3) given: Pasta making

piece starts in machine A w/ prob. $\frac{1}{2}$

machine B w/ prob. $\frac{1}{2}$

Initial length of the piece is B.V. X

In machine A $f_A(X) \sim U(0,1)$
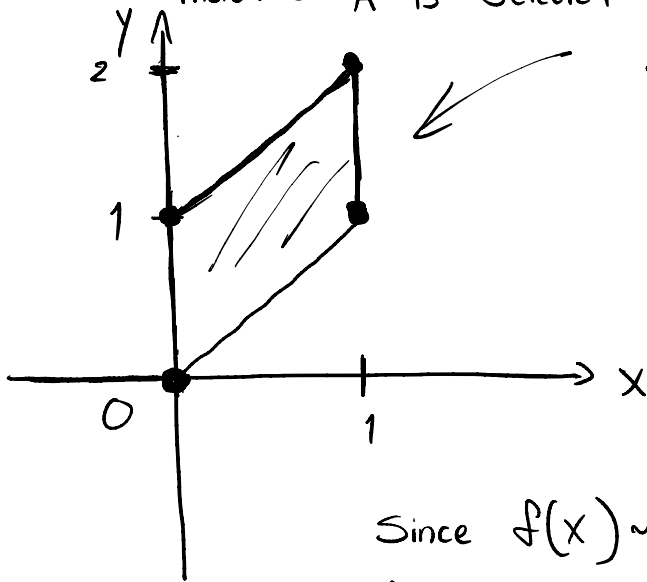
B $f_B(x) \sim U(0,2)$

stretch $f_S(Y) \sim U(X, X+1)$

a) Sketches

1) $f_{X,Y|A}(x,y \mid A)$ 2) $f_{X,Y|B}(x,y \mid B)$

3.1.1) Graph of joint pdf of X & Y, given that machine A is selected

$$f_{X,Y|A}(x,y \mid A)$$

Since $f(x) \sim U(0,1)$ for machine A being selected

$$f_X(x) = \begin{cases} 1; & 0 \le x \le 1 \\ 0; & \text{otherwise} \end{cases}$$

$$f(y) \sim U(x, x+1)$$

⇒ implies that $X=0$ $Y=0$
   or $Y = X$

⇒ y can never be less than X
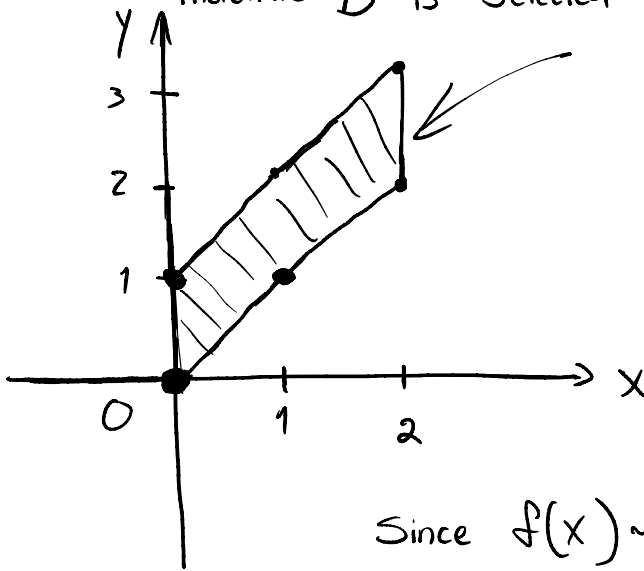
   $Y \ge X$

⇒ Let's say $X = 1$

   $Y \sim U(x, x+1)$

   $\min[Y] = 1$

   $\max[Y] = 2$

   Same logic for $X = 0$

3.1.1) Graph of joint pdf of X & Y, given that machine B is selected

$$f_{X,Y|B}\left(x, y \mid B\right)$$

Since $f(x) \sim U(0, 2)$

for machine B being selected

$$f_X(x) = \begin{cases} \frac{1}{2}; & 0 \le x \le 2 \\ 0; & \text{otherwise} \end{cases}$$

$$f(y) \sim U(x, x+1)$$

⟹ implies that X=0 Y=0
   or $Y = X$

⟹ y can never be less than X
   $Y \ge X$

⟹ Let's say X= 1
   $Y \sim U(x, x+1)$ so max of
   Y is 2

= Let's say X=2
   max of Y is 3 by
   same logic @ above