2021R1-MATH1510 HW 5

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TOTAL POINTS

18 / 20

QUESTION 1

1 Q1 **3 / 3**

✓ **- 0 pts** correct

QUESTION 2

2 Q2 **2 / 4**

✓ **- 2 pts**

`![b.png](/files/b4e3ea08-613c-4ea1-85eb-afb855bf02dc)`

QUESTION 3

3 Q3 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 4

4 Q4 **3 / 3**

✓ **- 0 pts** Correct

QUESTION 5

5 Q5 **3 / 3**

✓ **- 0 pts** Correct

QUESTION 6

6 Q6 **3 / 3**

✓ **- 0 pts** All Correct

Part A:

1. Evaluate the following definite integrals.

   (a) $\int_0^2 e^{\sqrt{x}} dx$.

   (b) $\int_{2/\sqrt{3}}^2 \frac{\sqrt{x^2-1}}{x} dx$;

(a) Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

$$\int_0^{\sqrt{2}} 2u e^u \, du$$

$$= 2\left[ue^u\right]_0^{\sqrt{2}} - 2\int_0^{\sqrt{2}} e^u \, du$$

$$= 2\left(\sqrt{2}e^{\sqrt{2}}\right) - 2\left(e^{\sqrt{2}} - 1\right)$$

$$= 2e^{\sqrt{2}}\left(\sqrt{2} - 1\right) + 2 \quad //$$

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(b) Let $u = \sqrt{x^2 - 1}$, $du = \frac{1}{2\sqrt{x^2-1}}(2x)\,dx$

$$= \frac{x}{\sqrt{x^2-1}}\,dx$$

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{u^2}{u^2+1}\,du$$

$$= \Big[u\Big]_{1/\sqrt{3}}^{\sqrt{3}} - \Big[\tan^{-1}(u)\Big]_{1/\sqrt{3}}^{\sqrt{3}}$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} - \frac{\pi}{3} + \frac{\pi}{6}$$

$$= \sqrt{3} - \frac{1}{\sqrt{3}} - \frac{\pi}{6} \quad //$$

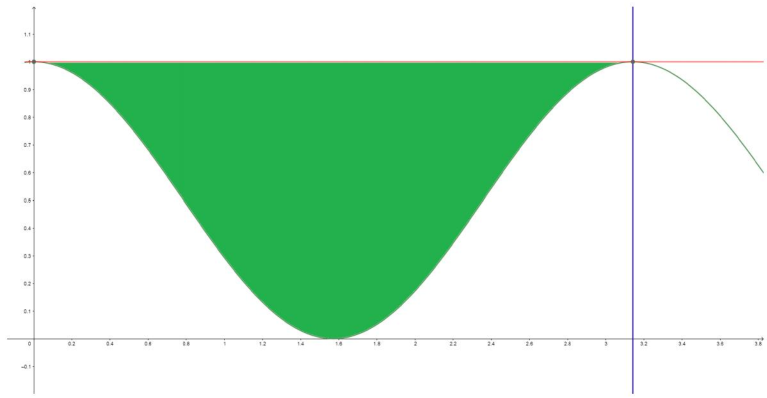
1 Q1 3 / 3

✓ - 0 pts correct

2. Let R be the region bounded between the curves $y = 1$ and $y = \cos^2 x$ for $0 \leq x \leq \pi$.

   (a) Find the volume of the solid generated by rotating the region R about the x-axis.

   (b) Find the volume of the solid generated by rotating the region R about the line $y = 1$.



(a) $$\int_0^{\pi} \pi (1)^2 \, dx - \int_0^{\pi} \pi (\cos^2 x)^2 \, dx$$

$$= \pi \left[x \right]_0^{\pi} - \pi \int_0^{\pi} \left(\frac{1 + \cos(2x)}{2} \right)^2 dx$$

$$= \pi \left[x \right]_0^{\pi} - \frac{\pi}{4} \int_0^{\pi} \left[1 + 2\cos(2x) + \cos^2(2x) \right] dx$$

$$= \pi^2 - \frac{\pi}{4} \int_0^{\pi} \left[1 + 2\cos(2x) + \frac{1 + \cos(4x)}{2} \right] dx$$

$$= \pi^2 - \frac{\pi}{8} \int_0^{\pi} \left[3 + 4\cos(2x) + \cos(4x) \right] dx$$

