THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1510 Calculus for Engineers (2020-2021)
Supplementary Exercise 8

Fundamental Theorem of Calculus

1. Find $\dfrac{d}{dx}\displaystyle\int_3^x \cos(e^t - e^{-t})\,dt$ by using the fundamental theorem of calculus.

2. Find $\dfrac{dy}{dx}$ if

   (a) $y = \displaystyle\int_1^x \cos(t^2)\,dt$

   (b) $y = \displaystyle\int_1^{x^2+1} \cos(t^2)\,dt$

   (c) $y = \displaystyle\int_x^{x^2+1} \cos(t^2)\,dt$

   (Hint: $\displaystyle\int_x^{x^2+1} \cos(t^2)\,dt = \int_1^{x^2+1} \cos(t^2)\,dt - \int_1^x \cos(t^2)\,dt$.)

3. Find $\lim\limits_{h\to 0} \dfrac{1}{h^3}\displaystyle\int_0^h \sin(t^2)\,dt$.

   (Hint: Using L'Hôpital rule.)

Area Bounded by Graphs

4. Consider the curves

$$\begin{array}{ll} C_1: & y = 1 - x^2 \quad \text{with } x \in \mathbb{R} \\ C_2: & y = x + 1 \quad \text{with } x \in \mathbb{R} \end{array}$$

   (a) Find the intersection(s) of C_1 and C_2.

   (b) Sketch the graphs of C_1 and C_2. Make sure to include their intersection(s) in your graphs.

   (c) Find the area of the region bounded by C_1 and C_2.

5. Express the area of the region bounded by the curves:

$$y = \frac{2}{x+1}$$

$$y = 2 - x^2$$

   as a definite integral (or a sum of definite integrals) along:

(a) the x-axis;

(b) the y-axis.

6. Find the area enclosed by the curve $y = x^2$, the x-axis and the line $x = 2$.

7. Find the area enclosed by the curve $y = \sin x$ for $0 \leq x \leq \pi$ and the x-axis.

8. Find the area enclosed by the curves $y = x$ and $y = x^2$.

9. Find the area enclosed by the curves $y = 2^x$, $y = 1 - x$ and $y = 4x - 4$.

10. Let $f(x) = |x|$. Recall that $f(x)$ can be expressed as

$$f(x) = \begin{cases} x & \text{if} \quad x \geq 0, \\ -x & \text{if} \quad x < 0. \end{cases}$$

By writing $\displaystyle\int_{-3}^{2} |x|\,dx = \int_{-3}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx = \int_{-3}^{0} -x\,dx + \int_{0}^{2} x\,dx,$

evaluate $\displaystyle\int_{-3}^{2} |x|\,dx.$

11. (a) Solve $x^2 - 5x + 6 > 0$.

(b) Let $f(x) = |x^2 - 5x + 6|$. Then, $f(x)$ can be expressed as

$$f(x) = \begin{cases} \_\_\_\_\_ & \text{if} \quad x > \_\_\_\_\_, \\ \_\_\_\_\_ & \text{if} \quad \_\_\_\_ \leq x \leq \_\_\_\_, \\ \_\_\_\_\_ & \text{if} \quad x < \_\_\_\_. \end{cases}$$

(c) Evaluate $\displaystyle\int_{0}^{5} |x^2 - 5x + 6|\,dx.$

12. Evaluate $\displaystyle\int_{-2}^{3} |2x - x^2|\,dx.$

13. Find the area enclosed by the curve $y^2 = -x + 6$, the x-axis and the line $y = x$ for $x \geq 0$.

14. Find the area bounded by the lower semi-circle defined by $x^2 + y^2 = 25$ $(y < 0)$ and the parabola $x^2 + 2y - 1 = 0$.

15. Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where $a, b > 0$.