請勿攜去

Not to be taken away

香 港 中 文 大 學

The Chinese University of Hong Kong

二〇一六至一七年度下學期科目考試

Course Examination 2nd Term, 2016-17

科目編號及名稱 Course Code & Title : **MATH1510J Calculus for Engineers**

時間 Time allowed : **2** 小時 hours **00** 分鐘 minutes

學號 Student I.D. No. : ____________ 座號 Seat No. : ____________

Please show the work with as much detail as possible for every step.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} \sin x & \text{if } \ x \geq 0, \\ 0 & \text{if } \ x < 0. \end{cases}$$

(a) (6 points) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\pi} f(x)$.

(b) (8 points) Show that $f(x)$ is continuous at $x = 0$.

(c) (6 points) Is $f(x)$ differentiable at $x = 0$? Justify your answer.

2. (a) (6 points) Find $\frac{dy}{dx}$ if

$$y = \frac{x^2+x+1}{x^2+1}.$$

(b) (6 points) Find $\frac{dy}{dx}$ if

$$y = \sin(\ln x + 3^x).$$

(c) (6 points) Find $\frac{dy}{dx}$ if

$$y = (\tan x)^{\sin x} \quad \text{where } 0 < x < \frac{\pi}{2}.$$

(d) (6 points) Find

$$\frac{d}{dx}\left\{\int_{x^2}^{x} \sqrt{t^3+1}\,dt\right\}.$$

3. Evaluate the following integrals:

   (a) (6 points) $\int \left(\sec x + \frac{1}{\sqrt[3]{x}} - 3^x\right) dx$;

   (b) (6 points) $\int \frac{1}{x - x^2}\, dx$;

   (c) (6 points) $\int e^{\cos x} \sin x\, dx$;

   (d) (6 points) $\int \sin^2 x \cos^2 x\, dx$;

   (e) (6 points) $\int x^3 \ln x\, dx$;

   (f) (6 points) $\int \frac{x}{\sqrt{x} - 1}\, dx$;

   (g) (6 points) $\int \frac{1}{\sqrt{x^2 + 2x + 2}}\, dx$.

4. Let

$$f(x) = xe^{-2x} \quad \text{over the interval } I = [0, 3].$$

   (a) (10 points) Find all the critical point(s) of $f(x)$ on the given interval. Then, find the interval(s) on which $f(x)$ is increasing and the interval(s) on which $f(x)$ is decreasing.

   (b) (6 points) For each critical point, determine whether it is a local maximum, minimum or neither.

   (c) (6 points) Find the global maximum and minimum of $f(x)$ on the given interval.

5. Solve the following problems separately. Justify your answers.

   (a) (8 points) Find the area of the region in the xy-plane bounded by the graphs of the functions:

$$\begin{cases} f(x) = x^2 + 5, \\ g(x) = 5 - 3x. \end{cases}$$

   (b) Let $\mathcal{R}$ be the region in the xy-plane bounded by the curve $y = 4(x-1)^2$ and the line $y = 4$.

   Express the volumes of the following solids as the integrals of functions (**You do not need to evaluate the integrals**):

   i. (4 points) The solid obtained by revolving $\mathcal{R}$ about the x-axis.

   ii. (4 points) The solid obtained by revolving $\mathcal{R}$ about the vertical line $x = 1$.

6. Solve the following problems separately. Justify your answers.

(a) (5 points) Find the Maclaurin polynomial of order 3 of

$$f(x) = \tan x.$$

(b) (5 points) Find the Maclaurin series of

$$f(x) = 2^x.$$

(c) (5 points) Find the Maclaurin series of

$$f(x) = \frac{1}{x+2}.$$

(d) Find the Maclaurin polynomial of $f(x)$ of order 3 where:

i. (5 points) $f(x) = (\sin x) \cdot \ln(1+x)$;

ii. (5 points) $f(x) = \sin(x + x^2)$.

7. Solve the following problems separately. Justify your answers.

(a) (8 points) Given that

$$f(x, y, z, w) = \sin(xyw) + e^{xz} \ln(y + w).$$

Find f_x, f_y, f_z and f_{xy}.

(b) (8 points) Suppose $g(u, s, t)$ is a differentiable function and

$$u = y - x, \quad s = z - y, \quad t = x - z,$$

where x, y, z are independent variables. Find a constant A such that

$$\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial g}{\partial z} = A.$$

(c) (7 points) Compute

$$\int_0^3 \int_0^\pi (x \sin y)\, dy dx.$$

(d) (8 points) Let

$$f(x, y) = y\sqrt{x}.$$

Compute the double integral of $f(x, y)$ over the domain

$$D = \{(x, y) \mid 0 \le x \le 1 \text{ and } x \le y \le 1\}.$$

8. Solve the following problems separately. Justify your answers.

(a) (4 points) Evaluate the following limit:

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right).$$

(b) (4 points) Let

$$f(x)=(x+1)\ln(x+1)-x.$$

Show that $f(x)\geq 0$ for all $x\geq 0$.

(c) (4 points) Show that the equation

$$2^x = x+\tan x$$

has a solution in $\mathbb{R}$.

(d) i. (3 points) Write down the Maclaurin polynomials of order 4 of

$$\begin{aligned} f(x) &= \ln(1+2x^2)-2x\sin x, \\ g(x) &= \sin^2 x - x^2. \end{aligned}$$

ii. (1 point) Hence, or otherwise, evaluate

$$\lim_{x\to 0^+}\frac{\ln(1+2x^2)-2x\sin x}{\sin^2 x - x^2}.$$

(e) (4 points) Approximate the value of $\int_0^1 \sin(x^2)\,dx$ with an error less than 0.001.

–End of Paper–