Lecture 8

Portfolio Diversification and Security Market Line

Instructor: Prof. Chen (Alison) Yao
CUHK Business School

Lecture Outline

- Portfolio Diversification
- Risk: Systematic and Unsystematic
- Measuring Risk-Expected Return Relationship
  - The Security Market Line (SML)
  - The Capital Asset Pricing Model (CAPM)

Portfolio Expected Return and Variance

Expected return and **variance** for a portfolio can be computed by forecasting the probabilities of future economy states

- $E(R) = \sum_{s=1}^{S} p_s R_s$
- $Var(R) = \sigma^2 = \sum_{s=1}^{S} p_s [R_s - E(R)]^2$
- where there are *S* possible states of the economy, *s=1,2,…,S*
- p_s is the probability that state *s* occurs
- R_s is the portfolio return in state *s*

Portfolio Expected Return and Variance

Expected return for a portfolio can also be computed as:

$$\mathrm{E}\left(R_p\right) = w_1 \times E(R_1) + w_2 \times E(R_2) + \cdots + w_n \times E(R_n)$$

- n is the total number of assets in the portfolio
- w_i is the portfolio weight of asset *i* (percentage of investment in asset *i*)
- $E(R_i)$ is the expected return of asset *i*
- $w_1 + w_2 + \cdots + w_n = 1$

Question: Can we compute portfolio variance as follows?

$$\mathrm{Var}\left(R_p\right) = w_1 \times \sigma_1^2 + w_2 \times \sigma_2^2 + \cdots + w_n \times \sigma_n^2$$

or

$$\mathrm{Var}\left(R_p\right) = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + \cdots + w_n^2 \times \sigma_n^2$$

or

$$\mathrm{Var}\left(R_p\right) = [w_1 \times \sigma_1 + w_2 \times \sigma_2 + \cdots + w_n \times \sigma_n]^2$$

- σ_i^2 is the variance and σ_i is the standard deviation for asset *i*

Portfolio Expected Return and Variance

Expected return for a portfolio can also be computed as:

$$\mathrm{E}(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2) + \cdots + w_n \times E(R_n)$$

- n is the total number of assets in the portfolio
- w_i is the portfolio weight of asset *i* (percentage of investment in asset *i*)
- $E(R_i)$ is the expected return of asset *i*
- $w_1 + w_2 + \cdots + w_n = 1$

Question: Can we compute portfolio variance as follows?

$$\mathrm{Var}(R_p) = w_1 \times \sigma_1^2 + w_2 \times \sigma_2^2 + \cdots + w_n \times \sigma_n^2$$

or

$$\mathrm{Var}(R_p) = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + \cdots + w_n^2 \times \sigma_n^2$$

or

$$\mathrm{Var}(R_p) = [w_1 \times \sigma_1 + w_2 \times \sigma_2 + \cdots + w_n \times \sigma_n]^2$$

- σ_i^2 is the variance and σ_i is the standard deviation for asset *i*

Portfolio Variance

Expected return of a two-asset portfolio:

$$\mathrm{E}(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2)$$

Variance of a two-asset portfolio:

$$\mathrm{Var}(R_p) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}$$

- $\rho_{1,2}$ is the correlation between asset 1 and 2
- w_i is the portfolio weight of asset *i* (percentage of investment in asset *i*)

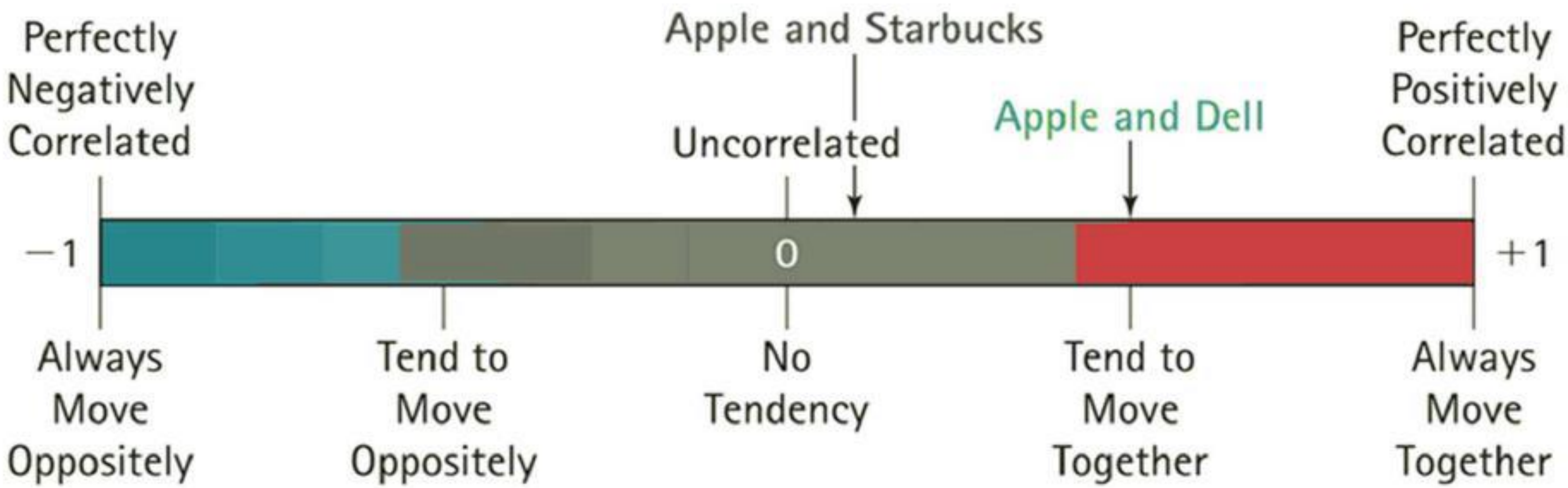
Correlation measure the extent to which returns on different assets tend to move together. It is defined as

