```
with(DirectSearch)
```

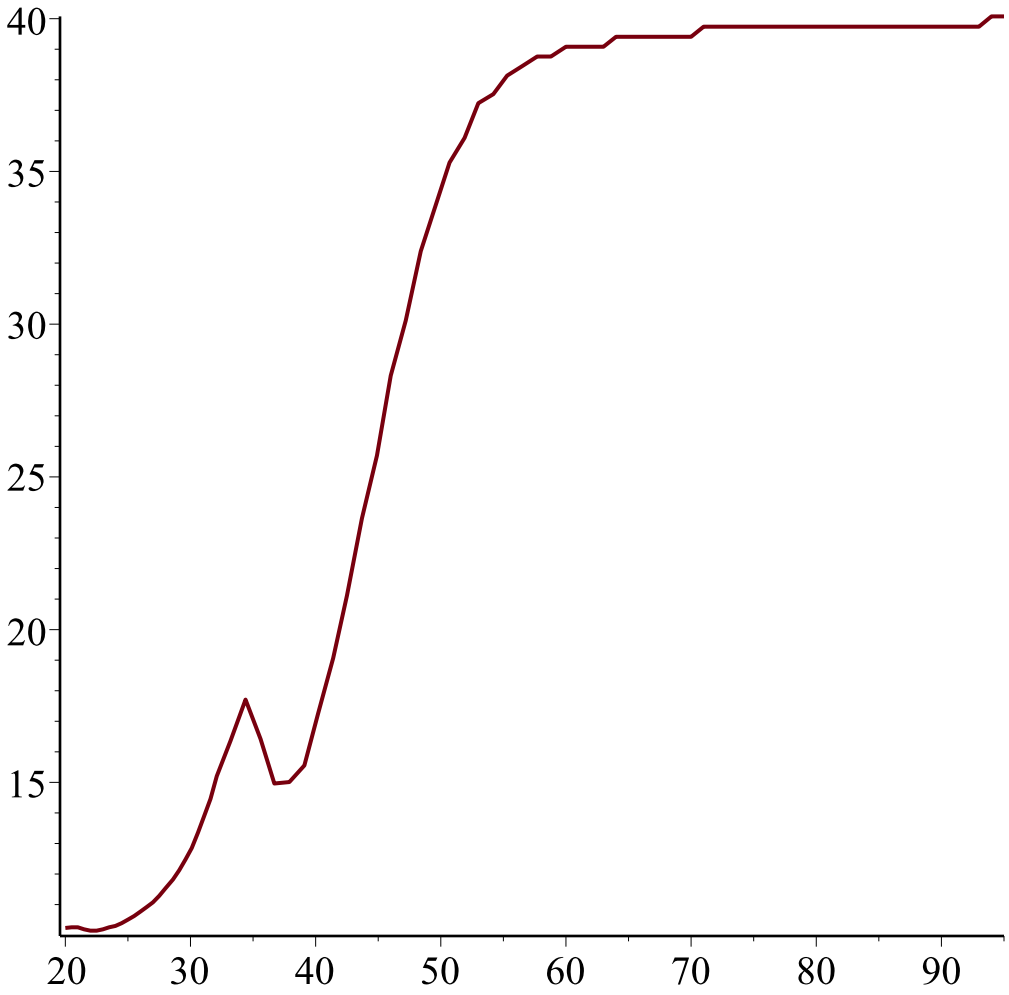
[*BoundedObjective*, *CompromiseProgramming*, *DataFit*, *ExponentialWeightedSum*, *GlobalOptima*, *GlobalSearch*, *Minimax*, *ModifiedTchebycheff*, *Search*, *SolveEquations*, *WeightedProduct*, *WeightedSum*]

[*BoundedObjective*, *CompromiseProgramming*, *DataFit*, *ExponentialWeightedSum*, *GlobalOptima*, *GlobalSearch*, *Minimax*, *ModifiedTchebycheff*, *Search*, *SolveEquations*, *WeightedProduct*, *WeightedSum*] **(2)**

