香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一一至一二年度 上學期科目考試
Course Examination 1st Term, 2011-12

科目編號及名稱
Course Code & Title : **MATH1510A University Mathematics for Engineering**

時間
Time allowed : 2 小時 hours 00 分鐘 minutes

學號
Student ID. No. : _____ 座號 Seat No.: _____

Answer ALL questions. Justify all your steps.

1. (a) (i) Sketch the function $f(x) = \dfrac{|x|+1}{x^2-4}$.

   (ii) Show that the function is not differentiable at $x = 0$.

   (b) Determine whether the following function is differentiable at $x = 0$,

   $$g(x) = \sqrt{|x|x^2}\ .$$

2. (a) Write down the Taylor's polynomial of degree 4 centered at 0 for the function

   $$f(x) = (1+x)^{\pi}.$$

   (b) Compute the value of

   $$\ln 0.3$$

   with an error less than 0.1 using Taylor's theorem.

3. (a) Let $f : \mathbb{R} \to \mathbb{R}$ satisfy for all real numbers x, y $(x \neq y)$

   $$|f(x) - f(y)| \leq |x-y|^{1+\beta}$$

   for some positive constant β. Show that $f(x) = c$, $\forall x \in \mathbb{R}$, where c is a constant.

   (b) Let a be any positive real number. Compute the limit

   $$\lim_{t \to 0} \frac{a^t - 1}{t}.$$

   where $t > 0$. (Hint: You can assume $\lim_{t \to 0} a^t = 1$.)

4. (a) Compute the indefinite integral

$$\int \frac{x+2}{(x+1)^2(x-2)}dx.$$

(b) Compute the indefinite integral

$$\int x^3 \cdot (\cosh x)^2 dx.$$

5. Show the following:

$$\int_0^{\pi/2} \sin^{2n} x dx = \left(\frac{\pi}{2}\right) \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n},$$

where $n = 1, 2, 3, \cdots$

End of Paper