4.3) Show $V[\partial] \geq V[\varepsilon]$

0 by property of BLP

$$V[\varepsilon] = E[\varepsilon^2] - E[\cancel{\varepsilon}]^2$$

0

$$V[\partial] = E[\partial^2] - E[\cancel{\partial}]^2$$

$$V[\partial] \geq V[\varepsilon]$$

$$E[\partial^2] \overset{?}{\geq} E[\varepsilon^2]$$

$$E[Y^2 - 2Yg(S) + g(S)^2] \overset{?}{\geq} E[Y^2 - 2Yf(X) + f(X)^2]$$

$$E[g(S)^2] - 2E[Yg(S)] - E[g(S)]^2 + E[g(S)]^2 \geq E[f(X)^2] - 2E[Yf(X)] - E[f(X)]^2 + E[f(X)]^2$$

0 0

$$E[g(S)]^2 - 2E[Yg(S)] \geq E[f(X)]^2 - 2E[Yf(X)]$$

$$E[g(S)]^2 - 2E[Yg(S)] \geq E[f(X)]^2 - 2E[Yf(X)]$$

It is noted here that

$$g(S) = \tilde{\alpha} + \tilde{\beta} S = \tilde{\alpha} + \beta(X + U)$$

$$= \tilde{\alpha} + \tilde{\beta} X + \tilde{\beta} U$$

$$E[\tilde{\alpha} + \tilde{\beta} X + \beta U]^2 > E[\alpha + \beta X]^2$$

& the second term

$$E[\tilde{\alpha} Y + \tilde{\beta} XY + \beta UY] > E[\alpha Y + \beta Y]$$

Since first term go by $c^2$ & second term go by $c$

the LHS is larger than the right handside

$$\therefore \quad V[\partial] \geq V[\varepsilon]$$