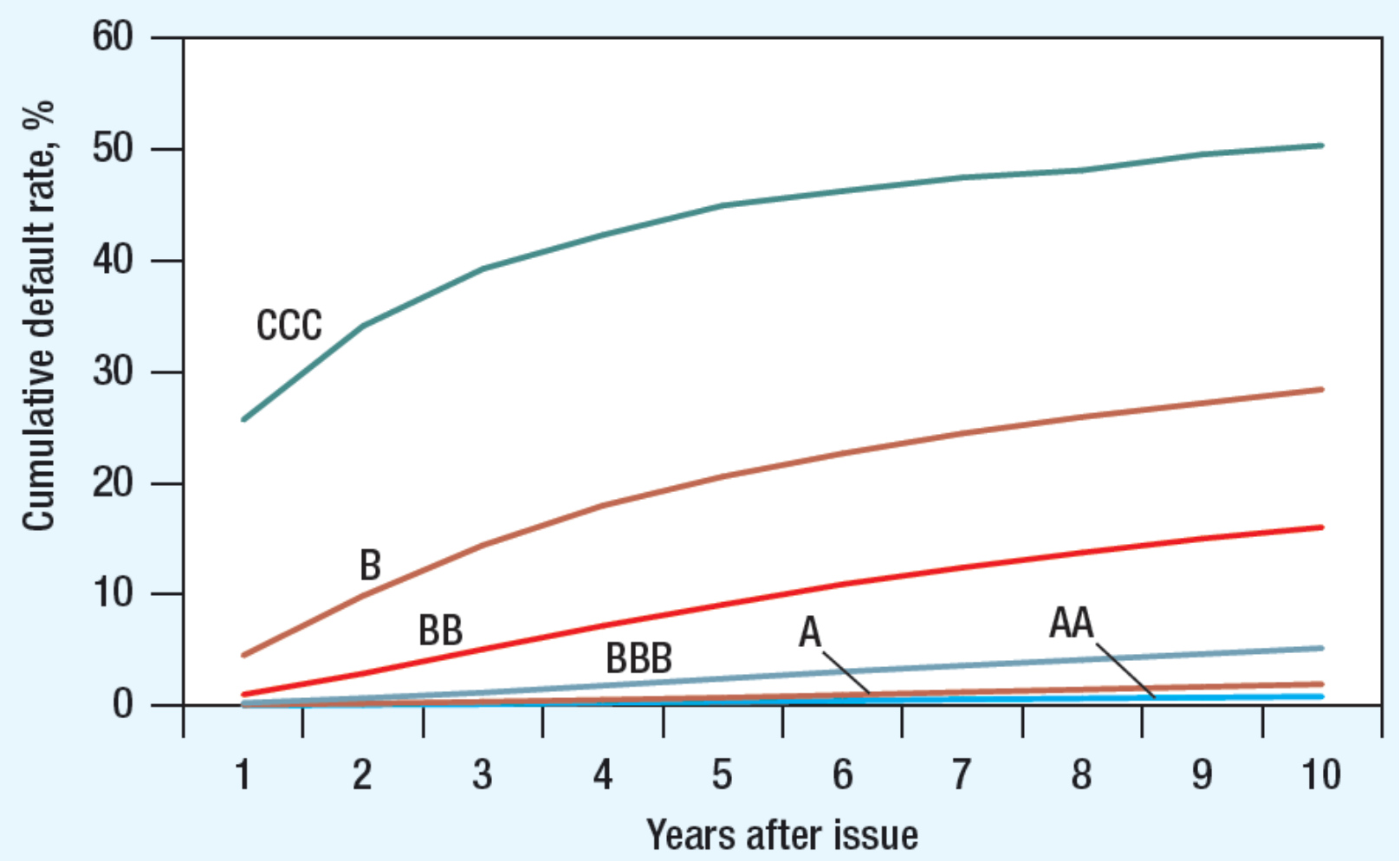
# Corporate Yield Spread During Financial Crisis

