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香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一八至一九年度上學期科目考試

Course Examination 1st Term, 2018-19

科目編號及名稱 Course Code & Title : **MATH1510A/B/C/D/E/F/G/H/I Calculus for Engineers**

時間 Time allowed : 2 小時 hours 00 分鐘 minutes

學號 Student I.D. No : ______ 座號 Seat No. : ______

- There are a total of 200 points and 31 questions. Question 1 to 10 are multiple choice questions, question 11 to 20 are short questions and question 21 to 31 are long questions.
- Write your answers of the multiple choice questions into the given boxes on page 2 **in the question paper.**
- Write your answers of the short questions into the given boxes below the corresponding questions **in the question paper.**
- Write your answers of the long questions **in the examination answer book.** If you need extra space to answer questions, raise your hands.
- You must return your question paper and examination answer book(s) at the end of the examination.

1-10		22a		23c		26a		30a	
11-20		22b		24a		26b		30b	
21a		22c		24b		27a		30c	
21b		22d		24c		27b		30d	
21c		22e		25a		28		31a	
21d		23a		25b		29		31b	
21e		23b		25c			Total		

Multiple choice questions

Each of question 1-10 is worth 2 points. No deduction for incorrect answer.

Multiple Choice Questions
Not to be provided
P.2 - P.5

Short questions

Each of question 11-20 is worth 2 points. No justification is needed.

11. Given the parametric equations,

$$\begin{cases} x &= -1 + \cos 2t \\ y &= 1 + \sin 2t \end{cases}$$

Write down an equation relating x and y without t.

Answer:

12. Suppose

$$f(x) = \begin{cases} e^x + a, & \text{if } x \leq 0 \\ \dfrac{\sin x}{x} + 1, & \text{if } x > 0 \end{cases}$$

is continuous at $x = 0$. Then the value of a is

Answer:

13. $\dfrac{d^{1001}}{dx^{1001}} \cos 2x =$

Answer:

14. For $x \in \left(0, \dfrac{\pi}{2}\right)$, $\displaystyle\int (\sec x + \tan x)\, dx =$

Answer:

15. The average value of $f(x) = \dfrac{1}{1+x^2}$ over the interval $[0, 2]$ is

Answer:

16. If $f(x) = \int_1^{\sin x} t \ln(1+t^2)\, dt$, then $f'(x) =$

Answer:

17. Let $\mathcal{R}$ be the region bounded by the curve $f(x) = x^2$ and $y = 1$ with $x \in [0, 1]$. The volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = 1$ is given by the definite integral (Do not evaluate)

Answer:

18. Suppose $f(x)$ and $g(x)$ have Maclaurin series

$$\sum_{n=0}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \cdots$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

respectively. Then the third order Maclaurin polynomial of $f(x)g(x)$ is

Answer:

19. Let $f(x, y) = (2 + \sin x)^y$. Then $\dfrac{\partial f}{\partial x} =$

Answer:

20. $\int_0^1 \int_0^y (2xy)\, dxdy =$

Answer:

Long questions

21. (10 points) Differentiate the following functions with respect to x:

(a) $y = \sqrt[3]{x} - \dfrac{1}{\sqrt{x}}$ where $x > 0$;

(b) $y = (3x-2)^3(3-x^2)^2$;

(c) $y = \sec(x^e) + \tan(e^x)$ where $x \in (0, \ln \frac{\pi}{2})$;

(d) $y = \dfrac{\arcsin x}{x^4+1} + \sec x \csc x$ where $x \in (0,1)$;

(e) $y = \sin(\cos(\ln(x^2+1)))$.

22. (20 points) Evaluate the following integrals:

(a) $\displaystyle\int \frac{2x^2}{x^2-1}\,dx$ where $x > 1$;

(b) $\displaystyle\int \frac{1}{\sqrt{-5+6x-x^2}}\,dx$ where $x \in (3,4)$;

(c) $\displaystyle\int (x^3+1)\cos(x^4+4x)\,dx$;

(d) $\displaystyle\int \frac{1}{1-\sin x+\cos x}\,dx$ where $x \in (0,1)$;

(e) $\displaystyle\int_0^1 3x^2 \arctan(x^3)dx$.

23. (10 points) Consider the function

$$f(x) = xe^{3-x}$$

with domain $(0, \infty)$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Find all the critical point(s) of $f(x)$. For each critical point, determine whether it's a local maximum, local minimum or neither.

