Spectral Analysis of Coupled Electrical Oscillations

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Abstract

This experiment investigates the behaviour of coupled electric oscillations using Fourier analysis in different circuit configurations. Task I, a low-point capacitively coupled resonant circuit was analysed. The eigenfrequencies remained at $f_{1,\text{Task 1}} = 8.7738$ kHz across all coupling values, while $f_{2,\text{Task 1}}$ decreased with increasing coupling capacitance $C_{K,\text{Task 1}}$. The coupling factor $k_{C,T}$ followed the expected inverse trend with $C_{K,\text{Task 1}}$, and curve fitting showed a capacitance of $C_{\text{Task 1}} = 0.0208\,\mu F$ with $r^2 = 0.9972$. In Task II, a high-point capacitively coupled configuration was studied. Unlike Task I, f_2 remained constant at $f_{2,\text{Task 2}} = 9.15527$ kHz, while $f_{1,\text{Task 2}}$ decreased with increasing $C_{K,\text{Task 2}}$. The coupling factor $k_{C,H}$ showed an increasing trend and the extracted capacitance was $C_{\text{Task 2}} = 0.0205\mu F$ with $r^2 = 0.9977$, showing excellent agreement with Task I (relative error $\approx 1.45\%$). The two configurations highlighted the symmetric but opposite behaviour of in-phase and out-of-phase modes. Task III focused on beat oscillations using $C_{K,\text{Task 3}} = 0.008\mu F$. The beat period measured from the time trace was $T_{S,\text{measured}} = 4.185 \cdot 10^{-4}$ s, compared to $T_{S,\text{calculated}} = 4.369 \cdot 10^{-4}$ s from the frequency spectrum, resulting in a small relative error of $\approx 4.03\%$. The eigenfrequencies matched exactly with those from Task II, validating both time- and frequency-domain approaches.

1 Introduction

The paper explores the resultant frequency spectrum for 3 different circuit configurations utilising 2 capacitors and 2 inductors with a switch connected across a power source and a variable coupling capacitance. The 3 said configurations are as follows;

(1) Capacitive coupling, low-point circuit; where from the voltage and the frequency spectrum is measured across and the coupling factor and the capacitance of the set up is determined from the in-phase and the out-of-phase frequencies for a range of 10 coupling capacitance.

(2)Capacitive coupling, high-point circuit; where from the voltage and the frequency spectrum is measured across and the respective coupling factor, the capacitance (which is then compared with the capacitance from configuration (1)) is determined from the in-phase and the out-of-phase frequencies for a range of 10 coupling capacitance.

(3)Short Circuit of capacitive coupling, high-point circuit: Where beat period of one selected coupling factor is determined and compared with the configuration (2).

Before conducting the experiment, the following 'Key Words' are used to prepare for the theory and application;

- Resonant Circuit, Coupled Resonant Circuits
- Differential Equation of damped electrical oscillations, solutions, natural frequency
- Fourier transformation, frequency spectra, Nyquist-Shannon theorem, aliasing

With the sources for preparation being mainly Electrodynamics and Optics (Undergraduate Lecture Notes in Physics) by Wolfgang Demtroder.

2 Theoretical Basis

The following report investigates the properties of coupled electric oscillations for 3 configurations of a circuit combination of capacitor, inductor, switch, coupling capacitance and a power source.

The low-point circuit of capacitively coupled oscillations is characterized by its respective angular eigenfrequencies. These are derived by applying Kirchhoff's laws and solving the resulting coupled differential equations, yielding the following expressions:

$$\omega_1 = \omega_0 = \frac{1}{\sqrt{LC}} \tag{1}$$

$$\omega_2 = \frac{1}{\sqrt{L(\frac{1}{C} + \frac{2}{C_K})^{-1}}} \tag{2}$$

Where with the relation that $f_1 = \frac{w_1}{2\pi}$ and $f_2 = \frac{w_2}{2\pi}$, with the standard definition of a coupling factor, one gets the coupling factor of a low-point circuit as:

$$k_{C,T} = \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 + \omega_2^2} = \frac{f_1^2 - f_2^2}{f_1^2 + f_2^2} = \frac{C}{C_K + C} \tag{3}$$

Where the above equation presents a inverse relation between the coupled capacitance and the coupling factor, as $k_{C,T} \propto \frac{1}{C_k}$. The f_1 and f_2 correspond to in-phase and an out-of-phase current flow respectively.

The high-point circuit of capacitively coupled oscillations is similarly characterized by its respective angular frequencies. These are also derived by applying Kirchhoff's laws and solving the coupled differential equations, resulting in:

$$\omega_- = \frac{1}{\sqrt{L(C + 2C_K)}} \tag{4}$$

$$\omega_+ = \frac{1}{\sqrt{LC}} \tag{5}$$

Where the symbols + and $-$ correspond to the current directions through the circuit branches being in-phase or out-of-phase, respectively, with $C_K > 0$ and $\omega_- > \omega_+$.

The coupling factor for the high-point circuit is given by:

$$k_{C,H} = \frac{\omega_-^2 - \omega_+^2}{\omega_-^2 + \omega_+^2} = 1 - \frac{2f_+^2}{f_-^2 + f_+^2} = \frac{C_K}{C_K + C} \tag{6}$$

The angular frequency of the beat phenomenon is expressed as:

$$\omega_S = \frac{2\pi}{T_S} = \omega_+ - \omega_- \tag{7}$$

The normal period T corresponds to the average frequency of the two modes:

$$T = \frac{1}{f_{average}} = \frac{2}{f_- + f_+} \tag{8}$$

In a coupled resonant circuit, such as two LC circuits connected with a coupling capacitor, there are different modes of oscillation can occur depending on how the system is initially set.

The in-phase oscillation mode is when the currents in both LC circuits oscillate in the same direction at every moment in time. This mode is typically achieved by applying the same initial voltage to both circuits at the same time. As a result, both circuits resonate together at the same angular frequency, corresponding to the lower of the two eigenfrequencies of the system. On the other hand, the out-of-phase oscillation mode corresponds to the two circuits oscillating with currents flowing in opposite directions. This mode happens when applying a voltage that creates opposite potentials across the two circuits. The resulting oscillation occurs at the higher eigenfrequency of the system.

