THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1510 Calculus for Engineers (Fall 2021)
Coursework 5

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Class: MATH1510G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/
Signature: David Date: 11-10-2021

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	3						
2	2						
3	2						
4	2						
5					Total	9	/ 10

Clarify: $\log x = \log_{10} x = \frac{\ln x}{\ln 10}$

Part A

1. Find $f'(x)$ if

   (a) $f(x) = ex^{\pi} + \sqrt{2}\pi^{x} + \pi^{\pi}$

   (b) $f(x) = \dfrac{\log x}{x+1}$

   (c) $f(x) = \sec(\tan x)$

   (d) $f(x) = \ln(\ln(e^{x} + x))$

   (e) $f(x) = \cos^2(2^x)$ $= [\cos(2^x)]^2$

(a) $f'(x) = e\pi x^{\pi-1} + \sqrt{2}\pi^{x} \ln\pi$ ✓ //

(b) $f'(x) = \dfrac{\frac{1}{x\ln 10}(x+1) - \log x}{(x+1)^2}$

$= \dfrac{x+1 - x\ln 10 \cdot \log x}{x \ln 10 \, (x+1)^2}$ ✓ //

(c) $f'(x) = \sec(\tan x)\tan(\tan x) \times (\sec x)^2$ ✓

(d) $f'(x) = \dfrac{1}{\ln(e^x+x)} \times \dfrac{1}{e^x+x} \times (e^x+1)$

$= \dfrac{e^x+1}{(e^x+x)\ln(e^x+x)}$ //

(e) $f'(x) = 2\cos(2^x) \times (-\sin(2^x))$
$\times\ 2^x \ln 2$

$= -2^{x+1}\ln 2 \cdot \cos(2^x)\sin(2^x)$ //

2. Find $\frac{dy}{dx}$ if

−0.5 (a) $y = \dfrac{3^x\sqrt[3]{x^2+4}}{\sqrt{e^x+1}}$

(b) $y = \sin^{-1}(3x+1)$

(c) $y = (\sin x)^x$

−0.5

(d) $x^3 - 2xy + 2y^2 = 5$
Express your answer in terms of x, y.

$[e^{(\ln 3)x}]' = e^{(\ln 3)x} \cdot \ln 3$

(a)

$\ln y = e^{x\ln 3} + \frac{1}{3}\ln(x^2+4) - \frac{1}{2}\ln(e^x+1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = 3^x \ln 3 + \frac{1}{3} \times \frac{1}{x^2+4}(2x) - \frac{1}{2} \times \frac{1}{e^x+1}(e^x)$

$\dfrac{dy}{dx} = \dfrac{3^x\sqrt[3]{x^2+4}}{\sqrt{e^x+1}}\left[\cancel{3^x}\ln 3 + \dfrac{2x}{3(x^2+4)} - \dfrac{e^x}{2(e^x+1)}\right]$

//

(b) $\frac{dy}{dx} = \frac{1}{\sqrt{1-(3x+1)^2}} (3)$

$= \frac{3}{\sqrt{(-3x)(3x+2)}}$ //

$(1-3x-1)(1+3x+1)$

(c) $\cancel{\frac{dy}{dx}}$

$\ln y \cancel{\frac{1}{y}\left(\frac{dy}{dx}\right)} = x \ln(\sin x)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \ln(\sin x) + \frac{\cancel{1}\, x}{\sin x}(\cos x)$

$\frac{dy}{dx} = (\sin x)^x \left[\ln(\sin x) + \frac{x\cos x}{\sin x} \cancel{}\right]$ //

(d) By implicit differentiation, we have:

$3x^2 - 2y - 2x\left(\frac{dy}{dx}\right) + \cancel{4\left(\frac{dy}{dx}\right)}\; 4y\cdot\left(\frac{dy}{dx}\right) = 5$

$\frac{dy}{dx}(4-2x) = 5 - 3x^2 + 2y$

$\frac{dy}{dx} = \cancel{\frac{5-3x^2+2y}{4-2x}}$ //

$\frac{dy}{dx} = \frac{3x^2 - 2y}{2x - 4y}$

$$\frac{\cancel{4} - \cancel{2}(\cancel{4})(x+2)^{\cancel{2}}}{(x+2)^{6}}$$

1 1 2

Part B

3. Let $f(x) = \ln(2x+4)$ for $x > -2$.
   (a) Find $f'(x)$, $f''(x)$ and $f'''(x)$.
   (b) Let n be a positive integer. Write down $f^{(n)}(x)$.

