請勿攜去
Not to be taken away

香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一七至一八年度下學期科目考試
Course Examination 2nd Term, 2017-18

科目編號及名稱 Course Code & Title : **MATH1510J Calculus for Engineers**

時間 Time allowed : 2 小時 hours 00 分鐘 minutes

學號 Student I.D. No : ____________ 座號 Seat No. : ____________

Answer All Questions.

1. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} \dfrac{1}{1+x} & \text{if } \quad x > 0; \\ 1 - x & \text{if } \quad x \le 0. \end{cases}$$

(a) Find $\lim\limits_{x \to 0^+} f(x)$.

(b) Show that $f(x)$ is continuous at $x = 0$.

(c) Is $f(x)$ differentiable at $x = 0$? Justify your answer.

2. (12 pts) Find $\dfrac{dy}{dx}$ for the following y.

(a) $y = x^4 - \cos x + 3\log_2 x + 7$.

(b) $y = \sqrt{1 + e^{\sin x}}$.

(c) $y = 5x^5 \sec x$.

(d) $y = \displaystyle\int_{-x^2}^{x} (t^5 + t + 1)^9 \, dt$.

3. (18 pts) Compute the following integrals :

   (a) $\displaystyle\int (x - \sec x \tan x + 2^x)\, dx.$

   (b) $\displaystyle\int \sin^5 x \cos x\, dx.$

   (c) $\displaystyle\int_1^{e^2} \frac{1 + \ln x}{x}\, dx.$

   (d) $\displaystyle\int_0^{\pi} \sin^2 x\, dx.$

   (e) $\displaystyle\int \frac{\sqrt{x}}{x+1}\, dx.$

   (f) $\displaystyle\int x \sin x\, dx.$

4. (16 pts) Answer the following questions.

   (a) Let
   $$f(x) = x^2 e^x.$$

      i. Compute $f'(x)$ and $f''(x)$.

      ii. Find the interval(s) on which f is increasing, and those on which f is decreasing.

      iii. Find all the inflection points of f.

   (b) Consider the function
   $$g(x) = x - 2\sin x.$$
   on the interval $[0, \pi]$.

      i. Solve $g'(x) = 0$ on the interval $[0, \pi]$.

      ii. Find the absolute maximum and absolute minimum of $g(x)$ on the interval $[0, \pi]$.

5. (8 pts) Answer the following questions.

   (a) Find the area of the region on the xy-plane bounded by the following graphs:

$$\begin{cases} y = x + 10 \\ y = x^2 - 2x \end{cases}$$

   (b) Let $\mathcal{R}$ be the region on the xy-plane bounded by the following graphs:

$$\begin{cases} y = x^2 \\ y = x^3 \end{cases}$$

   Express the volumes of the following solids as integrals of functions. (You do not need to evaluate the integrals)

   i. The solid obtained by revolving $\mathcal{R}$ about the x-axis.

   ii. The solid obtained by revolving $\mathcal{R}$ about the y-axis.

6. (12 pts) Answer the following questions.

   (a) Let $f(x, y) = x^3e^y - y\cos x - y^2 - 2$.

   i. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

   ii. Suppose

$$x = s - t \quad \text{and} \quad y = s + t.$$

   Find $\frac{\partial f}{\partial t}$ at $(s, t) = (1, 1)$.

   (b) Evaluate the following double integrals.

   i. $\displaystyle\int_0^1 \int_0^2 (6xy^2 - 4y + 1)\, dy\, dx.$

   ii. $\displaystyle\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy.$

7. (12 pts) Answer the following questions.

   (a) Find the Taylor polynomial of order 3 of
   $$f(x) = e^x \sin x$$
   at $x_0 = 0$.

   (b) By differentiating the Taylor series
   $$\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k = 1 + x + x^2 + x^3 + \cdots, \quad -1 < x < 1,$$
   find the Taylor series of
   $$g(x) = \frac{1}{(1-x)^2}$$
   at $x_0 = 0$.

   (c) Using part (a) and (b) above, find the Taylor polynomial of order 2 of
   $$h(x) = \frac{e^x \sin x}{(1-x)^2}$$
   at $x_0 = 0$.

8. (12 pts) Solve the following problems separately. Justify your answers.

   (a) Evaluate
   $$\lim_{x \to 1} x^{\frac{1}{1-x}}.$$

   (b) Let n be a non-negative integer and $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n x \, dx$.

      i. Show that $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geq 2$.

      ii. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$.

   (c) Let $a > 0$ and $f : [0, a] \to \mathbb{R}$ be a continuous function.

      i. Show that
      $$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx.$$

      ii. Hence, find $\displaystyle\int_0^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x} \, dx$.

~ End of Examination ~