

# Answers to Odd Problems

## 1. Basic Concepts

1.  $\Omega = \{ABC, ACB, BAC, BCA, CAB, CBA\}$ , 3.  $1/4$ , 5. (a)  $16/36$ , (b)  $20/36$ , 7.  $3/4$ , 9.  $1:4/57$ ,  $2:16/57$ ,  $3:15/57$ ,  $4:12/57$ , 11. Number of outcomes out of 216: (a) 1, (b) 3, (c) 6, (d) 10, (e) 15, (f) 21, (g) 25, (h) 27, 13. 90, 15. 77, 17. 0.2, 19. Yes, 21. Yes, 25. 0.44, 27.  $19/27$ , 29. (a) 0.048, (b) 0.296, (c) 0.464, (d) 0.192, 31. 0.4032, 33. 0.4914, 35.  $0:6/36$ ,  $1:10/36$ ,  $2:8/36$ ,  $3:6/36$ ,  $4:4/36$ ,  $5:2/36$ , 37. The same as for the sum of two ordinary dice, 39. 7, 41.  $1/4$ , 43.  $-17/216$ , 45. Flag or Joker:  $-13/54$ , 20:  $-12/54$ , 10:  $-10/54$ , 5:  $-12/54$ , 2:  $-9/54$ , 1:  $-6/54$ , 47.  $32/10$ , 49. 26, 51. mean 5.8125, variance 1.03, 53. Mean 3.888, variance 0.432. 55. No.

## 2. Combinatorial Probability

1.  $9!$ , 3. 3,360, 5. (a) 151,200, (b) 210, 7. 55, 9. (a)  $26^3 10^3$  (b) 0.6391, 11. 0.0605, 13. (a) 540, (b) 372, 15. 118,800, 17. (a) 240, (b) 480, 19.  $(P_{15,5})^4 P_{15,4}$ , 21. (a) 120, (b) 60, (c) 504,000, (d) 34,650, 23. (a)  $50!/(8!7!)$ , (b)  $C_{50,8} C_{50,7}$ , 25. 103,680, 27. (a)  $4!=24$ , (b) 576, (c) 105, (d) 2,520, (e) 40,320, (f) 384, 29.  $60/1024$ , 31. (a) 0.152, (b) 0.618, 33. (a) 0.05469, (b) 0.92981, 35.  $21/32$ , 37. (a) 0.3874, (b) 0.3773, (c) 0.3725, 39. 0.3297, 41. Poisson: 0.778801, Exact: 0.778703, 43. 0.1889, 45. 0.9735, 47. 0.4232, 49. 0.11192, 51. 0.6664, 53.  $10/21$ . 55. 0.3343, 57.  $C_{26,5} C_{4,1} / C_{30,6}$ , 59. 0.3684, 61. (a) 0.4085, (b) 0.5107, (c) 0.0766, 63.  $C_{12,5} \cdot C_{10,5} \cdot 5!$ , 65.  $5.47 \times 10^{-4}$ , 67.  $1/2$ , 69. 0.5814, 71. 0.2285, 73. (a) 0.7969, (b) 0.1992, (c) 0.0038, 75. (a) Five of a kind: 0.000771, (b) four of a kind: 0.019290, (c) full house: 0.038580, (d) three of a kind: 0.154320, (e) two pair: 0.231481, (f) one pair: 0.462962, (g) no pair: 0.092592, 77.  $5/32$ , 79. (a) 0.9, (b) 0.3, 81. 0.3439, 83. 0.028, 85.  $P(A) = 0.1655$ ,  $P(A)/P(B) = 3.9971$ , 87. First bound: 0.1667, second: 0.1551, third: 0.1555213, exact: 0.1555124

## 3. Conditional Probability

1.  $2/3$ , 3.  $4/25$ , 5. (a) 0.4385, (b) 0.4561, (c) 0.1052, 7.  $6/7$ , 9.  $2/3$ , 11.  $1/3$ , 13.  $P(A \cap B) = 1/8$ ,  $P(B) = 3/8$ ,  $P(A \cup B) = 1/2$ , 15. 0.6, 17. Put 1 white ball in the first urn and all the other balls in the second. 19.  $11/216$ , 21. 14: 146/1296,

23. 0.9568, 25. 0.17, 27.  $2/5$ , 29. 0.7, 31. 0.4918, 33.  $1/4$ ,  $1/3$ , 35.  $1/51$ , 37. (a) 0.9, (b)  $1/730$ , 39.  $12/29$ , 41.  $8/9$ , 43.  $56/65$ , 45.  $4/5$ , 47.  $1/7$ , 49. (a)  $1/2$ , (b)  $5/9$ , 51.  $8/9$ , 53.  $5/6$ , 55.  $4/7$ , 57.  $4/7$ ,  $2/7$ ,  $1/7$ , 59. 0.062, 0.3387, 61.  $P(N_1 = k, N_6 = j) = P(N_1 = j, N_6 = k) = (4/6)^{j-1}(5/6)^{k-j-1}(1/6)^2$ , 63. (a)  $1/6$  (b)  $1/2$ ,

