ME-639

Assignment-2

23210047

Task-1:

Show that $RS(a)R^T = S(Ra)$, where R is a rotation matrix

$\Rightarrow S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$ $\Rightarrow$ Skew symmetric matrix for vector $\bar{a} = [a_x \ a_y \ a_z]^T$

- let R be a rotation matrix about z-axis with angle θ

$$R = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow RS(a)R^T = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_z s\theta & -a_z c\theta & a_y \\ a_z c\theta & a_z s\theta & -a_x \\ -a_y c\theta - a_x s\theta & -a_y s\theta + a_x c\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a_z s\theta c\theta - a_z s\theta c\theta & -a_z s^2\theta - a_z c^2\theta & a_y c\theta + a_x s\theta \\ a_z s^2\theta + a_z c^2\theta & -a_z s\theta c\theta + a_z s\theta c\theta & a_y s\theta - a_x c\theta \\ -a_y c\theta - a_x s\theta & -a_y s\theta + a_x c\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_z & a_x s\theta + a_y c\theta \\ a_z & 0 & -(a_x c\theta - a_y s\theta) \\ -(a_x s\theta + a_y c\theta) & a_x c\theta - a_y s\theta & 0 \end{bmatrix}$$

$$\Rightarrow Ra = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} a_x c\theta - a_y s\theta \\ a_x s\theta + a_y c\theta \\ a_z \end{bmatrix}$$

$$\text{So, } S(Ra) = \begin{bmatrix} 0 & -a_z & a_x s\theta + a_y c\theta \\ a_z & 0 & -(a_x c\theta - a_y s\theta) \\ -(a_x s\theta + a_y c\theta) & a_x c\theta - a_y s\theta & 0 \end{bmatrix}$$

$\Rightarrow$ So, proved $\boxed{RS(a)R^T = S(Ra)}$

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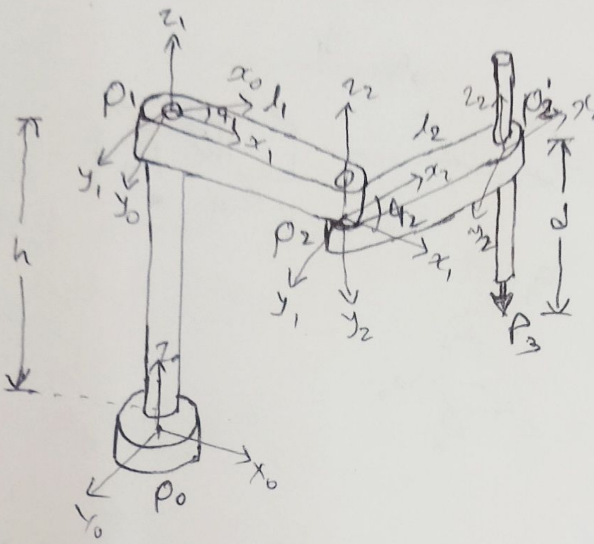
Task-2: RRP SCARA



P_0 - base coordinate frame at origin

P_3 - Tip coordinate frame.

$P_3 = [0 \quad 0 \quad -d]^T$

$$\Rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

where,

$$H_0^3 = H_0^1 H_1^2 H_2^3$$

① $H_0^1 = \left[\begin{array}{c|c} Rot(z, q_1) & Trans(z, h) \\ \hline 0\,0\,0 & 1 \end{array}\right]$, Rotation about z with 'q_1' angle & Translation about frame height 'h' in z-direction.

② $H_1^2 = \left[\begin{array}{c|c} Rot(z, q_2) & Trans(x, l_1) \\ \hline 0\,0\,0 & 1 \end{array}\right]$, Rotation z with '$q_2$' angle & Translation about link-1 length 'l_1' in x-direction

③ $H_2^3 = \left[\begin{array}{c|c} Rot(z, 0) & Trans(x, l_2) \\ \hline 0\,0\,0 & 1 \end{array}\right]$, Only translation about link-2 length 'l_2' in x-dir.

$$\boxed{\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}}$$

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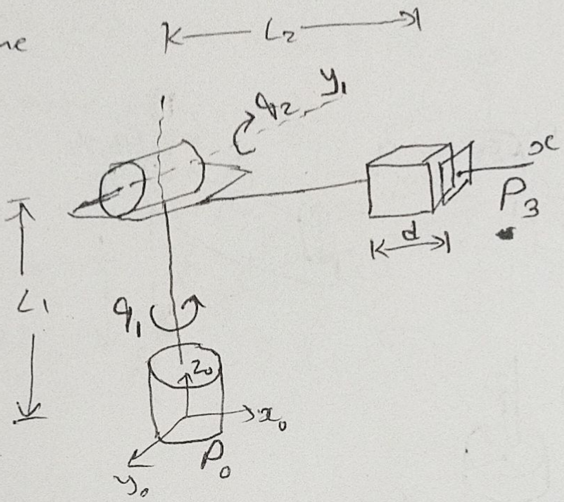
Task-4: Stanford Type RRP Configuration.