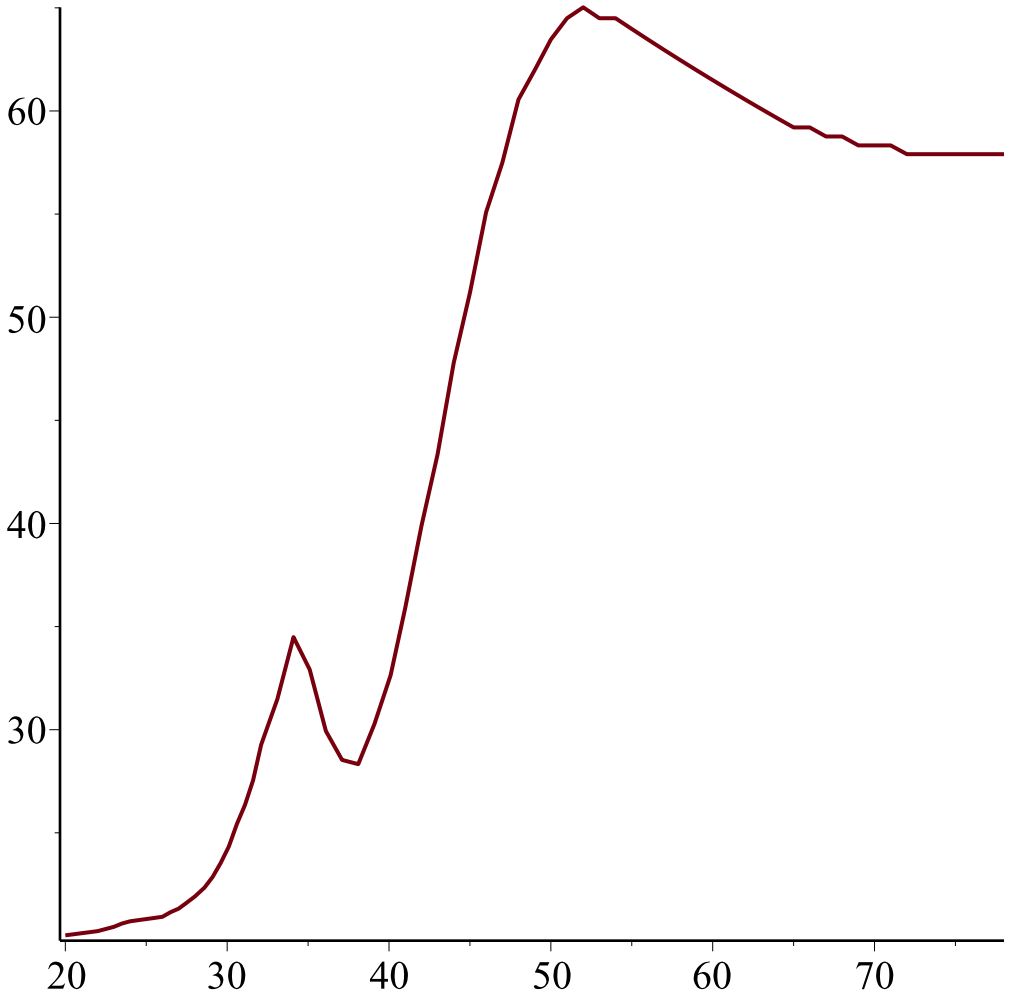
```
with(ExcelTools)
```

[*Export*, *Import*, *WorkbookData*] **(3)**

```
X40 := Import("C:\\Users\\Chenting\\Documents\\CurvesData40.xlsx", "Sheet1", "Z3:Z86") :
A40 := Import("C:\\Users\\Chenting\\Documents\\CurvesData40.xlsx", "Sheet1", "AD3:AD86") :
plot(X40[1..84, 1], A40[1..84, 1])
```




```
X60 := Import("C:\\Users\\Chenting\\Documents\\CurvesData60.xlsx", "Sheet1", "Z3:Z105") :
A60 := Import("C:\\Users\\Chenting\\Documents\\CurvesData60.xlsx", "Sheet1", "AD3:AD105") :
plot(X60[1..71, 1], A60[1..71, 1])
```




```
X100 := Import("C:\\Users\\Chenting\\Documents\\CurvesData100.xlsx", "BVS120-100Strain",
    "D17:D112") :
A100 := Import("C:\\Users\\Chenting\\Documents\\CurvesData100.xlsx", "BVS120-100Strain",
    "M17:M112")
```

$$\begin{bmatrix} 1..96 \text{ x } 1..1 \text{ Array} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(4)}$$

```
X60
```

$$\begin{bmatrix} 73 \text{ x } 1 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(5)}$$

```
X80 := Import("C:\\Users\\Chenting\\Documents\\CurvesData80.xlsx", "Sheet1", "Z13:Z105") :
A80 := Import("C:\\Users\\Chenting\\Documents\\CurvesData80.xlsx", "Sheet1", "AD13:AD105")
```

$$\begin{bmatrix} 1..93\ x\ 1..1\ Array \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix} \tag{6}$$

$X120 := Import(\text{"C:\\Users\\Chenting\\Documents\\CurvesData120.xlsx"}, \text{"Sheet1"}, \text{"Z3:Z105"})$:
$A120 := Import(\text{"C:\\Users\\Chenting\\Documents\\CurvesData120.xlsx"}, \text{"Sheet1"}, \text{"AD3:AD105"})$

$$\begin{bmatrix} 1..103\ x\ 1..1\ Array \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{bmatrix} \tag{7}$$

$whattype(X40)$

$$Array \tag{8}$$

$X40 := Matrix(X40)$:
$A40 := Matrix(A40)$:
$X60 := Matrix(X60)$:
$A60 := Matrix(A60)$:
$X100 := Matrix(X100)$:
$A100 := Matrix(A100)$:
$X80 := Matrix(X80)$:
$A80 := Matrix(A80)$:
$X120 := Matrix(X120)$:
$A120 := Matrix(A120)$:
$interface(showassumed = 0)$

$$1 \tag{9}$$

$$x \to piecewise\left(x < x1, a1, x < \frac{40 - a1}{b1}, a1 + b1\ x, \frac{40 - a1}{b1} \le x, 30\right) \tag{10}$$

$g := x \to eval(f40(x), \{x1 = 30, a1 = 10, b1 = 1.5\})$

$$x \to f40(x) \Big| a1 = 10, b1 = 1.5, x1 = 30 \tag{11}$$

$g(x)$

$$\begin{cases} 10 & x < 30 \\ 1.5\ x + 10 & x < 20.00000000 \\ 30 & 20.00000000 \le x \end{cases} \tag{12}$$

