Category Abstract

Mikita Kukavenka

November 18, 2019

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Definition 0.1. A *category* **C** consists of

- a collection of objects: A, B, C, …
- a collection of arrows: f, g, h, …
- f o r each arrow f o b j e c t s $\mathrm{dom}(f)$ and $\mathrm{cod}(f)$ called the *domain* and *codomain* of f. If $\mathrm{dom}(f) = A$ and $\mathrm{cod}(f) = B$, we also write $f : A \to B$,
- given $f : A \to B$ and $g : B \to C$, so that $\mathrm{dom}(g) = \mathrm{cod}(f)$, there is an arrow $g \circ f : A \to C$,
- an arrow $\mathbf{Id}_A : A \to A$ for every object A of **C** ,

such that

(Associative law) for every $f : A \to B$, $g : B \to C$ and $h : C \to C$ we have

$$h \circ (g \circ f) = (h \circ g) \circ f, \tag{1}$$

(Unit laws) for every $f : A \to B$ we have

$$f \circ \mathbf{Id}_A = f = \mathbf{Id}_B \circ f. \tag{2}$$

$$\begin{array}{ccccc} A & \xrightarrow{f} & B & & \\ & \searrow^{g\circ f} & \downarrow{\scriptstyle g} & \searrow^{g\circ h} & \\ & & C & \xrightarrow[h]{} & D \end{array} \tag{3}$$

$$\mathrm{id}_A \circlearrowright A \longrightarrow B \times C \circlearrowleft \mathrm{id}_{B\times C}$$