The Exponential Distribution in R versus the Central Limit Theorem

Overview

The goal of this project is to investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. A `lambda = 0.2` is set for all simulations in this report. This report will investigate the distribution of averages of 40 exponentials.

This report will illustrate via simulation and associated explanatory text, the properties of the distribution of the mean of 40 exponentials. This report shows:

1. The sample mean as compared to the theoretical mean of the distribution.
2. How variable the sample is (via variance) when compared to the theoretical variance of the distribution.
3. How the distribution is approximately normal.

Simulations

Sample Mean versus Theoretical Mean

A series of 1000 simulations will be performed to create a dataset for which to compare the Central Limit Theorem. Each simulation contains 40 observations and the exponential distribution function will be set to `rexp(40, 0.2)`.

```
lambda = 0.2
n = 40
nosim = 1000
set.seed(349)
```

The simulations are then carried out to collect the necessary data, and the data is then plotted.

```
exp_sim <- function(n, lambda) {
        mean(rexp(n,lambda))
}

sim <- data.frame(ncol=2,nrow=1000)
names(sim) <- c("Index","Mean")

for (i in 1:nosim) {
        sim[i,1] <- i
        sim[i,2] <- exp_sim(n,lambda)
}
```

The mean of the simulation data, n=1000

```
sample_mean <- mean(sim$Mean)
sample_mean
```

```
## [1] 4.983227
```

Theoretical exponential mean of the exponential distribution

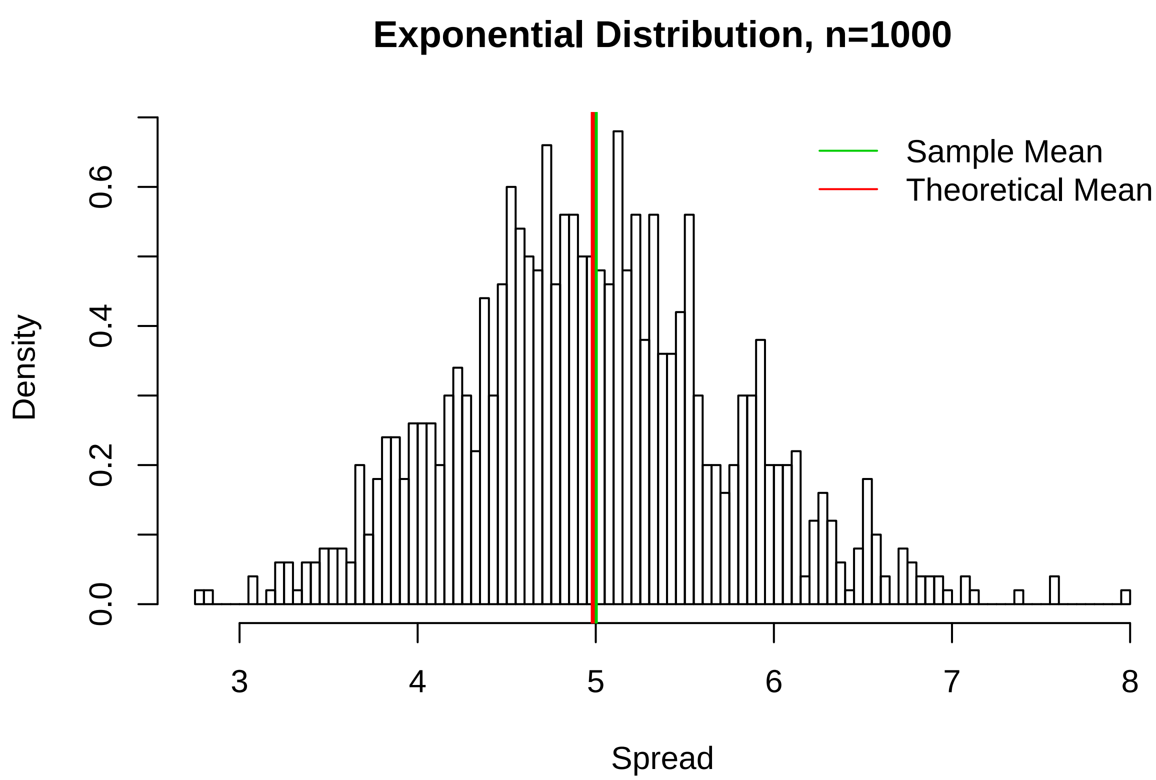
```
theor_mean <- 1/lambda
theor_mean
```

```
## [1] 5
```

The simulation mean is virtually identical to the theoretical mean.

