Extended experiment results for Modeling the Emergence of Polarized Communities Paper

1 Polarization experiments on Twitter and Reddit data

We show our results on Twitter and Reddit at Figure 1. For each graph we uniformly at random sample 50 nodes from each class (i.e., sets of nodes with positive, and negative views respectively) as labeled training data and run ITP++ and BOF. Our results confirm the same remarks we made for the synthetic experiments.

An interesting by-product of our model is the improved visualizations we obtain. We use tSNE to visualize the embeddings of the adjacency matrix of the original graph, and the graph at equilibrium [2]. Figure 3 shows the two visualizations. We visualize the graph structure using TSNE embedding of the graph adjacency matrix. Figures 3(a),(b) show the result on Twitter dataset. The plot on the left describes the original graph with nodes colored based on their original opinions (red positive, blue negative). In Figure 3(b) we observe that ITP++ generates a visible pattern of separation between the nodes. The same remark holds for Reddit, see Figures 3(c), (d).

2 Tackle flat value issue on MNIST

We test our models on the MNIST dataset in two different ways. First we perform binary classification: given a pair of digits, we sample 1 000 pictures for each digit and randomly pick two examples as labeled data, and start the learning. The experiment for each pair is repeated for five times. Results are reported in Table 1 as average accuracy and average absolute prediction value. We highlight the cells with highest accuracy in each row. The ITP model outperforms other methods with higher accuracy and relative high average absolute prediction value, showing the its power in "few shot" classification scenario. We also notice the overall accuracy gets lower when dealing with pairs that share similar shape, such as 7 and 9.

Second, we test the models on the dataset contain all ten digits. We use one-vs-rest strategy for all the models to deal with this multi-class task. Figure 2(c) shows the resulting metrics. ITP model still has the best accuracy when labeled data is limited. We also see a drop of accuracy from $10/n$ to 1%, because the these two ratios happen to be relatively close, and $10/n$ ensures a balanced training set while for 1% we may get labels only from one class. Also, we cannot observe the flat values issue in this case, since the $10/n$ setting provides more labeled data for multi-class task and n is not large enough to apply the asymptotic results of [3]. Even in this case, we still can see our models outperform label propagation under "few shot" scenario.

3 Convergence

We provide empirical evidence for the speed of convergence of our method. Figure 4 shows the infinity norm of the difference between current label $x(k)$ and equilibrium x^* of four iterative methods on a synthetic block model with 400 nodes in semi-log scale.. The average running time per iteration is: 0.4ms for label propagation, 4.5ms for BOF, 4.2ms for ITP and 8.9ms for ITP++. Note that the convergence rates for ITP and label propagation decrease rapidly, indicating we can terminate faster by easily adjusting the tolerence ϵ. Overall we observe that our methods ITP and ITP++ require relatively more time to convergence due to the increasing amount of parameter updating, but nonetheless the amount of extra time is reasonable compared to the existing methods of label propagation and BOF.

