Risk and Return

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Motivation

NPV and other valuation techniques need required rate of return

- opportunity cost
- risk-adjusted discount rate
- determined by “the market”
- how?

Introduce risk into the valuation process

- what the stock returns have been historically?
- how to measure risk and how risky are stocks?
- how to estimate the required rate of return for a given level of risk?

Two Lessons from Market History

- Lessons from capital market history
  - There is a reward for bearing risk
  - The greater the potential reward, the greater the risk
  - This is called the risk-return trade-off

Lecture Outline

- Historical Returns
  - Returns
    - risky assets on average earn a risk premium
  - Risk
    - The greater the potential reward, the greater is the risk
  - More about returns
    - Arithmetic vs. Geometric returns

- Expected Returns
  - Single asset
  - Portfolio

Investment Returns

Return on your investment: gain (or loss) from that investment

Historical return (*realized return*): the past gain (or loss) of an investment that actually occurred

Expected return: the gain or loss that an investor anticipates on an investment

Returns can be expressed in:

- Dollar returns: Amount received – Amount invested
- Percentage returns: (Amount received – Amount invested) / Amount invested

Dollar Returns

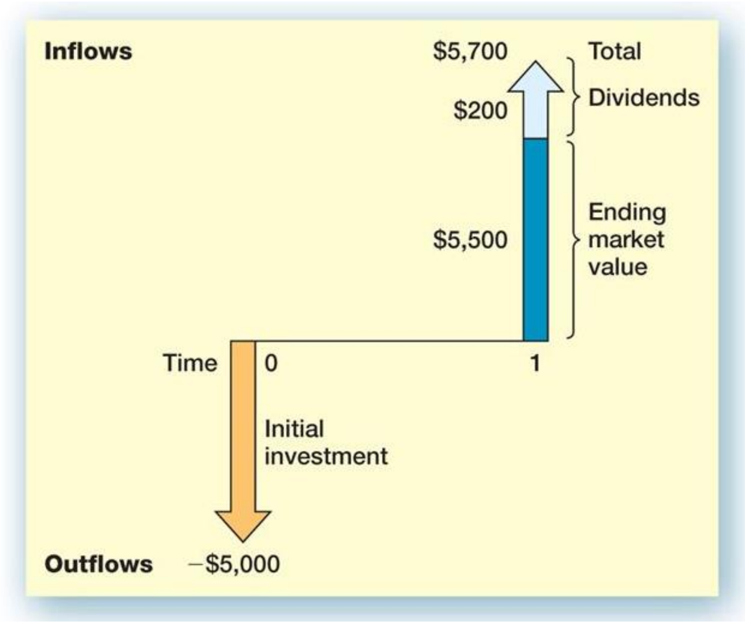
- Total dollar return =

  income from investment

  + capital gain (loss) due to change in price

Dollar Return Example

Suppose at the beginning of the year, you purchased 1,000 shares of at $5 per share. Over the year, the stock paid a dividend of $0.2 per share. It is now year-end, the value of the stock has risen to $5.5 per share. What is your total dollar return?