Histogram plot of the exponential distribution, n=1000

```
hist(sim$Mean, breaks = 100, prob = TRUE,
     main="Exponential Distribution, n=1000", xlab="Spread")
        abline(v = theor_mean, col= 3, lwd = 2)
        abline(v = sample_mean, col = 2, lwd = 2)
        legend('topright', c("Sample Mean", "Theoretical Mean"), bty = "n",
                lty = c(1,1), col = c(col = 3, col = 2))
```



Sample Variance versus Theoretical Variance

Let us now compare the variance present in the sample means of the 1000 simulations to the theoretical varience of the population.

The variance of the sample means estimates the variance of the population by using the varience of the 1000 entries in the means vector times the sample size, 40. In other words, `σ2=Var(samplemeans)×N`.

```
sample_var <- var(sim$Mean)
theor_var <- ((1/lambda)^2)/40
```

The theoretical variance of the population is given as `σ2=(1/lambda)2`

```
sample_var
```

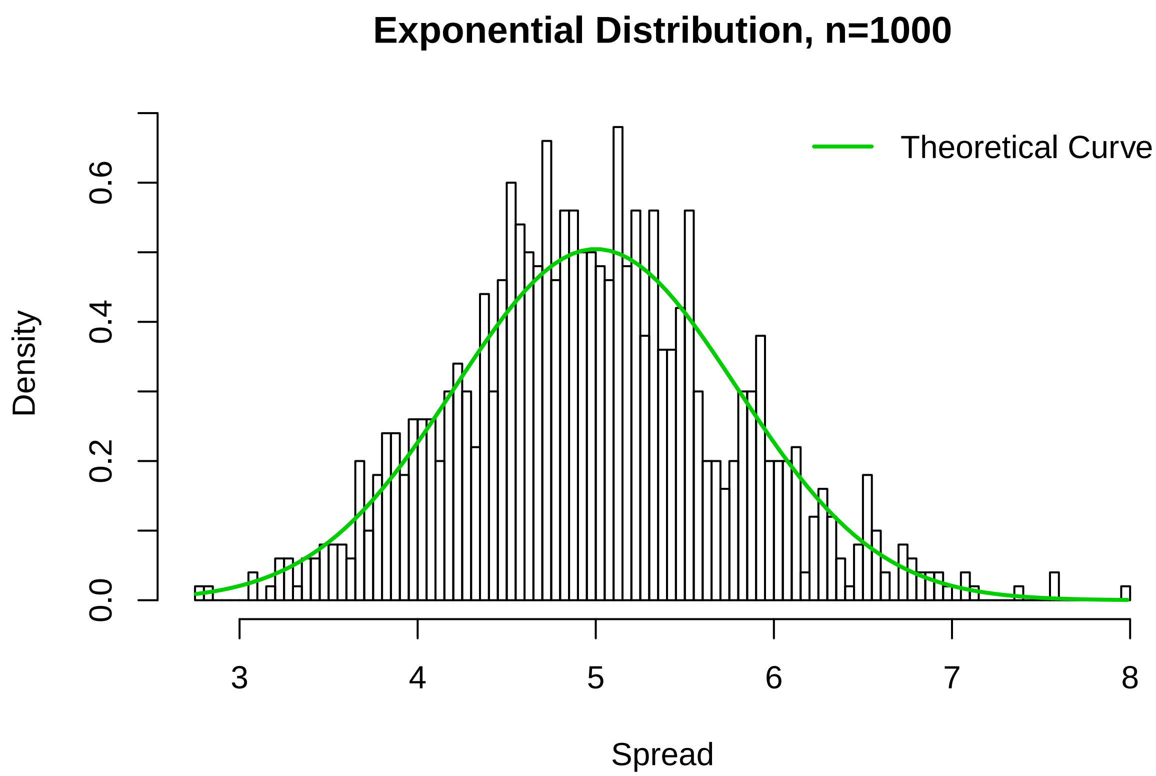
```
## [1] 0.6010593
```

```
theor_var
```

```
## [1] 0.625
```

