Homework Assignment 1
Fall 2021

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Question 1

(a) The cost of a new automobile is $25,000. If the interest rate is 6%, how much would you have to set aside now to provide this sum in five years? Assume interests are compounded annually and the cost of the car does not appreciate over those five years.

(b) How much will you have at the end of 30 years if you invest $200 today at 12% annually compounded? How much will you have if you invested at 12% continuously compounded?

Question 2

565000

The Murphy family are planning to buy a new home. The house costs $800,000, but the family have $235,000 in savings that they can use as down-payment. The remainder is to be financed by a mortgage. Their bank offers a 30-year loan, with fixed interest at 9.6% (per annum), with fixed monthly payments.

(a) What are the monthly payments that the Murphy's will have to make so that the entire mortgage is paid off in 30 years?

(b) If the Murphy's can afford to pay $4,500 per month, can they afford the house? If not, how much additional down-payment would they require?

(c) Now suppose the family pays $5,600 per month instead. Can they pay off their mortgage in 20 years without increasing their down-payment?

Question 3

The Murphy family, whose acquaintance we have made earlier, are setting up a retirement plan. They will make fixed monthly contributions to a pension fund, until Mr and Mrs Murphy retire 30 years from now. After retirement, the family are planning to withdraw a fixed amount "C" each month for the next 20 years. Assume a fixed 7.2% annual discount rate.

(a) If the Murphy's plan to withdraw $2,500 each month, how much would they have to pay into the fund each month before they retire?

(b) How much can the family withdraw each month after retirement, if they can only afford to contribute $300 each month to the fund now?

Q1. (a) Amount needed:

$$\$25000 \div (1+6\%)^5$$

$$= \$18681.45 \mathbin{/\!/}$$

(b) For 12% annually compounded:

$$\$200 \times (1+12\%)^{30}$$

$$= \$5991.98 \mathbin{/\!/}$$

For 12% continuously compounded:

$$\$200 \times e^{0.12 \times 30}$$

$$= \$7319.65 \mathbin{/\!/}$$

Q2. (a) Monthly payments:

$$\text{Annuity PV} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

$$\$565000 = C \times \left[\frac{1}{0.008} - \frac{1}{0.008(1+0.008)^{30\times12}}\right]$$

$$C = \$4792.10$$

(b) $\because \$4500 < \$4792.10$
(The required payment)

$\therefore$ No, they cannot.

$\Rightarrow$ The annuity PV when the monthly payments $= \$4500$:

$$\text{Annuity PV} = \$4500 \times \left[\frac{1}{0.008} - \frac{1}{0.008(1+0.008)^{360}}\right]$$

$$\text{Annuity PV} = \$530560.29$$

$\therefore$ The additional down-payment required:

$\$565000 - \$530560.29$

$= \$34439.71$ //

(c) If the monthly payments = \$5600 :

$$\$565000 = \$5600 \times \left[\frac{1}{0.008} - \frac{1}{0.008(1+0.008)^t} \right]$$

$$\frac{2825}{28} = 125 - \frac{125}{1.008^t}$$

$$\frac{125}{1.008^t} = \frac{675}{28}$$

$$1.008^t = \frac{140}{27}$$

$$t = 206.5475\ldots \text{ months}$$

$$< 240 \text{ months (20 years)}$$

$\therefore$ Yes, they can. //

Q3. (a) The amount they have when they retire:

$$\text{Annuity PV} = \$2500 \times \left[\frac{1}{0.006} - \frac{1}{0.006(1+0.006)^{20\times12}}\right]$$

$$= \$317521.08$$

∴ Let \$m be their monthly payments.

$$\$317521.08 = \$m \times \left[\frac{(1+0.006)^{30\times12}}{0.006} - \frac{1}{0.006}\right] \times (1+0.006)$$

$$m = 248.68 \text{ //}$$

(b) The amount they have when they retire:

$$\text{Annuity Due FV} = \$300 \times \left[\frac{(1+0.006)^{30\times12}}{0.006} - \frac{1}{0.006}\right] \times (1+0.006)$$

$$\text{Annuity Due FV} = \$383052.24$$

∴ Find the following "C" value:

$$\$383052.24 = \$C \times \left[\frac{1}{0.006} - \frac{1}{0.006(1+0.006)^{20\times12}}\right]$$

$$C = 3015.96 \text{ //}$$

∴ They will receive \$3015.96 each month.