$$\rho_{1,2} = \frac{Cov(R_1, R_2)}{\sigma_1\ \sigma_2}$$

where $Cov(R_1, R_2) = \sigma_{1,2} = \sum_{s=1}^{S} p_s[(R_{1,s} - E[R_1])(R_{2,s} - E[R_2])]$ is the **covariance** between asset 1 return and asset 2 return

Correlation

- $\rho_{1,2}$ is the correlation between asset *1* and asset *2*
- $-1 \leq \rho_{1,2} \leq +1$
  - if $\rho_{1,2} = +1$, returns always move in the same direction (perfect positive correlation)
  - if $\rho_{1,2} = -1$, returns always move in opposite direction (perfect negative correlation)



Portfolio Variance Example

Form a portfolio: invest 75% in gold stock, 25% in auto stock

			gold stock			auto stock
scenario	prob		return			return
recession	0.25	×	+ 13%	=	+ 3.25%	– 13%
normal	0.50	×	+ 7%	=	+ 3.50%	+ 17%
boom	0.25	×	– 11%	=	– 2.75%	+ 27%
				=	+ 4.00%	
expected return			+ 4%			+ 12%

Portfolio Variance Example

Example Cont'd

Form a portfolio: invest 75% in gold stock, 25% in auto stock

scenario	prob		gold stock return		auto stock return
recession	0.25	×	(+ 13% – 4%)^2	= 0.002025	– 13%
normal	0.50	×	(+ 7% – 4%)^2	= 0.000450	+ 17%
boom	0.25	×	(– 11% – 4%)^2	= 0.005625	+ 27%
				= 0.008100	
expected return			+ 4%		+ 12%
variance			0.0081		0.0225
St. deviation			$\sqrt{0.0081}$	= 0.09	0.15

Portfolio Variance Example

Example Cont'd

Form a portfolio: invest 75% in gold stock, 25% in auto stock

		gold stock	auto stock
scenario	prob	return	return
recession	0.25	+ 13%	– 13%
normal	0.50	+ 7%	+ 17%
boom	0.25	– 11%	+ 27%
expected return		+ 4%	+ 12%

covariance

$$0.25 \times (13\% - 4\%) \times (-13\% - 12\%) + 0.50 \times (7\% - 4\%) \times (17\% - 12\%) + 0.25 \times (-11\% - 4\%) \times (27\% - 12\%) = -0.0105$$

correlation

$$-0.0105/(0.09 \times 0.15) = -0.7778$$

Portfolio Variance Example

Example Cont'd

Form a portfolio: invest 75% in gold stock, 25% in auto stock

		gold stock	auto stock
scenario	prob	return	return
recession	0.25	+ 13%	– 13%
normal	0.50	+ 7%	+ 17%
boom	0.25	– 11%	+ 27%

variance

$$0.75^2 \times 0.0081 + 0.25^2 \times 0.0225 + 2 \times 0.75 \times 0.25 \times 0.09 \times 0.15 \times (-0.7778) = 0.002025$$

Portfolio Formed by Different Weights

	Portfolio Weights		Portfolio Expected Return	Portfolio Standard Deviation (σ_p)	$w_1\sigma_1 + w_2\sigma_2$
	Gold (w_1)	Auto (w_2)			
A	0%	100%	12%	15.0%	15.0%
B	20%	80%	10.4%	10.7%	13.8%
C	40%	60%	8.8%	6.6%	12.6%
D	60%	40%	7.2%	3.8%	11.4%
E	80%	20%	5.6%	5.2%	10.2%
F	100%	0%	4%	9.0%	9.0%

- $\mathrm{Var}(R_p) = \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2} \leq (w_1\sigma_1 + w_2\sigma_2)^2$ as $-1 \leq \rho_{1,2} \leq +1$
- $\mathrm{Std}(R_p) = \sigma_p \leq w_1\sigma_1 + w_2\sigma_2$
- The risk of the portfolio is lower than the weighted average of individual stock risk

Diversification

- **Diversification** works because losses in one asset may be offset by gains in some of the other assets

- This is only possible if the returns on the assets do not always move together in the same direction (i.e.the correlation is less than perfect ($\rho_{1,2} < 1$)). Hence, the portfolio standard deviation is less than the weighted average.

