**Simplified Analytical Model for Bolt Connections**

Introduction

The bolted connection is a common type of connection in steel structure. Bolted connections can be simulated by continuum finite element method but this kind of model takes too long time for computation and is not practical to be integrated into the FE model of the whole structure. In order to simplify the model, usually a rotational spring is used for modelling the bolted connections, linking the other structural components like the beams or columns. However, the shear strength of bolted connection of a bolted connection is usually depends on the clamping force between plates and the rotational spring model cannot take the coupling of the shear strength and the clamping force into account.

In order to propose a simplified model that can accurately simulate the bolted connections, understanding the behavior of the basic forms of bolted connections is necessary. The figure below is a double-lap joint in which bolts clamp 3 plates. Some of the practical bolted connections, like fig, has similar shear behavior with double-lap joint due to the bolts also clamp 3 plates.

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| **Fig. 1** double-lap joint | **Fig. 2** A beam-column connection with go-through bolts |

When load is applied at the two ends of a double-lap joint, the load-displacement relationship is show in **Fig. 3**.

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| **Fig. 3** Load-displacement relationship of double-lap joint |

The 3 turning points on the curve are named as A, B and C. At OA stage, Shear is transmitted by static friction force between the core plates and cover plates. There’s little relative displacement between core plates and cover plate. AB stage is the slip stage. After point A, relative displacement between core plate and cover is significantly increased. The slip is due to the gap between bolt shank and hole wall. BC is the confined stage in which the bolt shank contacts with the hole wall.

The method for determining each point on the curve

Point A

According to GB50017-2003(2003), *Ay* can be determined by the formula below:

**(1)**

For a double lap joint in which there are 2 contact surfaces between plates, *nf* is 2. *µ* is the friction coefficient and *Fx*is the bolt axial force. According to continuum FE analysis, the core plate and cover plates are not fully contact after the preload is applied to the bolt, instead, the only contact area between core plates and cover plates is the adjacent area of the bolt hole. When the core plates are subject to tensile load, the load conveys from core plates to cover plates from the contact area, and according to Saint-Venant's Principle, the tensile stress is concentrated at the contact area near the bolt hole but is uniformly distributed at the place far enough from the bolt hole.

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| **Fig. 4** Undeformed Shape | **Fig. 5** Deformed Shape after applying preload |
| \\ad.monash.edu\home\User044\cdin0003\Pictures\CoverPlateContactArea.PNG |  |
| **Fig. 6** Contact Area around the Bolt Hole |  |

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| **Fig. 7** Stress of core plate in axial direction | **Fig. 8** Stress of cover plate in axial direction |
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| **Fig. 9** Tensile area and non-tensile area for one-row bolted connection | |
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| **Fig. 10** Tensile area and non-tensile area for two-row bolted connection | |
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| **Fig. 11** Core plate and cover plate as beam element with variable cross section | |
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| **Fig. 12** 4-element model of double-lap joint with variable cross section | |

For simplicity, an assumption is made that the cover plate and core plate have limited tensile area, as is shown in **Fig. 9** and **Fig. 10**. The core plate and the cover plate can be seen as beams with variable cross sections (see **Fig. 11**) and the double-lap joint can be seen as a 4-beam model (see **Fig. 12**).

The axial displacement of core plate  can be determined by the following formula:

 **(2)**

The area of the cross section of the core plate can be determined by formula:

 **(3)**

In which is the thickness of the core plate, is the width of the core plate, is the diameter of the bolt hole and , are denoted in the figure.

Substituting (2) into (1) yields:

 **(4)**

Similarly, the axial displacement of the cover plate can be determined as:

 **(5)**

Due to there’s no relative slip between core plate and cover plate, *Ax* is the sum of the 2 axial displacements:

 **(6)**

Point B

AB is the slipping stage in which the kinetic friction remains constant. So at this stage, shear force remains constant. The horizontal distance between B and A is the clearance of the bolt hole. So the formula for determining point B is:

 **(7)**

 **(8)**

In the formula above, is the diameter of the bolt hole and is the diameter of the bolt shank.