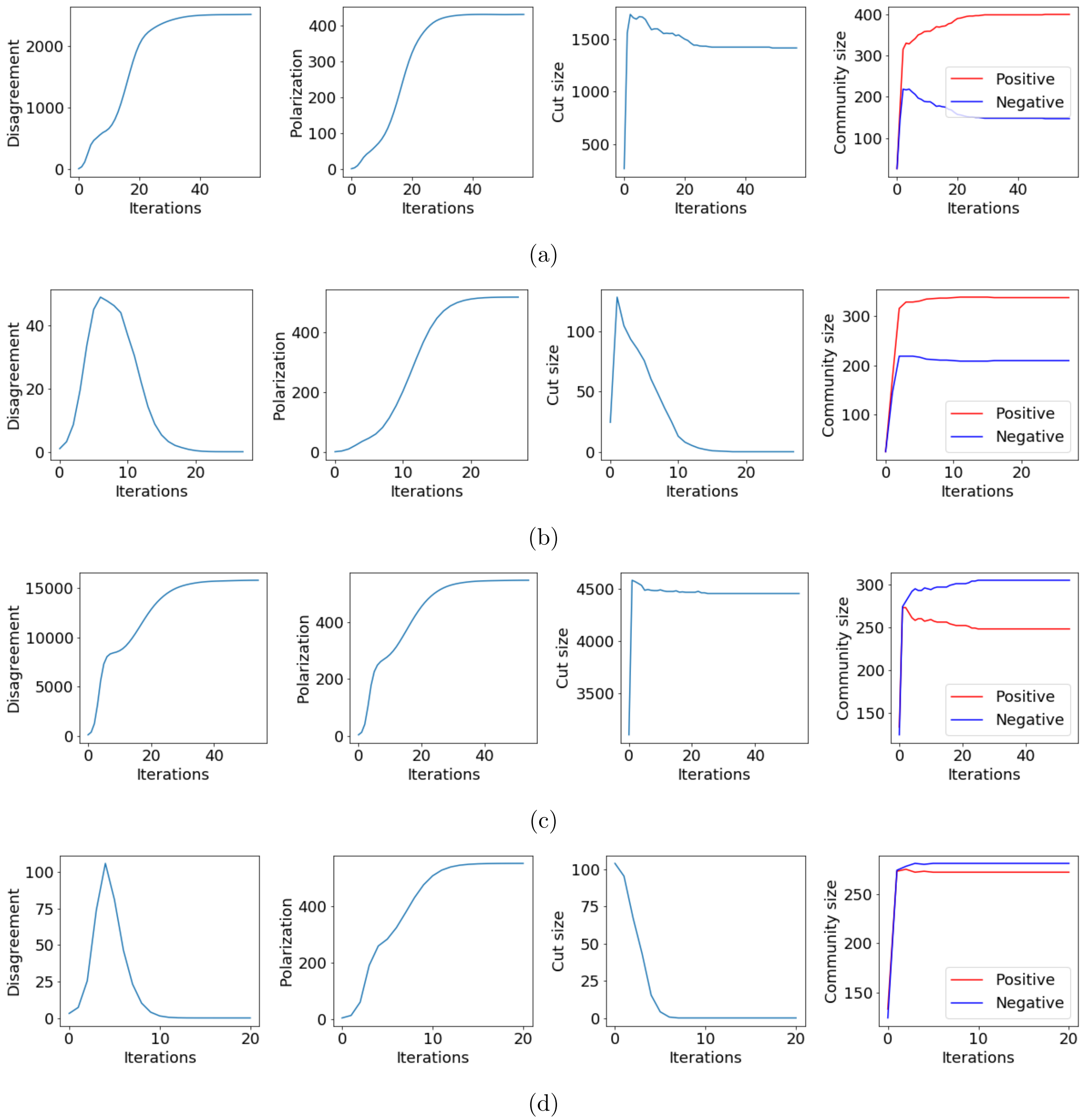

Figure 1: From left to right: Disagreement, polarization, cut size and predicted community size per iteration for (a) BOF [1], and (b) ITP++ on the Twitter graph. Similarly, (c) BOF [1], and (d) ITP++ on the Reddit graph.

Digit pairs	**Lex**		**Friedkin-Johnsen**		**BOF**		**Label Prop**		**ITP**		**ITP++**		**SVM**
	Acc	**Abs**	**Acc**	**Abs**	**Acc**	**Abs**	**Acc**	**Abs**	**Acc**	**Abs**	**Acc**	**Abs**	**Acc**
0 and 1	**0.948**	**0.444**	**0.911**	**0.001**	**0.906**	**0.999**	**0.928**	**0.183**	**0.955**	**0.744**	**0.906**	**0.999**	**0.943**
2 and 4	**0.969**	**0.349**	**0.991**	**0.002**	**0.974**	**0.999**	**0.975**	**0.187**	**0.990**	**0.934**	**0.962**	**0.999**	**0.843**
3 and 6	**0.984**	**0.413**	**0.986**	**0.001**	**0.987**	**0.999**	**0.562**	**0.318**	**0.999**	**0.836**	**0.980**	**0.999**	**0.865**
7 and 9	**0.711**	**0.216**	**0.773**	**0.001**	**0.726**	**0.999**	**0.598**	**0.126**	**0.774**	**0.921**	**0.716**	**0.999**	**0.749**

Table 1: Pairwise MNIST digit classification result.

