Exercice 3: Mécanique du solide TD 2

$R_0\,(O, \vec{x}_0, \vec{y}_0, \vec{z}_0)$

$R_1\,(O, \vec{x}_1, \vec{y}_1, \vec{z}_1)$

$R_2\,(B, \vec{x}_2, \vec{y}_2, \vec{z}_2)$

$\vec{\Omega}_e = \dot{\alpha}\,\vec{z}_0$ $\quad$ $\vec{\Omega}_r = \dot{\theta}\,\vec{x}_1$

$\vec{OB} = a\,\vec{x}_1$ $\quad$ $\vec{BG} = b\,\vec{z}_2$

$$\vec{V}_a(G) = \vec{V}_e(G) + \vec{V}_r(G)$$

$$\vec{V}_e(G) = \underbrace{\vec{V}_a(O)}_{0} + \vec{\Omega}_e \wedge \vec{OG}$$

$$= \dot{\alpha}\,\vec{z}_0 \wedge (\vec{OB} + \vec{BG}) = \dot{\alpha}\,\vec{z}_1 \wedge a\,\vec{x}_1 + \dot{\alpha}\,\vec{z}_1 \wedge b\,\vec{z}_2$$

$$\boxed{\vec{V}_e(G) = a\dot{\alpha}\,\vec{y}_1 + b\dot{\alpha}\sin\theta\,\vec{x}_1}$$

$\vec{z}_2 = \cos\theta\,\vec{z}_1 - \sin\theta\,\vec{y}_1$

$$\vec{V}_r(G) = \left.\frac{d\vec{OG}}{dt}\right/_{R_1} = \frac{d}{dt}\left(a\,\vec{x}_1 + b\,\vec{z}_2\right)/_{R_1}$$

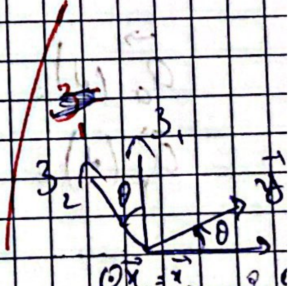
$$= b\left.\frac{d\vec{z}_2}{dt}\right/_{R_1}$$

$$= b\,\dot{\theta}\,\vec{x}_2 \wedge \vec{z}_2 = \boxed{-b\dot{\theta}\,\vec{y}_2}$$

$$\vec{a}_a(G) = \vec{a}_e(G) + \vec{a}_c(G) + \vec{a}_r(G)$$

$$\vec{a}_r(G) = \left.\frac{d\vec{V}_r(G)}{dt}\right/_{R_1} = -b\ddot{\theta}\,\vec{y}_2 - b\dot{\theta}\,\dot{\theta}\,\vec{x}_2 \wedge \vec{y}_2$$

$$\boxed{\vec{a}_r(G) = -b\ddot{\theta}\,\vec{y}_2 - b\dot{\theta}^2\,\vec{z}_2}$$



$\vec{y}_2 = \cos\theta\,\vec{y}_1 + \sin\theta$

$$\vec{a}_c(G) = 2\,\vec{\Omega}_e \wedge \vec{V}_r(G) = 2\dot{\alpha}\,\vec{z}_1 \wedge -b\dot{\theta}\,\vec{y}_2$$

$$\boxed{\vec{a}_c(G) = 2\dot{\alpha}\,b\dot{\theta}\cos\theta\,\vec{x}_1}$$

$$\vec{a}_e(G) = \underbrace{\vec{a}_a(O)}_{0} + \dot{\vec{\Omega}}_e \wedge \vec{OG} + \vec{\Omega}_e \wedge (\vec{\Omega}_e \wedge \vec{OG})$$

$$= \ddot{\alpha}\,\vec{z}_1 \wedge (a\,\vec{x}_1 + b\,\vec{z}_2) + \dot{\alpha}\,\vec{z}_1 \wedge \left(\dot{\alpha}\,\vec{z}_1 \wedge (a\,\vec{x}_1 + b\,\vec{z}_2)\right)$$

$$= a\ddot{\alpha}\,\vec{y}_1 + b\ddot{\alpha}\sin\theta\,\vec{x}_1 + \dot{\alpha}\,\vec{z}_1 \wedge (a\dot{\alpha}\,\vec{y}_1 + b\dot{\alpha}\sin\theta\,\vec{x}_1)$$

$$\boxed{\vec{a}_e(G) = a\ddot{\alpha}\,\vec{y}_1 + b\ddot{\alpha}\sin\theta\,\vec{x}_1 - a\dot{\alpha}^2\,\vec{x}_1 + b\dot{\alpha}^2\sin\theta\,\vec{y}_1}$$

Exercice 5 :

$R_0(O, \vec{x}_0, \vec{y}_0, \vec{z})$

$R_1(O, \vec{x}_1, \vec{y}_1, \vec{z})$ $\quad \vec{\Omega}_e = \dot{\theta}_1 \vec{z}$ $\quad \vec{OC} = R\vec{x}_1$

