THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1510 Calculus for Engineers (2020-2021)
Supplementary Exercise 9

Definite Integration

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^2 x^2 - 3x + 4\,dx$

   (b) $\displaystyle\int_{-2}^5 |x^2 - 3x + 2|\,dx$

   (c) $\displaystyle\int_0^4 xe^{|2-x|}\,dx$

   (d) $\displaystyle\int_0^{\pi/6} (\sec x + \tan x)^2\,dx$

   (e) $\displaystyle\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x}\,dx$

   (f) $\displaystyle\int_{\ln(3/4)}^{\ln(4/3)} \frac{e^x}{(1+e^{2x})}\,dx$

   (g) $\displaystyle\int_1^e \frac{1}{x\sqrt{1+(\ln x)^2}}\,dx$

   (h) $\displaystyle\int_0^{\ln 2} e^{-x}\ln(1+e^x)\,dx$

   (i) $\displaystyle\int_0^{\pi/2} e^{3x}\sin x\,dx$

   (j) $\displaystyle\int_1^3 \ln x\,dx$

   (k) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}(1+\sqrt{x})^4\,dx$

   (l) $\displaystyle\int_0^1 \frac{x^2+4x}{\sqrt[3]{x^3+6x^2+1}}\,dx$

   (m) $\displaystyle\int_{-2}^1 (x+1)\sqrt{x+3}\,dx$

   (n) $\displaystyle\int_1^2 \frac{\ln x}{x}\,dx$

   (o) $\displaystyle\int_1^2 \frac{e^{2x}}{e^x-1}\,dx$

   (p) $\displaystyle\int_{-1}^{\sqrt{3}} \frac{1}{(1+x^2)^{3/2}}\,dx$

   (q) $\displaystyle\int_{-1}^2 x^2\sqrt{4-x^2}\,dx$

2. Evaluate the following improper integrals.

   (a) $\displaystyle\int_0^\infty e^{-x}\,dx$

   (b) $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$

   (c) $\displaystyle\int_3^\infty \frac{1}{9+x^2}\,dx$

   (d) $\displaystyle\int_{-\infty}^0 xe^x\,dx$

3. (a) By using integration by parts, find $\displaystyle\int \sin(\ln x)\,dx$.

   (b) Hence, evaluate $\displaystyle\int_1^{e^\pi} \sin(\ln x)\,dx$.

4. Given that $I_n = \displaystyle\int_0^1 (1-x^3)^n\,dx$, where n is an nonnegative integer. Show that for $n \geq 1$,

   $$(3n+1)I_n = 3nI_{n-1}.$$

Hence, find I_5.

5. Let p and q be positive integers. Show that

$$\int_0^1 x^p(1-x)^q\,dx = \frac{q}{p+1}\int_0^1 x^{p+1}(1-x)^{q-1}\,dx.$$

Hence, find $\displaystyle\int_0^1 x^4(1-x)^3\,dx$.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function,

- if $f(x) = f(-x)$ for all $x \in \mathbb{R}$, $f(x)$ is called an even function;
- if $-f(x) = f(-x)$ for all $x \in \mathbb{R}$, $f(x)$ is called an odd function.

(a) Show that x^2 and $\cos x$ are even functions.

(b) Show that x^3 and $\sin x$ are odd functions.

(Remark: The graph of an even function must be symmetric along the y-axis and the graph of an odd function must be symmetric about the origin.)

7. (Harder Problem) Let $a > 0$ and let $f(x)$ be an even function. Show that

$$\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx.$$

Hence, evaluate $\displaystyle\int_{-\pi}^{\pi} |x|\sin|x|\,dx$.

8. (Harder Problem) Let $a > 0$ and let $f(x)$ be an odd function. Show that

$$\int_{-a}^a f(x)\,dx = 0.$$

Hence, evaluate $\displaystyle\int_{-\pi}^{\pi} x^4\tan 3x\,dx$.

Volumes of Solids of Revolution

9. Find the volume of the solid generated by revolving the area bounded by the graph of $y = \sin x$ and the x-axis between $x = 0$ and $x = \pi$ about the x-axis.

10. Find the volume of the solid generated by revolving the area bounded by the graph of $y = x^2$ and the line $x + y - 6 = 0$ about the x-axis.

11. Find the volume of the solid generated by revolving the area bounded by the curves $y = x^2$ and $y^2 = x$ about

(a) the x-axis.

(b) the line $y = 1$.

12. Find the volume of the solid generated by revolving the area bounded by the curve $x - 2y + 4 = 0$ and $y^2 = x + 4$ about

    (a) the x-axis.

    (b) the y-axis.

    (c) the line $x = -4$.