Week 9

Louise Carlo Salomon

Excercise 1:A

1.$f(x) = x^3 - 5x^2 - 7x + 4$

$$f(1) = -(1)^3 - 5(1)^2 - 7(1) + 4 = -7$$
$$f(-2) = (-2)^3 - 5(-2)^2 - 7(-2) + 4 = -10$$
$$f(\frac{1}{2}) = (\frac{1}{2})^3 - 5(\frac{1}{2})^2 - 7(\frac{1}{2}) + 4 = -0.0625$$

2.$g(x) = 2x^6 + 3x^4 - x^2 + 3$

$$g(2) = 2(2)^6 + 3(2)^4 - (2)^2 + 3 = 175$$
$$g(3) = 2(3)^6 + 3(3)^4 - (3)^2 + 3 = 1695$$
$$g(-1) = 2(-1)^6 + 3(-1)^4 - (-1)^2 + 3 = 7$$

3.$h(x) = 2x^3 - 7x + 3$

$$h(-3) = 2(-3)^3 - 7(-3) + 3 = -30$$
$$h(5) = 2(5)^3 - 7(5) + 3 = 218$$
$$h(-10) = 2(-10)^3 - 7(-10) + 3 = -1927$$

Excercise 1:B

4.Determine if $x - 3$is a factor of $P(x)$ where $P(x) = x^4 - 3x^3 - x + 3$.

$$\begin{array}{r|rrrrr} & 1 & -3 & 0 & -1 & 3 \\ 3 & & 3 & 0 & 0 & -3 \\ \hline & 1 & 0 & 0 & -1 & 0 \end{array} \tag{1}$$

therefore $x - 3$ is a factor of $P(x)$.

5.Determine if $x - 1$ is a factor of $P(x)$ where $P(x) = x^{25} - 4$.

$$P(1) = (1)^{25} - 4 = -3$$

therefore $x - 1$ is not a factor of $P(x)$.

6. Find k so that $x-2$ is a factor of $P(x) = x^3 - kx^2 - 4x + 20$.

$$\begin{aligned} P(2) &= (2)^3 - k(2)^2 - 4(2) + 20 \\ 0 &= 8 - 4k - 8 + 20 \\ 0 &= 20 - 4k \\ -20 &= -4k \\ \frac{-20}{-4} &= \frac{-4k}{-4} \\ 5 &= k \end{aligned}$$

therefore, k should be 5 so that $x-2$ will be a factor of $P(x)$

Excercise 2. Answer is asked.

1. $P(2)$ in $P(x) = x^4 + 4x^3 - x^2 - 16x - 4$

$$\begin{aligned} P(2) &= (2)^4 + 4(2)^3 - (2)^2 - 16(2) - 4 \\ P(2) &= 16 + 32 - 4 - 32 - 4 \\ P(2) &= 8 \end{aligned}$$

2. Prove $y-3$ is a factor of $3y^3 - 7y^2 - 20$. $y = 3$

$$0 = 3(3)^3 - 7(3)^2 - 200 \qquad = -2$$

therefore, $y-3$ is not a factor of $3y^3 - 7y^2 - 20$.

3. Evaluate $P(4)$ where $P(x) = 3y^3 - 7y^2 - 20$.

$$\begin{aligned} P(4) &= 3(4)^3 - 7(4)^2 - 20 \\ P(4) &= 60 \end{aligned}$$

therefore $P(4) = 60$.

4. Prove $x-1$ is a factor of $P(x) =$.

$$P(1) = (1)^2 + 2(1) + 5P(1) = 8$$

using synthetic division,

$$\begin{array}{r|ccc} & 1 & 2 & 5 \\ 1 & & 1 & 3 \\ \hline & 1 & 3 & 8 \end{array} \qquad (2)$$

therefore, knowing that,

If $P(a) = 0$, then $x - a$ is factor of $P(x)$. Conversely, if $x - a$ is a factor of $P(x)$, then $P(a) = 0$.

and plugging in our case, we find that,

$P(1) \neq 0$ then $x - 1$ is not a factor of $P(x)$. Conversely, $x - 1$ is not a factor of $P(x)$, then $P(1) \neq 0$.

5.

$$\begin{aligned} P(x) &= 5x^3 + 3x^2 - 8 \\ P(4) &= 5(4)^3 + 3(4)^2 - 8 \\ P(4) &= 360 \end{aligned}$$

therefore, remainder $R = 360$.