- The lower the correlation between assets in our portfolio, the more we can *reduce* our risk

- This is the effect of *diversification*

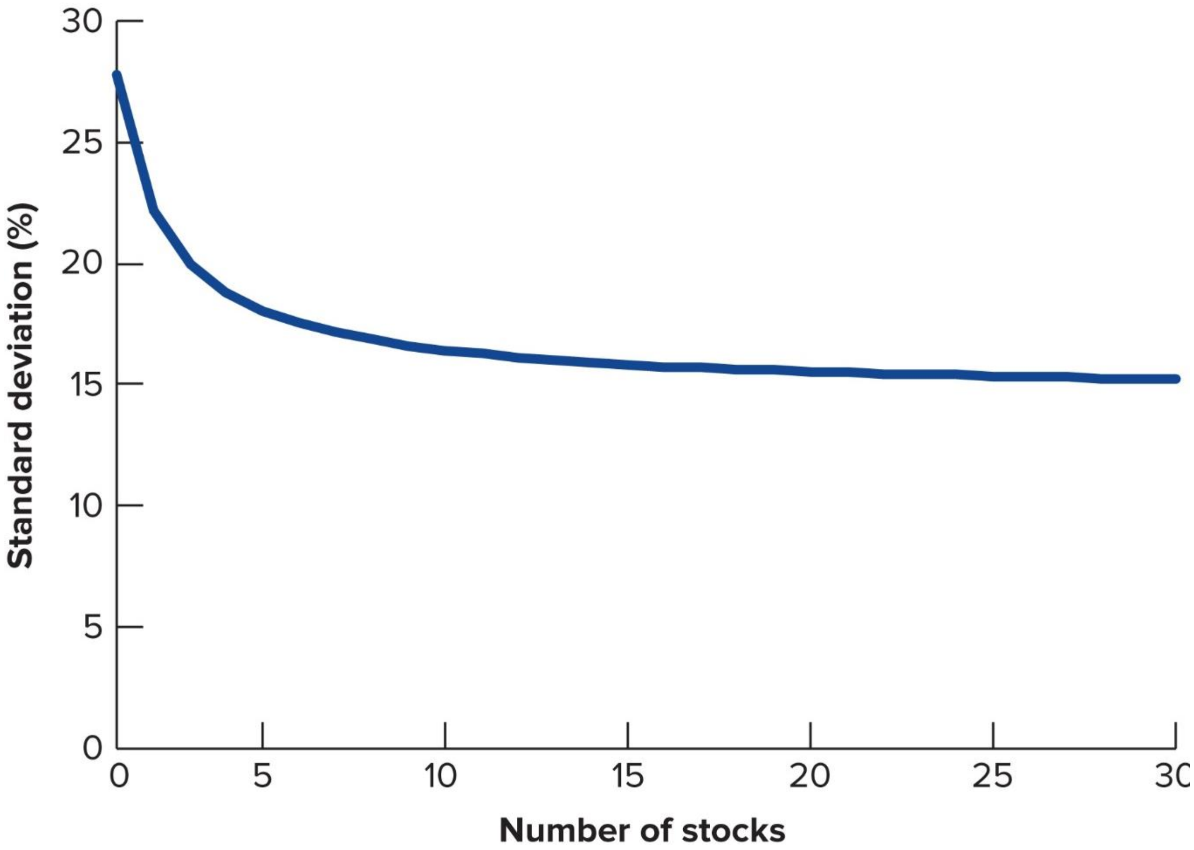
Lecture Outline

- Portfolio Diversification

- Risk: Systematic and Unsystematic

- Measuring Risk-Expected Return Relationship
  - The Security Market Line (SML)
  - The Capital Asset Pricing Model (CAPM)

Diversification and Number of Stocks



The standard deviation declines as the number of securities is increased

Some of the riskiness associated with individual assets can be eliminated by forming portfolios

There is a minimum level of risk that cannot be eliminated simply by diversifying. This minimum level is ‘non-diversifiable risk’