The slope of BC

BC is the bearing stage, in which the bolt shank contact with the bolt hole. Both of the bolt shank and the bolt hole deform. The bolt under bearing can be seen as a 3-spring model shown in the **Fig. 13** below:

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| **Fig. 13** Contact Area around the Bolt Hole |

Assume a force F is applied on the spring on the right side of the shank, the 2 springs on the left side are subject to force with magnitude of F/2. The deformation of the spring on the right side is:

 **(9)**

The deformation of the spring on the left side is:

 **(10)**

Primož Može and Darko Beg have done a research about bearing stiffness. They compared the “Normalized force” which is the bearing force over hole diameter, plate thickness and tensile strength, and the “Normalized displacement” which is the ratio between axial deformation of the bolt hole and the diameter of the bolt hole.

The bolt shank can be seen as beam element with pinned ends on both sides, due to the bolt shank is very thick and short, Timoshenko Beam Theory should be used. The bending deformation of the bolt can be determined by:

 **(11)**

In which *lb* is the length of the bolt, *G* is the shear modulus, *As* is the shear area, *E* is the Young’s Modulus of the bolt, and *I* is the moment of inertia, for circular cross section, *As* is 9/10 of cross section area.

Therefore the stiffness of bearing stage is:

 **(12)**

Method for Finite Element Modelling

Formulae (1)-(10) can be used for predicting the behavior of a simple double-lap joint, but when analyzing the whole structure considering the actual behavior of double-lap joint, or analyze the behavior of double-lap joint that is subject to more complex load, for example, the cover plates are subject to vertical loads which results in variable bolt preload, FE analysis is required. A simplified FE modelling approach using an open source FE package named OpenSees is introduced in this section.

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| **Fig. 14** Contact Area around the Bolt Hole |

Modelling of Steel Plates with Force-based Beam-column element

Core plates and cover plates are modelled with Force-based Beam-column elements (see **Fig. 14**). 10 integration points are placed along the element and the integration method is Fixed Location Integration. Fiber sections are used for cross sections of the element. Cross sections placed at different integration points are different so that a variable cross section is formed.

Modelling of the contact and friction of steel plates

SingleFPBearing elements are places where the core plate and the cover plate contact (see **Fig. 14**). SingleFPBearing element is originally used for modelling the concave friction pendulum. In order to model the contact behavior of the bolted connection, modification of OpenSees source code is required. The modified SingleFPBearing element should have the following features: 1. Simulation of contact behavior between the core plate and the cover plate, including the behavior when the plates are in contact and not in contact. 2. Simulation of the bearing behavior when the bolt shank and plates are in contact. When core plates and cover plates are in contact, the shear force – shear deformation relationship of the element is like **Fig. 15.**

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| **Fig. 15** shear force – shear deformation relationship of the element |

This Curve includes 3 stages, the first stage is static friction stage in which the shear force increases but no shear deformation occurs, in this stage, the slope should be infinitely large and in FE program a very large value is set. The second stage is slip stage, the ordinate value of this stage depends on the friction coefficient and normal force. If the normal force changes during a multi-step analysis, this stage will not be a constant, instead, it may increase or decrease. The third stage is bearing stage, the slope of this stage is determined by formulae **(9)** – **(12)**. The compression stiffness is infinitely large and tensile stiffness equals to the axial stiffness of the bolt.

When the core plates and cover plates are not in contact, the bending, shear, axial stiffness equal to the bending, shear, axial stiffness of the bolt.

Method for applying initial preload to the bolts

Pretension of the bolt is applied by a truss element. The material model used in this element is Steel02 material with a very small Young’s modulus and the actual preload applied to the bolt.

Method for modelling the connection consisting through bolt.

Verification of the simplified model

Compare the FE result of normal double-lap joint to experimental results (Chen Y. Y. 2004)

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| **Fig.**  Load-Displacement Curve of Specimen HBS1 | **Fig.**  Load-Displacement Curve of Specimen HBS2 |
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| **Fig.**  Load-Displacement Curve of WM’s FE |  |

Compare the FE result of beam-column connection with experimental results

Simulation of structure using proposed modelling method

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6. Conclusion

The theoretical load-displacement curve is accurate

The simplified is accurate and computational efficient.

, G. (2003). 钢结构设计规范 [S].