A more complex situation arises when the system is setted in such a way that both normal modes are present simultaneously. This happens, for example, when only one of the capacitors in the circuit is initially charged while the other is left uncharged. In this case, the two modes interfere, producing what is known as a beat oscillation. The energy alternately flows between the two circuits, resulting in periodic phenomenon where the observed oscillation grows and diminishes in a repeating cycle. This is characterized by a beat frequency, which is the difference between the two eigenfrequencies. In the frequency domain, this behaviour manifests as two distinct peaks in the frequency spectrum, corresponding to the frequencies of the in-phase and out-of-phase modes.

3 Experimental Exploration

3.1 Task I: Capacitively coupled resonant circuits, low-point circuit

3.1.1 Materials

- Two capacitors with the same unknown capacitance, $C_{\text{Task 1}}$
- Capacitor Decade used as a Variable Capacitor, $C_{k,\text{Task 1}}$
- 2 Coils with the same unknown inductance but 750 windings each, $L_{\text{Task 1}}$
- Standard Breadboard
- PicoScope 2000 Series; Used as a Voltmeter

- Electronic Switch with a connected Power Supply (unknown model)
- Residual Items; Wires, Computer, Notebook, PicoScope 7 T&M Software.

3.1.2 Methodology and Set up

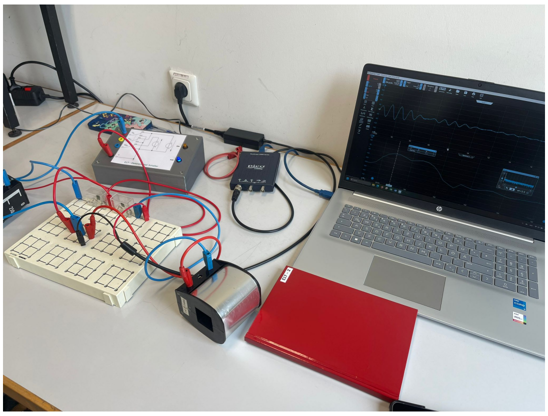

Image 1.a Setup for Task I

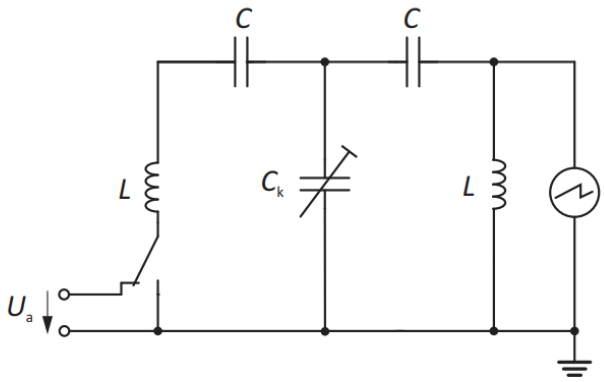

Image 1.b Low-Point-Circuit Scheme

In reference to the above schematic, Image 1.b, the circuit has to be set up with the Electric Switch connected across the U_a configuration. The Picoscope's software, the PicoScope 7 T&M Software, is set to showcase voltage and the voltage frequency spectrum for a set value of $C_{k \text{ Task 1}}$. The said voltage measurement and the frequency spectrum is saved for further analysis. The experiment is repeated for 10 different values of the coupling capacitance, from which the coupling constant, $k_{c,T}$, and the in phase and out of phase frequencies ,$f_{1 \text{ Task 1}}$ and $f_{2 \text{ Task 1}}$ respectively, are determined. The range of $C_{k \text{ Task 1}}$ ran from $[0.01\ \mu F,\ 0.1\ \mu F]$ at an interval of $0.01\ \mu F$.

3.1.3 Data and Analysis

From the measurements, the following data presents the results of the frequency spectrum FFT, the Fast Fourier Transform, with the Amplitude, A_1 (dBu), against the frequency, $F_{1, \text{ Task 1}}$ (MHz) of all 10 different $C_{k \text{ Task 1}}$ in one graph.

The below figure, Figure 1, presents the frequency spectrum for Low Point circuit configuration. The graph presents a clear split between two frequencies, as corresponding to the $f_{1 \text{ Task 1}}$ and $f_{2 \text{ Task 1}}$. With $f_{1 \text{ Task 1}}$ consistently being the same for all the values of $C_{k \text{ Task 1}}$ - corresponding to in-phase oscillation mode. With the second frequency (corresponding to out-phase oscillation mode) spike gradually increasing and approaching closer to $f_{1 \text{ Task 1}}$ with an increasing coupling capacitance. This correlates to a gradual decoupling of the capacitors, as it is approaching a limit of one frequency spike. Additionally, the amplitude of $f_{1 \text{ Task 1}}$ is constant for all values of $C_{k \text{ Task 1}}$, which is line with how the supplied the voltage was constant and from the same power source. Individual frequencies for each $C_{k \text{ Task 1}}$, with the found coupling factor $k_{C,T}$ utilising equation 3 from Section 2, is presented in the below Table.

The below table, Table 1, presents a clear quantitative representation of the behaviour discussed above - a clear constant $f_{1 \text{ Task 1}}$ with an decreasing $f_{2 \text{ Task 1}}$. The found $k_{C,T}$

