3.2) given that $Y \leq 1$

what is $P(A \mid Y < 1)$

- Using conditional probability, Law of total probability & Bayes Rule
- For less writting, I use $P(Y)$ short for $P((x,y))$. Since Y is a R.V. depending on X, I'll shorten it to just $P(Y)$

$$P(A \mid Y \leq 1) = \frac{P(Y \leq 1 \mid A)\,P(A)}{P(Y \leq 1 \mid A)\,P(A) + P(Y \leq 1 \mid B)\,P(B)}$$

Known :

$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$

- Here we integrate $f(X, Y \mid A)$ over the interval $[0, 1]$ to get $P((x,y) \leq 1 \mid A)$

$\Rightarrow$ But this integration is just the area so here I take the area of a triangle

$$P(x, y \leq 1 \mid A) = \frac{1}{2}(1)(1) = \frac{1}{2}$$

Scale by the pdf of A $\quad \frac{1}{2}(1) = \frac{1}{2}$

Similarly for $P((X,Y) \leq 1 \mid B)$

$$P((x,y) \leq 1 \mid B) = \frac{1}{2}(1)(1) = \frac{1}{2}$$

$\Rightarrow$ here I scale by pdf $= \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ to make this geometrically sounds...

$\Rightarrow$ Putting this together

$$P(A \mid Y \leq 1) = \frac{P(Y \leq 1 \mid A)\, P(A)}{P(Y \leq 1 \mid A)\, P(A) + P(Y \leq 1 \mid B)\, P(B)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{1}{\cancel{4}}}{\frac{3}{\cancel{8}\,2}} = \frac{2}{3}$$

$$P(A \mid Y \leq 1) = \frac{2}{3}$$