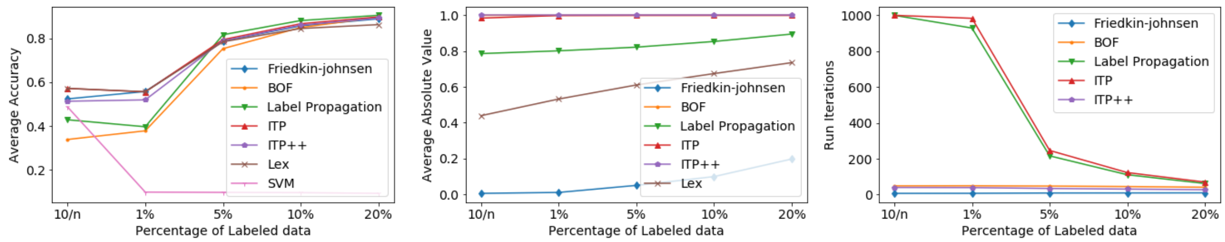

Figure 2: From left to right: Average accuracy, average absolute value of node labels, and number of iterations to convergence for the MNIST Dataset.

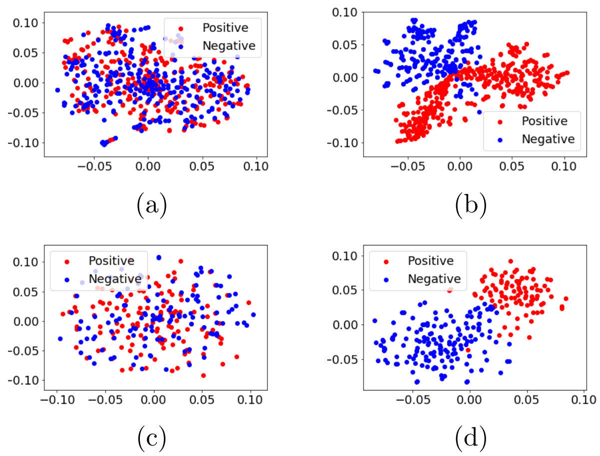

Figure 3: tSNE plots of (a) original Twitter graph, and (b) graph at equilibrium for ITP++. Same plots for (c) original Reddit graph, and (d) graph at equilibrium for ITP++. Node are labeled according to their opinion (red for positive, blue for negative). We observe that ITP++ is able to amplify the existing signals and generate visible patterns for both graphs.

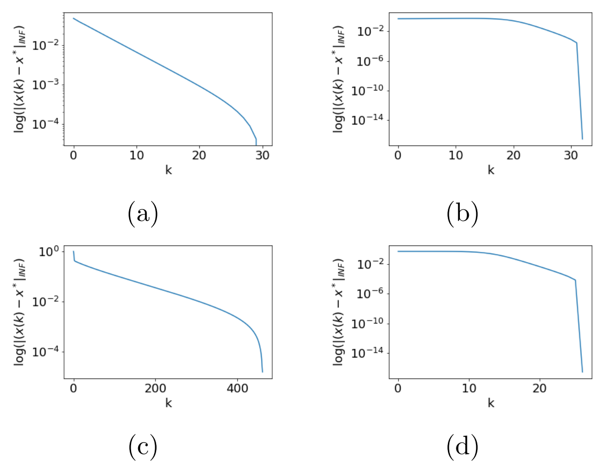

Figure 4: Convergence plots (a) Label propagation, (b)BOF [1], (c) ITP, and (d) ITP++.

References

[1] P. Dandekar, A. Goel, and D. T. Lee. Biased assimilation, homophily, and the dynamics of polarization. *Proceedings of the National Academy of Sciences*, 110(15):5791–5796, 2013.

[2] L. v. d. Maaten and G. Hinton. Visualizing data using t-sne. *Journal of machine learning research*, 9(Nov):2579–2605, 2008.

[3] B. Nadler, N. Srebro, and X. Zhou. Semi-supervised learning with the graph laplacian: The limit of infinite unlabelled data. *Advances in neural information processing systems*, 22:1330–1338, 2009.