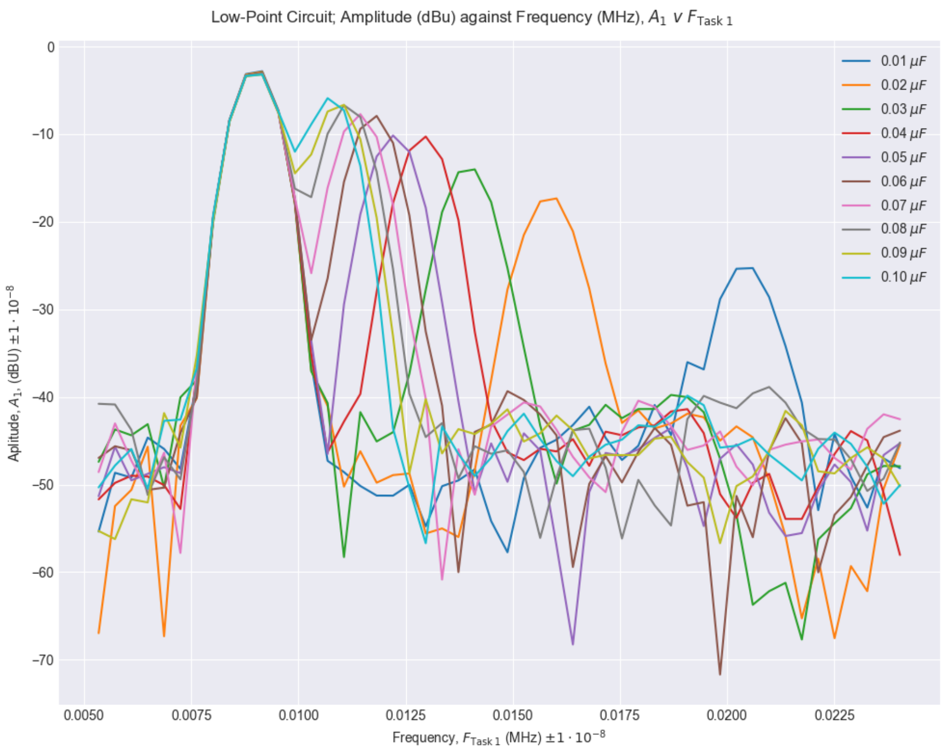

Figure 1: Low-Point Circuit; Amplitude (dBu) against Frequency (MHz), A_1 vs $F_{\text{Task 1}}$

presents a decreasing trend with an increasing $C_{k \text{ Task 1}}$, leading to the comparison that the coupling factor is smaller with when two frequency spikes are closer to each other. Furthermore, utilising the equation 3, a fit of the calculated kC, T and the range of $C_{k \text{ Task 1}}$ is made to find the capacitance of the capacitors, $C_{\text{Task 1}}$. As such, it follows,

C_K (μF)	$f_{1,\text{Task 1}}$ (kHz)	$f_{2,\text{Task 1}}$ (kHz)	$k_{C,T}$
0.01	8.7738	20.21790	0.683044
0.02	8.7738	15.64026	0.521267
0.03	8.7738	13.73291	0.420274
0.04	8.7738	12.58850	0.346107
0.05	8.7738	11.82556	0.289933
0.06	8.7738	11.44409	0.259622
0.07	8.7738	11.06262	0.227737
0.08	8.7738	10.68115	0.194212
0.09	8.7738	10.68115	0.194212
0.10	8.7738	10.29968	0.158983

Table 1: Low-Point Circuit; factor $k_{C,T}$ as a function of the coupling capacitor $C_{k,\text{Task 1}}$ and eigenfrequencies $f_{1,\text{Task 1}}$ and $f_{2,\text{Task 1}}$.

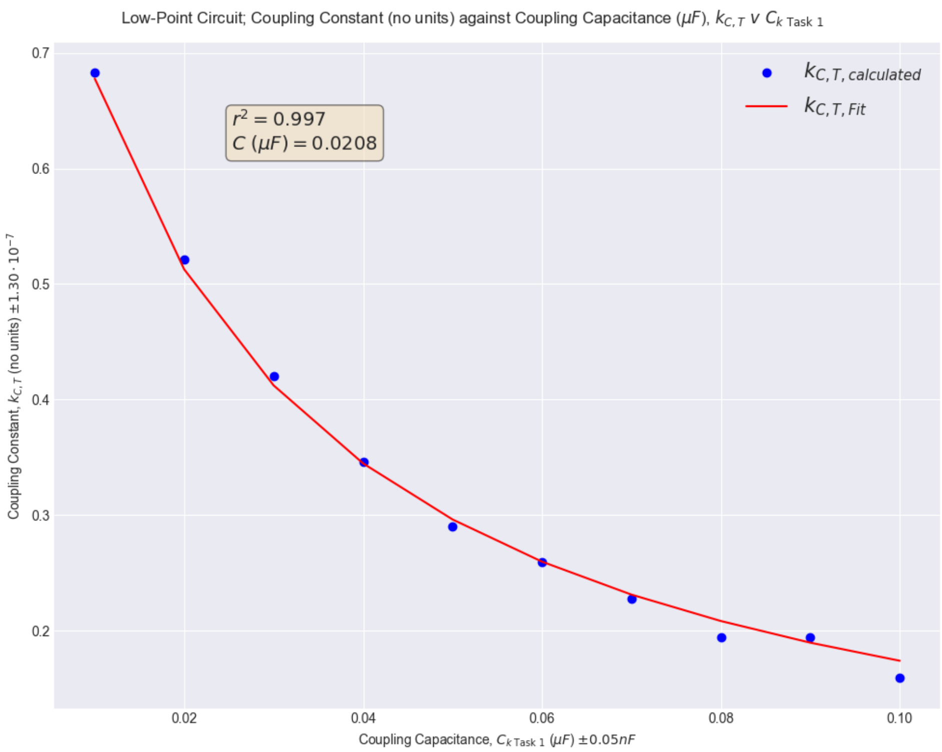

Figure 2: Low-Point Circuit; Coupling Constant (no units) against Coupling Capacitance (μF), $k_{C,T}$ vs $C_{k\ \text{Task 1}}$

The above trend of Figur 2, explicitly presents the relation $k_{C,T} \propto \frac{1}{C_k}$, and with a r^2 value of $0.997216120379656 \approx 0.997$ (where a score of 1 correlated to a perfect match between the measured data and the fit of $k_{C,T}$), the found Capacitance came out to $C_{\text{Task 1}} = 0.0208223341783957 75 \approx 0.0208\mu F$. As the fit is of high accuracy and the value of $C_{\text{Task 1}} = 0.0208\mu F$ is within a reasonable range for standard capacitors ("Standard Capacitor Values: Essential Guide for Beginner"), it can be said the data / measured and found values are ***accurate and precise***. More details in the Discussion