Corporate Yield Spread = Corporate Bond Yield –Treasury Yield




**1953 - 2010 Average Spread: Aaa 0.82%, Baa 1.80%**

# Default Rates of Corporate Bonds

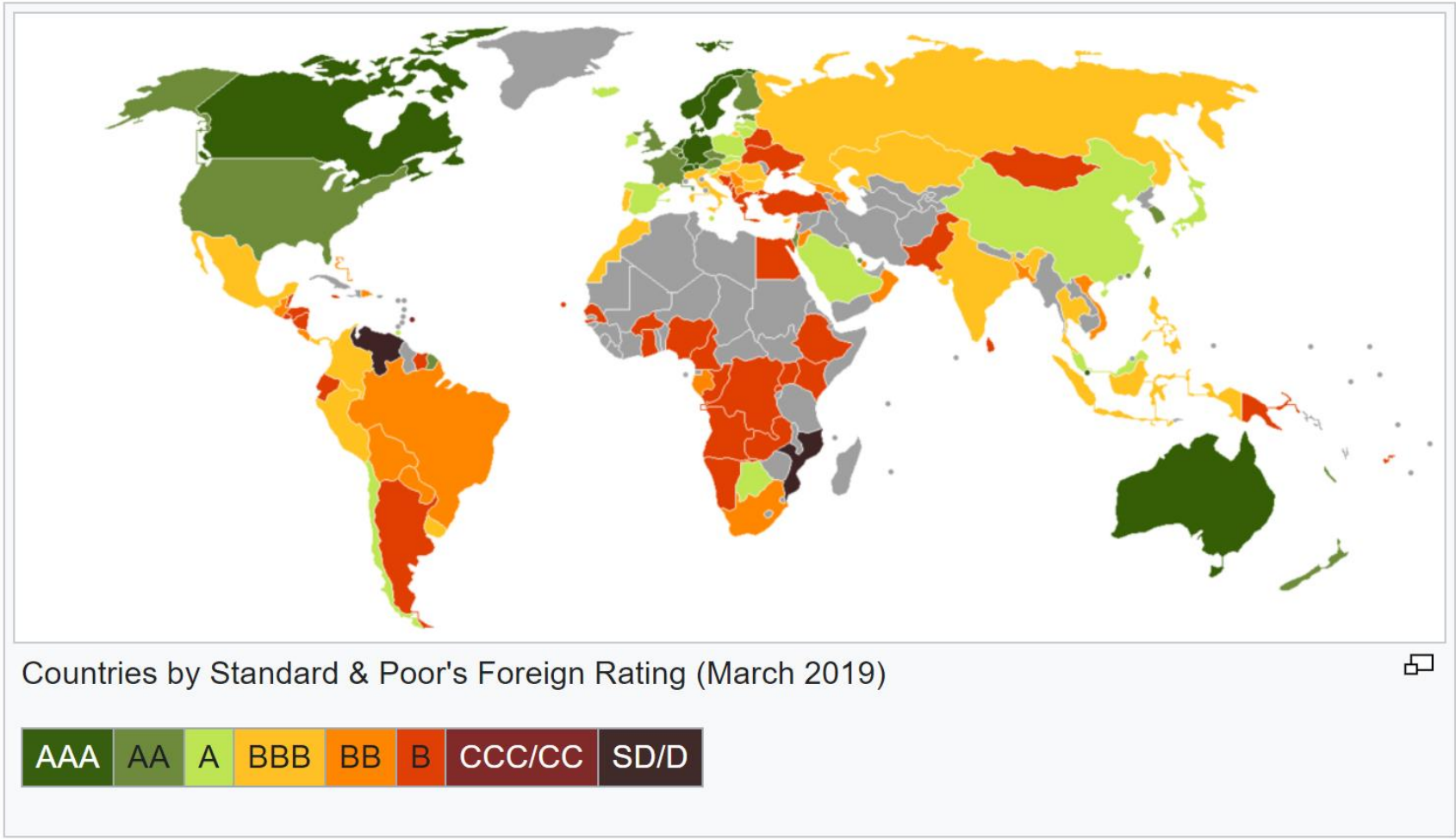



***1981-2010***

# Sovereign Bonds

- Sovereign Bonds issued by national governments
  - U.S. Treasury securities are generally considered to be default free.
  - sovereign bonds are not default free

# Are Sovereign Bonds Riskless?



# Summary

- Determining bond prices and yields is an application of basic discounted cash flow principles.

- Bonds are subject to interest rate risk
  - The longer the time to maturity, the greater the interest rate risk
  - The greater the coupon rate, the lower the interest rate risk

- Bonds are subject to default risk

- When investors price bonds (and thus yield), they take all risks that bonds are exposed to into account