$$= \pi^2 - \frac{\pi}{8} \left[3x + 2\sin(2x) + \frac{1}{4}\sin(4x) \right]_0^{\pi}$$

$$= \frac{5\pi^2}{8} \quad //$$

(b) $\int_0^{\pi} \pi (1-1)^2 \, dx - \int_0^{\pi} \pi (\cos^2 x - 1)^2 \, dx$

$= \pi \int_0^{\pi} (\sin^2 x) \, dx$

$= \pi \int_0^{\pi} \left(\frac{1 - \cos 2x}{2} \right) dx$

$= \frac{\pi}{2} \int_0^{\pi} (1 - \cos 2x) \, dx$

$= \frac{\pi}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\pi}$

$= \frac{\pi^2}{2}$ //

2 Q2 2 / 4

✓ - 2 pts

`![b.png](/files/b4e3ea08-613c-4ea1-85eb-afb855bf02dc)`

$-6y^2+7y=0$

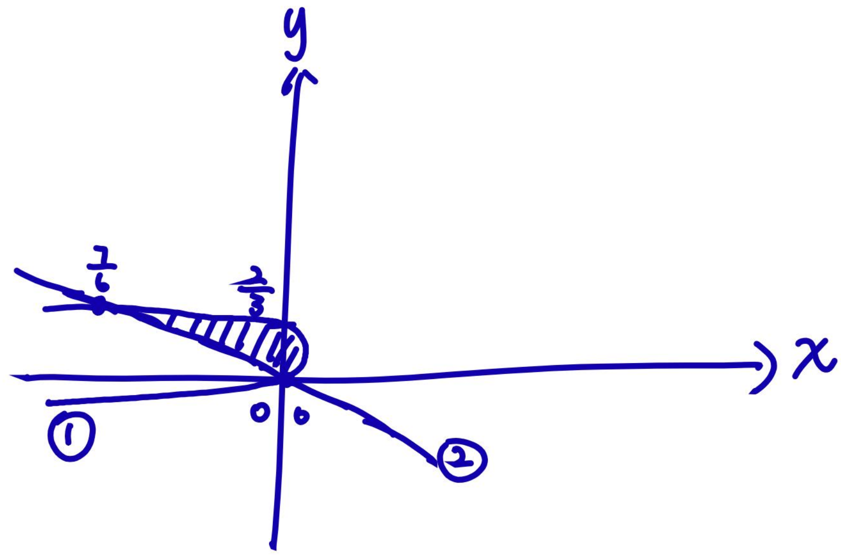
Part B:

3. Let R be the region bounded by curve $x=-6y^2+4y$ and the line $x+3y=0$ on the $xy-$plane. Find the area of R.

Required area :

$\int_0^{\frac{7}{6}}(-6y^2+4y)\,dy$

$-\int_0^{\frac{7}{6}}(-3y)\,dy$



$= \int_0^{\frac{7}{6}}(7y-6y^2)\,dy$

$= \left[\frac{7}{2}y^2-2y^3\right]_0^{\frac{7}{6}}$

$= \frac{343}{216}$ //

3 Q3 **4 / 4**

✓ **- 0 pts** Correct

4. A particle moves in a straight line with speed $v(t) = t^2 + 2t$, where $t \in [0, 9]$ is the time.

   (a) Find the average speed v^* of the particle between $t = 0$ and $t = 9$.

   (b) Find the time $t^* \in [0, 9]$ when the particle moves in the average speed v^*.

(a) Average speed $v^* : \frac{1}{9}\int_0^9 (t^2+2t)\,dt$

$$= \frac{1}{9}\left[\frac{1}{3}t^3 + t^2\right]_0^9$$

$$= 36 \quad //$$

(b) The required time $t^* =$ $t^2 + 2t = 36$

$$t^2 + 2t - 36 = 0$$

$$t = \frac{-2 \pm \sqrt{4 - (4)(-36)}}{2}$$

$$= -1 + \sqrt{37} \quad //$$

or $-1 - \sqrt{37}$ (rejected.)

