4.2 Maximum & Minimum of $P(X<Y)$

1) Y takes on value $Y=1$ & $Y=0$

$\Rightarrow$ theres only 1 case when $X<Y$

$X=0,\ Y=1$

$\Rightarrow$ Here we want ;

$$P(X=0 \text{ and } Y=1) = P((X=0)\cap(Y=1))$$

• Using the Addition Rule

$$P((X=0)\cap(Y=1)) = P(X=0) + P(Y=1) - P((X=0)\cup(Y=1))$$

Case of minimum:

$$P((X=0)\cup(Y=1)) \leq 1$$

Since both events resulted in the entire probability space

$$P((X=0)\cap(Y=1)) = P(X=0) + P(Y=1) - 1$$

$$P((X=0)\cap(Y=1)) = \frac{1}{3} + \frac{1}{2} - 1$$

$$P((X=0)\cap(Y=1)) = -\frac{1}{6}$$

$\Rightarrow$ P cannot be negative

Case of Minimum Continue

Prob ≥ 0 By axiom 2

$P((x=0) \cap (y=1)) = -\frac{1}{6}$ for $P((x=0) \cup (y=1)) = 1$

$\Rightarrow$ So the minimum value for $P(x<y) = 0$

<u>Case of maximum:</u>

$P(y=1) = \frac{1}{2}$ $P(x=0) = \frac{1}{3}$ $P(x=0) \subseteq P(y=1)$

By Monotonicity:

$P(x=0) \leq P(y=1)$

$P((x=0) \cup (y=1)) \geq P(y=1) = \frac{1}{2}$

$\Downarrow$ expand

$P(x=0) + P(\cancel{y=1}) - P((x=0) \cap (y=1)) \geq P(\cancel{y=1})$

$P((x=0) \cap (y=1)) \leq P(x=0)$

$$\boxed{P((x=0) \cap (y=1)) \leq \frac{1}{3}}$$