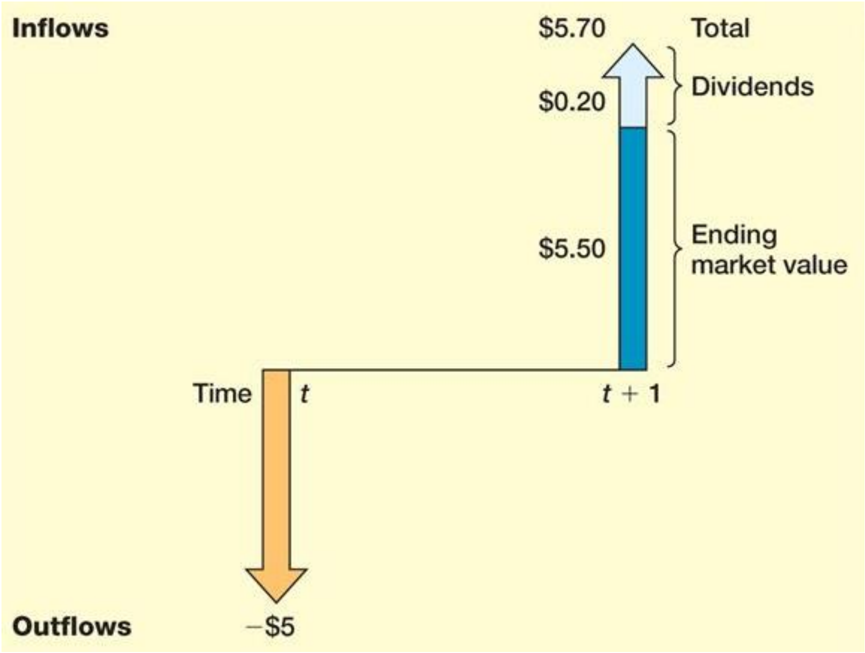


- $\text{Dividend} = \$0.20 \times 1000 = \$200$
- $\text{Capital Gain} = (\$5.50 - \$5) \times 1000 = \$500$
- Total dollar return = Dividend income + Capital Gain = $200 + $500 = $700

Percentage Return

- Total percentage return = dividend yield + capital gains yield
- Dividend yield = income / beginning price
