2021R1-MATH1510 HW 2

Cho Kit CHAN

TOTAL POINTS

20 / 20

QUESTION 1

1 Q1 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 2

2 Q2 **4 / 4**

✓ **- 0 pts** **Click here to replace this description.**

QUESTION 3

3 Q3 **4 / 4**

✓ **- 0 pts** Correct

QUESTION 4

4 Q5 **4 / 4**

(a)

✓ **- 0 pts** Correct

(b)

✓ **- 0 pts** Correct

QUESTION 5

5 Q7 **4 / 4**

✓ **- 0 pts** Correct

Please show more steps next time.

Part A:

1. Let $f(x) = \sin(2x + \pi)$. Use definition (first principle) to find $f'(x)$ for any $x \in \mathbb{R}$.

By definition of first principle, we have:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Let $f(x) = \sin(2x+\pi)$, we have:

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sin(2x + 2\Delta x + \pi) - \sin(2x+\pi)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\cos(2x+\Delta x+\pi)\sin(\Delta x)}{\Delta x}$$

$$= 1 \times \lim_{\Delta x \to 0} 2\cos(2x + \Delta x + \pi)$$

$$= 1 \times 2\cos(2x+\pi)$$

$$= 2\cos(2x+\pi) \;//$$

1 Q1 **4 / 4**

✓ **- 0 pts** Correct

2. Let $\mathcal{C}$ be the curve defined by the equation $xy = \ln x + y^3$. Given that $A = (1, 0)$ is a point on $\mathcal{C}$,

   (a) Find $\dfrac{dy}{dx}$ in terms of x and y.

   (b) Find $\left.\dfrac{d^2y}{dx^2}\right|_A$.

(a)

$$xy = \ln x + y^3$$

$$xy - y^3 = \ln x$$

$$y + \frac{dy}{dx}(x - 3y^2) = \frac{1}{x}$$

$$\frac{dy}{dx}(x - 3y^2) = \frac{1 - xy}{x}$$

$$\frac{dy}{dx} = \frac{1 - xy}{x(x - 3y^2)} \quad //$$

(b)

$$\frac{dy}{dx} = \frac{1}{x(x-3y^2)} - \frac{y}{x - 3y^2}$$

$$\left.\frac{dy}{dx}\right|_A = \frac{1}{(1-0)} - \frac{0}{1-0}$$

$$= 1$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{-(x-3y^2) - x\left[1 - \frac{dy}{dx}(6y)\right]}{x^2(x-3y^2)^2}$$

$$- \frac{\frac{dy}{dx}(1)(x-3y^2) - y\left[1 - \frac{dy}{dx}(6y)\right]}{(x-3y^2)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_A = \frac{-(1) - (1-0)}{1^2(1-0)^2} - \frac{1-0}{(1-0)^2}$$

$$= -2 - 1$$

$$= -3 \quad //$$

2 Q2 4 / 4

✓ - 0 pts Click here to replace this description.

Part B:

3. Determine the point(s) of discontinuity of the function:

$$f(x) = \begin{cases} x^2 + 3x - 1, & \text{if } x \leq 0, \\ \dfrac{\sin x}{x}, & \text{if } 0 < x \leq \pi, \\ \cos x + 1, & \text{if } \pi < x. \end{cases}$$

$\because \lim_{x \to 0^-} f(x) = 0^2 + 3(0) - 1$

$= -1$

$f(0) = -1$

$\lim_{x \to 0^+} f(x) = 1 \neq \lim_{x \to 0^-} f(x)$ & $f(0)$.

$\therefore$ The function is discontinuous when $x = 0$.

$\because \lim_{x \to \pi^-} f(x) = \dfrac{\sin \pi}{\pi}$

$= 0$

$f(\pi) = 0$

$\lim_{x \to \pi^+} f(x) = \cos \pi + 1$

$= 0$

$\because \lim_{x \to \pi^-} f(x) = f(\pi) = \lim_{x \to \pi^+} f(x)$,

$\therefore$ The function is continuous when $x = \pi$.

$\therefore$ There is one point of discontinuity when $x = 0$.

3 Q3 **4 / 4**

✓ **- 0 pts** Correct

5. Find $\frac{dy}{dx}$ by logarithmic differentiation if

(a) $y = \dfrac{(x^2+5)^4}{(e^{-x}+2)\sqrt{x^4+1}}$;

(b) $y = x^{x+1}$, for $x > 0$.

(a) $\ln y = 4\ln(x^2+5) - \ln(e^{-x}+2)$

$- \frac{1}{2}\ln(x^4+1)$

$\frac{d}{dx}(\ln y) = 4 \times \frac{1}{x^2+5}(2x) - \frac{1}{e^{-x}+2}(-e^{-x})$

$- \frac{1}{2} \times \frac{1}{x^4+1}(4x^3)$

$\frac{dy}{dx}\left(\frac{1}{y}\right) = \frac{8x}{x^2+5} + \frac{e^{-x}}{e^{-x}+2} - \frac{2x^3}{x^4+1}$

$\frac{dy}{dx} = \left(\frac{8x}{x^2+5} + \frac{e^{-x}}{e^{-x}+2} - \frac{2x^3}{x^4+1}\right)\left[\frac{(x^2+5)^4}{(e^{-x}+2)\sqrt{x^4+1}}\right]$ //

(b) $y = x^{x+1}$

$\ln y = (x+1)\ln x$

$\frac{dy}{dx}\left(\frac{1}{y}\right) = \ln x + (x+1)\frac{1}{x}$

$\frac{dy}{dx} = (x\ln x + x + 1)\, x^x$ //

4 Q5 **4 / 4**

(a)

✓ **- 0 pts** **Correct**

(b)

✓ **- 0 pts** **Correct**

7. * Let u, v be functions of x. The first order derivative of uv can be obtained by the product rule:

$$(uv)' = u'v + uv'.$$

The general formula for n-th order derivative of uv was derived by the German mathematician Gottfried Wilhelm Leibniz:

$$(uv)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} \underline{u^{(k)}} \underline{v^{(n-k)}},$$

where $\binom{n}{k} = C_k^n = \dfrac{n!}{k!(n-k)!}$, the symbol $u^{(k)} = \dfrac{d^k u}{dx^k}$ means the k-th order derivative of u and $u^{(0)} = u$.

By Leibniz's formula, compute $f^{(100)}(x)$ if

$$f(x) = (2x^3 + 5x^2 - x + 3)\cos x.$$

$$f^{(100)}(x) = \sum_{k=0}^{100} C_k^{100} (2x^3+5x^2-x+3)^{(k)} (\cos x)^{(100-k)}$$

$$= (2x^3+5x^2-x+3)(\cos x)$$
$$+ 100(6x^2+10x-1)(\sin x)$$
$$+ 4950(12x+10)(-\cos x)$$
$$+ 161700(12)(-\sin x)$$
$$+ 3921225(0)(\cos x)$$
$$+ \ldots$$
$$= \cos x(2x^3+5x^2-59401x-49497)$$
$$+ \sin x(600x^2+1000x-1940500)$$

5 Q7 **4 / 4**

✓ **- 0 pts** **Correct**

Please show more steps next time.