2021R1-MATH1510 HW 4

Cho Kit CHAN

TOTAL POINTS

20 / 20

QUESTION 1

1 Q1 **2 / 2**

1a

✓ **- 0 pts** Correct

1b

✓ **- 0 pts** Correct

QUESTION 2

2 Q2 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 3

3 Q4b **5 / 5**

✓ **- 0 pts** Correct

QUESTION 4

4 Q5a **3 / 3**

✓ **- 0 pts** Correct

QUESTION 5

5 Q7cd **6 / 6**

7c

✓ **- 0 pts** Correct

7d

✓ **- 0 pts** Correct

Part A:

1. Evaluate the following indefinite integrals by substitutions.

   (a) $\displaystyle\int (2021x+1)(x-1)^{1510}dx$;

   (b) $\displaystyle\int \frac{(\ln x)^3}{x}dx$.

(a) Let $u = x-1$, then $du = dx$

& $2021x = 2021u + 2021$

$$\int (2021u+2022)\, u^{1510}\, du$$

$$= (2021u+2022)\frac{u^{1511}}{1511} - \int \frac{u^{1511}}{1511}(2021)\, du.$$

$$= \frac{2021u+2022}{1511}\left(u^{1511}\right) - \frac{2021}{2284632}\left(u^{1512}\right) + \text{Constant}$$

$$= \frac{2021x+1}{1511}(x-1)^{1511} - \frac{2021}{2284632}(x-1)^{1512} + \text{Constant} \,//$$

(b) Let $u = \ln x$, then $du = \frac{1}{x}dx$

$$\int u^3\, du$$

$$= \frac{1}{4}u^4 + \text{Constant}$$

$$= \frac{(\ln x)^4}{4} + \text{Constant} \,//$$

1 Q1 **2 / 2**

1a

✓ **- 0 pts** **Correct**

1b

✓ **- 0 pts** **Correct**

2. Evaluate the following indefinite integrals by integration by parts.

(a) $\displaystyle\int \overset{u}{x^2}\, \overset{dv}{\sin x}\, dx$;

(b) $\displaystyle\int \overset{u}{\ln(x + x^2)}\, \overset{dv}{dx}$.

(a) $\displaystyle\int x^2 \sin x \, dx = \overset{dv}{(-\cos x)}\, \overset{u}{x^2} - \int (-\cos x)\, 2x \, dx$

$\displaystyle = -x^2 \cos x + (2x)(\sin x) - 2\int \sin x \, dx$

$= -x^2 \cos x + 2x \sin x + 2\cos x + \text{Constant}$ //

(b) $\displaystyle\int \ln(x + x^2)\, dx = x \ln(x+x^2) - \int x \cdot \frac{1}{x+x^2} \cdot (2x+1)\, dx$

$\displaystyle = x \ln(x+x^2) - \int \frac{2x+1}{x+1}\, dx$

$\displaystyle = x \ln(x+x^2) - \int \left(2 - \frac{1}{x+1}\right) dx$

$= x \ln(x+x^2) - 2x + \ln|x+1|$

$+ \text{Constant}$ //

2 Q2 4 / 4

✓ **- 0 pts** Correct

4. Evaluate the following indefinite integrals by partial fraction decomposition.

(a) $\displaystyle\int \frac{8}{(x-1)(x+1)(x+3)}\,dx$;

(b) $\displaystyle\int \frac{3x^2+7x}{(x-1)(x^2+4x+5)}\,dx$.

(a) By partial decomposition, we have:

$$\int \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3}\,dx$$

where A, B, C are integers.

$A(x^2+4x+3)$
$B(x^2+2x+3)$
$C(x^2-1)$

$$A+B+C = 0 \quad \ldots \text{①}$$
$$4A+2B = 0 \quad \ldots \text{②}$$
$$3A+3B-C = 8 \quad \ldots \text{③}$$

①+③ : $4A+4B = 8 \quad \ldots$ ④

④−② : $2B = 8$

$B = 4$, $A = -2$, $C = -2$ //

∴ We have:

$$-2\int \frac{1}{x-1}\,dx + 4\int \frac{1}{x+1}\,dx - 2\int \frac{1}{x+3}\,dx$$

$$= -2\ln|x-1| + 4\ln|x+1| - 2\ln|x+3| + \text{Constant}$$

$$= 2\ln\left|\frac{(x+1)^2}{(x-1)(x+3)}\right| + \text{Constant} \quad //$$

(b) By partial decomposition, we have:

$$\int \frac{A}{x-1} + \frac{Bx+C}{x^2+4x+5}\,dx$$, where A, B, C are integers.

$$\begin{aligned} A+B \quad &= 3 \quad \ldots (1) \\ 4A-B+C &= 7 \quad \ldots (2) \\ 5A \quad -C &= 0 \quad \ldots (3) \end{aligned}$$

$(2)+(3)$: $9A - B = 7 \ldots (4)$

$(4)+(1)$: $10A = 10$

$A = 1, \quad B = 2, \quad C = 5$

$\therefore$ We have $\displaystyle\int \frac{1}{x-1} + \frac{2x+5}{x^2+4x+5}\,dx$

$$= \int \frac{1}{x-1}\,dx + \int \frac{2x+4+1}{x^2+4x+5}\,dx$$

$$= \ln|x-1| + \ln(x^2+4x+5) + \int \frac{1}{(x+2)^2+1}\,dx$$

$$= \ln|x-1| + \ln(x^2+4x+5) + \tan^{-1}(x+2)$$

$$+ \text{Constant} \quad //$$

3 Q4b **5 / 5**

✓ **- 0 pts** Correct

5. Evaluate the following indefinite integrals by t-substitution.

(a) $\displaystyle\int \frac{1}{2\sin x + \cos x + 1}\,dx$;

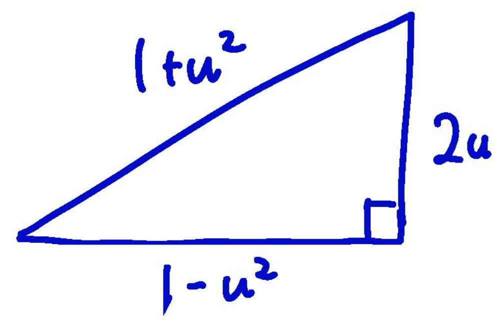
(b) $\displaystyle\int \frac{1}{2+\cos x}\,dx$.