$g(35)$

$$30 \tag{13}$$

x

$$x\sim \tag{14}$$

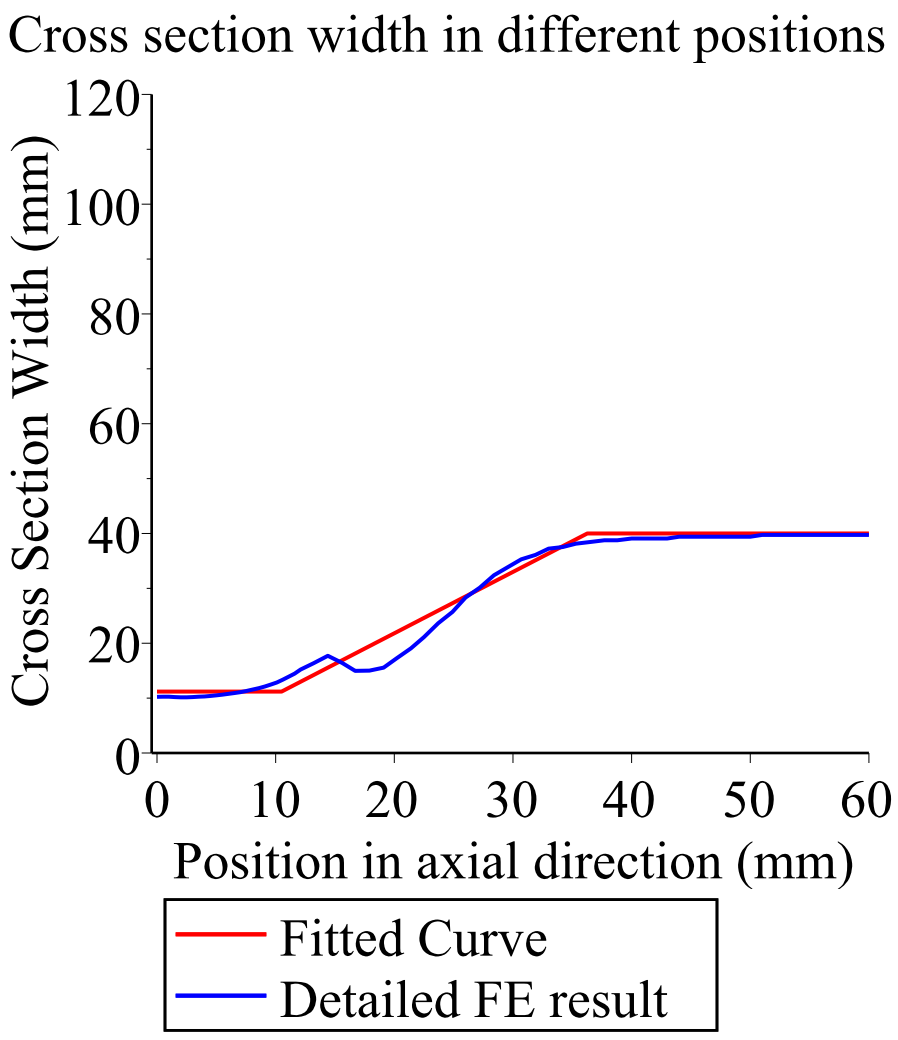
$sol40 := DataFit(f40, X40[1..73, 1], A40[1..73, 1], x, fitmethod = lts)$

$$\left[1.35316934565547, \begin{bmatrix} 11.1901607879367 \\ 1.11816890744122 \end{bmatrix}, 97\right] \quad (15)$$

$x := 'x'$

$$x \quad (16)$$

$p1 := plot('f40(x, sol40[2][1], sol40[2][2])', x = 20..80, y = 0..120, color = red, legend = "Fitted Curve ", legendstyle = [font = ["TimesNewRoman", 16]]) :$
$p2 := plot(X40[1..69, 1], A40[1..69, 1], color = blue, legend = "Detailed FE result ", legendstyle = [font = ["TimesNewRoman", 16]]) :$
$plots[display]({p1, p2}, labels = ["Position in axial direction (mm)", "Cross Section Width (mm)"], labeldirections = [horizontal, vertical], title = "Cross section width in different positions", font = ["Times New Roman", Roman, 16], size = [350, 400], tickmarks = [[20 = "0", 30 = "10", 40 = "20", 50 = "30", 60 = "40", 70 = "50", 80 = "60"], default], labelfont = ["Times New Roman", 16])$

