# STA 522, Spring 2021
# Introduction to Theoretical Statistics II

Lecture 6

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# AGENDA

- Method of maximum likelihood
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# Review: Method of Estimation

- **Method of Moments:** Equate population moments with the sample moments, then solve for parameters.

- **Method of Maximum Likelihood:** For each sample point $\underline{x}$, let $\hat{\theta}(\underline{x})$ be a parameter value at which the likelihood function $L(\theta \mid \underline{x})$ attains its maximum as a function of $\theta$, with $\underline{x}$ held fixed. A **maximum likelihood estimator (MLE)** of the parameter $\theta$ based on a sample $\underline{X}$ is $\hat{\theta}(\underline{X})$.

- **Note:** since the logarithm function is strictly increasing on $(0, \infty)$ (and so one-to-one), the value which maximizes $\log L(\theta \mid \underline{x})$ is the same value that maximizes $L(\theta \mid \underline{x})$.

- **Example:** $X_1, X_2, \ldots, X_n \sim$ iid Bernoulli $(p)$, for $0 \leq p \leq 1$. The MLE of $p$ is $\hat{p} = \dfrac{1}{n} \sum_{i=1}^{n} X_i$

**Example:** Let $X_1, X_2, \ldots, X_n \sim$ iid $\mathrm{N}(\theta, 1)$, where $\theta$. Find the MLE of $\theta$.

The likelihood function for $\theta$ is given by

$$L(\theta \mid \underline{x}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x_i - \theta)^2\right] = \left(\frac{1}{2\pi}\right)^{n/2} \exp\left[-\frac{1}{2}\sum_{i=1}^{n}(x_i - \theta)^2\right]$$

Therefore the log likelihood is:

$$\log L(\theta \mid \underline{x}) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n}(x_i - \theta)^2 = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n}(\theta - x_i)^2$$

which implies

$$\frac{d \log L(\theta \mid \underline{x})}{d\theta} = -\frac{1}{2}\sum_{i=1}^{n} 2\,(x_i - \theta) \gtreqless 0 \quad \text{according as} \quad \theta \lesseqgtr \frac{1}{n}\sum_{i=1}^{n} x_i = \overline{x}$$

Thus the MLE of $\theta$ is $\hat{\theta} = \overline{x}$.

**Example (Restricted Range MLE):** Let $X_1, X_2, \ldots, X_n \sim$ iid $\mathrm{N}(\theta, 1)$, where $\theta \geq 0$. Find the MLE of $\theta$.

# Homework

- Read p. 282 – 291.
- Exercises: TBA.