- Capital gains yield = (ending price – beginning price) / beginning price

Percentage Return Example



- $\text{Dividend yeild} = \frac{0.20}{5} = 4\%$
- $\text{Capital gain yeild} = \frac{(5.50 - 5)}{5} = 10\%$
- Total percentage return =

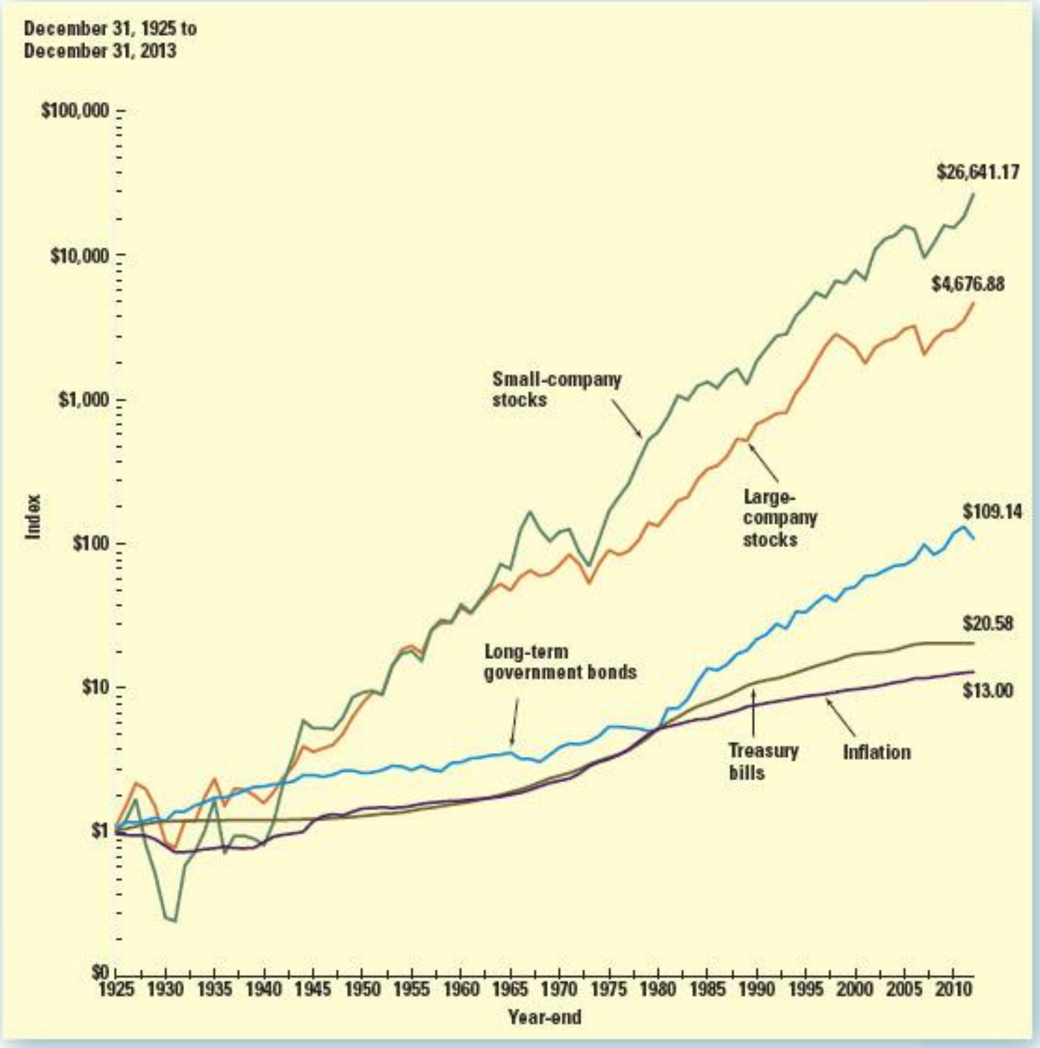
Dividend yield + Capital gain yield = 4% + 10% = 14%

