1. Definitions of the Function Suite

In this section, 28 constrained optimization problems are proposedwhich can be transformed into the following format:

$$\text{Minimize: } f(X), \quad X = (x_1, x_2, ..., x_n) \text{ and } X \in S \tag{1}$$

$$\text{Subject to: } \begin{matrix} g_i(X) \le 0, & i = 1, ..., p \\ h_j(X) = 0, & j = p+1, ..., m \end{matrix} \tag{2}$$

Usually equality constraints are transformed into inequalities of the form

$$\left|h_j(X)\right| - \varepsilon \le 0 \text{ , for } \quad j = p+1, ..., m \qquad \ldots (3)$$

A solution X is regarded as feasible if $g_i(X) \le 0$, *for* $i = 1, \ldots, p$ and $\left|h_j(X)\right| - \varepsilon \le 0$, for $j = p+1, ..., m$. In this special session ε is set to 0.0001.

C01: $\text{Min } f(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2 \quad z = x - o$

$$g(x) = \sum_{i=1}^{D} \left[z_i^2 - 5000\cos(0.1\pi z_i) - 4000 \right] \le 0$$

$$x \in [-100, 100]^D$$

C02: $\text{Min } f(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2 \quad z = x - o, \; y = M * z$

$$g(x) = \sum_{i=1}^{D} \left[y_i^2 - 5000\cos(0.1\pi y_i) - 4000 \right] \le 0$$

$$x \in [-100, 100]^D$$

C03: $\text{Min } f(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2 \quad z = x - o$

$$g(x) = \sum_{i=1}^{D} \left[z_i^2 - 5000\cos(0.1\pi z_i) - 4000 \right] \le 0$$

$$h(x) = -\sum_{i=1}^{D} z_i \sin(0.1\pi z_i) = 0$$

$$x \in [-100, 100]^D$$

C04: $\text{Min } f(x) = \sum_{i=1}^{D} \left[z_i^2 - 10\cos(2\pi z_i) + 10 \right] \quad z = x - o$

$$g_1(x) = -\sum_{i=1}^{D} z_i \sin(2 z_i) \le 0$$

$$g_2(x) = \sum_{i=1}^{D} z_i \sin(z_i) \le 0$$

$$x \in [-10, \; 10]^{\mathrm{D}}$$

C05: $\text{Min } f(x) = \sum_{i=1}^{D-1}(100(z_i^2 - z_{i+1})^2 + (z_i - 1)^2)$ $z = x - o,\ y = M_1 * z,\ w = M_2 * z$

$$g_1(x) = \sum_{i=1}^{D}\left[y_i^2 - 50\cos(2\pi y_i) - 40\right] \le 0$$

$$g_2(x) = \sum_{i=1}^{D}\left[w_i^2 - 50\cos(2\pi w_i) - 40\right] \le 0$$

$$x \in [-10,\ 10]^D$$

C06: $\text{Min } f(x) = \sum_{i=1}^{D}\left[z_i^2 - 10\cos(2\pi z_i) + 10\right]$ $z = x - o$

$$h_1(x) = -\sum_{i=1}^{D} z_i \sin(z_i) = 0$$

$$h_2(x) = \sum_{i=1}^{D} z_i \sin(\pi z_i) = 0$$

$$h_3(x) = -\sum_{i=1}^{D} z_i \cos(z_i) = 0$$

$$h_4(x) = \sum_{i=1}^{D} z_i \cos(\pi z_i) = 0$$

$$h_5(x) = \sum_{i=1}^{D} (z_i \sin(2 * \sqrt{|z_i|})) = 0$$

$$h_6(x) = -\sum_{i=1}^{D} (z_i \sin(2 * \sqrt{|z_i|})) = 0$$

$$x \in [-20,\ 20]^D$$

C07: $\text{Min } f(x) = \sum_{i=1}^{D}(z_i \sin(z_i))$ $z = x - o$

$$h_1(x) = \sum_{i=1}^{D}(z_i - 100\cos(0.5 z_i) + 100) = 0$$

$$h_2(x) = -\sum_{i=1}^{D}(z_i - 100\cos(0.5 z_i) + 100) = 0$$

$$x \in [-50,\ 50]^D$$

C08: $\text{Min } f(x) = \max(z)$ $z = x - o,\ y_l = z_{(2l-1)},\ w_l = z_{(2l)}$ *where* $l = 1, \ldots, D/2$