Systematic and Unsystematic Risk

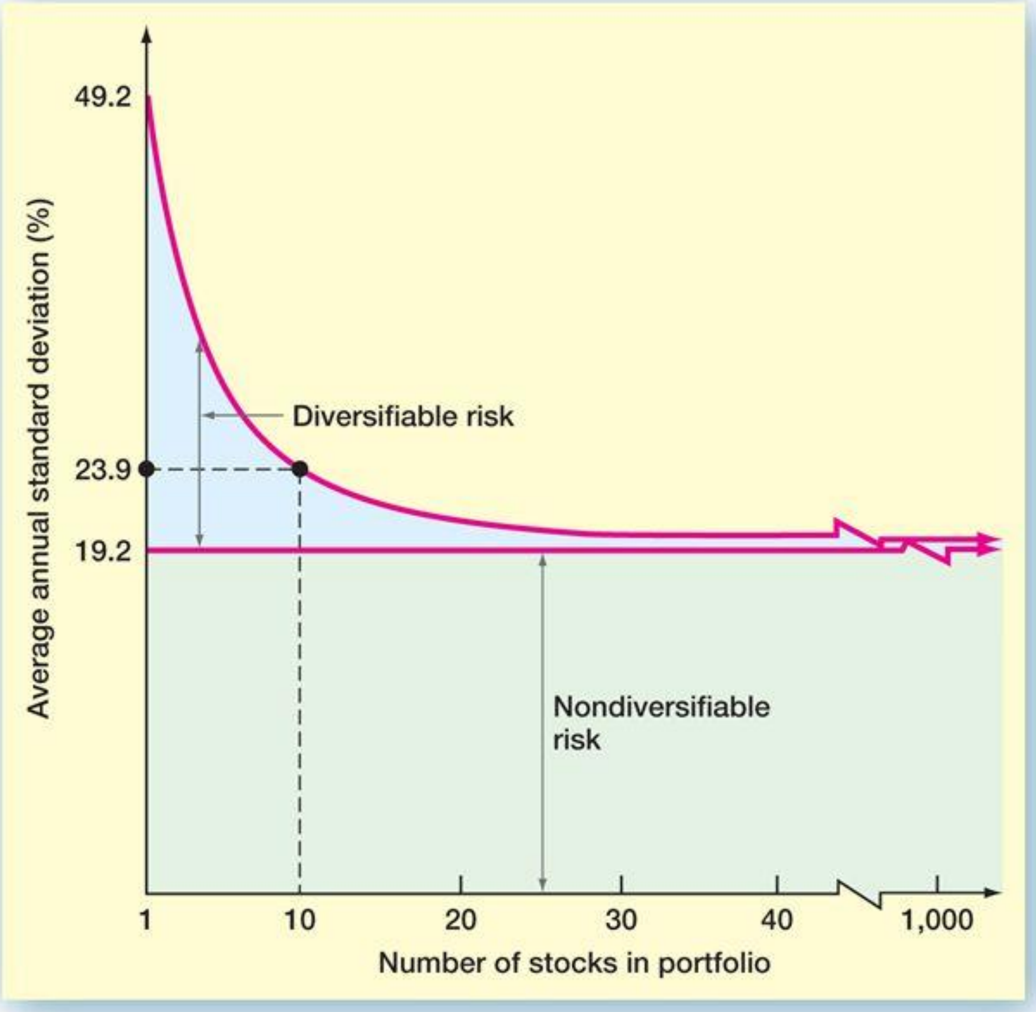
- There are two types of risk associated with each security.

- **Systematic Risk** (market risk, non-diversifiable risk):
  - a risk that influences a large number of assets
  - E.g., changes in GDP, recessions, inflation, etc.

- **Unsystematic Risk**(unique risk, asset-specific risk, diversifiable risk):
  - a risk that affects at most a small number of assets.
  - E.g., labor strikes, part shortages, etc.

- **Total risk = Systematic risk + Unsystematic risk**

Portfolio Diversification



Total Risk

- Total risk = systematic risk + unsystematic risk
- The standard deviation of returns is a measure of total risk
- For well-diversified portfolios, unsystematic risk is very small
- Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk

The Systematic Risk Principle

- There is a reward for bearing risk
- There is not a reward for bearing risk unnecessarily
- The expected return on a risky asset depends only on that asset's systematic risk since unsystematic risk can be diversified away

Measuring Systematic Risk

- How do we measure systematic risk?
  - We use the beta coefficient (β)
  - Beta coefficient (β): the amount of systematic risk present in a particular risky asset *i* relative to that in the **market portfolio**.

- What is the market portfolio?
  - The market portfolio is a bundle of investments that includes every type of asset available in the world financial market
    - S&P500 index is commonly used as a proxy for the market portfolio
  - The market portfolio is completely diversified, it has only systematic risk, and not unsystematic risk
  - The market portfolio has a beta of $\beta_{\mathrm{M}} = 1$

What Does Beta Tell Us?