3.1.4 Discussion

The resulting behaviour of the Low Point Circuit configuration is of expected results, clearly presenting an in-phase and an out-of-phase oscillation modes. Where there is a clear decoupling with an increasing $C_{k \text{ Task 1}}$, where it leads from Equation 3 showing a $k \propto 1/C_K$ relation between the coupling constant and $C_{k,\text{Task 1}}$, where a $k_{\text{Task 1}}$ of 0 would correspond to a full decoupling. Additionally the constant frequency and amplitude of the found in-phase frequency is an align with the definition of the in-phase angular frequency, where $w_1 = 2\pi f_1 = \frac{1}{\sqrt{LC}}$, is a constant for a circuit where all the respective components have an inductance of L and capacitance of C. While the amplitude and frequency of $f_{2,\text{Task 1}}$, from equation 2, has the relation $2\pi f_2 = 1/\sqrt{L(\frac{1}{C} + \frac{2}{C_k})^{-1}}$, leads to correlation how an increasing C_k would correspond to dominating capacitance C, and lead to the approximation $\propto 1/\sqrt{L(\frac{1}{C})^{-1}} = \frac{1}{\sqrt{LC}}$, at the limit of a large C_k. Which is in line with the seen behaviour from Figure 1. As the graphs present behaviour in line with the theoretical expected results, there is level of accuracy presented in the data and such the data can be said to be of at a reasonable limit of an error.

Even if the values are within reasonable expected range, and there is no clear large deviation away for any one value set - which re-affirms the accuracy and precision of the data, and as such hints towards a set up of of most ideality one can perform in this experimental environment, possible deviations away from an ideal system can be attributed to issues developing due to capacitor stress. In comparison to the capacitor's size, consistent application of voltage puts on electric stress which jams and disturbs the flow of current to some extent. This creates, minor if the capacitor has not been over stimulated for longer periods of time but major issues if it has, some deviations of what the current flow would be for an ideal system. Especially if the capacitor had been functioning close to its maximum rated conditions, it would result in visible deviations from unstressed current flow. Additionally, the fluctuations in temperature would affect the capacitors' dissipation factor, dielectric strength, insulation resistance, and capacitance. These results would lead to not only a lose of energy (which in itself adds on to the non-ideality of the system) but lead to a difference in voltage flow - creating a constant deviation away from a close to ideal voltage spectrum. Which in turn would affect the found frequency spectrum and $C_{\text{Task 1}}$. There is also the effects of the lose of energy and resistance due to wires, inductors, and other components in the board.

3.1.5 Error Analysis

The uncertainty in the Amplitude / Intensity, Frequency and in $C_{k,\text{ Task 1}}$ correspond to the smallest measurable value from the the respective measurement tool, while the uncertainty in $C_{\text{Task 1}}$ and $k_{C,T}$ is found by propagating the measured data sets through the square root of the partial derivative of each variable parameter, as follows

$$\Delta k_{C,T} = \sqrt{\left(\frac{\partial k_{C,T}}{\partial f_{1,\text{ Task 1}}}\Delta f_{1,\text{ Task 1}}\right)^2 + \left(\frac{\partial k_{C,T}}{\partial f_{2,\text{ Task 1}}}\Delta f_{2,\text{ Task 1}}\right)^2} \tag{9}$$

$$\Delta C_{\text{ Task 1}} = \sqrt{\left(\frac{\partial C_{\text{ Task 1}}}{\partial C_{k \text{ Task 1}}}\Delta C_{k \text{ Task 1}}\right)^2 + \left(\frac{\partial C_{\text{ Task 1}}}{\partial k_{C,T}}\Delta k_{C,T}\right)^2} \tag{10}$$

coming out to uncertainties as follows

Parameter	Mean	Maximum
$\Delta f_{1,2 \text{ Task 1}}$ (MHz)	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
ΔA_1 (dBu)	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
$\Delta C_{k, \text{ Task 1}}$ (nF)	0.05	0.05
$\Delta k_{C,T}$ (no units)	$1.298... \cdot 10^{-7} \approx 1.30 \cdot 10^{-7}$	$1.434... \cdot 10^{-7} \approx 1.43 \cdot 10^{-7}$
$\Delta C_{\text{ Task 1}}$ (μF))	$0.000311.. \approx 0.0003$	$0.001077... \approx 0.0012$

Table 2: Mean and Maximum Uncertainty Parameters of Task 1

3.2 Task II: Capacitively coupled resonant circuits, high-point circuit

3.2.1 Materials

- Two capacitors with the same unknown capacitance, C_2
- Capacitor Decade used as a Variable Capacitor, $C_{k,\text{Task 2}}$
- 2 Coils with the same unknown inductance but 750 windings each, L_2
- Standard Breadboard
- PicoScope 2000 Series; Used as a Voltmeter
- Electronic Switch with a connected Power Supply (unknown model)
- Residual Items; Wires, Computer, Notebook, PicoScope 7 T&M Software.

3.2.2 Methodology and Set up

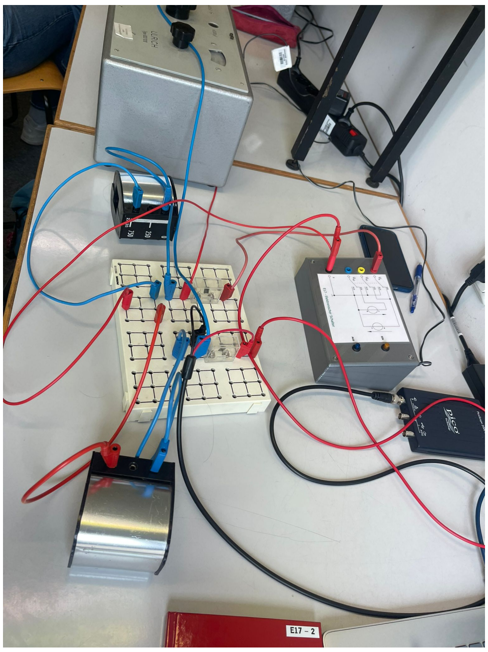

Image 2.a Setup for Task II

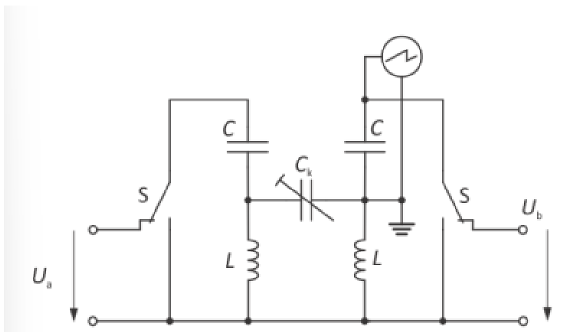

Image 2.b High-Point Circuit Scheme

The Picoscope is connected to the laptop and it is used to measure the voltage oscillations across one of the capacitors on the breadboard. As seen in Image 2.a, the wiring follows the configuration of the high-point circuit shown in Image 2.b. The Capacitor Decade, though not visible in the setup, is connected between the two capacitors on the breadboard and allows for capacitance values of $C_{k,\text{Task 2}}$ ranging from 0.004 to 0.1 μF.