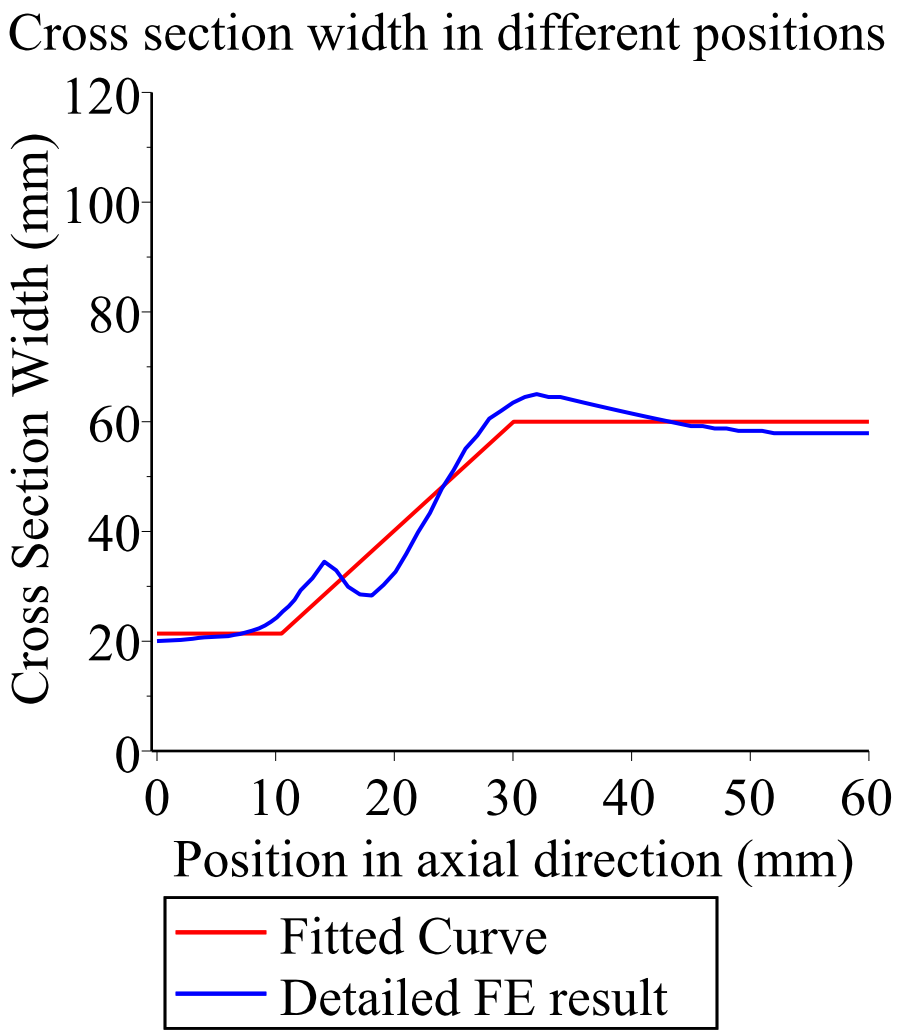


$$-(-sol40[2][2] * 30.5 + sol40[2][1] - 40) / sol40[2][2]$$
$$56.2651943461660 \quad (17)$$

$sol60 := DataFit(f60, X60[1..71, 1], A60[1..71, 1], x, fitmethod = lms)$

$$\left[10.0130952619645, \begin{bmatrix} 21.3935844605646 \\ 1.97390004638918 \end{bmatrix}, 105\right] \quad (18)$$

```
p1 := plot('f60(x, sol60[2][1], sol60[2][2])', x = 20..80, y = 0..120, color = red, legend
      = "Fitted Curve   ", legendstyle = [font = ["TimesNewRoman", 16]]) :
p2 := plot(X60[1..73, 1], A60[1..73, 1], color = blue, legend = "Detailed FE result   ", legendstyle
      = [font = ["TimesNewRoman", 16]]) :
plots[display]({p1, p2}, labels = ["Position in axial direction (mm)", "Cross Section Width (mm)"],
     labeldirections = [horizontal, vertical], title = "Cross section width in different positions", font
      = ["Times New Roman", Roman, 16], size = [350, 400], tickmarks = [[20 = "0", 30 = "10", 40
      = "20", 50 = "30", 60 = "40", 70 = "50", 80 = "60"], default], labelfont = ["Times New Roman", 16])
```



$$-(-sol60[2][2]*30.5+sol60[2][1]-60)/sol60[2][2]$$

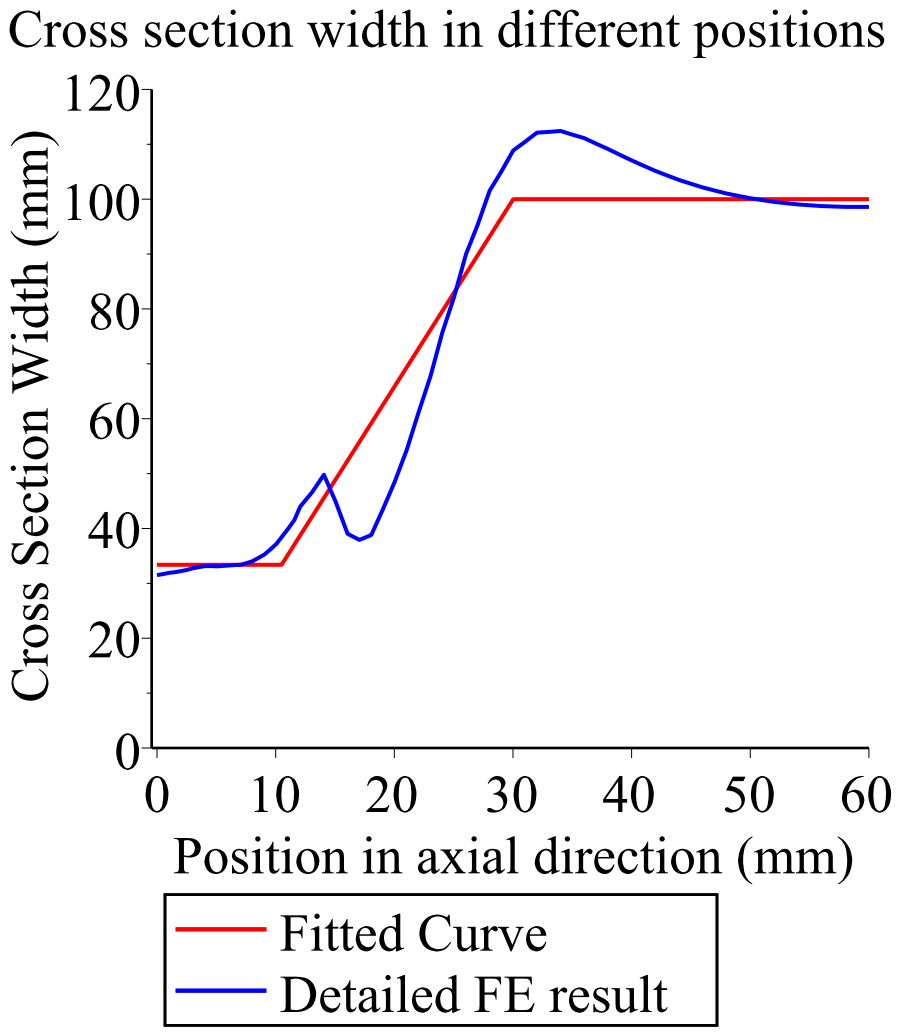
$$51.2387198256464 \qquad \textbf{(19)}$$

$$sol100 := DataFit(f100, X100[1..72, 1], A100[1..72, 1], x, fitmethod = lts)$$

$$\left[34.2315163891587, \begin{bmatrix} 33.3791030897281 \\ 3.41190704315782 \end{bmatrix}, 271\right] \qquad \textbf{(20)}$$

```
p1 := plot('f100(x, sol100[2][1], sol100[2][2])', x = 20..80, y = 0..120, color = red, legend
      = "Fitted Curve   ", legendstyle = [font = ["TimesNewRoman", 16]]) :
p2 := plot(X100[1..72, 1], A100[1..72, 1], color = blue, legend = "Detailed FE result   ", legendstyle
```