$R_2(C, \vec{x}_2, \vec{y}_2, \vec{z})$ $\quad \vec{\Omega}_r = \dot{\theta}_2 \vec{z}$ $\quad \vec{CM} = r\vec{x}_2$

$$\vec{V}_a(M) = \vec{V}_e(M) + \vec{V}_r(M)$$

$$\vec{V}_e(M) = \underbrace{\vec{V}_a(O)}_{=0} + \vec{\Omega}_e \wedge \vec{OM}$$

$$= \dot{\theta}_1\vec{z} \wedge (R\vec{x}_1 + r\vec{x}_2)$$

$$= \dot{\theta}_1\vec{z}_1 \wedge R\vec{x}_1 + \dot{\theta}_1\vec{z}_2 \wedge r\vec{x}_2$$

$$\boxed{\vec{V}_e(M) = R\dot{\theta}_1\vec{y}_1 + r\dot{\theta}_1\vec{y}_2}$$

$$\vec{V}_r(M) = \frac{d\vec{OM}}{dt}\Big/_{R_1} = \frac{d}{dt}(\cancel{R\vec{x}_1} + r\vec{x}_2)\Big/_{R_1}$$

$$= r\frac{d\vec{x}_2}{dt}\Big/_{R_1}$$

$$\boxed{\vec{V}_r(M) = r\dot{\theta}_2\vec{y}_2}$$

$$\vec{a}_a(M) = \vec{a}_e(M) + \vec{a}_c(M) + \vec{a}_r(M)$$

$$\vec{a}_r(M) = \frac{d\vec{V}_r(M)}{dt}\Big/_{R_1} = r\dot{\theta}_2\dot{\theta}_2\vec{z}_2 \wedge \vec{y}_2$$

$$\boxed{\vec{a}_r(M) = -r\dot{\theta}_2^2\vec{x}_2}$$

$$\vec{a}_c(M) = 2\vec{\Omega}_e \wedge \vec{V}_r = 2\dot{\theta}_1\vec{z}_2 \wedge r\dot{\theta}_2\vec{y}_2 = \boxed{-2r\dot{\theta}_1\dot{\theta}_2\vec{x}_2}$$

$$\vec{a}_e(M) = \underbrace{\vec{a}_a(O)}_{=0} + \dot{\vec{\Omega}}_e \wedge \vec{OM} + \vec{\Omega}_e \wedge (\vec{\Omega}_e \wedge \vec{OM})$$

$$= R\ddot{\theta}_1\vec{y}_1 + r\ddot{\theta}_1\vec{y}_2 + \dot{\theta}_1\vec{z} \wedge (R\dot{\theta}_1\vec{y}_1 + r\dot{\theta}_1\vec{y}_2)$$

$$\boxed{\vec{a}_e(M) = R\ddot{\theta}_1\vec{y}_1 + r\ddot{\theta}_1\vec{y}_2 - R\dot{\theta}_1^2\vec{x}_1 - r\dot{\theta}_1^2\vec{x}_2}$$

Exercice 6:

$\vec{V} = v\,\vec{u_x}$

$x = v\,\theta\, x_x$

$R_0(O, \vec{u_x}, \vec{u_y}, \vec{u_z})$

$R_1(O_1, \vec{u_x}, \vec{u_y}, \vec{u_z})$ $\quad \vec{\Omega_e} = \vec{0}$

$R_2(O_1, \vec{u_x}, \vec{u_r}, \vec{u_\theta})$ $\quad \vec{\Omega_r} = \dot{\theta}\,\vec{u_x}$ $\quad \vec{O_1M} = R\,\vec{u_r}$

$\vec{v_a}(M) = \vec{v_e}(M) + \vec{v_r}(M)$

$\vec{v_e}(M) = \vec{v_a}(O_1) + \underbrace{\vec{\Omega_e} \wedge \vec{OM}}_{=0}$

$\boxed{\vec{v_e}(M) = v\,\vec{u_x}}$

$\vec{v_r}(M) = \frac{d\vec{O_1M}}{dt}\Big/_{R_1} = R\,\dot{\theta}\vec{u_x} \wedge \vec{u_r}$

$\boxed{\vec{v_r}(M) = R\dot{\theta}\,u_\theta}$

$\vec{a_a}(M) = \vec{a_e}(M) + \vec{a_c}(M) + \vec{a_r}(M)$

$\vec{a_r}(M) = \frac{d\vec{V_r}(M)}{dt}\Big/_{R_1} = R\ddot{\theta}\,\vec{u_\theta} + R\dot{\theta}\dot{\theta}\,\vec{u_x} \wedge \vec{u_\theta}$

$\boxed{\vec{a_r}(M) = R\ddot{\theta}\,\vec{u_\theta} - R\dot{\theta}^2\,\vec{u_r}}$

$\vec{a_c}(M) = 2\vec{\Omega_e} \wedge \vec{v_r}(M)$

$\boxed{\vec{a_c}(M) = \vec{0}}$

$\vec{a_e}(M) = \vec{a_a}(O_1) + \underbrace{\dot{\vec{\Omega_e}} \wedge \vec{O_1M} + \vec{\Omega_e} \wedge (\vec{\Omega_e} \wedge \vec{O_1M})}_{=0}$

$\boxed{\vec{a_e}(M) = \dot{v}\,\vec{u_x}}$

Scanned with CamScanner