The switch is connected to the breadboard via the red and blue cables. For each value set on the Capacitor Decade, a different cable was connected to either the Ub_1 or Ub_2 position, corresponding to in-phase or out-of-phase oscillations, respectively. The coils are connected accordingly to match the high-point circuit configuration.

3.2.3 Data and Analysis

The frequencies of the in-phase and out-of-phase oscillation modes are found by identifying the global maximum in the frequency spectrum of each value of the capacitor decade value as seen in Table 3. The $f_{2\ \text{Task 2}}$ value corresponds to the eigenfrequency associated with ω_-, and $f_{1\ \text{Task 2}}$ corresponds to ω_+. Based on Eqs. 4 and 5, the results shown in the table are as expected: $f_{2\ \text{Task 2}}$ remains constant, because it does not depend on $C_{k,\text{Task 2}}$, while $f_{1\ \text{Task 2}}$ decreases as $C_{k,\text{Task 2}}$ increases, indicating an inverse proportional relationship.

$C_{k\ \text{Task 2}}$ (μF)	$f_{1,\text{Task 2}}$ (kHz)	$f_{2,\text{Task 2}}$ (kHz)	$k_{C,H}$
0.004	7.62939	9.15527	0.18032
0.008	6.86646	9.15527	0.27999
0.01	6.48499	9.15527	0.33179
0.02	5.34058	9.15527	0.49222
0.03	4.57764	9.15527	0.59999
0.04	4.19617	9.15527	0.65279
0.05	3.8147	9.15527	0.70414
0.06	3.43323	9.15527	0.75342
0.07	3.43323	9.15527	0.75342
0.08	3.05176	9.15527	0.80000
0.09	3.05176	9.15527	0.80000
0.1	2.67029	9.15527	0.84320

Table 3: Coupling factor $k_{C,H}$ as a function of the coupling capacitor $C_{k\ \text{Task 2}}$ and eigenfrequencies $f_{1\ \text{Task 2}}$ and $f_{2\ \text{Task 2}}$.

In Fig. 3, the frequency spectra are plotted separately for the in-phase, out-of-phase, and combined cases in order to compare the different behaviours and observe the trends more clearly. In both the in-phase and out-of-phase cases, there are two main peaks for each value of the coupling capacitance $C_{k,\text{Task 2}}$.

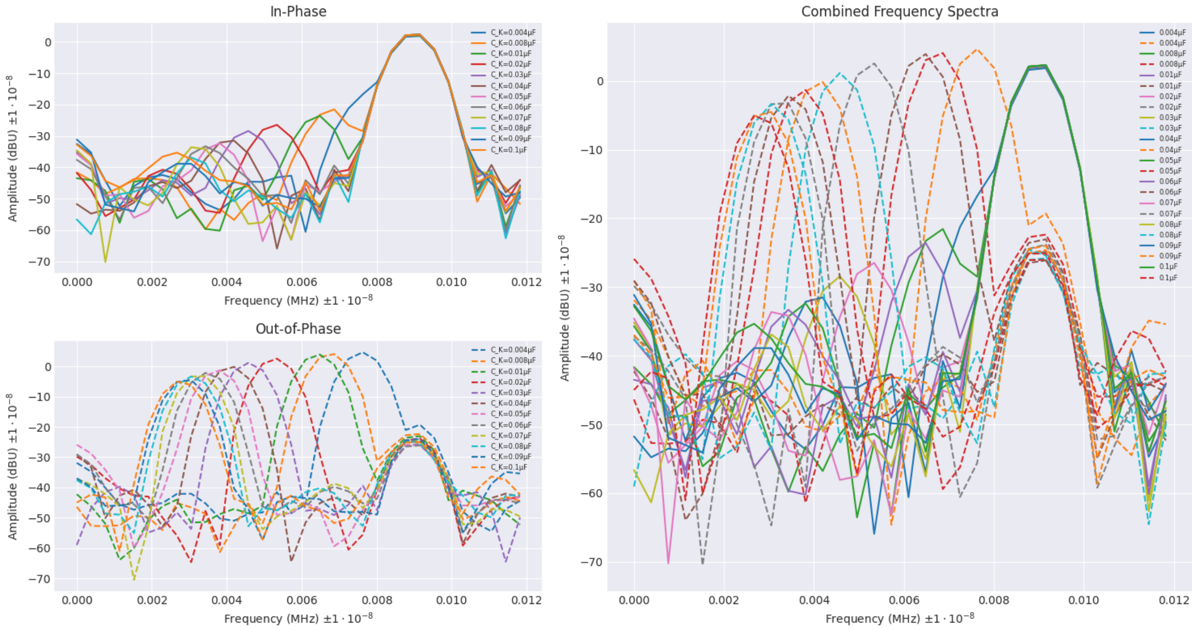

Figure 3: High-Point Circuit; Amplitude (dBu) against Frequency (MHz)

In the in-phase plot, the second peak becomes harder to distinguish for higher values of $C_{k,\text{Task 2}}$, such as 0.1 or 0.09 μF, as it lies closer to the noisy region of the graph. However, the main peak remains prominently visible and identical for all values of the coupling capacitance, consistently occurring at $f_{2\ \text{Task 2}} = 9.15527 kHz$. This trend aligns with the expectation, where an increase in $C_{k,\text{Task 2}}$ results in a decrease in both the frequency $f_{1\ \text{Task 2}}$ and the corresponding amplitude. Another important observation is the relative position of the main peak to the secondary peak. In the in-phase case, the smaller (secondary) peak is located to the left of the main peak.