(c) Find all the point(s) of inflection of $f(x)$.

24. (20 points)

(a) Find the area of the region bounded by the curves

$$C_1: \quad y = \sin 2x \quad \text{with } x \in [0, \pi/2]$$
$$C_2: \quad y = \cos x \quad \text{with } x \in [0, \pi/2]$$

(b) Let $\mathcal{R}$ be the bounded region in the xy-plane bounded by $y = e^x - 1$, $y = 1$ and the y-axis. Express the volumes of the following solids as definite integrals (Do not evaluate):

(i) The solid obtained by revolving $\mathcal{R}$ about the x-axis.

(ii) The solid obtained by revolving $\mathcal{R}$ about the y-axis.

(c) Suppose the displacement of an object at time t is given by

$$s(t) = 3t^2 - t^3$$

Find the average **speed** of the object over the period $t = 0$ to $t = 3$.

25. (18 points) Given that $f(x) = \dfrac{1}{2 - 3x}$ and its Maclaurin series is

$$T_\infty(x) = \sum_{n=0}^{\infty} \frac{3^n}{2^{n+1}} x^n$$

(a) Determine the radius of convergence of $T_\infty(x)$.

(b) Find the second order Maclaurin polynomial of $g(x) = \dfrac{\cos x}{2 - 3x}$.

(c) Find the Maclaurin series of $h(x) = \dfrac{1}{(2 - 3x)^3}$. Be sure to include a formula for the coefficient of x^n in your expression.

26. (12 points)

(a) Suppose

$$f = \ln(3x^2 + y)$$

Find $\dfrac{\partial f}{\partial s} = f_s$ and $\dfrac{\partial f}{\partial t} = f_t$ if

(i) $x = s + t$, $y = s - t$;

(ii) $x = 3s - \cos t$, $y = e^t$.

(b) Find the value(s) of k such that $w = (x^2 + y^2 + z^2)^k$ satisfies

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0 \quad \text{for any } (x, y, z) \neq (0, 0, 0).$$

27. (10 points)

(a) Given that

$$\lim_{x\to -1^+} f(x) = 1, \quad \lim_{x\to -1^-} f(x) = 2, \quad \lim_{x\to 0^+} f(x) = 3, \quad \lim_{x\to 0^-} f(x) = 4,$$
$$\lim_{x\to 1^+} f(x) = 5, \quad \lim_{x\to 1^-} f(x) = 6, \quad \lim_{x\to \infty} f(x) = 7, \quad \lim_{x\to -\infty} f(x) = 8.$$

Evaluate $\lim_{x\to 0^+} f(x^3 - x)$ and $\lim_{x\to 0^+} f(x^2 - x^4)$.

(b) Suppose f is a continuous function with domain $[0,1]$ and that $0 \le f(x) \le 1$ for every x in $[0,1]$. Show that f must have a fixed point.

28. (10 points) Let

$$f(x) = \int_0^x \frac{t^2-9}{1+\cos^2(t)}\,dt$$

with domain $\mathbb{R}$. Find all the critical point(s) of $f(x)$. For each critical point, determine if it's a local maximum, local minimum or neither.

29. (10 points) Find the volume of the solid under the graph of the function

$$w(x,y) = x^2 e^{(y^2)}$$

over the region

$$\mathcal{D} = \{(x,y) \mid 0 \le x \le 1, x^3 \le y \le 1\}.$$

30. (28 points) For any integer $n \ge 1$, define

$$\begin{aligned} f_n(x) &= (x - x^n)e^{(-x^2)} \quad \text{with domain } [0,\infty) \\ F_n(x) &= \int_0^{x^2} f_n(t)\,dt \quad \text{with domain } [0,\infty) \end{aligned}$$

(a) Evaluate $\lim_{h\to 0^+} \frac{F_n(h)}{h^4}$ for any $n > 1$.

(b) Show that there exists $x^* > 0$ that attains the global maximum of $F_n(x)$ for any $n > 1$. Find x^*.

(c) Find a reduction formula for $F_n(x^*)$ and hence, find $F_3(x^*)$.

(d) Approximate $F_2(x^*)$ with an error less than 0.005.

