THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 7

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Class: MATH1510G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

David — Signature

1-11-2021 — Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	2						
2	2						
3							
4	3						
5	3				Total	10	/ 10

$\frac{\sqrt{2}-1}{2-1}$

$\sqrt{2}^{x} \ln\sqrt{2}$.

Part A

1. Evaluate the following indefinite integrals.

   (a) $\int (x^{\sqrt{2}} + \sqrt{2}^{x})\, dx$

   (b) $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}\right) dx$

(a) $\int (x^{\sqrt{2}} + \sqrt{2}^{x})\, dx$

$= \left(\frac{1}{\sqrt{2}+1}\right) x^{\sqrt{2}+1} + \frac{1}{\frac{1}{2}\ln 2}\sqrt{2}^{x} + \text{Constant}$.

$= (\sqrt{2}-1)\, x^{\sqrt{2}+1} + \frac{2\sqrt{2}^{x}}{\ln 2} + \text{Constant}$ //

(b) $\int (x^{\frac{1}{3}} + x^{-\frac{1}{3}})\, dx$

$= \frac{3}{4} x^{\frac{4}{3}} + \frac{3}{2} x^{\frac{2}{3}} + \text{Constant}$ //

2. Evaluate the following indefinite integrals.

(a) $\int (2x-3)^{1510}\,dx$

(b) $\int x\sqrt{x+1}\,dx$

(a) Let $u = 2x-3$

$du = 2dx$

$\therefore$ We have $\frac{1}{2}\int u^{1510}\,du$

$= \frac{1}{2}\left(\frac{u^{1511}}{1511}\right) + \text{Constant}$

$= \dfrac{(2x-3)^{1511}}{3022} + \text{Constant}$.

(b) Let $u = x+1$

$du = dx$

$\int (u-1)\sqrt{u}\,du$.

$= \int (u^{\frac{3}{2}} - \sqrt{u})\,du$

$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + \text{constant}$

$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + \text{constant}$.

$\frac{\pi}{4} = 45°$

Part B

3. A particle is moving on the xy-plane and its position at time t is

$$\vec{r}(t) = (x(t), y(t)) = (e^{-t}\cos t, e^{-t}\sin t) \quad \text{for } t \geq 0.$$

(a) Find the velocity and acceleration of the particle at $t = \frac{\pi}{4}$.

(b) Find the position of the particle in the long run, i.e., $\lim_{t\to\infty} \vec{r}(t)$.

(a) Velocity $= \vec{r}'(t) = (-e^{-t}\cos t + e^{-t}(-\sin t), -e^{-t}\sin t + e^{-t}\cos t)$

$= (-e^{-t}(\cos t + \sin t), e^{-t}(\cos t - \sin t))$ //

Acceleration $= \vec{r}''(t)$

$= (e^{-t}(\cos t + \sin t) - e^{-t}(\cos t - \sin t),$

$-e^{-t}(\cos t - \sin t) - e^{-t}(\cos t + \sin t))$

$= (2e^{-t}\sin t, -2e^{-t}\cos t)$ //

$\therefore$ We have $\vec{r}'(\frac{\pi}{4}) = (-\sqrt{2}e^{-\frac{\pi}{4}}, 0)$