In contrast, for the out-of-phase graph, the second peak remains nearly identical across all values of $C_{k,\text{Task 2}}$, around the same amplitude as the second peaks in the in-phase case. The main peaks have amplitude approximately the same as the main peaks in the in-phase graph. The main peaks in this case vary with $C_{k,\text{Task 2}}$, showing a decreasing trend in frequency as the capacitance increases. Moreover, the second peak is now situated to the right of all main peaks, opposite to what is observed in the in-phase case. In order to determine the effective capacitance on the breadboard, Eq. 6 is used with the values measured from the high-coupling configuration to fit the data shown in Fig. 4. The capacitance is obtained through a non-linear fit, resulting in $C_{\text{Task 2}} = 0.0205 \mu F$ with a $r^2 = 0.9977$.

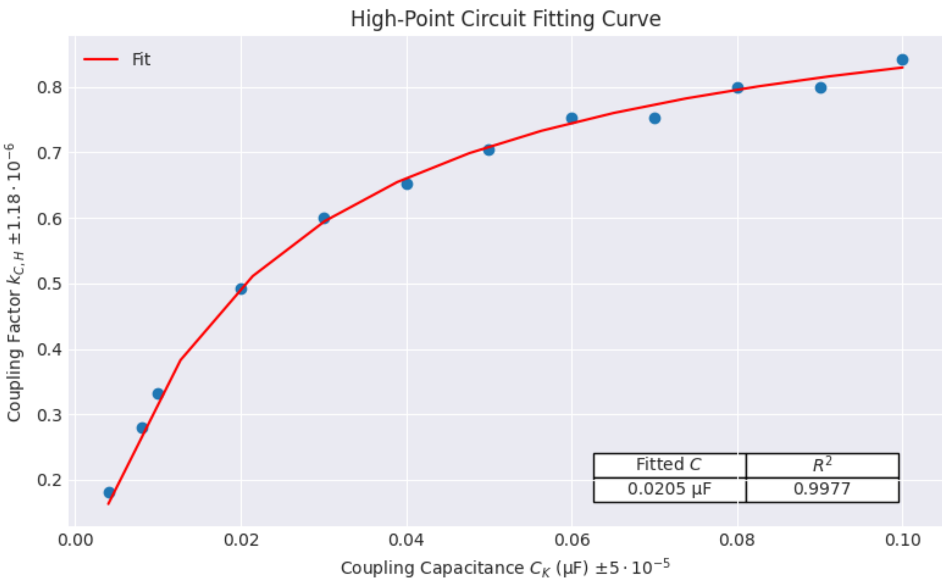

Figure 4: Fitted Curve to determine the value of the capacitance C.

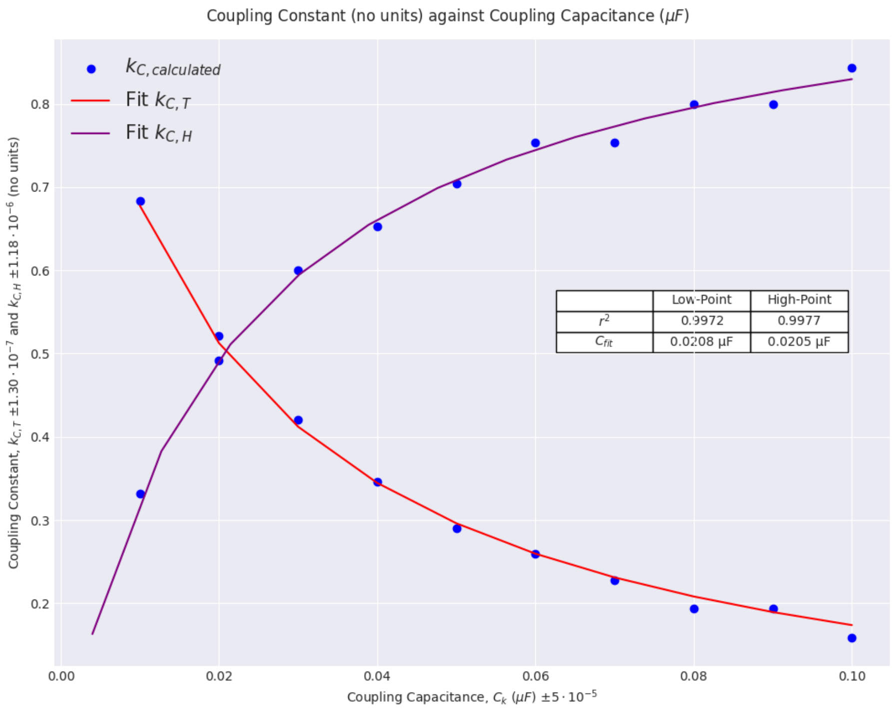

Figure 5: Fitted Curve to determine the value C from calculated coupling factors $k_{C,H}\&k_{C,T}$ as functions of C_k

In Fig. 5, the calculated values of the coupling factors as functions of the coupling capacitance $C_{k\text{ Task 2}}$, for both low-point and high-point circuits in order to analyse the difference and the values of $C_{\text{ Task 2}}$ obtained from the curve fitting of the values. With really good coefficients of determination $r^2_{low} = 0.9972$ for the low-point circuit and $r^2_{high} = 0.9977$ for the high-point circuit, the fitted capacitance values are $C_{fit,low} = 0.0208\,\mu F$ and $C_{fit,high,\text{ Task 2}} = 0.0205\,\mu F$ with a relative error of $\approx 1.45\%$, corresponding to accurate measurement of the voltage time trace and frequency. The closeness of the capacitance values, and the consistent data that matches with the theoretical expectations, it can be said that the data is ***accurate and precise***

3.2.4 Discussion

The graph presented in this task highlights the key characteristics of in-phase and out-of-phase oscillations and illustrates how they are related. As mentioned earlier, for each value of the coupling capacitance, the two dominant frequency peaks behave consistently: for the in-phase configuration, the secondary peak appears on the left (lower frequency) of the main peak, whereas for the out-of-phase configuration, it shifts to the right (higher frequency). This reversal is not due to a fundamental change in system behaviour, but rather due to the symmetry and phase relationship of the two modes.