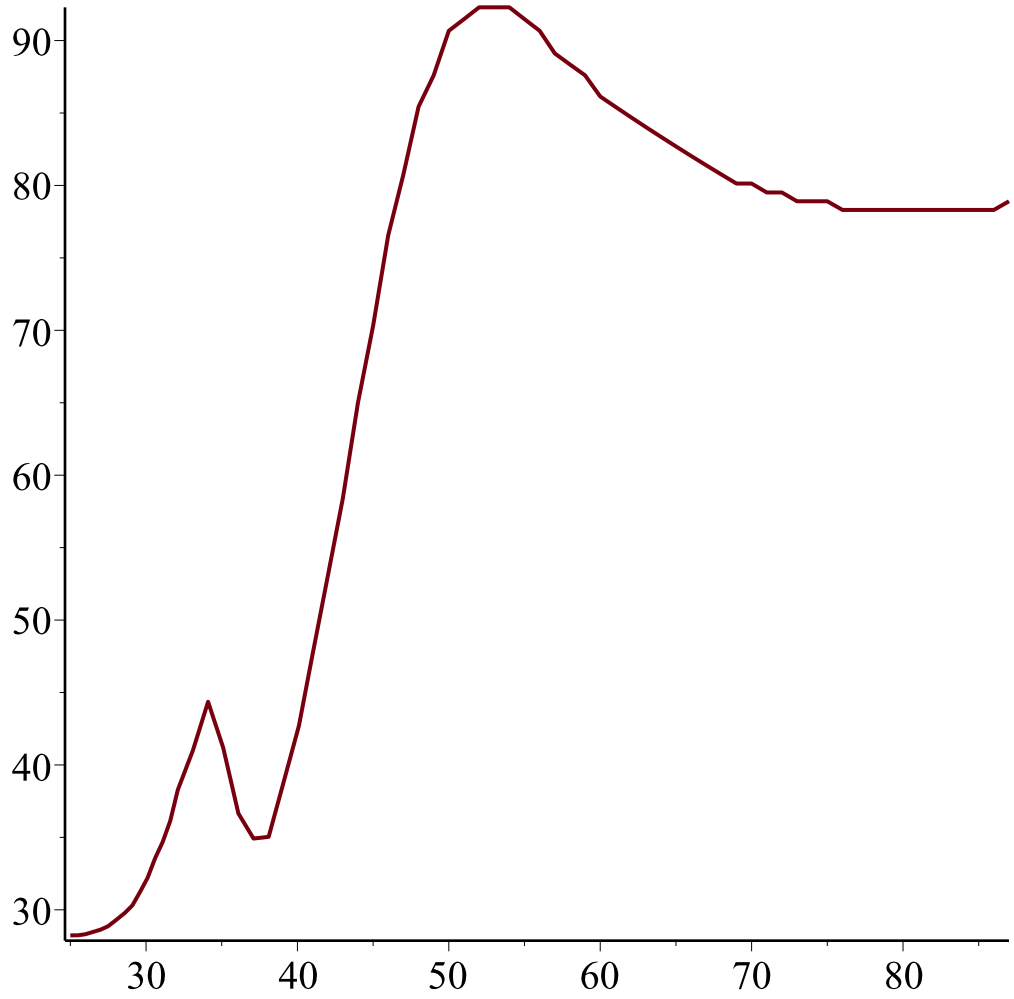
```
= [font = ["TimesNewRoman", 16]]) :
plots[display]({p1, p2}, labels = ["Position in axial direction (mm)", "Cross Section Width (mm)"],
    labeldirections = [horizontal, vertical], title = "Cross section width in different positions", font
    = ["Times New Roman", Roman, 16], size = [350, 400], tickmarks = [[20 = "0", 30 = "10", 40
    = "20", 50 = "30", 60 = "40", 70 = "50", 80 = "60"], default], labelfont = ["Times New Roman", 16])
```



$$-(-sol100[2][2]*30.5+sol100[2][1]-100)/sol100[2][2]$$

$$50.0259999957714 \qquad \textbf{(21)}$$

```
plot(X80[1..70, 1], A80[1..70, 1])
```



