THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 10

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Class: MATH1510 G1

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/
David — Signature; 22-11-2021 — Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	2						
2	3						
3	2						
4	2						
5					Total	9	/ 10

Part A

1. Evaluate each of the following definite integrals.

(a) $\int_0^2 x\ln(x^2+1)\,dx$

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(b) $\int_0^5 |-x^2+7x-10|\,dx$

(a) Let $u = x^2+1$

$du = 2x\,dx$

dv u

$\frac{1}{2}\int_1^5 \ln(u)\,du$

$= \frac{1}{2}\left[u\ln(u)\right]_1^5 - \frac{1}{2}\int_1^5 u\cdot\left(\frac{1}{u}\right)du$

$= \frac{5}{2}\ln 5 - \frac{1}{2}\left[u\right]_1^5$

$= \frac{5}{2}\ln 5 - 2$ //

(b)

$\int_2^5 (-x^2+7x-10)\,dx + \int_0^2 (x^2-7x+10)\,dx$

$= \left[-\frac{1}{3}x^3+\frac{7}{2}x^2-10x\right]_2^5 + \left[\frac{1}{3}x^3-\frac{7}{2}x^2+10x\right]_0^2$

$= \frac{26}{3} - \frac{25}{6} + \frac{26}{3}$

$= \frac{79}{6}$ //

2. Let

$$f(x) = \frac{1}{(1+x)\sqrt{x}}.$$

Evaluate each of the following improper integrals.

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(a) $\displaystyle\int_1^\infty f(x)\,dx$

(b) $\displaystyle\int_0^1 f(x)\,dx$

(a) $\displaystyle\int_1^\infty \frac{1}{(1+x)\sqrt{x}}\,dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$$2\int_1^\infty \frac{1}{u^2+1}\,du.$$

$$= 2\left[\tan^{-1}(u)\right]_1^\infty$$

$$\pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

(b) $\displaystyle\int_0^1 \frac{1}{(1+x)\sqrt{x}}\,dx$

$$= 2\int_0^1 \frac{1}{u^2+1}\,du$$

$$= 2\left[\tan^{-1}(u)\right]_0^1$$

$$= \frac{\pi}{2}$$

Part B

3. (a) Find $\frac{d}{dx}\int_0^x e^{(t^2)}\,dt$.

   (b) Find $\frac{d}{dx}\int_0^{\sin 2x} e^{\sin t}\,dt$.

   (c) By L'Hôpital's rule and parts (a),(b), evaluate

$$\lim_{x\to 0}\frac{\int_0^x e^{(t^2)}\,dt}{\int_0^{\sin 2x} e^{\sin t}\,dt}$$

2

(a) e^{x^2} //

(b) $\frac{d}{d\sin 2x}\int_0^{\sin 2x} e^{\sin t}\,dt \cdot \frac{d\sin 2x}{dx}$

$= e^{\sin(\sin 2x)} \cdot 2\cos(2x)$ //

(c) $\lim_{x\to 0}\frac{e^{x^2}}{2\cos(2x)\,e^{\sin(\sin 2x)}}$

$=$?

4. Let

$$F(x) = \int_0^x |t|\, dt.$$

(a) $F(x)$ can be stated explicitly in the form

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$$F(x) = \begin{cases} g(x) & \text{if } x \geq 0 \\ h(x) & \text{if } x < 0, \end{cases}$$