P_0 - base coordinate frame at origin

P_3 - tip coordinate frame

$P_3 = [d \ 0 \ 0]^T$



$$\Rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}, \text{ where } H_0^3 = H_0^1 H_1^2 H_2^3$$

① $H_0^1 = \left[\begin{array}{c|c} Rot(z, q_1) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0\,0\,0 & 1 \end{array}\right]$, Rotation about z with 'q_1' & No translation

② $H_1^2 = \left[\begin{array}{c|c} Rot(y, q_2) & Trans(z, L1) \\ \hline 0\,0\,0 & 1 \end{array}\right]$, Rot. y with q_2 & Trans. in z about height L1

③ $H_2^3 = \left[\begin{array}{c|c} Rot(x, 0) & Trans(x, L2) \\ \hline 0\,0\,0 & 1 \end{array}\right]$, No rotation & Trans. L2 in x-dir

$$\boxed{\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}}$$

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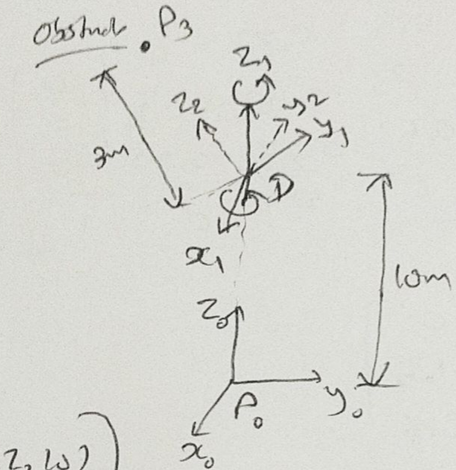
Task-5 : Drone Problem.

$\Rightarrow$ Obstacle (from base coordinates)

$$\left\{\begin{array}{ll} 10\text{ m in } z \text{ dir} & \rightarrow \text{Trans}(z, 10) \\ 30^\circ \text{ rot @ } x\text{-axis} & \rightarrow \text{Rot}(x, 30) \\ 60^\circ \text{ rot. @ } z\text{-axis} & \rightarrow \text{Rot}(z, 60) \\ 3\text{ m above drone} & \rightarrow \text{Trans}(z, 3) \end{array}\right\}$$

$$P_3 = [0 \ \ 0 \ \ 3]^T$$

$\Rightarrow$ Showing combine rotation for obstacle P_3



$$H_0^3 = H_0^1 H_1^2 H_2^3,$$

where,

$$H_0^1 = \left[\begin{array}{c|c} \text{No rot.} & \text{Trans}(z, 10) \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$H_1^2 = \left[\begin{array}{c|c} \text{Rot}(z, 30) & \text{No Trans} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$H_2^3 = \left[\begin{array}{c|c} \text{Rot}(x, 60) & \text{No Trans} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$\Rightarrow$ Obstacle position in base coordinate frame.

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}, \quad \text{where } P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

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$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.87 & -0.5 & 0 \\ 0 & 0.5 & 0.87 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -0.87 & 0 & 0 \\ 0.87 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & -0.87 & 0 & 0 \\ 0.75 & 0.43 & -0.5 & 0 \\ 0.43 & 0.25 & 0.87 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

{matrix calculation shown in Colab Notebook}

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \\ 1 \end{bmatrix}$$

So, $P_0 = [0 \ -1.5 \ 12.6]^T$

Obstacle position in base coordinate frame.

Task-6:- gearbox in drone application

→ Ussually not preferred as it reduces efficiency, increases inertia & increases weight.

→ Coreless DC motor & BLDC motor used as they provide less friction, weight & higher rpm

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Task-6: Motor-gearbox in robotics
&
drones

⇒ Spur gear – most common & simple
– power transfer b/w parallel lined axis (no rotation of axis)
– Cylindrical in shape

⇒ Bevel gear – Conical inshape
– Similar to spur gears, but power transfer b/w perpendicular axis of rotation.
– b/w two intersecting shafts

⇒ Worm gear – Similar to bevel gears, but allows higher gear ratio
– Low cost, low noise & vibration
– retaining heat & less efficient

⇒ Planetary gearbox
– gear in sun & ring gear format 
– allows higher gearratio with better grip
– Higher efficiency & low powerloss
– but axially not compact.

⇒ Cycloidal
– toothed gear with cycloidal profile
– follows eccentric motion in cycloid path
– axially compact, Higher gear ratio

⇒ Leadscrew & Nut
– Ussually preffered for linear motion
– Leadscrew driven by motor gives linear motion to T-nut

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Task-7: Manipulator Jacobian for RRP SCARA configuration

$$\Rightarrow J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

{For RRP Scara Joint-1&2 is Revolute joint & Joint-3 is Prismatic Joint}

→ Joint origins

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ h \end{bmatrix}, \quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ h \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ h - d_3 \end{bmatrix}$$

→ Joint-axis Vectors:-

$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

{Note! z_3 is for tip-coordinate reference.}

$$\Rightarrow J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & -1 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

{Note:- {Vector cross product}

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{bmatrix} a_2 b_3 - b_2 a_3 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

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Task-9: Manipulator Jacobian for Planar RRR

$$\Rightarrow J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

→ Joint origins

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 C_1 \\ l_1 S_1 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \\ 0 \end{bmatrix},$$

$$O_3 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} \\ l_1 S_1 + l_2 S_{12} + l_3 S_{123} \\ 0 \end{bmatrix}$$

Note:-
$C_1 = \cos(q_1)$
$C_{12} = \cos(q_1 + q_2)$
$C_{123} = \cos(q_1 + q_2 + q_3)$
– Similar for S_i.

→ Joint axis vectors:

$$Z_0 = Z_1 = Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow J = \begin{bmatrix} -(l_1 S_1 + l_2 S_{12} + l_3 S_{123}) & -(l_2 S_{12} + l_3 S_{123}) & -l_3 S_{123} \\ l_1 C_1 + l_2 C_{12} + l_3 C_{123} & l_2 C_{12} + l_3 C_{123} & l_3 C_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$