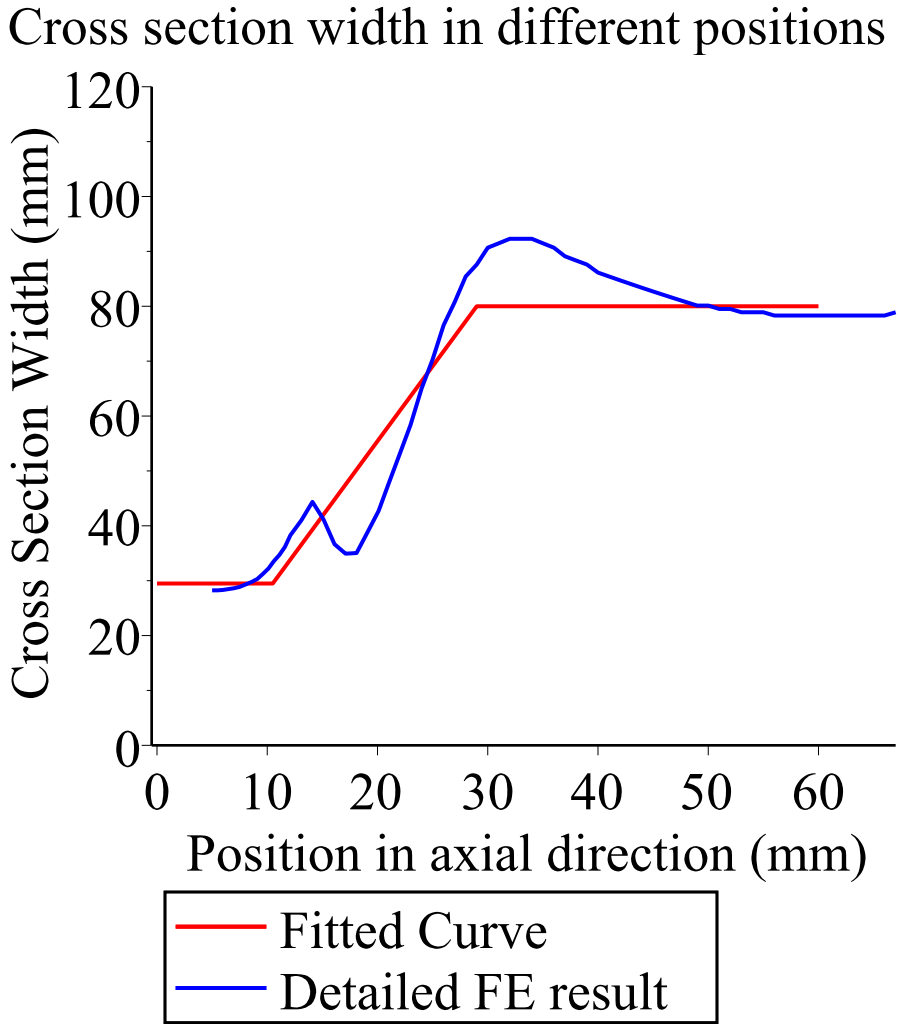
$sol80 := DataFit(f80, X80[1..70, 1], A80[1..70, 1], x, fitmethod = lts)$

$$\left[30.7483610391694, \begin{bmatrix} 29.4828014600004 \\ 2.73065943254411 \end{bmatrix}, 235\right] \quad \textbf{(22)}$$

$p1 := plot('f80(x, sol80[2][1], sol80[2][2])', x = 20..80, y = 0..120, color = red, legend = "Fitted Curve ", legendstyle = [font = ["TimesNewRoman", 16]]) :$

$p2 := plot(X80[1..70, 1], A80[1..70, 1], color = blue, legend = "Detailed FE result ", legendstyle = [font = ["TimesNewRoman", 16]]) :$

$plots[display]({p1, p2}, labels = ["Position in axial direction (mm)", "Cross Section Width (mm)"], labeldirections = [horizontal, vertical], title = "Cross section width in different positions", font = ["Times New Roman", Roman, 16], size = [350, 400], tickmarks = [[20 = "0", 30 = "10", 40 = "20", 50 = "30", 60 = "40", 70 = "50", 80 = "60"], default], labelfont = ["Times New Roman", 16])$



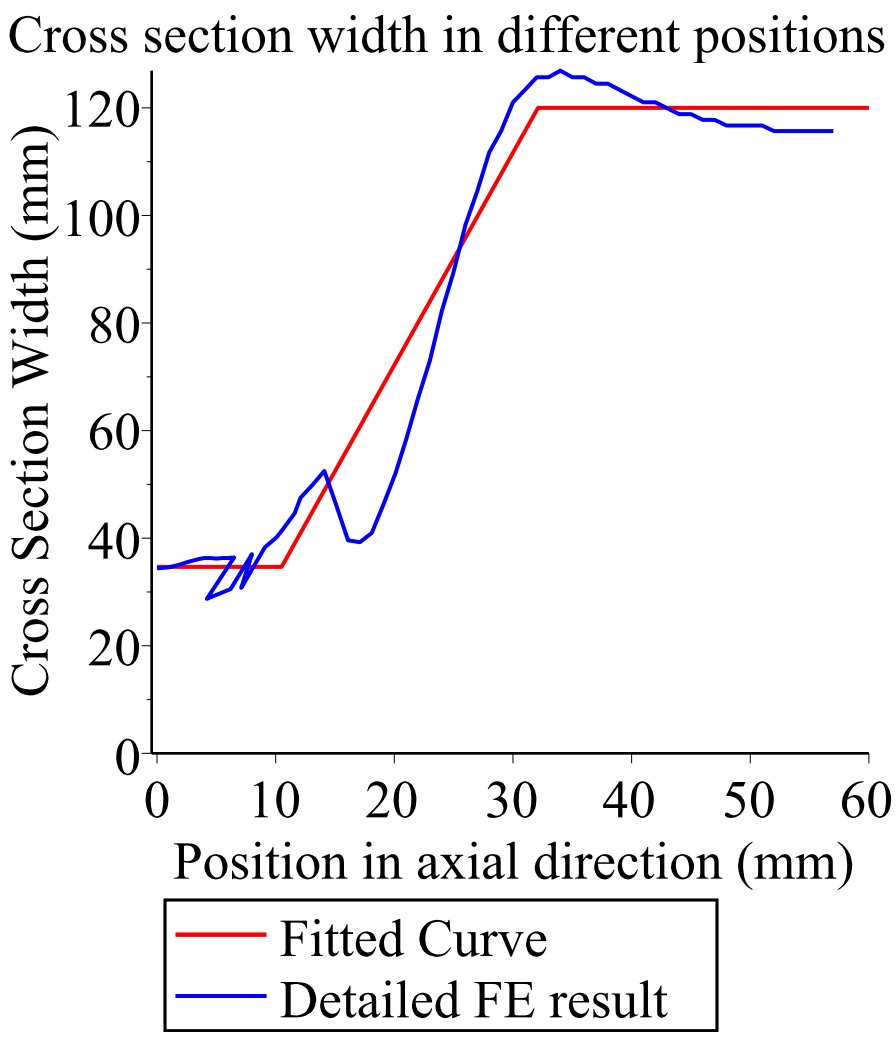
$$-(-sol80[2][2]*30.5+sol80[2][1]-80)/sol80[2][2]$$