The Importance of Financial Markets

- Financial markets allow companies, governments and individuals to increase their utility

- Financial markets also provide us with information about the returns that are required for various levels of risk

Return of Various Investments



Average Return

We use the *historical data* on an asset:

- ***Arithmetic Average Return***: the return earned in an average period over a multiple periods.

$$Arithmetic\ Average\ Return = \bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots R_T)$$

where R_t is the historical return of a security in period *t*, and *T* is the total number of historical periods

Average Return Example

Year End	S&P 500 Index	Dividends Paid*	S&P 500 Realized Return
2001	1148.08		
2002	879.82	14.53	−22.1%
2003	1111.92	20.80	28.7%
2004	1211.92	20.98	10.9%
2005	1248.29	23.15	4.9%
2006	1418.30	27.16	15.8%
2007	1468.36	27.86	5.5%
2008	903.25	21.85	−37.0%
2009	1115.10	27.19	26.5%
2010	1257.64	25.44	15.1%
2011	1257.60	26.59	2.1%
2012	1426.19	32.67	16.0%
2013	1848.36	39.75	32.4%
2014	2058.90	42.47	13.7%

$$\bar{R} = \frac{1}{13}(-0.221 + 0.287 + 0.109 + 0.109 + 0.158 + 0.055 - 0.370$$
$$+ 0.265 + 0.151 + 0.021 + 0.160 + 0.324 + 0.137) = 8.7\%$$

Risk Premium

Risk premium: the return difference between a risk-bearing security and a risk-free security

Risk-free return: return on Treasury bills

Investment	Average Return	Risk Premium
Large Stocks	12.1%	8.6%
Small Stocks	16.9%	13.4%
Long-term Corporate Bonds	6.3%	2.8%
Long-term Government Bonds	5.9%	2.4%
U.S. Treasury Bills	3.5%	0.0%

Based on 1926-2013

Our first lesson: risky assets on average earn a risk premium

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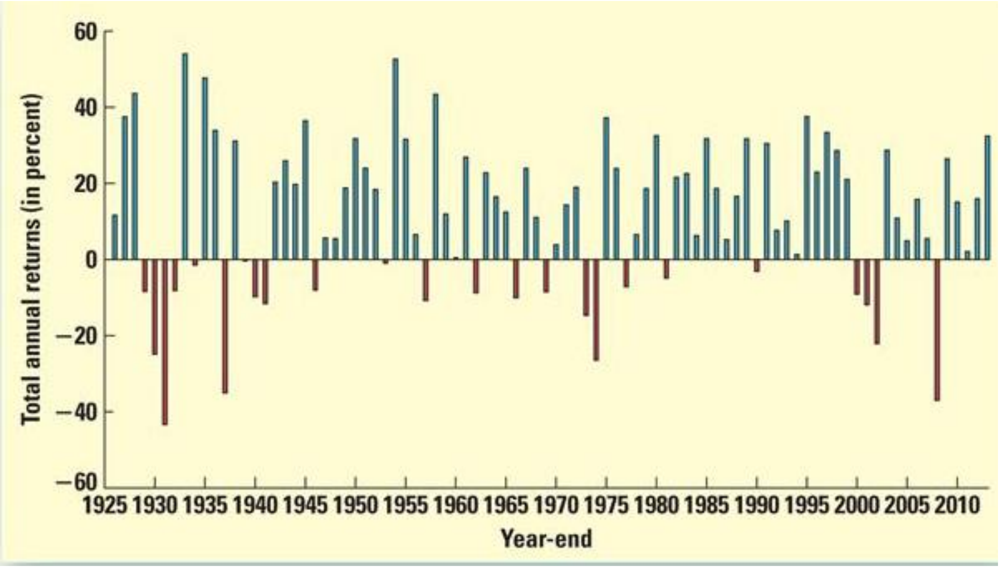
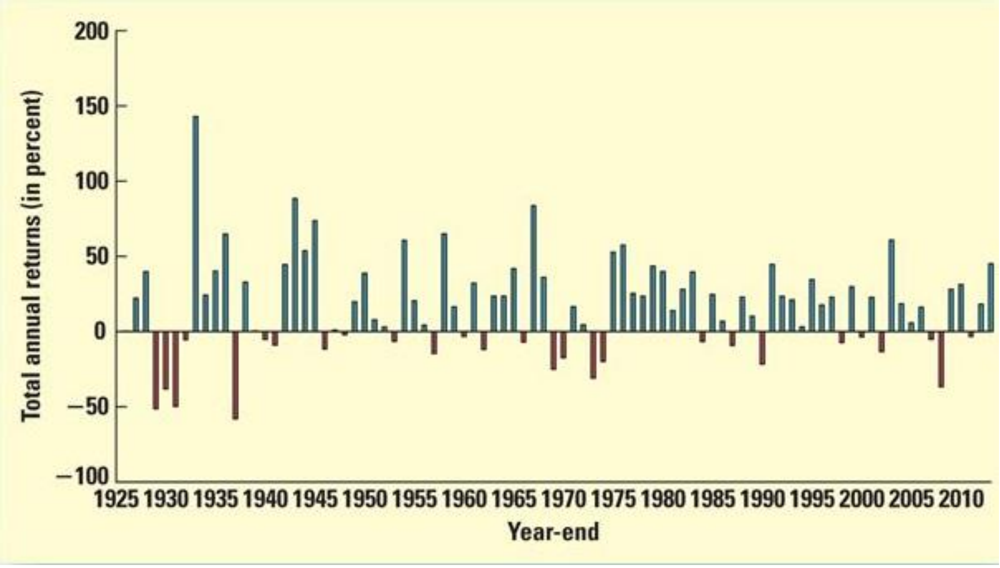
Historical Year-by-Year Return