4 Q4 **3 / 3**

✓ **- 0 pts** Correct

5. Evaluate

$$\lim_{x\to 0} \frac{\int_0^{2x} \sin(e^t - e^{-t})\,dt}{x \sin x}.$$

$\frac{?}{0}$

By L'hôpital's Rule, we have:

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left[\int_0^{2x} \sin(e^t - e^{-t})\,dt\right]}{\sin x + x\cos x}$$

$$= \lim_{x\to 0} \frac{2\sin(e^{2x} - e^{-2x})}{\sin x + x\cos x}$$

$$= \lim_{x\to 0} \frac{2\cos(e^{2x} - e^{-2x}) \times (2e^{2x} + 2e^{-2x})}{2\cos x - x\sin x}$$

$$= \frac{2 \times (2+2)}{2}$$

$$= 4 \quad //$$

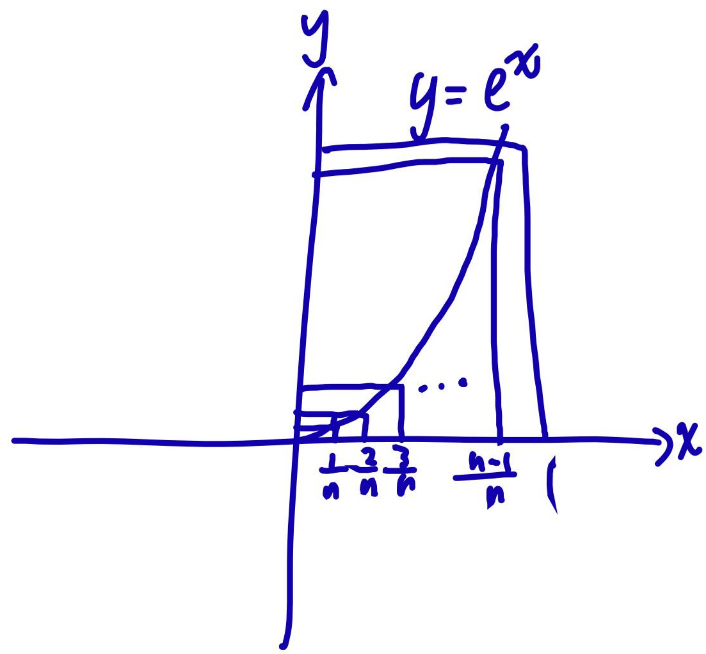
5 Q5 3 / 3

✓ - 0 pts Correct

6. By considering Riemann sum of a suitable integral, evaluate each of the following limits.

(a) $\lim_{n\to\infty}\left(\frac{1}{n}e^{1/n}+\frac{1}{n}e^{2/n}+\cdots+\frac{1}{n}e^{n/n}\right)$

(b) $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right)$



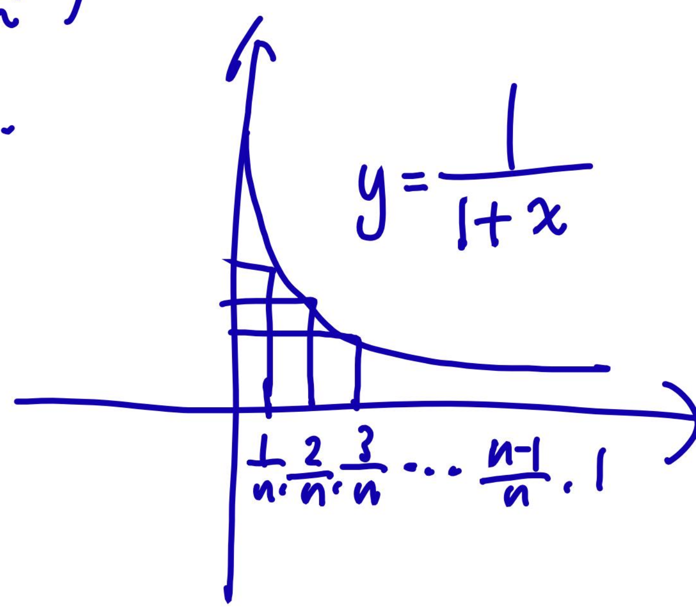
(a) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right)e^{\frac{k}{n}}$

$= \int_0^1 (e^x)\,dx$

$= \left[e^x\right]_0^1$

$= e-1$ //

(b) $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right)\left(\frac{1}{1+\frac{k}{n}}\right)$



$= \int_0^1\left(\frac{1}{1+x}\right)dx$

$= \left[\ln|1+x|\right]_0^1$

$= \ln 2$ //

6 Q6 **3 / 3**

✓ **- 0 pts** **All Correct**