- A beta of 1 implies the asset has the same systematic risk as the overall market

- A beta < 1 implies the asset has less systematic risk than the overall market

- A beta > 1 implies the asset has more systematic risk than the overall market

Note:

- Beta is defined as follows:

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)} = \frac{\sigma_i}{\sigma_M}\rho_{i,M}$$

- Beta of risk-free assets = 0

Example for Company Beta

Company	Ticker	Industry	Equity Beta
General Mills	GIS	Packaged Foods	0.20
Consolidated Edison	ED	Utilities	0.28
The Hershey Company	HSY	Packaged Foods	0.28
Abbott Laboratories	ABT	Pharmaceuticals	0.31
Newmont Mining	NEM	Gold	0.32
Wal-Mart Stores	WMT	Superstores	0.35
Nike	NKE	Footwear	0.91
Microsoft	MSFT	Systems Software	1.01
Southwest Airlines	LUV	Airlines	1.09
Intel	INTC	Semiconductors	1.09
Whole Foods Market	WFM	Food Retail	1.10
Foot Locker	FL	Apparel Retail	1.11
Advanced Micro Devices	AMD	Semiconductors	2.24
Ford Motor	F	Automobile Manufacturers	2.38
Sotheby's	BID	Auction Services	2.39
Wynn Resorts Ltd.	WYNN	Casinos and Gaming	2.41
United States Steel	X	Steel	2.52
Saks	SKS	Department Stores	2.57

Source: CapitalIQ

Note: Betas with Respect to the S&P 500 for Individual Stocks (based on monthly data for 2007–2012)

Total vs. Systematic Risk

Consider the following information:

	Standard Deviation	Beta
Security C	20%	1.25
Security K	30%	0.95

- Which security has more total risk?
- Which security has more systematic risk?
- Which security should have the higher expected return?

Beta and Risk Premium

- Remember that

$$\text{Risk premium of a risky asset } i = E(R_i) - R_f$$

- The higher the beta, the greater the risk premium should be

- Can we define the relationship between the risk premium and beta so that we can estimate the expected return?