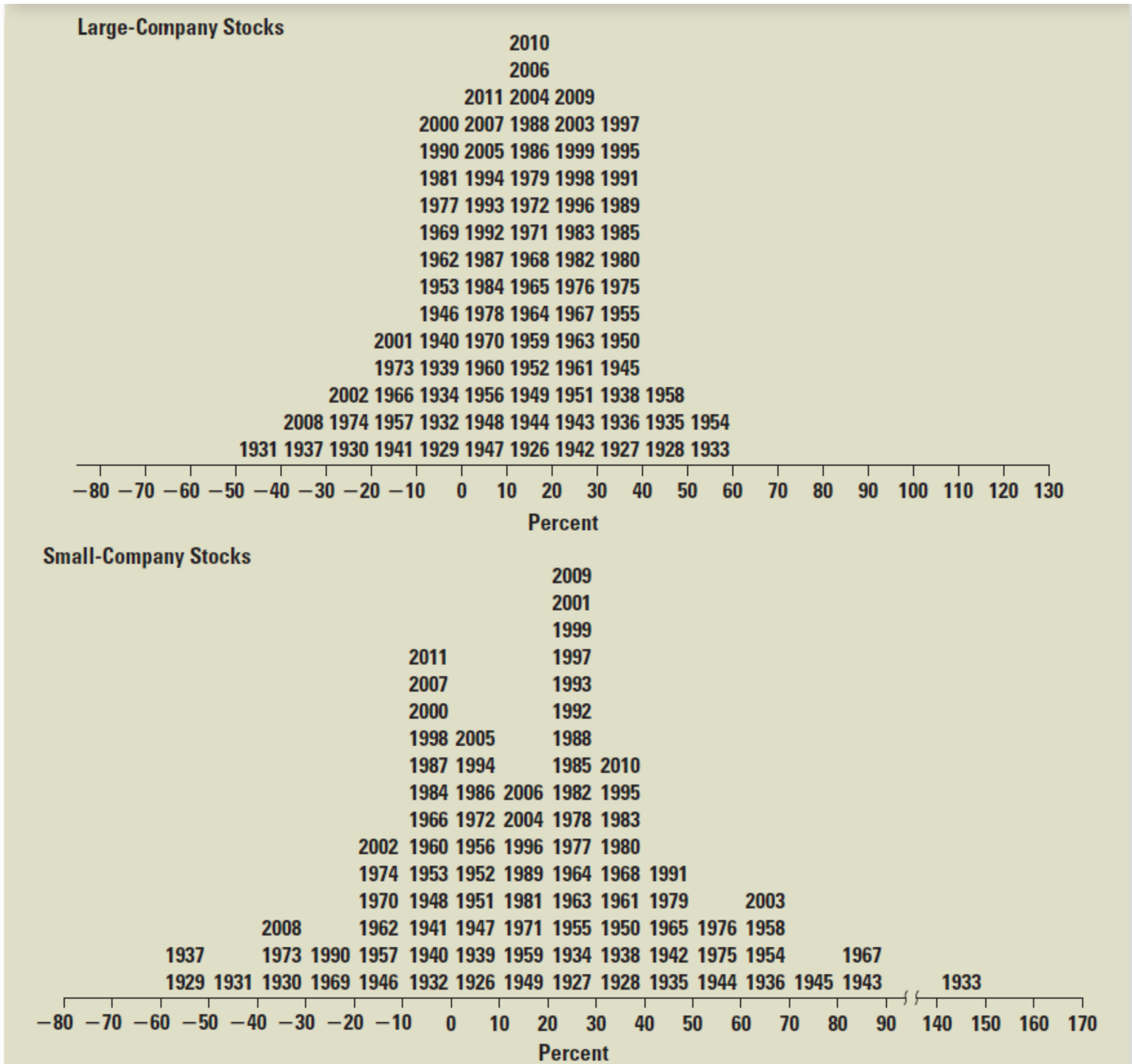


Large Company Common Stocks



Small Company Common Stocks

Frequency Distribution of Returns



Volatility

We need to qualify the dispersion of returns

We use ***variance*** or its square root, the ***standard deviation***, as measures for volatility

Variance:

$$Var(R) = \sigma^2 = \frac{1}{T-1}\left[(R_1 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2\right]$$

where $\bar{R}$ is the arithmetic average return

Standard Deviation:

$$SD(R) = \sigma = \sqrt{Var(R)}$$

Variance and Standard Deviation Example

Year End	S&P 500 Index	Dividends Paid*	S&P 500 Realized Return
2001	1148.08		
2002	879.82	14.53	−22.1%
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2014	2058.90	42.47	13.7%

From Page 13, we already calculated that $\bar{R} = 8.7\%$.

$$\begin{aligned} Var(R) &= \frac{1}{T-1}\left[(R_1-\bar{R})^2 + \cdots + (R_T-\bar{R})^2\right] \\ &= \frac{1}{13-1}\left[(-0.221-0.087)^2 + (0.287-0.087)^2 + \cdots + (0.137-0.087)^2\right] \\ &= 0.038 \end{aligned}$$

The volatility or standard deviation is therefore $SD(R) = \sqrt{Var(R)} = \sqrt{0.038} = 19.5\%$

Risk

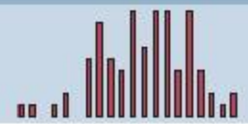
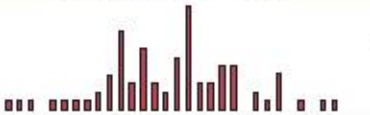
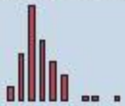
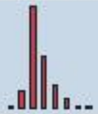
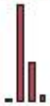
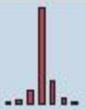
- Risk is the *uncertainty* associated with *future possible outcomes*.

- Investment risk refers to the potential for your investment return to fluctuate (go up or down) in value from period to period.

