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香 港 中 文 大 學
The Chinese University of Hong Kong

二0一五至一六年度下學期科目考試
Course Examination 2nd Term, 2015-16

科目編號及名稱
Course Code & Title : **MATH1510H Calculus for Engineers**

時間 小時 分鐘
Time allowed : **2** hours **00** minutes

學號 座號
Student I.D. No : ..... Seat No. : .....

Answer ALL Questions.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ x & \text{if } x < 0 \end{cases}$$

(a) (4 points) Find $\lim_{x \to -1} f(x)$ and $\lim_{x \to 1} f(x)$

(b) (4 points) Show that $f(x)$ is continuous at $x = 0$

(c) (4 points) Is $f(x)$ differentiable at $x = 0$? Justify your answer.

2. Find $\dfrac{dy}{dx}$ if

(a) (3 points) $y = \dfrac{3}{x^3} + \sqrt{x} + e^2$

(b) (3 points) $y = \dfrac{1}{(x^2 + 4x + 3)^{100}}$

(c) (3 points) $y = \sec(\ln x + 2^x)$

(d) (3 points) $y = \displaystyle\int_x^{x^2} \sqrt{t^4 + 1}\, dt$

3. Evaluate each of the following integrals :

(a) (3 points) $\int \left(\frac{x^{-2} + x}{x} + \sqrt{x} \right) dx$

(b) (3 points) $\int \sin^3 x \cos^2 x \, dx$

(c) (3 points) $\int \frac{\sqrt{x}}{\sqrt{x} - 1} dx$

(d) (3 points) $\int \sqrt{1 - x^2} \, dx$

(e) (3 points) $\int \frac{1}{1 - x^2} dx$

(f) (3 points) $\int x \cos x \, dx$

4. Solve the following problems separately. Justify your answers.

(a) (4 points) Let

$$f(x) = x + \frac{2}{x}$$

Find all the critical points and determine whether each critical point corresponds to a local min or max (or neither).

(b) (7 points) Let $A = (x, y)$ be a point on the curve $y = \sqrt{x}$ and $B = (2, 0)$ be a fixed point.

i. Show that the distance between A and B can be expressed as a function

$$f(x) = \sqrt{x^2 - 3x + 4}$$

ii. Suppose, among all points on $y = \sqrt{x}$, A and B has the shortest distance. Find the x-coordinate of A.

5. Solve the following problems separately. Justify your answers.

(a) (4 points) Sketch the region in the xy-plane bounded by the graphs of the functions :

$$\begin{cases} f(x) = 4 - x^2 \\ g(x) = 6x + 9 \end{cases}$$

and express its area as integral(s) of function(s).

(You do not need to evaluate the integrals)

(b) (4 points) Let $\mathcal{R}$ be the region in the xy-plane bounded by the curve $y = (x+2)^2$ and the line $y = 1$.

Express the volumes of the following solids as the integrals of functions.

(You do not need to evaluate the integrals)

i. The solid obtained by revolving $\mathcal{R}$ about the x-axis.

ii. The solid obtained by revolving $\mathcal{R}$ about the vertical line $x = -2$.

6. Solve the following problems separately. Justify your answers.

(a) (6 points) Given that

$$\begin{cases} z &= f(x,y) \\ x &= r\cos\theta \\ y &= r\sin\theta \end{cases}$$

where r, θ are independent variables. Express $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$ in terms of r, θ, f_x, f_y

(b) (5 points) Let

$$f(x,y) = x\sqrt{y}$$

Compute the double integral of $f(x,y)$ over the domain

$$D = \{(x,y) \mid 0 \le x \le 1 \text{ and } x \le y \le 1\}$$

7. Solve the following problems separately. Justify your answers.

(a) (4 points) Find the Taylor polynomial of order 3 of

$$f(x) = \tan x$$

at $x_0 = 0$.

(b) (4 points) Find the Taylor series of

$$f(x) = e^x$$

with center $a = -1$.

(c) (5 points) Find the Maclaurin polynomial of order 3 of

$$f(x) = \frac{\cos x}{2x+1}$$

(d) (5 points) Find the Maclaurin polynomial of order 4 of

$$f(x) = \sin(x + x^2)$$

8. Solve the following problems separately. Justify your answers.

(a) (2 points) Evaluate

$$\lim_{x\to 0^+}\left(\frac{1}{\ln(x+1)}-\frac{1}{x}\right)$$

If the limit does not exist but diverges to $\pm\infty$, please indicate so and determine the correct sign.

(b) (2 points) Let

$$f(x)=|x| \quad \text{when } x\in(-1,1]$$

and $f(x+2)=f(x)$ for all x. Find the Fourier series of f.

(c) (2 points) Let

$$f(x)=x-\arctan x$$

Show that $f(x)\geq 0$ for all $x\geq 0$.

(d) (4 points)

i. Write down the Maclaurin polynomials of order 4 of

$$\begin{aligned} f(x) &= \ln(1+2x^2)-2x\sin x \\ g(x) &= \sin^2 x - x^2 \end{aligned}$$

ii. Hence, or otherwise, evaluate

$$\lim_{x\to 0^+}\frac{\ln(1+2x^2)-2x\sin x}{\sin^2 x-x^2}$$

If the limit does not exist but diverges to $\pm\infty$, please indicate so and determine the correct sign.