(a) Let $u = \tan\frac{x}{2}$,

$$du = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}\,dx$$

We have
$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\left(\frac{x}{2}\right)}$$

$$= \frac{2u}{1-u^2}$$



$$\sin x = \frac{2u}{1+u^2}$$

$$\cos x = \frac{1-u^2}{1+u^2}$$

$$\therefore\ 2\int \frac{1}{4u+1-u^2+1+u^2}\,du$$

$$= \int \frac{1}{2u+1}\,du$$

$$= \frac{1}{2}\ln|2u+1| + \text{Constant}$$

$$= \frac{1}{2}\ln\left|2\tan\frac{x}{2}+1\right| + \text{Constant.}\ //$$

(b) Let $u = \tan\frac{x}{2}$.

$du = \sec^2\left(\frac{x}{2}\right)\cdot\frac{1}{2}\,dx$

By obtaining (a), we have:

$$\int\frac{1}{2+\cos x}\,dx = 2\int\frac{1}{2+2u^2+1-u^2}\,du$$

$$= 2\int\frac{1}{u^2+3}\,du$$

$$= \frac{2\sqrt{3}}{3}\int\frac{1}{\frac{u^2}{3}+1}\,d\left(\frac{u}{\sqrt{3}}\right)$$

$$= \frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + \text{Constant}$$

$$= \frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{\sqrt{3}\tan\frac{x}{2}}{3}\right) + \text{Constant} \quad //$$

4 Q5a 3 / 3

✓ - 0 pts Correct

7. * Evaluate the following indefinite integrals.

(a) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}\cos^3\sqrt{x}}dx$;

(b) $\displaystyle\int \frac{3\sin x}{2-\cos x-\cos^2 x}\,dx$;

(c) $\displaystyle\int \frac{2-\sqrt{x}}{x+1}dx$;

(d) $\displaystyle\int \frac{2}{x(x^{1/3}+2)}\,dx$;

(e) $\displaystyle\int \frac{\sqrt{x}}{e^{\sqrt{x}}}dx$.

(a) Let $u=\sqrt{x}$, $du=\dfrac{1}{2\sqrt{x}}dx$

We have: $2\displaystyle\int \frac{\sin u}{\cos^3 u}\,du$

$$= -2\int \frac{1}{\cos^3 u}\, d(\cos u)$$

$$= \frac{1}{\cos^2 u} + \text{Constant}$$

$$= \frac{1}{\cos^2\sqrt{x}} + \text{Constant} \quad //$$

(b) $-3\displaystyle\int \frac{\sin x}{\cos^2 x+\cos x-2}\,dx$

Let $u=\cos x$, $du=-\sin x\,dx$

$3\displaystyle\int \frac{1}{u^2+u-2}\,dx$

$$= 3\int \frac{1}{(u+2)(u-1)}\, dx$$

$$= \int \frac{-1}{u+2} + \frac{1}{u-1}\, dx$$

$$= \ln\left|\frac{u-1}{u+2}\right| + \text{Constant}$$

$$= \ln\left|\frac{\cos x - 1}{\cos x + 2}\right| + \text{Constant} \quad //$$

(c) $\displaystyle\int \frac{2-\sqrt{x}}{x+1}\, dx$

Let $u = \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\, dx$

$$\int \frac{2-u}{u^2+1} \cdot (2u)\, du$$

$$= 2\int \frac{2u}{u^2+1} - \frac{u^2}{u^2+1}\, du$$

$$= 2\int \frac{1}{u^2+1}\, d(u^2+1) - 2\int 1 - \frac{1}{u^2+1}\, du$$

$$= 2\ln(u^2+1) - 2u + \tan^{-1}(u) + \text{Constant}$$

$$= 2\ln(x+1) - 2\sqrt{x} + \tan^{-1}(\sqrt{x}) + \text{Constant} \quad //$$

(d) $\displaystyle\int \frac{2}{x(x^{\frac{1}{3}}+2)}\,dx$

Let $u = x^{\frac{1}{3}}$, $du = \frac{1}{3}x^{-\frac{2}{3}}\,dx$

$$6\int \frac{1}{u(u+2)}\,du$$

$$= 3\int \frac{1}{u} + \frac{-1}{u+2}\,du$$

$$= 3\ln\left|\frac{u}{u+2}\right| + \text{Constant} \quad //$$

(e) $\displaystyle\int \frac{\sqrt{x}}{e^{\sqrt{x}}}\,dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$$2\int u^2 e^{-u}\,du$$

$$= 2\left[u^2(-e^{-u}) - \int 2u(-e^{-u})\,du\right]$$

$$= 2\left[-u^2e^{-u} + 2ue^{-u} - \int 2e^{-u}\,du\right]$$

$$= 2\left(-u^2e^{-u} + 2ue^{-u} + 2e^{-u}\right) + \text{Constant}$$

$$= 4e^{-\sqrt{x}} + 4\sqrt{x}\,e^{-\sqrt{x}} - 2x\,e^{-\sqrt{x}} + \text{Constant} \;//$$

5 Q7cd 6 / 6

7c

✓ **- 0 pts** Correct

7d

✓ **- 0 pts** Correct