- The greater the volatility, the greater the uncertainty

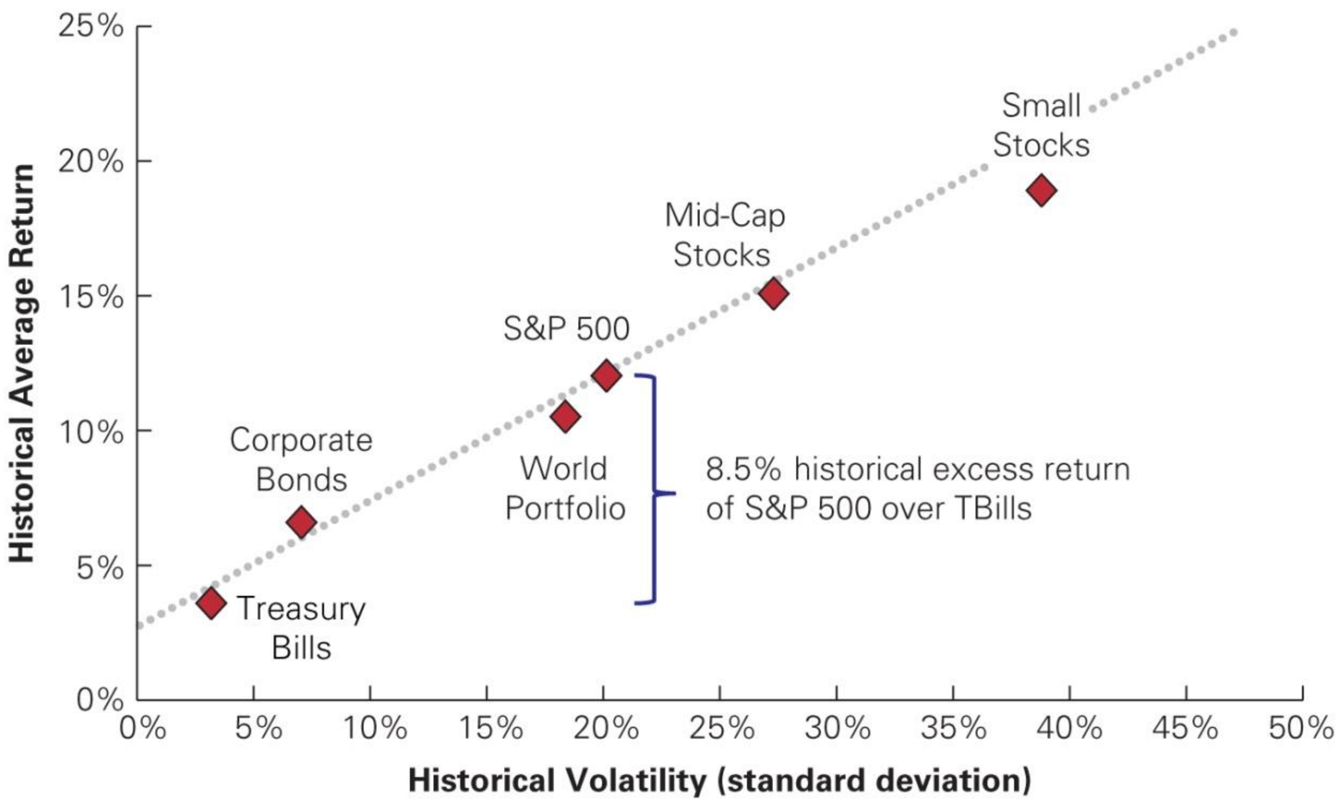
- We can use *historical* variance or standard deviation as a measure for uncertainty
  - $Var(R) = \sigma^2 = \frac{1}{T-1}[(R_1 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2]$
  - how much on average the realized returns tend to deviate from the historical mean
  - how much we should **expect to be surprised**
  - a measure of **uncertainty** = **risk**

Risk and Return

SERIES	AVERAGE RETURN	STANDARD DEVIATION	DISTRIBUTION
Large-company stocks	12.1%	20.2%	
Small-company stocks	16.9	32.3	
Long-term corporate bonds	6.3	8.4	
Long-term government bonds	5.9	9.8	
Intermediate-term government bonds	5.4	5.7	
U.S. Treasury bills	3.5	3.1	
Inflation	3.0	4.1	

Our second lesson: The greater the potential reward, the greater is the risk

Trade-off Between Risk and Return



Source: CRSP, Morgan Stanley Capital International

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  - Single asset
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More About Returns

Suppose two years ago you and a friend both invested $100 each in two different investment funds.

While your own fund earned a steady return on investment of 4% in both years, your friend's fund lost 16% in the first year, but managed to gain 24% over the second year.

What are the arithmetic average returns for the two funds? Will both of you walk away with the same amount of money?

Arithmetic Average Return

Here is what happened:

		end of year 1		end of year 2
your fund	$100	$\xrightarrow{4\%}$ 104.00	$\xrightarrow{4\%}$	108.16

arithmetic average return = (4% + 4%) / 2 = 4%

your friend's fund	$100	$\xrightarrow{-16\%}$ 84.00	$\xrightarrow{24\%}$	104.16

arithmetic average return = (–16% + 24%) / 2 = 4%

- A large loss, followed by an even bigger gain is not the same as two moderate gains of average size!

Geometric Average Return

How can we capture this? Consider a hypothetical fund:

- the fund earns the same return R in both years
- what value must R take so that the final fund value is the same as that of your friend's fund?

your friend's fund: \$100 $\xrightarrow{-16\%}$ 84.00 $\xrightarrow{24\%}$ 104.16

hypothetical fund: \$100 $\xrightarrow{R}$ $100(1+R)$ $\xrightarrow{R}$ $100(1+R)^2$