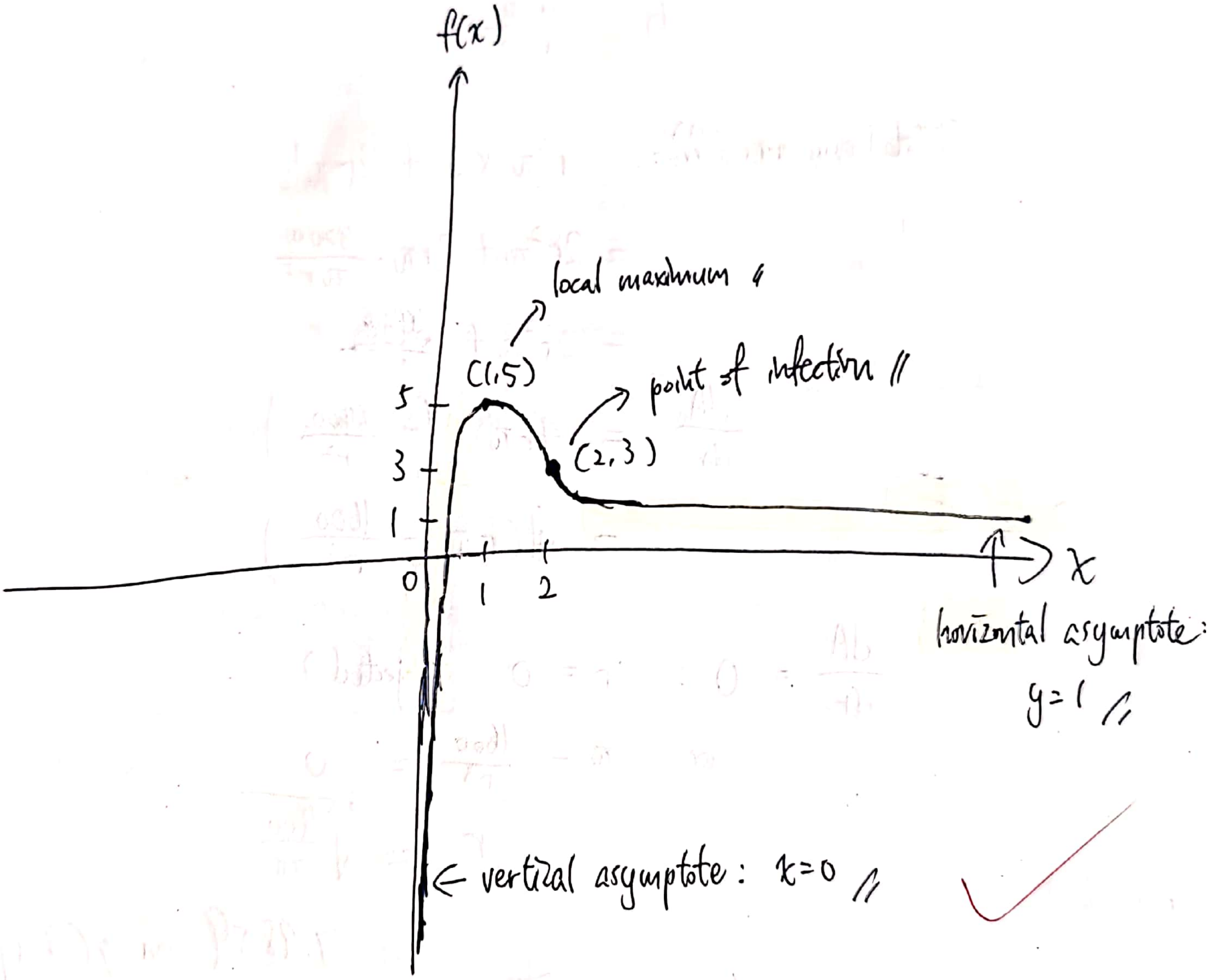
$\vec{r}''(\frac{\pi}{4}) = (\sqrt{2}e^{-\frac{\pi}{4}}, -\sqrt{2}e^{-\frac{\pi}{4}})$ //

(b) $\lim_{t\to\infty} \vec{r}(t) = (0, 0)$ //

4. Sketch a graph of a twice-differentiable function $f:(0,\infty)\to\mathbb{R}$ which satisfies the followings:

- $f(1)=5$ and $f(2)=3$
- $\lim_{x\to 0^+} f(x) = -\infty$ (DNE) and $\lim_{x\to\infty} f(x) = 1$
- $f'(x) > 0$ over $(0,1)$ and $f'(x) < 0$ over $(1,\infty)$
- $f''(x) < 0$ over $(0,2)$ and $f''(x) > 0$ over $(2,\infty)$

On your graph, label any local maximum(s), local ~~maximum~~(s) minimum, point of inflection(s) and asymptote(s) (if any).



$V = r^2 \pi h = 3200$

$6 \leq r \leq 10$

5. A new cylindrical container will be built in a factory to store hazardous chemicals. The capacity of the container should be 3200 m^3. Due to the safety regulation, the radius of the base of the container must be at least 6 m and at most 10 m. It is known that the building cost of the container is directly proportional to the total surface area (including both the top and bottom).

   Find the base radius of the container that minimizes the building cost (correct to 4 decimal places).

(r = radius ; h = height)

Volume: $r^2 \pi h = 3200$

$$h = \frac{3200}{\pi r^2}$$

Total surface area (A): $r^2 \pi \times 2 + 2r\pi h$

$$= 2r^2\pi + 2r\pi \cdot \frac{3200}{\pi r^2}$$

$$= 2r^2\pi + \frac{6400}{r}$$

$$\frac{dA}{dr} = 4r\pi + \left(-\frac{6400}{r^2}\right)$$

$$= 4r\left(\pi - \frac{1600}{r^3}\right)$$

$\frac{dA}{dr} = 0$: $r = 0$ (rejected)

or $\pi - \frac{1600}{r^3} = 0$

$$r = \sqrt[3]{\frac{1600}{\pi}}$$

$$= 7.9859 \text{ m} \ // \text{(4 d.p.)}$$

r	$6 \leq r \leq \sqrt[3]{\frac{1600}{\pi}}$	$r = \sqrt[3]{\frac{1600}{\pi}}$	$\sqrt[3]{\frac{1600}{\pi}} < r \leq 10$
A	$\searrow$		$\nearrow$
$\frac{dA}{dr}$	$-$ve	0	$+$ve