$$h_1(x) = \sum_{i=1}^{D/2}\left(\sum_{j=1}^{i} y_j \right)^2 = 0$$

$$h_2(x) = \sum_{i=1}^{D/2}\left(\sum_{j=1}^{i} w_j \right)^2 = 0$$

$$x \in [-100,\ 100]^D$$

C09: $\text{Min } f(x) = \max(z) \quad z = x - o,\ y_l = z_{(2l-1)},\ w_l = z_{(2l)} \ \textit{where}\ l = 1,...,D/2$

$$g(x) = \prod_{i=1}^{D/2} w_i \le 0$$

$$h(x) = \sum_{i=1}^{D/2-1} (y_i^2 - y_{i+1})^2 = 0$$

$$x \in [-10,\ 10]^D$$

C10: $\text{Min } f(x) = \max(z) \qquad z = x - o$

$$h_1(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2 \ = \ 0$$

$$h_2(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 = 0$$

$$x \in [-100,\ 100]^D$$

C11: $\text{Min } f(x) = \sum_{i=1}^{D} (z_i) \qquad z = x - o$

$$g(x) = \prod_{i=1}^{D} z_i \le 0$$

$$h(x) = \sum_{i=1}^{D-1} (z_i - z_{i+1})^2 = 0$$

$$x \in [-100,\ 100]^D$$

C12: $\text{Min } f(x) = \sum_{i=1}^{D} (y_i^2 - 10\cos(2\pi y_i) + 10),\ y = x - o$

$$g_1(x) = 4 - \sum_{i=1}^{D} |y_i| \le 0$$

$$g_2(x) = \sum_{i=1}^{D} y_i^2 - 4 = 0$$

$$x \in [-100,\ 100]^D$$

C13: $\text{Min } f(x) = \sum_{i=1}^{D-1} (100(y_i^2 - y_{i+1})^2 + (y_i - 1)^2),\ y = x - o$

$$g_1(x) = \sum_{i=1}^{D} (y_i^2 - 10\cos(2\pi y_i) + 10) - 100 \le 0$$

$$g_2(x) = \sum_{i=1}^{D} y_i - 2D \le 0$$

$$g_3(x) = 5 - \sum_{i=1}^{D} y_i \le 0$$

$$x \in [-100,\ 100]^D$$

C14: $\text{Min } f(x) = -20 \cdot \exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} y_i^2}) + 20 - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi y_i)) + e,\ y = x - o$

$g(x) = \sum_{i=2}^{D} y_i^2 + 1 - |y_1| \le 0$

$h(x) = \sum_{i=1}^{D} y_i^2 - 4 = 0$

$x \in [-100,\ 100]^{\mathrm{D}}$

C15: $\text{Min } f(x) = \max\{|y_i|, 1 \le i \le D\},\ y = x - o$

$g(x) = \sum_{i=1}^{D} y_i^2 - 100D \le 0$

$h(x) = \cos f(x) + \sin f(x) = 0$

$x \in [-100,\ 100]^{\mathrm{D}}$

C16: $\text{Min } f(x) = \sum_{i=1}^{D} |y_i|,\ y = x - o$

$g(x) = \sum_{i=1}^{D} y_i^2 - 100D \le 0$

$h(x) = (\cos f(x) + \sin f(x))^2 - \exp(\cos f(x) + \sin f(x)) - 1 + \exp(1) = 0$

$x \in [-100,\ 100]^{\mathrm{D}}$

C17: $\text{Min } f(x) = \frac{1}{4000}\sum_{i=1}^{D} y_i^2 + 1 - \prod_{i=1}^{D}\cos(\frac{y_i}{\sqrt{i}}),\ y = x - o$

$g(x) = 1 - \sum_{i=1}^{D}\text{sgn}(|y_i| - \sum_{j=1,2..D, j\ne i}^{D} y_j^2 - 1) \le 0$

$h(x) = \sum_{i=1}^{D} y_i^2 - 4D = 0$

$x \in [-100,\ 100]^{\mathrm{D}}$

C18: $\text{Min } f(x)=\sum_{i=1}^{D}(z_i^2-10\cos(2\pi z_i)+10)$, $z_i=\begin{cases} y_i, if\ |y_i|<0.5 \\ 0.5*round(2*y_i), otherwise \end{cases}$, $y=x-o$

$$g_1=1-\sum_{i=1}^{D}|y_i|\le 0$$

$$g_2=(x)=\sum_{i=1}^{D}y_i^2-100D\le 0$$

$$h(x)=\sum_{i=1}^{D}100(y_i^2-y_{i+1})^2+\prod_{i=1}^{D}\sin^2(y_i-1)\pi=0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C19: $\text{Min } f(x)=\sum_{i=1}^{D}(|y_i|^{0.5}+2\sin y_i^3)$, $y=x-o$