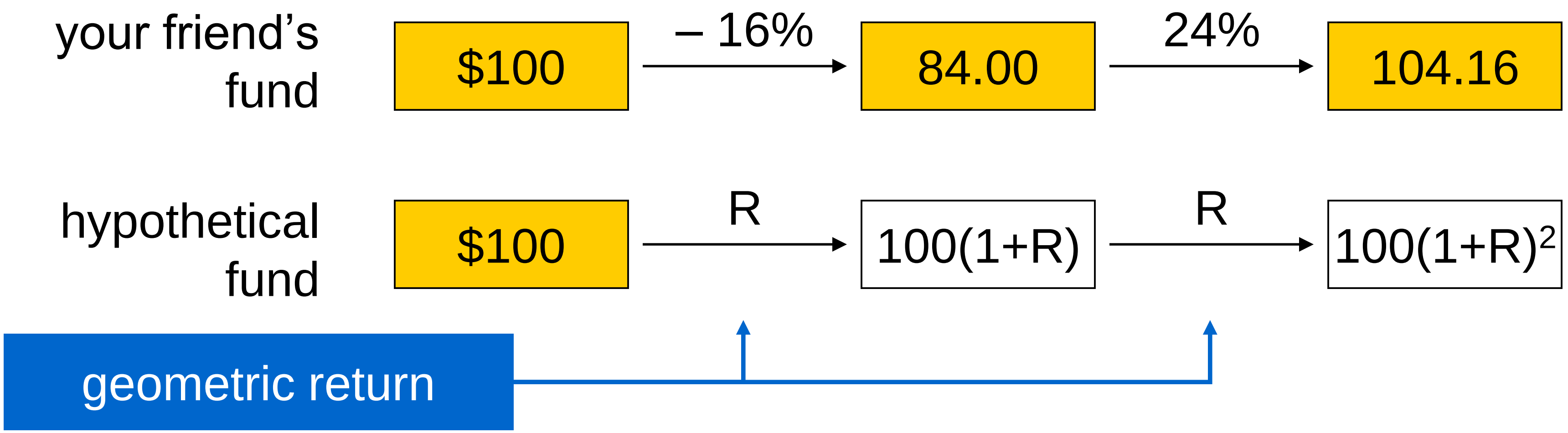
geometric return

$$100 \times (1 - 16\%) \times (1 + 24\%) = 100 \times (1 + R)^2 \qquad R = 2.06\%$$

Geometric Average Return

How can we capture this? Consider a hypothetical fund:

- the fund earns the same return R in both years
- what value must R take so that the final fund value is the same as that of your friend's fund?



$$(1 + R_1) \times (1 + R_2) = (1 + R)^2$$

Geometric Average Return

Geometric Average Return: the average *compound* return earned per period over multiple periods

$$Geometric\ Average\ Return = [(1+R_1)\times(1+R_2)\times\cdots\times(1+R_T)]^{1/T}-1$$

Arithmetic vs Geometric Return

- Arithmetic average – return earned in an average period over multiple periods

- Geometric average – average compound return per period over multiple periods

- Which is better?
  - The arithmetic average is overly optimistic for long horizons
  - The geometric average will be less than the arithmetic average unless all the returns are equal (i.e. zero volatility)
  - The greater the volatility the greater the difference between arithmetic and geometric returns
  - When it comes to investment returns and retirement planning it is compounded (geometric) returns that matter

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Expected Return

Which stock to invest on?

- It depends on the *expected* return, which is the return that an investor anticipate in the *future*.

How to estimate expected return?

Method 1

- Use historical returns
  - $E(R) = \frac{1}{T}(R_1 + R_2 + \cdots R_T)$

Method 2

- Forecast future states of the economy, probability of each state, and asset return in each state

Expected Return and Variance

Using Probabilities

The **expected return** is defined as the probability-weighted average of all possible returns

- $E(R) = \sum_{s=1}^{S} p_s R_s$
- where there are S possible states of the economy, *s=1,2,…,S*
- p_s is the probability that state *s* occurs
- R_s is the return in state *s*

Variance can be computed as:

- $Var(R) = \sigma^2 = \sum_{s=1}^{S} p_s [R_s - E(R)]^2$

Expected Return and Variance Example

		gold stock	auto stock
scenario	prob	return	return
recession	0.25	+ 13%	– 13%
normal	0.50	+ 7%	+ 17%
boom	0.25	– 11%	+ 27%

Expected Return and Variance Example

Example Cont' d

			gold stock			auto stock
scenario	prob		return			return
recession	0.25	×	+ 13%	=	+ 3.25%	– 13%
normal	0.50	×	+ 7%	=	+ 3.50%	+ 17%
boom	0.25	×	– 11%	=	– 2.75%	+ 27%
				=	+ 4.00%	
expected return			+ 4%			+ 12%

