Homework Assignment 2 Solution
Fall 2020

Question 1

Consider the following two mutually exclusive projects:

Year	Cash Flow(A)	Cash Flow(B)
0	-$470,000	-$40,000
1	30,000	21,000
2	60,000	15,000
3	60,000	18,000
4	670,000	11,500

Whichever project you choose, if any, you require a 16 percent return on your investment.

a) If you apply the NPV criterion, which investment will you choose? Why?

b) If you apply the IRR criterion, which investment will you choose? Why?

c) Based on your answers in (a) through (b), which project will you finally choose? Why?

Solution:

a) The NPV for each project is:

$$\text{A: NPV} = -\$470{,}000 + \frac{\$30{,}000}{1.16} + \frac{\$60{,}000}{1.16^2} + \frac{\$60{,}000}{1.16^3} + \frac{\$670{,}000}{1.16^4} = \$8{,}926.34$$

$$\text{B: NPV} = -\$40{,}000 + \frac{\$21{,}000}{1.16} + \frac{\$15{,}000}{1.16^2} + \frac{\$18{,}000}{1.16^3} + \frac{\$11{,}500}{1.16^4} = \$7{,}134.08$$

NPV criterion implies we accept project A because project A has a higher NPV than project B.

b) The IRR for each project is:

$$\text{A: } \$470{,}000 = \frac{\$30{,}000}{1+\text{IRR}} + \frac{\$60{,}000}{1+\text{IRR}^2} + \frac{\$60{,}000}{1+\text{IRR}^3} + \frac{\$670{,}000}{1+\text{IRR}^4}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that: $\text{IRR} = 16.61\%$

$$\text{B: } \$40{,}000 = \frac{\$21{,}000}{1+\text{IRR}} + \frac{\$15{,}000}{1+\text{IRR}^2} + \frac{\$18{,}000}{1+\text{IRR}^3} + \frac{\$11{,}500}{1+\text{IRR}^4}$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that: $\text{IRR} = 25.50\%$.

IRR decision rule implies we accept project B because IRR for B is greater than IRR for A.

c) In this instance, the NPV criteria implies that you should accept project A, while IRR imply that you should accept project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept project A.

Question 2

Consider the following information about three stocks:

State of Economy	Probability of State of Economy	Rate of Return if State Occurs		
		Stock A	**Stock B**	**Stock C**
Boom	0.34	0.28	0.38	0.57
Normal	0.50	0.15	0.12	0.08
Bust	0.16	0.00	-0.27	-0.49

a) If your portfolio is invested 35 percent each in A and B and 30 percent in C, what is the portfolio expected return? The variance? The standard deviation?

b) If the expected T-bill rate is 3.20 percent, what is the expected risk premium on the portfolio?

Solution:

a) We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: $E(R_P) = .35(.28) + .35(.38) + .3(.57) = .4020$ or 40.20%

Normal: $E(R_P) = .35(.15) + .35(.12) + .3(.08) = .0947$ or 9.47%

Bust: $E(Rp) = .35(.00) + .35(-.27) + .3(-.49) = -.2415$ or -24.15%

And the expected return of the portfolio is:

$E(R_P) = .34(.4020) + .50(.0947) + .16(-.2415) = .1454$ or 14.54%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, than add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio is:

$$\sigma_p^2 = .34(.4020 - .1454)^2 + .50(.0947 - .1454)^2 + .16(-.2415 - .1454)^2 = .04762$$

$$\sigma_p = (.04762)^{1/2} = .2182 \text{ or } 21.82\%$$

b) The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:

$$\boldsymbol{RP_i = \mathrm{E}(R_P) - R_f = .1454 - .0320 = .1134 \text{ or } 11.34\%}$$

Question 3

Suppose Johnson & Johnson and the Walgreen Company have expected returns and volatilities shown below, with a correlation of 24%.

	Expected Return	**Standard Deviation**
Johnson & Johnson	9%	15%
Walgreen Company	12%	22%

a) What is the expected return a portfolio that is equally invested in Johnson & Johnson's and Walgreen's stock?

b) What is the volatility (standard deviation) of this portfolio?

If the correlation between Johnson & Johnson's and Walgreen's stock were to increase,

c) Would the expected return of the portfolio rise or fall?

d) Would the volatility of the portfolio rise or fall?

Solution:

a) In this case, the portfolio weights are $\boldsymbol{x_j = x_w = 0.5}$

$$\mathrm{E}[R_P] = x_j E[R_j] + x_w E[R_w] = 0.5(0.09) + 0.5(0.12) = 0.105$$

b) $\mathrm{SD}(R_P) = \sqrt{x_j{}^2 Var(R_j) + x_w{}^2 Var(R_w) + 2x_j x_w Cov(R_j, R_w)}$

$$= \sqrt{0.5^2 0.15^2 + 0.5^2 0.22^2 + 2(0.5)(0.5)(0.24)(0.15)(0.22)} = 0.147$$

c) The expected return would remain constant, assuming only the correlation changes,

$0.5 \times 0.09 + 0.5 \times 0.12 = 0.105.$

d) The volatility of the portfolio would increase (due to the correlation term in the equation for the volatility of a portfolio).

Question 4

Consider the following information about Stocks I and II:

State of Economy	Probability of State of Economy	Rate of Return if State Occurs	
		Stock I	Stock II
Recession	0.16	0.24	-0.28
Normal	0.60	0.21	0.10
Irrational exuberance	0.24	0.09	0.45

The market risk premium is 8.5 percent, and the risk-free rate is 5 percent.

a) Which stock has the most systematic risk?

b) Which one has the most unsystematic risk?

c) Which stock is "riskier"? Explain.

Solution:

The amount of systematic risk is measured by the $\boldsymbol{\beta}$ of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the $\boldsymbol{\beta}$ of an asset. The expected return of Stock I is:

$$\mathrm{E}(R_{\mathrm{I}}) = .16(.24) + .60(.21) + .24(.09) = .1860, \text{or } 18.6\%$$

Using the CAPM to find the β of Stock I, we find:

$$.1860 = .05 + .085\beta_{\mathrm{I}}$$
$$\beta_{\mathrm{I}} = 1.6$$

The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock's variance, we find:

$$\sigma_{\mathrm{I}}^2 = .16(.24 - .1860)^2 + .60(.21 - .1860)^2 + .24(.09 - .1860)^2 = .003024$$
$$\sigma_{\mathrm{I}} = (.003024)^{1/2} = .05499, \text{or } 5.499\%$$

Using the same procedure for Stock II, we find the expected return to be:

$$\mathrm{E}(R_{\mathrm{II}}) = .16(-.28) + .60(.10) + .24(.45) = .1232$$

Using the CAPM to find the β of Stock II, we find:

$$.1232 = .05 + .085\beta_{\mathrm{II}}$$
$$\beta_{\mathrm{II}} = .86$$

And the standard deviation of Stock II is:

$$\sigma_{\mathrm{II}}^2 = .16(-.28 - .1232)^2 + .60(.10 - .1232)^2 + .24(.45 - .1232)^2 = .05197$$
$$\sigma_{\mathrm{II}} = (.05197)^{1/2} = .2280, \text{or } 22.80\%$$

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I's. Thus,

a) I has more systematic risk.

b) II has more unsystematic and more total risk.

c) Since unsystematic risk can be diversified away, I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.