While the underlying eigenfrequencies remain the same for both configurations, the measured amplitude and prominence of each peak differ depending on how the oscillators are driven and how the signals are combined (in-phase or out-of-phase). This is because each configuration selectively emphasizes one of the normal modes, causing one peak to dominate while the other becomes less visible. The experiment clearly demonstrates the theoretical expectation: two coupled oscillators exhibit two resonant frequencies, and the system's response depends on the phase relationship between them.

Comparing the trends of the coupling factors $k_{C,H}$ and $k_{C,T}$ an opposite behaviour is clearly observed. This contrast arises both mathematically and physically. Mathematically, the behaviour is dictated by the formula used to calculate them. One behaves in relation to $\frac{1}{x}$, while the other follows a $1 - \frac{1}{x}$ trend, leading to opposite slopes as the coupling capacitance increases. Physically, this difference is a result of how the high-point and low-point circuits selectively describe different normal modes of the coupled system. The high-point circuit highlights the in-phase mode (lower frequency), while the low-point circuit describes the out-of-phase mode (higher frequency). Although the underlying system is the same, the way each configuration combines the signals-either adding or subtracting voltages determines which mode becomes dominant in the measurement. As a result, the calculated coupling factors appear to behave in opposite ways, even though they reflect the same physical interaction. This symmetry between the two modes and measurement setups explains the mirrored trends.

3.2.5 Error Analysis

The uncertainty in frequency, time, amplitude and voltage is the smallest measurable increment from the digital device. The uncertainty in the found values, $k_{C,H}$ and $C_{\text{ Task 2}}$ are found by propagating the measured values through their respective square root of partial derivatives of all input values and taking the mean, as follows:

$$\Delta k_{C,H} = \sqrt{\left(\frac{\partial k_{C,H}}{\partial f_{1,\text{ Task 2}}}\Delta f_{1,\text{ Task 2}}\right)^2 + \left(\frac{\partial k_{C,H}}{\partial f_{2,\text{ Task 2}}}\Delta f_{2,\text{ Task 2}}\right)^2} \tag{11}$$

$$\Delta C_{\text{Task 2}} = \sqrt{\left(\frac{\partial C_{\text{Task 2}}}{\partial C_{k,\text{Task 2}}}\Delta C_{k,\text{Task 2}}\right)^2 + \left(\frac{\partial C_{\text{ Task 2}}}{\partial k_{C,H}}\Delta k_{C,H}\right)^2} \tag{12}$$

Parameter	Mean	Maximum
$\Delta f_{1,2\ \text{Task 2}}(MHz)$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
$\Delta t_{\text{ Task 2}}(s)$	$1 \cdot 10^{-11}$	$1 \cdot 10^{-11}$
$\Delta C_{k,\ \text{Task 2}}(nF)$	0.05	0.05
$\Delta k_{C,H}$ (no units)	$1.18495 \cdot 10^{-6} \approx 1.18 \cdot 10^{-6}$	$1.244865 \cdot 10^{-6} \approx 1.24 \cdot 10^{-6}$
$\Delta C_{\text{ Task 2}}(\mu F)$	$0.0098663 \approx 0.01$	$0.016647 \approx 0.017$

Table 4: Mean and Maximum Uncertainty Parameters of Task 2

3.3 Task III: Beat period of the high-point circuit for one selected Coupling Factor

3.3.1 Methodology and Set up

The setup remains the same as in Task 2, with the exception of the cable previously connected to Ub_1/Ub_2, which is now connected to the ground of the switch box. As beat oscillations can be observed and measured by short-circuiting the right switch of the high-point circuit, charging the left capacitor, and then initiating the measurement by closing the left switch.

This setup allows the energy to oscillate between the two coupled LC circuits, producing a characteristic beat pattern. The chosen coupling capacitor value is $C_k = 0.008\mu F$, for which both the frequency spectra and time traces were recorded. The oscillation period was determined from the average of the frequency peaks, while the beat frequency was obtained from the frequency spectrum as the difference between the two dominant peaks.

The directly measured values are obtained from the time traces of the voltage oscillation. For the period, the time difference between two peaks is measured (marked with red dots in Fig. 7), and the period of the beat oscillation is measured as the time interval between two points where the wave packet envelope reaches zero (marked with orange dots in the same figure).

