## THE CHINESE UNIVERSITY OF HONG KONG

## Department of Mathematics MATH1510 Calculus for Engineers (Fall 2021) Coursework 8

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Total

$$(\tan^2\theta + 1)^3 = (\sec^2\theta)^3$$

## Part A

1. Evaluate  $\int \frac{x^3}{(x^2+1)^3} dx$  by using the given substitution.

(a) 
$$x = \tan \theta$$

(b) 
$$u = 1 + x^2$$

(a) 
$$x = \tan \theta$$
  
(b)  $u = 1 + x^2$ 

$$M$$
 (a)  $\chi = \tan \theta$ 

$$\frac{sh0}{\cos \theta} = \tan \theta$$

$$\frac{sh0}{\tan \theta} = \cos \theta$$

$$=\int \frac{5h^3\theta}{\cos^2\theta} \cdot \cos^2\theta$$

$$= \int \frac{5 \ln 30}{\cos^2 \theta} \cdot \cos^2 \theta \, d\theta$$

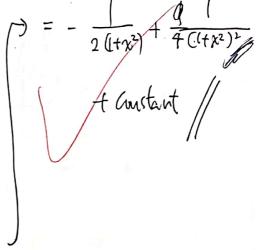
$$= \int \left(1 - \frac{5 \ln^2 \theta}{20}\right) \left(\frac{1}{2} \cdot \frac{5 \ln 2\theta}{2}\right)^3 d\theta$$

$$\frac{1}{2} \left( \frac{u-1}{u^3} \right)$$
 du

$$=\frac{1}{2}\int (u^{-2}-u^{-3}) du$$

$$= \frac{1}{2} \left( -\alpha^{-1} + \frac{1}{2} \alpha^{-\nu} \right) -$$

$$= \frac{1}{2} \left( -u^{-1} + \frac{1}{2} u^{-1} \right) + C$$



$$\frac{6(x+2)+5}{(x+2)^2+164^2}$$

2. Evaluate the following indefinite integrals.

(a) 
$$\int \frac{x^2 + 4x + 1}{x - 2} dx$$
(b) 
$$\int \frac{9x - 1}{3x^2 + x - 2} dx$$

$$(5) \int \frac{9x - 1}{3x^2 + x - 2} dx$$

(b) 
$$\int \frac{9x-1}{3x^2+x-2} \, dx$$

(c) 
$$\int \frac{6x + 17}{x^2 + 4x + 20} \, dx$$

(c) 
$$\int \frac{3x^2 + x - 2}{x^2 + 4x + 20} dx$$
 | 3 = 3 | A = 3

A+3B = 9

(a) If By leng divition, we have:

$$\left(\frac{(2t)(x+6)}{x-2} + \frac{13}{x-2}\right) dx$$

$$= \int (x+6) dx + (3) \int \frac{x+\nu}{x+\nu} dx$$

= 
$$\frac{1}{2}x^2 + 6x + 13 \ln|x-2| + Constant //$$

(6) 
$$\int \frac{4x-1}{(3x-2)(x+1)} dx$$

$$= \ln \left| \frac{2}{(x+1)^2(3x-2)} \right|$$

$$+ constant /$$

$$= \int \frac{3}{3x-1} dx + \int \frac{2}{x+1} dx$$

$$= \int \frac{3(2x+8)}{x^2+8+10} dx - \int \frac{7}{x^2+8x+10} dx$$

Let 
$$u = x^2 + \delta x + 20$$

$$\int u = (2x + \delta) dx$$

$$3\int \frac{1}{u} du - \int \frac{7}{(x+2)^2 + 16} dx$$

$$= 3 lm u - \frac{7}{16} \int \frac{\left( \frac{ktL}{4} \right)^2}{\left( \frac{ktL}{4} \right)^2} d\chi$$

$$= 3 \ln \left(x^2 + \sqrt{x} + 2\omega\right) - \frac{7}{16} \tan^{-1}\left(\frac{\kappa + 2}{4}\right)$$

$$cos(atb) = sha cosh + cosa shb$$

$$sh(atb) = sha cosh + cosa shb$$

$$sh^{2}\chi = \frac{1-cox}{2}$$
Part B  $-sh(a-b) = sha cosb - cosa shb = \frac{2}{2}$ 

3. Evaluate the following indefinite integrals.

(a) 
$$\int \sin^6 x \cos^3 x \, dx$$

(b) 
$$\int \sin^4 x \cos^4 x \, dx$$

(Hint: Consider the double angle formula for sine.)

(a) 
$$\int sih^6 x \cos^2 x d(sih x)$$

$$= \int sh^{2} x \left( 1 - sh^{2} x \right) d(shx)$$

$$= \int_{-\infty}^{\infty} \sin^6 x - \sin^8 x - d(\sin x)$$

(6) 
$$\int (shx cosx)^4 dx$$

$$= \int \left( \frac{1}{2} \sinh 2x \right)^{4} dx$$

$$=$$
  $\frac{1}{16}$   $\int_{0}^{\infty} sih^{\dagger}C2x^{\dagger} dx$ 

$$= \frac{1}{16} \frac{1 - \cos^2(4x)}{2} dx$$

$$= \frac{1}{32} \int_{32}^{1} \left| - \frac{1 + \sin 4x}{2} \right| dx$$

$$= \frac{1}{64} \int_{32}^{1} \left| - \frac{1 + \sin 4x}{2} \right| dx$$

= 
$$\frac{\chi}{64} + \frac{\cos \theta x}{512} \chi = \frac{\cos \theta x}{\sin \theta x}$$

4. Evaluate the following indefinite integrals.

(a) 
$$\int \frac{x}{\sqrt[3]{1+x^2}} \, dx$$

(b) 
$$\int \frac{1}{\sqrt{2x+3} - \sqrt{2x+1}} dx$$

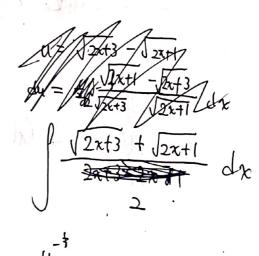
(4) Let 
$$u = 1 + 1 x^2$$

$$du = 2x dx$$

$$= \frac{1}{2} \int \frac{1}{u^{\frac{2}{3}}} du$$

$$= \frac{1}{2} \cdot \frac{3}{2} u^{\frac{2}{3}} + constant$$

$$= \frac{3}{4} \left( (+x^2)^{\frac{1}{3}} + constant \right)$$



(b) 
$$\int \frac{\sqrt{2x+3} + \sqrt{2x+1}}{2} dx$$

$$=\frac{1}{4}\int \sqrt{2\pi t^3} d(2\pi t^3) + \frac{1}{4}\int \sqrt{2\pi t^3} d(2\pi t^3)$$

$$= \frac{1}{6} (2\pi t^3)^{\frac{3}{2}} + \frac{1}{6} (2\pi t^4)^{\frac{3}{2}} + constant$$

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