3.3) Compute $V[D]$

Using Law of total variance

$$V[D] = V[E[D|C]] + E[V[D|C]]$$

Note: $E[D|C] = \frac{3}{2}c$

$$V[D|C] = \frac{1}{12}c^2$$

$$V[E[D|C]] = V\left[\frac{3}{2}c\right] = E\left[\left(\frac{3}{2}c\right)^2\right] - \left(E\left[\frac{3}{2}c\right]\right)^2$$

$$E\left[\left(\frac{3}{2}c\right)^2\right] = \int_0^2 \frac{9}{4}c^2 \frac{1}{2}\,dc = \frac{\cancel{9}^{3}}{8}\frac{c^3}{\cancel{3}_1}\Big|_0^2 = \frac{3c^3}{8}\Big|_0^2$$

$$= \frac{3\cdot\cancel{8}}{\cancel{8}} = 3$$

$$E\left[\frac{3}{2}c\right] = \int_0^2 \frac{3}{2}c\,\frac{1}{2}\,dc = \frac{3}{4}\frac{c^2}{2}\Big|_0^2 = \frac{3\cdot\cancel{4}}{\cancel{4}\cdot 2} = \frac{3}{2}$$

$$V[E[D|C]] = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$

$$E[V[D|C]] = E\left[\frac{1}{12}c^2\right]$$

$$= \int_0^2 \frac{1}{12}c^2 f_C(c)\, dc = \int_0^2 \frac{1}{12}c^2 \frac{1}{2}\, dc$$

$$= \int_0^2 \frac{1}{24} c^2 dc = \left.\frac{c^3}{24 \cdot 3}\right|_0^2 = \frac{\cancel{8}^{1}}{\cancel{24}_{3} \cdot 3} = \frac{1}{9}$$

$$V[D] = V[E[D|C]] + E[V[D|C]]$$

$$= \frac{3}{4} + \frac{1}{9}$$

$$\boxed{V[D] = \frac{31}{36}}$$