3.3.2 Data and Analysis

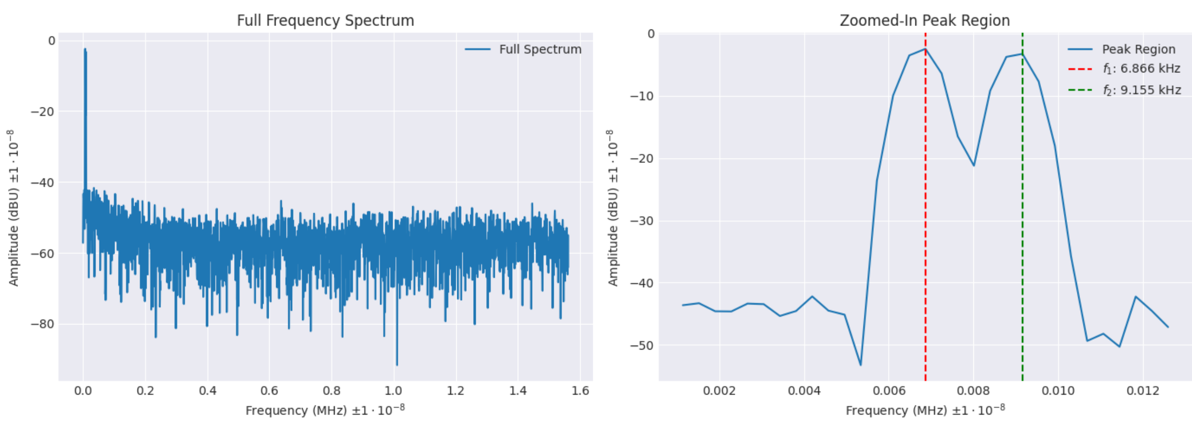

Figure 6: Frequency Spectra of the selected value of $C_K = 0.008\mu F$

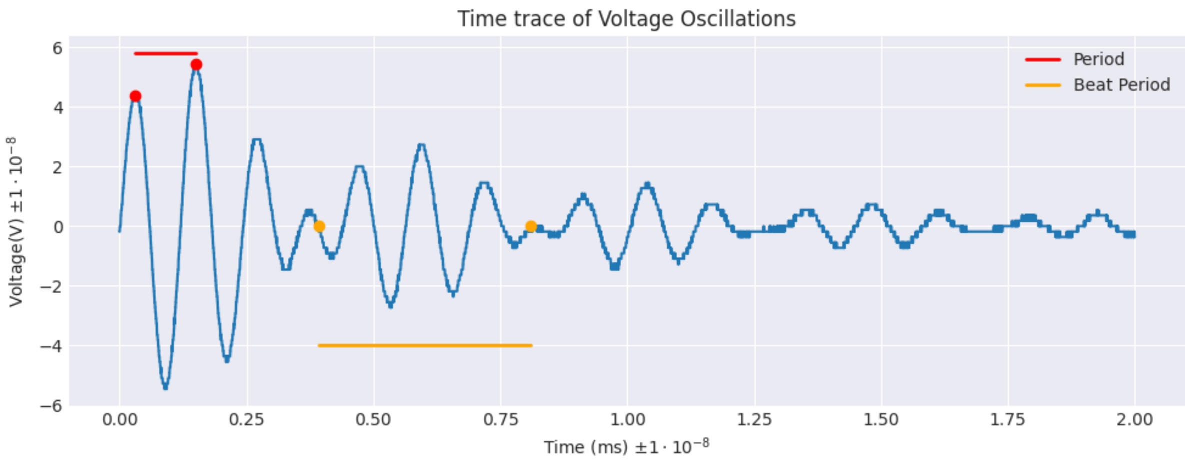

Figure 7: Time traces of the selected value of $C_K = 0.008\mu F$

The values obtained are for $f_{1,\text{Task 3}} = 6.86kHz$ and $f_{2,\text{Task 3}} = 9.155kHz$, beat frequency $f_{S,\text{Calculated}} = 2.2881kHz$, with $T_{S,\text{Calculated}} = 4.369 \cdot 10^{-4}s$ and $T_{\text{Calculated}} = 1.2483 \cdot 10^{-4}s$ from the frequency spectra, and from time traces $T_{S,\text{Measured}} = 4.185 \cdot 10^{-4}s$ and $T_{\text{Measured}} = 1.19 \cdot 10^{-4}s$.

Comparing with the values obtained in Task 2, for the coupling capacitance $C_k = 0.008, \mu F$, the results match **exactly**, validating both methods used for determining the eigenfrequencies and confirming the consistency of the experimental approach.

3.3.3 Discussion

Although the resulting time graph may not appear elegant, the oscillation period and beat period can still be accurately determined directly from the time traces. These measured values are then compared with those calculated from the frequency spectrum, which is based on the difference and average of the two eigenfrequencies.

Since the period derived from the frequency spectrum corresponds to an average frequency between the two modes, a perfect match with the manually measured period from the time traces is not expected. However, the relative error between them is only about $\approx 4.0322\%$, which demonstrates a good consistency in the results. One limitation arises from the use of automated peak detection in the code. This method can struggle with identifying peaks in periodic signals, especially when estimating the beat period. In this case, the envelope minimum, similar to nodes in standing waves, were identified manually.

A particularly strong confirmation of the data's accuracy is that the eigenfrequencies obtained in this task match exactly, even down to the first three decimals, with those calculated in Task 2. This agreement validates both experimental methods. While frequency domain analysis offers a more elegant and precise approach, especially when working with closely spaced peaks, the time domain method still proves reliable and offers intuitive insight into the dynamic energy exchange between the oscillators.

3.3.4 Error Analysis

The uncertainty in time and voltage is the smallest measurable increment from the digital device. The uncertainty in the found values, $T_{\text{S, Calculated}}$ and $T_{S,\text{Calculated}}$ are found by propagating the measured values through their respective square root of partial derivatives of all input values and taking the mean, as follows:

$$\Delta T_{S,\text{Calculated}} = \sqrt{\left(\frac{\partial T_S}{\partial f_{S,\text{Calculated}}}\Delta f_{S,\text{ Calculated}}\right)^2} \tag{13}$$

$$\Delta T_{\text{Calculated}} = \sqrt{\left(\frac{\partial T}{\partial f_{1,\text{Task 3}}}\Delta f_{1,\text{Task 3}}\right)^2 + \left(\frac{\partial T}{\partial f_{2,\text{Task 3}}}\Delta f_{2,\text{Task 3}}\right)^2} \tag{14}$$

Parameter	Mean	Maximum
$\Delta f_{1,2,\text{Task 3}}(MHz)$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
$\Delta t_{\text{ Task 3}}(ns)$	$1 \cdot 10^{-2}$	$1 \cdot 10^{-2}$
$\Delta f_{S,\text{Calculated}}(MHz)$	$2 \cdot 10^{-8}$	$2 \cdot 10^{-8}$
$\Delta T_{S,\text{Calculated}}(ns)$	3.82	3.82
$\Delta T_{\text{Calculated}}(ns)$	$0.15637016 \approx 0.15$	$0.20225 \approx 0.2$
$\Delta T_{S,\text{Measured}}(s)$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$
$\Delta T_{\text{Measured}}(s)$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-8}$

Table 5: Mean and Maximum Uncertainty Parameters of Task 3

4 Conclusion

This experiment successfully demonstrated the behaviour of coupled electric oscillations in various circuit configurations. Frequency Spectrum presented consistent expected result, as par with the theory. Possible improvements would be to measure and test fro higher or lower range of values of respective C_k and distance d. And experimenting with the same circuit configuration but with different capacitance (as compared to the same capacitance for all the configuration above).

5 Citation

"Standard Capacitor Values: Essential Guide for Beginner." *Leading Electronic Component Supplier from China* Weishielectronics, 24 Oct. 2024 Link