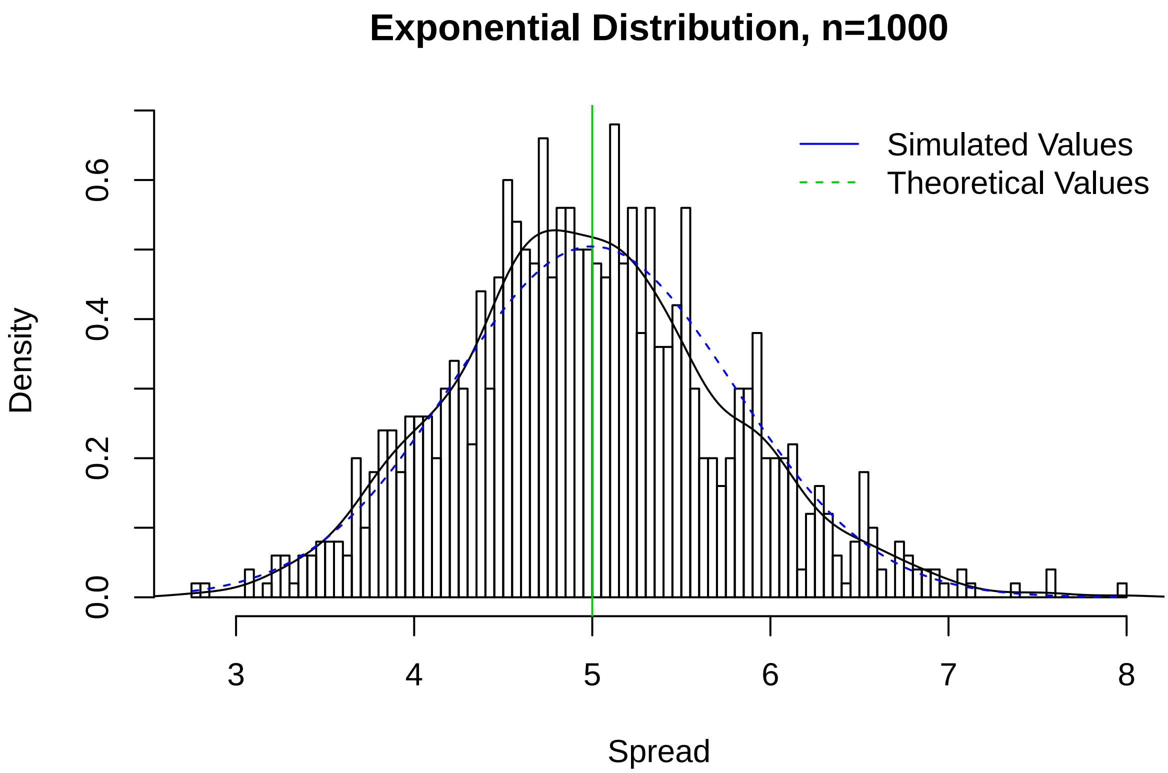
```
hist(sim$Mean, breaks = 100, prob = TRUE,
     main = "Exponential Distribution, n=1000", xlab = "Spread")

xfit <- seq(min(sim$Mean), max(sim$Mean), length = 100)
yfit <- dnorm(xfit, mean = 1/lambda, sd = (1/lambda/sqrt(40)))
        lines(xfit, yfit, pch = 22, col = 3, lwd = 2)
        legend('topright', c("Theoretical Curve"), lty = 1,lwd = 2,
               bty = "n", col = 3)
```




```
hist(sim$Mean, breaks = 100, prob = TRUE,
     main = "Exponential Distribution, n=1000", xlab = "Spread")
        lines(density(sim$Mean))
        abline(v = 1/lambda, col = 3)
```