$$48.9999996476798$$

$$sol120 := DataFit(f120, X120[1..70,1], A120[1..70,1], x, fitmethod=lts)$$

$$\left[37.2395504283801, \begin{bmatrix} 34.6649048165661 \\ 3.94891112568132 \end{bmatrix}, 130\right] \quad \textbf{(24)}$$

```
p1 := plot('f120(x, sol120[2][1], sol120[2][2])', x=20..80, y=0..120, color=red, legend
      ="Fitted Curve   ", legendstyle=[font=["TimesNewRoman", 16]]) :
p2 := plot(X120[1..70, 1], A120[1..70, 1], color=blue, legend="Detailed FE result   ", legendstyle
      =[font=["TimesNewRoman", 16]]) :
plots[display]({p1, p2}, labels=["Position in axial direction (mm)", "Cross Section Width (mm)"],
     labeldirections=[horizontal, vertical], title="Cross section width in different positions", font
      =["Times New Roman", Roman, 16], size=[350, 400], tickmarks=[[20="0", 30="10", 40
      ="20", 50="30", 60="40", 70="50", 80="60"], default], labelfont=["Times New Roman", 16])
```



$$SlopeMatrix := Vector([sol40[2][2], sol60[2][2], sol80[2][2], sol100[2][2], sol120[2][2]])$$

$$\begin{bmatrix} 1.11816890744122 \\ 1.97390004638918 \\ 2.73065943254411 \\ 3.41190704315782 \\ 3.94891112568132 \end{bmatrix} \quad \textbf{(25)}$$

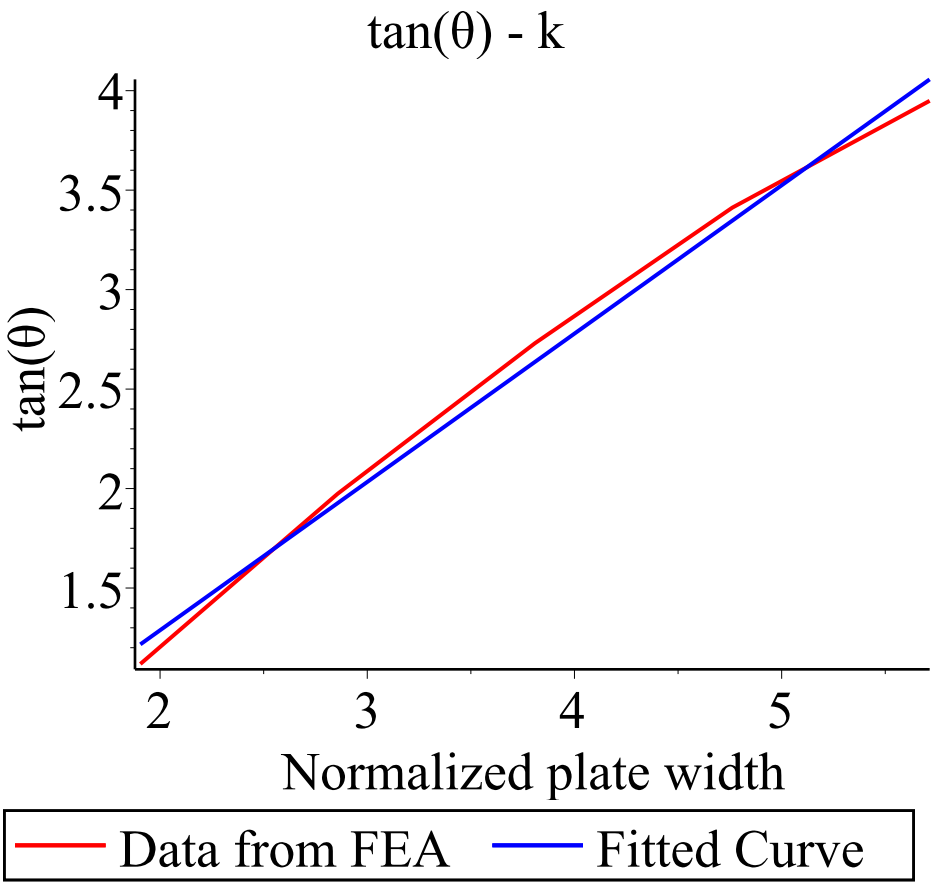
$$CaseMatrix := Vector\left(\left[\frac{40}{21}, \frac{60}{21}, \frac{80}{21}, \frac{100}{21}, \frac{120}{21}\right]\right)$$

$$\begin{bmatrix} \frac{40}{21} \\ \frac{20}{7} \\ \frac{80}{21} \\ \frac{100}{21} \\ \frac{40}{7} \end{bmatrix} \quad (26)$$

```
p1 := plot(CaseMatrix, SlopeMatrix, color = red, legend = "Fit  ", legendstyle = [font
      = ["TimesNewRoman", 16]]) :
regequation := Statistics[LinearFit]([1, x], CaseMatrix, SlopeMatrix, x)
```