	Y	X=1	2	3
65.	1	1/9	2/9	1/3
	2	1/18	0	1/6
	3	0	1/9	0

#### 4. Markov Chains

1. (a)  $x = 0.4$ ,  $y = 0.4$ ,  $z = 0.6$ , (b)  $x = 0.2$ ,  $y = 0.5$ ,  $z = 0.2$ , 3. 0.294, 5.  $13/32$ , 7. (b) 0.22, 0.166, 9. 51.2% in 1980, 56.8% in 1990, and 60.9% in 2000, 11. (a) 0.55, (b) 0.575, (c) 0.6, 13. 0.7825, 0.727291,  $8/11 = 0.727272$ , 15. 38%, 25%,  $8/33 = 24\%$ , 17. 0.211, 0.286, 0.502, 19.  $4/19$ , 21. (b)  $7/13$ , 23.  $4/5$ , 25. 0.25, 0.5, 0.25, 27. (a) 0.35, 0.34, 0.31, (b) 0.5294, 0.3235, 0.1470, 29. Long run frequencies are A:0.2, C:0.55, T:0.25, 31. (b) 0.2321, 0.1964, 0.5714, 33. 0.2817, 35.  $\pi(0) = 100/122$ ,  $\pi(1) = 10/122$ ,  $\pi(2) = 2/122$ ,  $\pi(3) = 10/122$ , 37.  $\pi(0) = 0.1$ ,  $\pi(1) = 0.4$ ,  $\pi(2) = 0.3$ ,  $\pi(3) = 0.2$ , 39.  $1/3$ , 41. 16.666, 43. (a)  $1/4$ , (b) 3.916, 45. 0.125/0.685

#### 5. Continuous Distributions

1.  $c = 1/3$ , 3. (a)  $1/2$ , (b) 0.3, (c) 0.05, 5. (a)  $15/28$ , (b)  $127/49$ , (c) 2.304, 7. Yes.  $x^{-2}e^{-1/x}$ , 9. (a)  $x^2/4$  for  $0 \leq x \leq 2$ , (b)  $1/4$ , (c)  $7/16$ , (d)  $\sqrt{2}$ , 11. (a)  $x^{1/2}$  for  $0 \leq x \leq 1$ , (b)  $1 - 1/\sqrt{3}$ , (c)  $1/6$ , (d)  $1/4$ , 13. any number in  $[0, 1]$ , 15. (a)  $1 - x^{-3}$  (b)  $(1 - u)^{-1/3}$ , 21.  $y^{-1/2}/2$ , 23.  $1 - \exp(-e^{-x})$ , 25. (a)  $f(y) + f(-y)$  for  $0 \leq y \leq 1$ , (b)  $\{f(\sqrt{z}) + f(-\sqrt{z})\}/2\sqrt{z}$  for  $0 \leq z \leq 1$ , 27. (a)  $c = 1$ , (b)  $3/8$ , 29.  $(1 - z)^2/2$  for  $0 \leq z \leq 1$ , 31.  $7/16$ , 33.  $F(x, y) = \min\{x, y\}$  if  $x, y > 0$  and  $\min\{x, y\} \leq 1$ ,  $F(x, y) = 1$  if  $\min\{x, y\} > 1$ , 0 otherwise, 35.  $F_X(x) = F_{X,Y}(x, \infty) = \lim_{y \rightarrow \infty} F_{X,Y}(x, y)$ , 37. (a)  $f_X(x) = (2/\pi)\sqrt{1 - x^2}$ , (b)  $f_Y(y|X = x) = 1/2\sqrt{1 - x^2}$  for  $-\sqrt{1 - x^2} < y < \sqrt{1 - x^2}$ , 39.  $f_X(x) = 3(1 - x)^2$  for  $0 < x < 1$ ,  $f_Y(y) = 6y(1 - y)$  for  $0 < y < 1$ , (b)  $f_X(x|Y = y) = 1/(1 - y)$ ,  $0 < x < 1 - y$

#### 6. Limit Theorems

1.  $-1$ , 3.  $-3/5$ , 5. 5.03, 7. 3.114, 9. 123.136, 11. Mean 6, variance 43, 13. (a) mean 2, variance 1.82, (b) 1 boy and 1 girl, 15. Mean 14.7, variance 38.99, 17. (a)  $P(X \geq 75) \leq 2/3$ , (b)  $P(40 < X < 60) \geq 0.75$ , 19. Chebyshev:  $\leq 1/4$ , normal approx: 0.0455, 21. 0.484, 23. 0.041, 25. 0.0016, 27. 24, 29. 0.0085, 31. 0.1056, 33. (a) 0.2658, (b) 0.2565, (c) 0.2475, 35. (a) 0.0566, (b) 0.0884, 37. 157, 39. 373, 41. 0.3897, 43. 527, 45. 0.0116, 47. [0.508, 0.528], 49. [0.0044, 0.0356], 51. 52.33, 53. 0.0081, 55. Yes, 57. No, 59. 23