31. (12 points)

(a) Let

$$f(x) = x^x(1-x)^{1-x}.$$

Show that $f(x) \ge f(\frac{1}{2})$ for all $0 < x < 1$.

(b) Given $a, b > 0$. Prove that

$$a^a b^b \ge \left(\frac{a+b}{2}\right)^{a+b}$$

and the equality holds if and only if $a = b$.

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Course Examination 1st Term, 2018-19

科目編號及名稱 Course Code & Title : **MATH1520A&B University Mathematics for Applications**

時間 Time allowed : **2** 小時 hours **00** 分鐘 minutes

學號 Student I.D. No : .... 座號 Seat No. : ....

- There are 10 questions. You have to answer all questions. The total score is 100.
- Please show your steps unless otherwise stated.

1. (6pts) Find the following limits or state that it does not exist. If the limit does not exist but diverges to plus or minus infinity, please indicate so, and determine the correct sign.

   (a) $\displaystyle\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{4+\sqrt{x}}-2}$

   (b) $\displaystyle\lim_{x\to +\infty} \frac{3x+2^x}{2x+3^x}$

2. (6pts) Find the (first) derivative of the following functions:

   (a) $y = \sqrt{\dfrac{1+x}{1-x}}$

   (b) $f(x) = \displaystyle\int_{1/x}^{x^2} \frac{1}{1+e^{t^2}}\, dt$

3. (24pts)

   (a) Evaluate the following integrals.

   i. $\int \left(\frac{5x^2 + 2\sqrt{x} - 1}{\sqrt{x}} \right) dx$ ii. $\int \frac{8x^3}{4x^2 - 4x + 1} dx$

   (b) Evaluate

   i. $\int_0^9 e^{\sqrt{x}} \, dx$

   ii. $\int_1^4 \sqrt{x} \ln x \, dx$

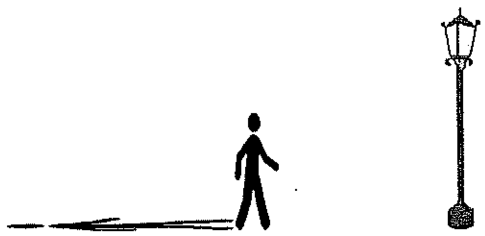
   iii. $\int_0^6 |2x^2 - 7x - 4| \, dx$

   iv. $\int_e^{+\infty} \frac{c}{x(\ln x)^{c+1}} dx$, where c is a positive constant

4. (8pts) Sketch the region enclosed by the following curves and find its area.

$$y = 16 - x^2, \quad y = -4x + 20, \quad \text{and} \quad x-\text{axis}.$$

5. (8pts) A street light is on top of a 12 feet pole. Mr. Bean, who is 6 feet tall, walks toward the pole at a rate of 5 feet per second along a straight path. At what speed is the tip of Mr. Bean's shadow moving from the base of the pole when he is 10 feet away from the pole?



6. (10pts) Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r.

7. **(8pts)** Find an explicit particular solution of the differential equation

$$x\frac{dy}{dx} - y = x\ln x, \quad y(1) = 4.$$

8. **(10pts)** A tank currently holds 200 gallons of brine that contains 4 pounds of salt per gallon. Pure water flows into the tank at the rate of 5 gallons per minute, while the mixture, kept uniform by stirring, runs out of the tank at the same rate.

   (a) Set up and solve a differential equation for the amount of salt $S(t)$ in tank at time t.

   (b) How much salt is in the tank at the end of 100 minutes?

9. **(10pts)** The amount of time (in minutes) that a person spends reading the editorial page of the newspaper is a random variable with the density function

$$f(x) = \begin{cases} \dfrac{k}{(x+1)^2}, & \text{if } 0 \le x \le 10 \\ 0, & \text{if } x < 0 \text{ or } x > 10 \end{cases}$$

where k is a positive constant.

   (a) Determine the value of the k.

   (b) Find the probability that a person spends at least 3 minutes reading the editorial page.

   (c) Find the average time spend reading the editorial page.

10. **(10pts)**

   (a) Find an integer n such that

$$n\int_0^1 xf''(3x)\,dx = \int_0^3 tf''(t)\,dt.$$

   (b) Compute

$$\int_0^1 xf''(3x)\,dx,$$

   given that $f(0) = 1, f(3) = 5, f'(3) = 7.$

——————— The End ———————