Practice Test

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Q 3.3 given:

$Y = \max[X_1, X_2, X_3]$ Find $P(Y \leq 0.5)$

implies finding the cdf of Y

=> Since $X_1\ X_2\ X_3$ are multually independent. We know that $Y \leq x$ iff every element of the sample is less than x

↳ implies: the CDF of Y as follow

$$P(Y \leq x) = P(X_1 \leq x, X_2 \leq x, X_3 \leq x)$$

$$= \prod_{i=1}^{3} P(X_i \leq x) = (F_X(x))^n$$

↳ from our pdf define previously

$$f_{X_1 X_2 X_3}(x_1, x_2, x_3) = \begin{cases} 8x^3, & 0 < x_1, x_2, x_3 < 1 \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow$ We integrate the pdf to get the cdf

$\Rightarrow$ But, from our deduction, we only need to integrate one of the cdf since at any given point only either X_1, X_2 or X_3 can be max

$$\int_0^y 2x\,dx = x^2 \Big|_0^y = y^2$$

$$F_Y(y) = \begin{cases} 0 ; & y \leq 0 \\ y^{2n} ; & 0 < y < 1 \\ 1 ; & y \geq 1 \end{cases}$$

where $n = 3$ ... since X_1 X_2 X_3

Now:

$$P(Y \leq 0.5) = \int_0^{0.5} y^6\,dy = \frac{y^7}{7}\Big|_0^{0.5} \approx 0.001116$$