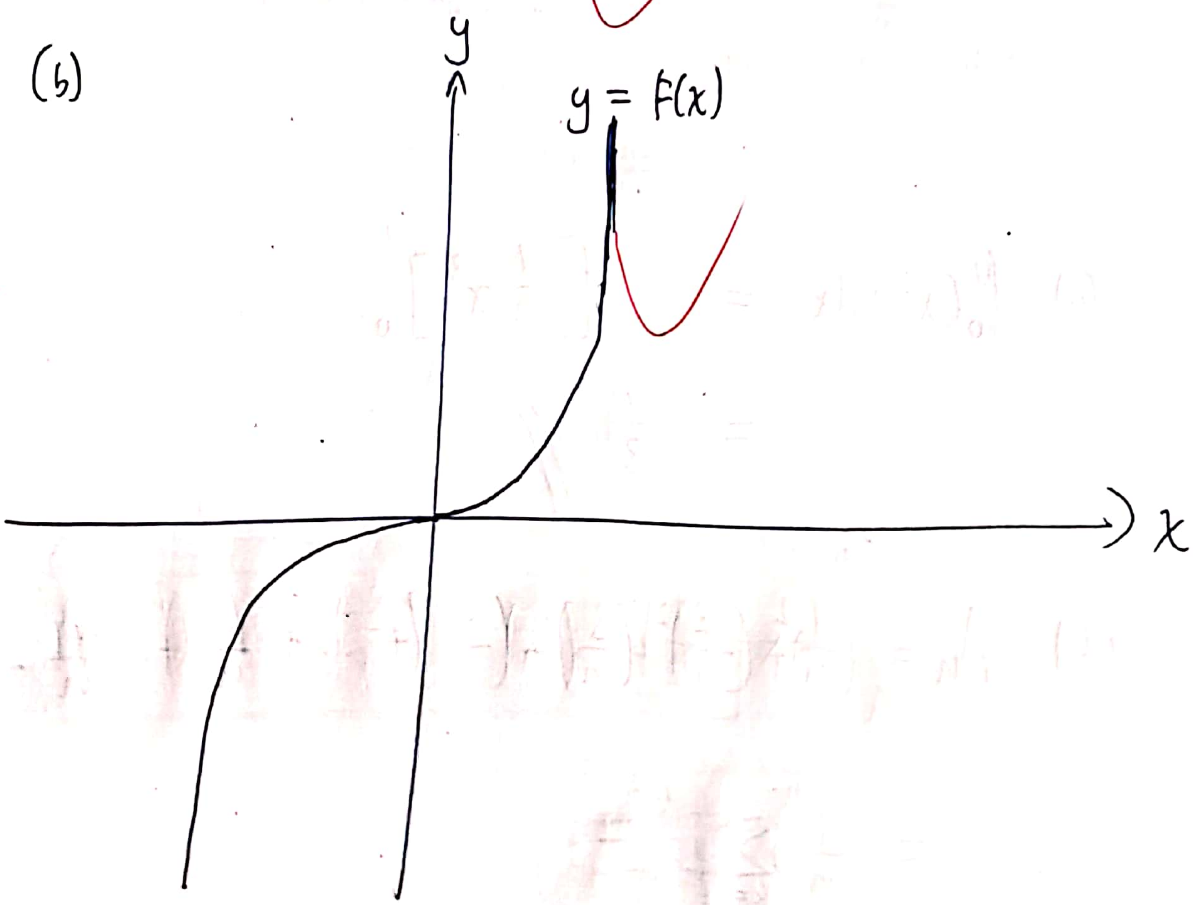
where g, h are polynomials. Find $g(x), h(x)$.

(b) Sketch the graph of $F(x)$.

(a) $F(x) = \begin{cases} \int_0^x (t)\, dt & \text{if } x \geq 0 \\ -\int_0^x (t)\, dt & \text{if } x < 0. \end{cases}$

$= \begin{cases} \frac{1}{2}x^2 & \text{if } x \geq 0 \\ -\frac{1}{2}x^2 & \text{if } x < 0 \end{cases}$

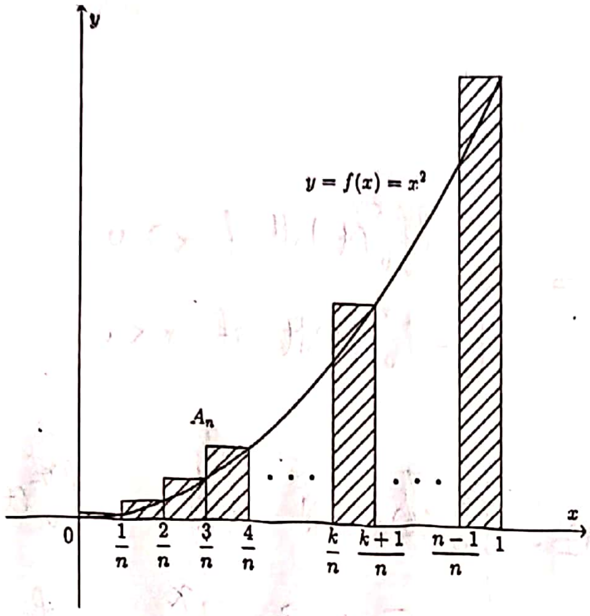
(b)



5. Let $f(x) = x^2$.

(a) Evaluate $\int_0^1 f(x)\,dx$.

(b) Suppose that the interval $[0, 1]$ ~~is~~ subdivided into n equal subintervals. Define A_n to be the Riemann sum of $f(x)$ as shown below.



Find A_n in terms of n.

(Hint: $\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$ and $\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$)

(c) By parts (a), (b), verify that

$$\lim_{n\to\infty} A_n = \int_0^1 f(x)\,dx$$

(a) $\int_0^1 (x^2)\,dx = \left[\frac{1}{3}x^3\right]_0^1$

$= \frac{1}{3}$ //

(b) $A_n = \frac{1}{n} \times \left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2 + 1^2 \right]$

$= \frac{1}{n} \times \sum_{k=1}^{n} \frac{k^2}{n^2}$

$$= \frac{1}{\cancel{n}} \times \frac{1}{6} \cancel{n}(n+1)(2n+1) \times \frac{1}{n^2}$$

$$= \frac{(n+1)(2n+1)}{6n^2} \quad //$$

$$= \frac{2n^2+3n+1}{6n^2} \quad //$$

(c) $$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\left(\frac{2n^2+3n+1}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2+\frac{3}{n}+\frac{1}{n^2}}{6}\right)$$

$$= \frac{1}{3} \quad //$$

$$= \int_0^1 f(x)\,dx \quad //$$