$\times 2 \quad \times 3 \quad \times 4$

$1 \to 1 \to 2 \to 6.$

(a) $$f'(x) = \frac{1}{2x+4}(2) = \frac{1}{x+2}$$ //

$$f''(x) = \frac{-(1)}{(x+2)^2} = -\frac{1}{(x+2)^2}$$

$$f'''(x) = -\frac{-2(x+2)}{(x+2)^4} = \frac{2}{(x+2)^3}$$ //

(b) $$f^{(n)}(x) = (-1)^{n+1} \times \frac{(n-1)!}{(x+2)^n}.$$ //

4. Let $f(x) = \sin 2x$.

(a) Find $f(0)$, $f'(0)$, $f''(0)$, $f'''(0)$ and $f^{(4)}(0)$.

(b) Let n be a positive integer. Write down $f^{(2n-1)}(0)$ and $f^{(2n)}(0)$.

(a) $f(0) = 0$ //

$f'(x) = 2\cos 2x$

$\rightarrow$ $f'(0) = 2(1) = 2$ //

$f''(x) = 2(-\sin 2x)(2) = -4\sin 2x$

$f''(0) = -4(0)$

$= 0$ //

$f'''(x) = -4\cos 2x\,(2) = -8 \cos 2x$

$f'''(0) = -8(1)$

$\rightarrow$ $= -8$ //

$f^{(4)}(x) = -8(-\sin 2x)(2)$

$= 16 \sin 2x$

$f^{(4)}(0) = 0$ //

(b) $f^{(2n-1)}(0) = (-1)^{n+1} \times 2^{2n-1}$ //

$f^{(2n)}(0) = 0$ //

5. Let $y = f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(a) Show that $\dfrac{dy}{dx} = 1 - y^2$.

(b) Show that

$$\frac{d^3y}{dx^3} + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = C$$

for some constant C. Also, find the value of C.

(a) $$\frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{(e^x+e^{-x})^2}$$

$$= \frac{(2e^x)(2e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{4}{(e^x+e^{-x})^2} \quad //$$

$$1-y^2 = (1-y)(1+y)$$

$$= \left(\frac{e^x+e^{-x}-e^x+e^{-x}}{e^x+e^{-x}}\right)\left(\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}}\right)$$

$$= \frac{(2e^{-x})(2e^x)}{(e^x+e^{-x})^2}$$

$$= \frac{4}{(e^x+e^{-x})^2}$$

$$= \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = 1-y^2 \quad //$$

(b) By $\frac{dy}{dx} = 1-y^2$,

We have $\frac{d^2y}{dx^2} = -2y\left(\frac{dy}{dx}\right)$

$$= -2y(1-y^2)$$

$$\frac{d^3y}{dx^2} = -2\left(\frac{dy}{dx}\right)(1-y^2) - 2y\left[-2y\left(\frac{dy}{dx}\right)\right]$$

$$= -2(1-y^2)^2 + 4y^2(1-y^2)$$

$$= -2(1-y^2)(1-3y^2)$$

$\therefore$ $\frac{d^3y}{dx^2} + 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2$

$$= -2(1-y^2)(1-3y^2) - 4y^2(1-y^2) + 2(1-y^2)^2$$

$$= (1-y^2)(-2+6y^2-4y^2+2-2y^2)$$

$$= (1-y^2)(0)$$

$$= 0 \quad //$$