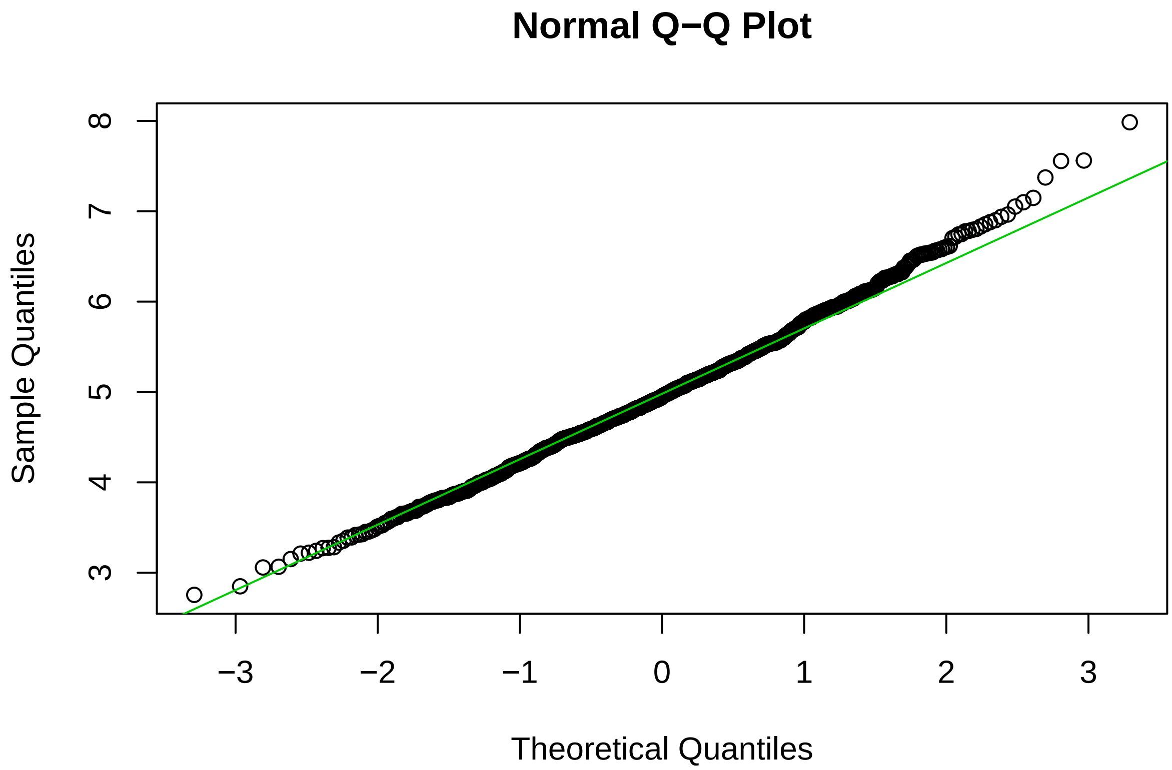
```r
xfit <- seq(min(sim$Mean), max(sim$Mean), length = 100)
yfit <- dnorm(xfit, mean = 1/lambda, sd = (1/lambda/sqrt(40)))
        lines(xfit, yfit, pch = 22, col = 4, lty = 2)
        legend('topright', c("Simulated Values", "Theoretical Values"),
                bty = "n", lty = c(1,2), col = c(4, 3))
```



Distribution

Due to the Central Limit Theorem, the averages of samples follow normal distribution. The figure above also shows the density computed using the histogram and the normal density plotted with theoretical mean and variance values. Also, the q-q plot below suggests the normality. The theoretical quantiles again match closely with the actual quantiles. These four methods of comparison prove that the distribution is approximately normal.

```r
qqnorm(sim$Mean, main ="Normal Q-Q Plot")
qqline(sim$Mean, col = "3")
```


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
3
4
5
6
7
8
−3
−2
−1
0
1
2
3