THE CHINESE UNIVERSITY OF HONG KONG

Department of Mathematics

MATH1510 Calculus for Engineers (Fall 2021)

Coursework 3

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Class: MATH1510G

I acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the website http://www.cuhk.edu.hk/policy/academichonesty/

David

Signature

27-9-2021

Date

General Guidelines for Coursework Submission.

- Please go to the class indicated by your registered course code via the CUSIS system. Failure to comply will result in a **2-point deduction** of the final score.
- Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Points will only be awarded for answers with sufficient justifications.
- All questions in **Part A** along with some selected questions in **Part B** will be graded. Question(s) labeled with * are more challenging.

For internal use only:

1	2	4					
2	2	5	2				
3a	2						
3b	2						
3c					Total	10	/ 10

$x+7-16$ $\sqrt{9+7} \to 4$

Part A

1. Without using L'Hôpital's rule, evaluate the following limits. Furthermore, if the limit does not exist but diverges to $\pm\infty$, please indicate so and determine the correct sign.

   (a) $\lim_{x\to 9} \frac{\sqrt{x+7}-4}{x-9}$

   (b) $\lim_{x\to\infty} x(x-\sqrt{x^2+1})$

(a) $\lim_{x\to 9} \frac{(\sqrt{x+7}-4)(\sqrt{x+7}+4)}{(x-9)(\sqrt{x+7}+4)}$

$= \lim_{x\to 9} \frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x+7}+4)}$

$= \frac{1}{\sqrt{9+7}+4}$

$= \frac{1}{8}$ //

(b) $\lim_{x\to\infty} \frac{x(x-\sqrt{x^2+1})(x+\sqrt{x^2+1})}{x+\sqrt{x^2+1}}$

$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+1}}$

$= \lim_{x\to\infty} - \frac{1}{1+\sqrt{1+\frac{1}{x^2}}}$

$= - \frac{1}{1+1}$

$= -\frac{1}{2}$ //

2. Suppose that

$$f(x) = \begin{cases} 1-x & \text{if } x < 1; \\ 2 & \text{if } x = 1; \\ \ln x & \text{if } x > 1. \end{cases}$$

(a) Find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$.

(b) Determine if f is continuous at $x = 1$.

(a) $\lim_{x\to 1^-} f(x) = 1-1$

$= 0$ //

$\lim_{x\to 1^+} f(x) = \ln 1$

$= 0$. //

(b) $\because f(1) = 2$

& $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 0$

$\neq f(1)$

$\therefore$ No. //

Part B

3. Without using L'Hôpital's rule, evaluate the following limits. Furthermore, if the limit does not exist but diverges to $\pm\infty$, please indicate so and determine the correct sign.

   (a) $\lim_{x\to 0} \frac{\tan 4x}{\sin 2x}$

   (b) $\lim_{x\to+\infty} \left(1+\frac{1}{3x-1}\right)^{2x+1}$

   (c) $\lim_{x\to\frac{\pi}{2}} \frac{\cos 3x}{\frac{\pi}{2}-x}$

   (Hint: Let $y=\frac{\pi}{2}-x$. You may use the formula

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta)$$

(a) $\lim_{x\to 0} \frac{\sin 4x}{\cos 4x} \times \frac{1}{\sin 2x}$

$= \lim_{x\to 0} \frac{\sin 4x}{4x} \times 4x \times \frac{2x}{\sin 2x} \times \frac{1}{2x} \times \frac{1}{\cos 4x}$

$= \lim_{x\to 0} \frac{2}{\cos 4x}$

$= \frac{2}{1}$

$= 2$ //

$-\frac{2}{3}+\frac{5}{3}$

(b) $\lim_{x\to\infty} (1+\frac{1}{3x-1})^{2x+1}$

$= \lim_{x\to\infty} (1+\frac{1}{3x-1})^{(3x-1)\frac{2}{3}+\frac{5}{3}}$

$= e^{\frac{2}{3}} \times \lim_{x\to\infty} (1+\frac{1}{3x-1})^{\frac{5}{3}}$

$$= e^{\frac{2}{3}} \quad //$$

(c) $\lim_{x \to \frac{\pi}{2}} \frac{\cos 3x}{\frac{\pi}{2} - x}$

If both $\lim_{x \to c} f(x)$, $\lim_{x \to c} g(x)$ exist,
then $\lim_{x \to c} f(x) + g(x) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$.
Lecture note page 22.

Let $y = \frac{\pi}{2} - x$, then $x = \frac{\pi}{2} - y$

$$= \lim_{y \to 0} \frac{\cos\left(\frac{3\pi}{2} - 3y\right)}{y}$$

$$= \lim_{y \to 0} \frac{\cos\frac{3\pi}{2} \cos 3y + \sin\frac{3\pi}{2} \sin 3y}{y} = \lim_{y \to 0} \frac{\sin\frac{3\pi}{2} \sin 3y}{y}$$

$$= -3$$

Try not splitting

$$= \lim_{y \to 0} \frac{\cos\frac{3\pi}{2} \cos 3y}{y} + \frac{\sin\frac{3\pi}{2} \quad \sin 3y}{3y} \times 3$$

$$= \lim_{y \to 0} \frac{\cos 3y}{y} \times \cos\frac{3\pi}{2} + (-1) \times 3 \times 1$$

$\because \lim_{y \to 0} \cos\frac{3\pi}{2} \cdot \frac{\cos 3y}{y}$ does not exists

as the limit approaches the positive infinity.
(ie $+\infty$)

$\therefore$ The limit ~~does not~~ exists.

$\left(\lim_{x \to 0^-} e^{\frac{1}{x}} = \lim_{y \to -\infty} e^{y} = 0 \right)$

4. Suppose that

$$f(x) = \begin{cases} 2 + e^{\frac{1}{x}} & \text{if } x < 0; \\ ax + 2 & \text{if } 0 \le x < 1; \\ x^2 & \text{if } x \ge 1. \end{cases}$$

where a is a real number.

(a) Show that f is continuous at $x = 0$ for any real number a.

(b) Given that f is continuous at $x = 1$, find the value(s) of a.

(a) $\because \lim_{x \to 0^-} f(x) = 2 + 0$

$= 2.$

$\lim_{x \to 0^+} f(x) = a(0) + 2.$

$= 2.$

& $f(0) = a(0) + 2$

$= 2.$

$\therefore$ We have $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$\therefore f$ is continuous at $x = 0$ for any real number of a.

(b) f is continuous at $x = 1$ ~~a~~ implies $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$:

$\therefore$ We have $= a + 2 = 1^2 = 1^2$

$a = -1$ //

Sub y = 0 .

5. Show that the equation $4^x = 3^x + 2^x$ has at least one real solution.
(Hint: Consider the function $f(x) = 4^x - 3^x - 2^x$.)

Let $f(x) = 4^x - 3^x - 2^x$.

① Take $x = 1$, we have $f(1) = 4-3-2 = -1 < 0$.

② Take $x = 2$, we have $f(2) = 4^2 - 3^2 - 2^2 = 16 - 9 - 4 = 3 > 0$,

& f is continuous on $[1, 2]$

By the Intermediate Value Theorem, there exists $c \in (1, 2)$ such that $f(c) = 4^c - 3^c - 2^c = 0$,
(i.e. $4^c = 3^c + 2^c$)

$\therefore$ The equation has at least one real solution.