Lecture Outline

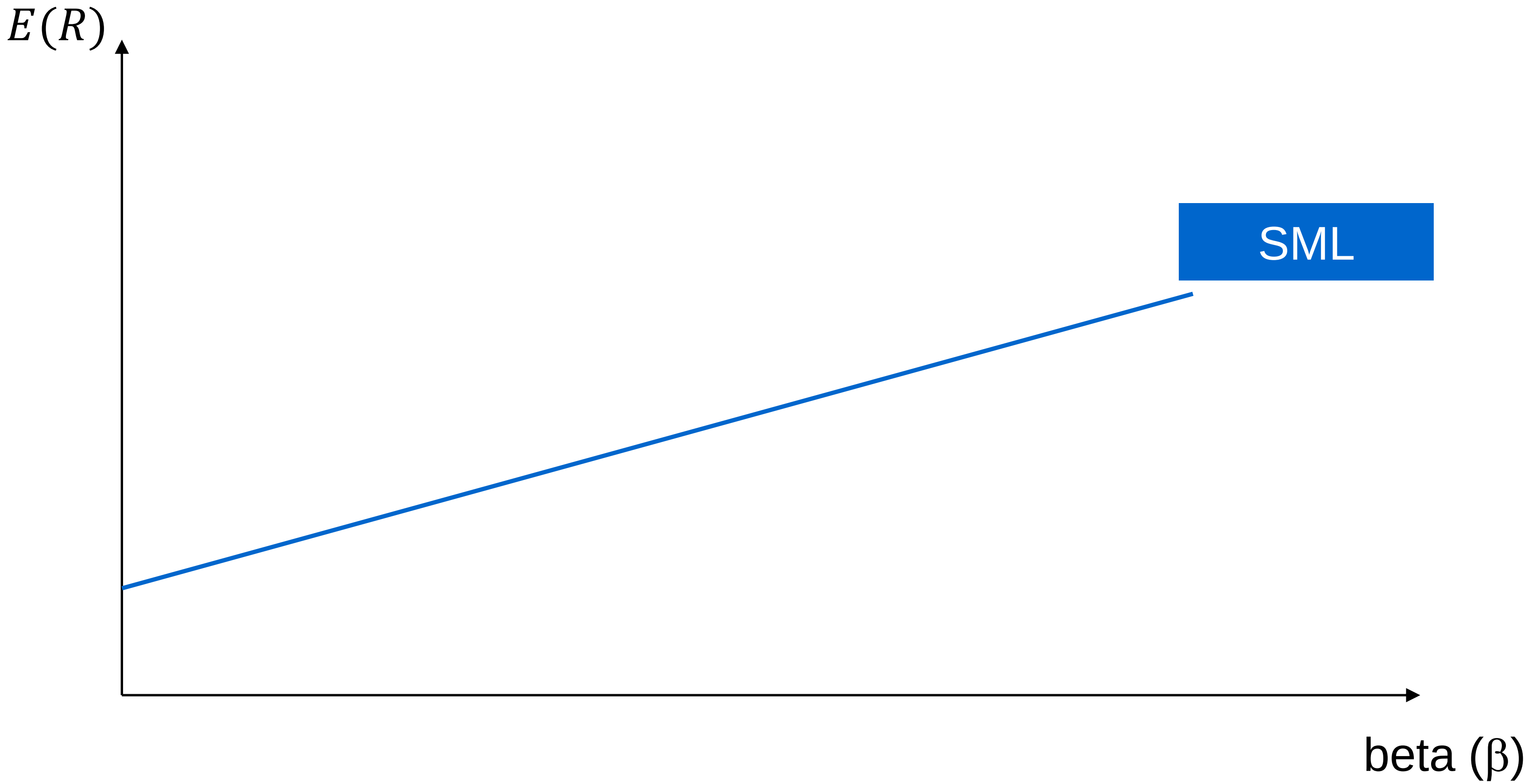
- Portfolio Diversification

- Risk: Systematic and Unsystematic

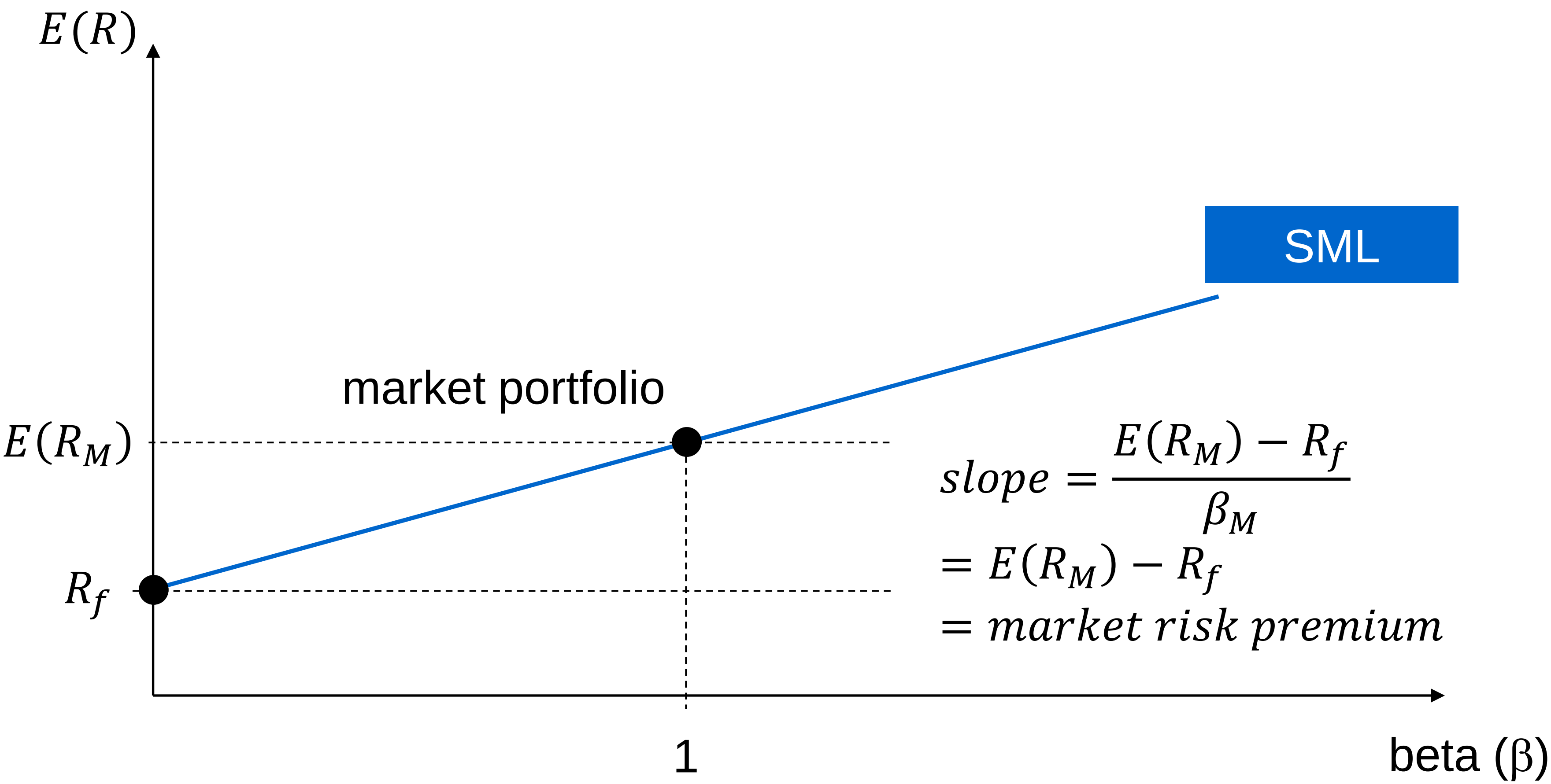
- Measuring Risk-Expected Return Relationship
  - The Security Market Line (SML)
  - The Capital Asset Pricing Model (CAPM)

Security Market Line

Security Market Line (SML): a positively sloped straight line displaying the linear risk-return relationship between the beta of a security and its expected return. SML directly links beta to the expected return.



Security Market Line



$$\boldsymbol{E(R_i) = R_f + \beta_i \times (E(R_M) - R_f)}$$

Reward-to-Risk Ratio

- The *reward-to-risk ratio* is the slope of the SML

$$slope = \frac{E(R_M) - R_f}{\beta_M} = E(R_M) - R_f = market\ risk\ premium$$

- For every unit of beta (systematic risk), the required additional return over the risk-free rate is $E(R_M) - R_f$.