$$g_1(x)=\sum_{i=1}^{D-1}(-10\exp(-0.2\sqrt{y_i^2+y_{i+1}^2}))+(D-1)\cdot 10/\exp(-5)\le 0$$

$$g_2(x)=\sum_{i=1}^{D}\sin^2(2y_i)-0.5D\le 0$$

$$x\in[-50,50]^{D}$$

C20 : $\text{Min } f(x)=\sum_{i=1}^{D-1}g(y_i,y_{i+1})+g(y_D,y_1), g(y_i,y_{i+1})=0.5+\frac{(\sin^2(\sqrt{y_i^2+y_{i+1}^2})-0.5)}{(1+0.001(\sqrt{y_i^2+y_{i+1}^2}))^2}$,

$y=x-o$

$$g_1(x)=\cos^2(\sum_{i=1}^{D}y_i)-0.25\cos(\sum_{i=1}^{D}y_i)-0.125\le 0$$

$$g_2(x)=\exp(\cos(\sum_{i=1}^{D}y_i))-\exp(0.25)\le 0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C21: $\text{Min } f(x)=\sum_{i=1}^{D}(y_i^2-10\cos(2\pi y_i)+10)$, $z=M(x-o)$

$$g_1(x)=4-\sum_{i=1}^{D}|z_i|\le 0$$

$$g_2(x)=\sum_{i=1}^{D}z_i^2-4=0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C22: Min $f(x)=\sum_{i=1}^{D}(100(z_i^2-x_{i+1})^2+(z_i-1)^2)$, $z=M(x-o)$

$$g_1(x)=\sum_{i=1}^{D}(z_i^2-10\cos(2\pi z_i)+10)-100\le 0$$

$$g_2(x)=\sum_{i=1}^{D}z_i-2D\le 0$$

$$g_3(x)=5-\sum_{i=1}^{D}z_i\le 0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C23: Min $f(x)=-20\cdot\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}z_i^2})+20-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i))+e$, $z=M(x-o)$

$$g(x)=\sum_{i=2}^{D}z_i^2+1-|z_1|\le 0$$

$$h(x)=\sum_{i=1}^{D}z_i^2-4=0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C24: Min $f(x)=\max\{|z_i|,1\le i\le D\}$, $z=M(x-o)$

$$g(x)=\sum_{i=1}^{D}z_i^2-100D\le 0$$

$$h(x)=\cos f(z)+\sin f(z)=0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C25: Min $f(x)=\sum_{i=1}^{D}|z_i|$, $z=M(x-o)$

$$g(x)=\sum_{i=1}^{D}z_i^2-100D\le 0$$

$$h(x)=(\cos f(z)+\sin f(z))^2-\exp(\cos f(z)+\sin f(z))-1+\exp(1)=0$$

$$x\in[-100,\ 100]^{\mathrm{D}}$$

C26: $\text{Min } f(x) = \frac{1}{4000}\sum_{i=1}^{D} y_i^2 + 1 - \prod_{i=1}^{D} \cos(\frac{y_i}{\sqrt{i}})$, $z = M(x-o)$

$$g(x) = 1 - \sum_{i=1}^{D} \text{sgn}(|z_i| - \sum_{j=1,2..D, j\neq i}^{D} z_j^2 - 1) \le 0$$

$$h(x) = \sum_{i=1}^{D} z_i^2 - 4D = 0$$

$$x \in [-100,\ 100]^{\text{D}}$$

C27: $\text{Min } f(x) = \sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10)$, $z_i = \begin{cases} y_i, if\ |y_i| < 0.5 \\ 0.5 * round(2 * y_i), otherwise \end{cases}$,

$z = M(x-o)$

$$g_1 = 1 - \sum_{i=1}^{D} |y_i| \le 0$$

$$g_2 = (x) = \sum_{i=1}^{n} y_i^2 - 100D \le 0$$

$$h(x) = \sum_{i=1}^{D} 100(y_i^2 - y_{i+1})^2 + \prod_{i=1}^{D} \sin^2(y_i - 1)\pi = 0$$

$$x \in [-100,\ 100]^{\text{D}}$$

C28: $\text{Min } f(x) = \sum_{i=1}^{D} (|z_i|^{0.5} + 2\sin z_i^3)$, $z = M(x-o)$

$$g_1(x) = \sum_{i=1}^{D-1} (-10\exp(-0.2\sqrt{z_i^2 + z_{i+1}^2})) + (D-1)\cdot 10 / \exp(-5) \le 0$$

$$g_2(x) = \sum_{i=1}^{D} \sin^2(2z_i) - 0.5D \le 0$$

$$x \in [-50,\ 50]^{\text{D}}$$