$$-0.203087262576613 + 0.745446600560169\,x \quad (27)$$

```
regequation:-Results( )
Error, `regequation` does not evaluate to a module
p2 := plot(regequation, x = 40/21 .. 120/21, color = blue, legend = "Fitted Curve   ", legendstyle = [font
      = ["TimesNewRoman", 16]]) :
plots[display]([p1, p2], labels = ["Ratio of plate width to hole diameter", "tan(θ)"], labeldirections
      = [horizontal, vertical], title = "tan(θ) - k", font = ["Times New Roman", Roman, 16], size = [350,
     400], labelfont = ["Times New Roman", 16])
```



$$InterceptMatrix := Vector\left(\left[\frac{sol40[2][1]}{21}, \frac{sol60[2][1]}{21}, \frac{sol80[2][1]}{21}, \frac{sol100[2][1]}{21}, \frac{sol120[2][1]}{21}\right]\right)$$

$$\begin{bmatrix} 0.532864799425558 \\ 1.01874211716974 \\ 1.40394292666668 \\ 1.58948109951086 \\ 1.65070975316981 \end{bmatrix} \quad \textbf{(28)}$$

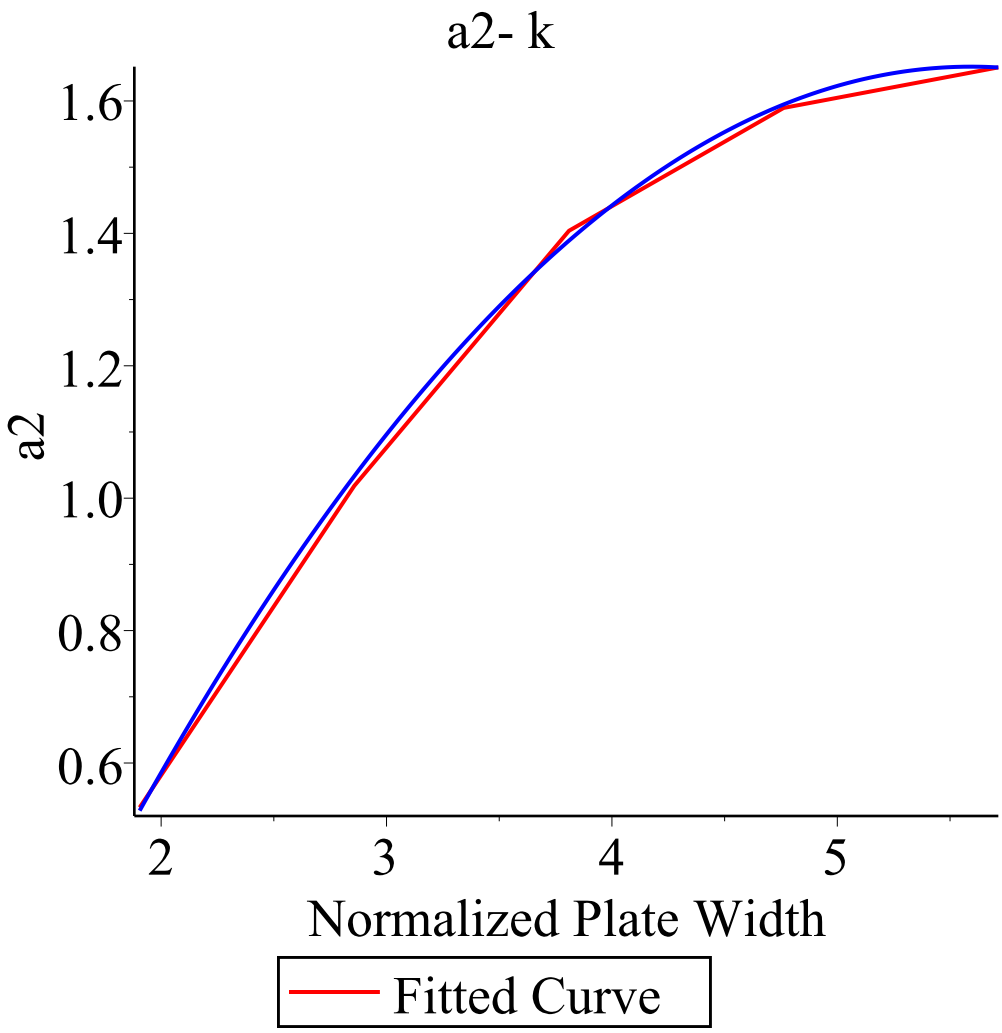
$p1 := plot(CaseMatrix, InterceptMatrix, color = red, legend = \text{"Fitted Curve "}, legendstyle = [font = [\text{"TimesNewRoman"}, 16]])$:

$regequation2 := Statistics[LinearFit]([1, x, x^2], CaseMatrix, InterceptMatrix, x)$

$$-0.932383382035580 + 0.924051012569408\,x - 0.0826055972634362\,x^2 \quad \textbf{(29)}$$

$p2 := plot\left(regequation2, x = \frac{40}{21} .. \frac{120}{21}, color = blue, legend = \text{"Fitted Curve "}, legendstyle = [font = [\text{"TimesNewRoman"}, 16]]\right)$:

$plots[display]([p1, p2], labels = [\text{"Ratio of plate width to hole diameter"}, \text{"a2"}], labeldirections = [horizontal, vertical], title = \text{"a2- k"}, font = [\text{"Times New Roman"}, Roman, 16], size = [350, 400], labelfont = [\text{"Times New Roman"}, 16])$


a2- k
1.6
1.4
1.2
1.0
0.8
0.6
a2
2
3
4
5
Normalized Plate Width
Fitted Curve