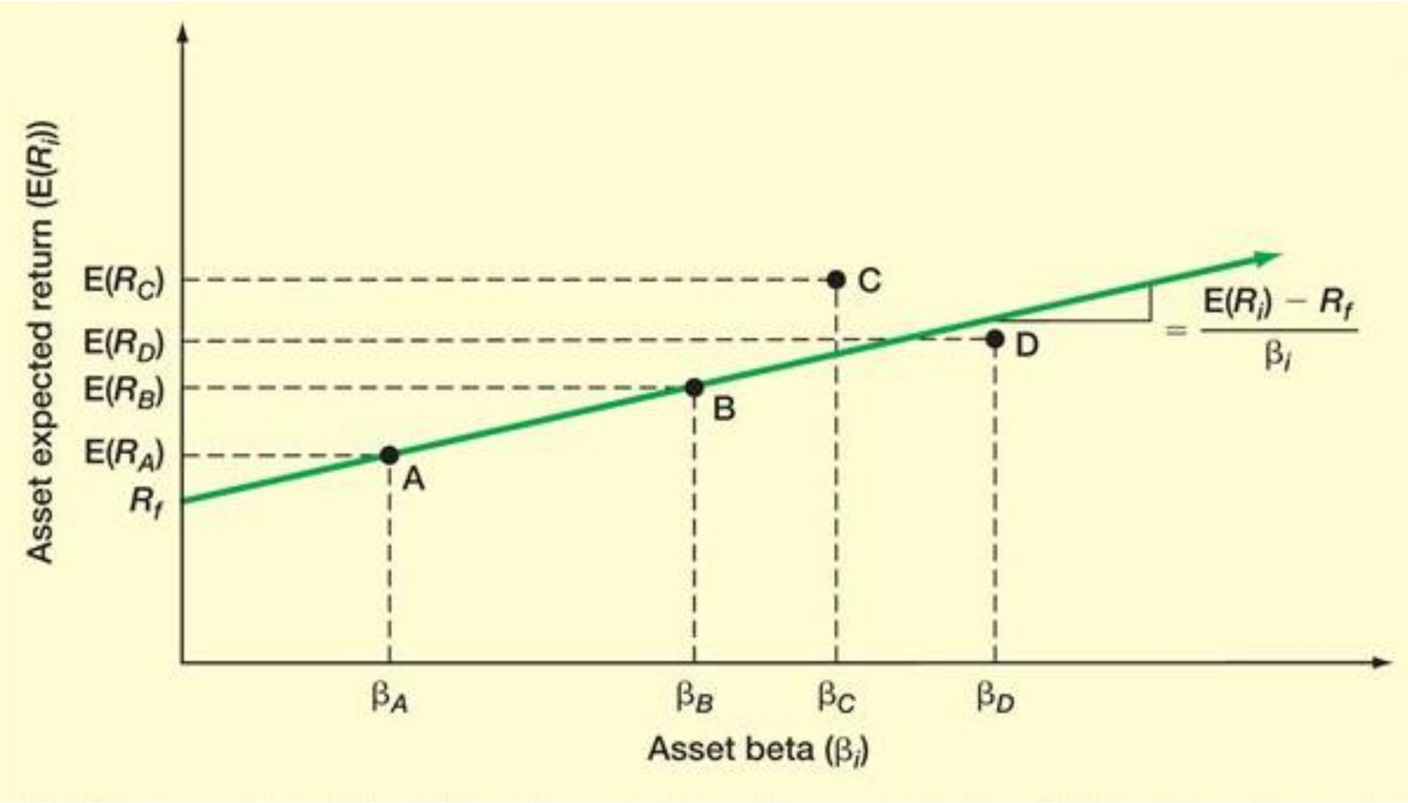
- **In equilibrium, all assets and portfolios must have the *same* reward-to-risk ratio:**

$$\frac{E(R_\mathrm{A}) - R_f}{\beta_\mathrm{A}} = \frac{E(R_\mathrm{B}) - R_f}{\beta_\mathrm{B}}$$

where *A*, *B* are any two assets on SML

Fundamental Result

In equilibrium, all assets must have the same reward-to-risk relation => they must plot on the same line



Question: Can C and D sustain in equilibrium?

Overvalued vs. Undervalued

Suppose you are observing the following situation:

Security	Beta	Expected return
SWMS Co.	1.3	14%
Insec Co.	0.8	10%

The risk-free rate is currently 6 percent. Is one of the two securities overvalued relative to the other?

Solution:

- We compute the reward-to-risk ratio for both:

  For SWMS, reward-to-risk ratio is $\frac{14\%-6\%}{1.3} = 6.15\%$

  For Insec, reward-to-risk ratio is $\frac{10\%-6\%}{0.8} = 5\%$

- Insec offers an insufficient expected return for its level of risk, at least relative to SWMS.