Expected Return and Variance Example

Example Cont' d

		gold stock		auto stock
scenario	prob	return		return
recession	0.25 ×	(+ 13% – 4%)^2	= 0.002025	– 13%
normal	0.50 ×	(+ 7% – 4%)^2	= 0.000450	+ 17%
boom	0.25 ×	(– 11% – 4%)^2	= 0.005625	+ 27%
			= 0.008100	
expected return		+ 4%		+ 12%
variance		0.0081		0.0225

Expected Return and Variance Example

Example Cont'd

		gold stock	auto stock
scenario	prob	return	return
recession	0.25	+ 13%	– 13%
normal	0.50	+ 7%	+ 17%
boom	0.25	– 11%	+ 27%
expected return		+ 4%	+ 12%
St. deviation		$\sqrt{0.0081}$ = 0.09	0.0225
		=9%	15%

Risk and Return Common Fallacies

- Is the expected return always the most likely outcome? → for some distributions yes, but not always (e.g. rolling a dice)

- Are extreme (high/low) returns always less likely? → usually yes, but not always, also depends on distribution

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Portfolio Expected Return

Portfolio is a group of assets held by an investor

Expected Return of a Portfolio:

$$\mathrm{E}\left(R_p\right) = w_1 \times E(R_1) + w_2 \times E(R_2) + \cdots + w_n \times E(R_n)$$

- n is the total number of assets in the portfolio
- w_i is the portfolio weight of asset *i* (percentage of investment in asset *i*)
- $E(R_i)$ is the expected return of asset *i*
- $w_1 + w_2 + \cdots + w_n = 1$

Portfolio Expected Return Example

Form a portfolio: invest 75% in gold stock, 25% in auto stock,

scenario	prob		gold return		auto return	portfolio return
recession	0.25		13%		– 13%	
normal	0.50		7%		+ 17%	
boom	0.25		– 11%		+ 27%	
expected return		0.75 ×	+ 4%	+ 0.25 ×	+ 12%	= 6.0%

Portfolio Expected Return Example

We can treat the portfolio as a single asset

Form a portfolio: invest 75% in gold stock, 25% in auto stock,

scenario	prob	gold return	auto return	portfolio return
recession	0.25	(0.75 × + 13%	+ 0.25 × – 13%) =	+ 6.5%
normal	0.50	(0.75 × + 7%	+ 0.25 × + 17%) =	+ 9.5%
boom	0.25	(0.75 × – 11%	+ 0.25 × + 27%) =	– 1.5%
expected return		0.25 × 6.5% + 0.50 × 9.5% + 0.25 × (– 1.5%)		= 6.0%

Portfolio Expected Return and Variance

We can treat the portfolio as a single asset

Form a portfolio: invest 75% in gold stock , 25% in auto stock

scenario	prob		gold return		auto return		portfolio return
recession	0.25	(0.75 ×	+ 13%	+ 0.25 ×	– 13%	) =	+ 6.5%
normal	0.50	(0.75 ×	+ 7%	+ 0.25 ×	+ 17%	) =	+ 9.5%
boom	0.25	(0.75 ×	– 11%	+ 0.25 ×	+ 27%	) =	– 1.5%

expected return: 0.25 × 6.5% + 0.50 × 9.5% + 0.25 × (– 1.5%) = 6.0%

variance: $0.25 \times (6.5\% - 6\%)^2 + 0.50 \times (9.5\% - 6\%)^2 + 0.25 \times (-1.5\% - 6\%)^2$ = 0.002025

standard deviation: $\sqrt{0.002025}$ = 4.5%

Summary

- Investors face a trade-off between risk and expected return.
  - The greater the potential reward, the greater the risk

- Expected return and risk can be estimated from historical averages or from forecasting the probabilities of future economy states
  - from historical averages:
    - $E(R) = \frac{1}{T}(R_1 + R_2 + \cdots R_T)$
    - $Var(R) = \sigma^2 = \frac{1}{T-1}[(R_1 - \bar{R})^2 + \cdots + (R_T - \bar{R})^2]$
  - from forecasting the probabilities of future economy states
    - $E(R) = \sum_{s=1}^{S} p_s R_s$
    - $Var(R) = \sigma^2 = \sum_{s=1}^{S} p_s [R_s - E(R)]^2$

- Expected return for a portfolio:
    - $\mathrm{E}(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2) + \cdots + w_n \times E(R_n)$