Overvalued vs. Undervalued

- An asset is said to be *overvalued* if its price is too *high*, given its expected return and risk.

- An asset is said to be *undervalued* if its price is too *low*, given its expected return and risk.

- Because Insec’s expected return is too low, its price is too high. Insec is overvalued relative to SWMS, and we would expect to see its price fall relative to SWMS’s.

- SWMS is undervalued relative to Insec.

Lecture Outline

- Portfolio Diversification
- Risk: Systematic and Unsystematic
- Measuring Risk-Expected Return Relationship
  - The Security Market Line (SML)
  - The Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model

- The **capital asset pricing model (CAPM)** defines the relationship between risk and return

$$E(R_i) = R_f + \beta_i \times (E(R_M) - R_f)$$

- If we know an asset's systematic risk, we can use the CAPM to determine its expected return

Factors Affecting Expected Return

CAPM: $E(R_i) = R_f + \beta_i \times (E(R_M) - R_f)$

- The *pure time value of money*: measured by the risk-free rate, R_f, this is the reward for merely waiting for your money, without taking any risk.

- The *reward for bearing systematic risk*: measured by the market risk premium, $E(R_M) - R_f$, this is the reward the market offers for bearing an average amount of systematic risk in addition to waiting.

- The *amount of systematic risk*: measured by β_i, this is the amount of systematic risk present in a particular asset or portfolio, relative to that in the market portfolio.

CAPM Example

A stock has a beta of 1.15, the expected return on the market is 10.3%, and the risk-free rate is 3.8%.

What must the expected return on this stock be?

Solution:

- Recall the CAPM

$$E(R_i) = R_f + \beta_i \times (E(R_M) - R_f)$$

- Substituting the values we are given, we find:

$$\begin{aligned} E(R_i) &= 3.8\% + 1.15 \times (10.3\% - 3.8\%) \\ &= 11.28\% \end{aligned}$$

CAPM Example

Suppose you observe the following situation:

Security	Beta	Expected return
Pete Co.	1.21	10.79%
Repete Co.	0.83	8.43%

Assume these securities are correctly priced. Based on the CAPM, what is the expected return on the market? What is the risk-free rate?

Solution:

- Recall: In equilibrium, all assets and portfolios must have the *same* reward-to-risk ratio:

$$\frac{E(R_{\mathrm{A}}) - R_f}{\beta_{\mathrm{A}}} = \frac{E(R_{\mathrm{B}}) - R_f}{\beta_{\mathrm{B}}}$$

$$\frac{10.79\% - R_f}{1.21} = \frac{8.43\% - R_f}{0.83}$$

$$R_f = 3.28\%$$

- Using CAPM $10.79\% = 3.28\% + 1.21 \times (E(R_M) - 3.28\%)$

$$E(R_M) = 9.49\%$$

Negative Beta Assets

Consider an asset with negative beta

- the returns on this asset are **negatively** correlated with the returns on the market
  - in a bull market, the asset performs badly
  - but in a bear market, the asset performs well
- if the CAPM holds, the asset's expected return should be less than R_f or could even be negative

Why hold such an asset?

- as an **insurance:**
  - a negative-beta asset generates high returns when the market is down. It can function as a hedge against the business cycle.
  - a classic example is gold:
    - average annual return on gold over the last 40 years was 2% lower than the risk-free rate

Portfolio Beta

Beta is additive:

- consider a portfolio with a fraction w_i (weight) of money invested in asset *i*
- what is the portfolio's beta?

$$\beta_p = \sum_{i=i}^{n} w_i \times \beta_i$$

- a simple weighted average of individual asset betas

A **portfolio** containing many different assets all with the same betas

- still has **the same beta** but
- has **less total risk** than any of the assets in the portfolio

Why?

- In the portfolio all unsystematic risks are diversified away

Portfolio Beta Example

Suppose you form a portfolio out of the following five stocks. What is the beta of this portfolio?

Stock	Proportions	Beta
A	20%	1.6
B	25%	1.2
C	10%	1.0
D	30%	0.9
E	15%	0.8

Solution:

$$\beta_p = \sum_{i=i}^{n} w_i \times \beta_i$$

$$= 0.20 \times 1.6 + 0.25 \times 1.2 + 0.10 \times 1.0 + 0.30 \times 0.9 + 0.15 \times 0.8$$

$$= 1.11$$

Summary

- Diversification lowers investment risks

- In equilibrium, all assets must have the same reward-to-risk relation

$$\frac{E(R_{\mathrm{A}}) - R_f}{\beta_{\mathrm{A}}} = \frac{E(R_{\mathrm{B}}) - R_f}{\beta_{\mathrm{B}}}$$

- The capital asset pricing model (CAPM) defines the relationship between risk and return

$$E(R_i) = R_f + \beta_i \times (E(R_M) - R_f)$$