In[ ]:=
```
ClearAll["Global`*"]
```

A multigait microrobot driven by powerful, high - frequency combustion actuators

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Auxiliary Functions

In[ ]:=
```
clean2[expr_] :=
 HoldForm[expr] /.
  h_Symbol[args___] /; Context@Unevaluated@h =!= "System`" :→ h
```

In[ ]:=
```
pdConv[f_] :=
 TraditionalForm[
   f /. Derivative[inds__][g_][vars__] :→  Apply[Defer[D[g[vars], ##]] &,
   Transpose[{{vars}, {inds}}] /. {{var_, 0} :→
     Sequence[], {var_, 1} :→ {var}}]]
```

In[ ]:=
```
TextbookEqOLD[expr_] := TableForm[TraditionalForm[{expr // pdConv }⟦1⟧⟦1⟧ // clean2]]
```

In[ ]:=
```
centering := CellPrint[ExpressionCell[#, "Output", TextAlignment → Center]] &
```

In[ ]:=
```
TextbookEq[expr_] :=
  TraditionalForm[{TableForm[expr] // pdConv }⟦1⟧⟦1⟧ // clean2] // centering
```

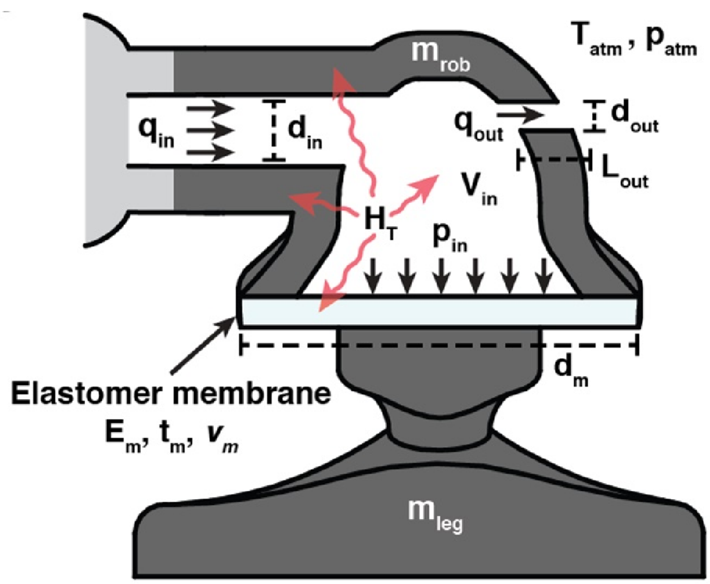
In[ ]:=
```
TextbookEqListNames[expr_] := TraditionalForm[
    {TableForm[expr, TableDirections → Row] // pdConv }⟦1⟧⟦1⟧ // clean2] // centering
```

In[ ]:=
```
$PrePrint = Style[#, "Output", TextAlignment → Center] &;
```

Overview

Nomenclature:

d_{in}	- Inlet diameter	d_{out}	- Outlet diameter
q_{in}	- Inlet volumetric flux	q_{out}	- Outlet volumetric flux
T_{in}	- Chamber temperature	T_{atm}	- Ambient temperature
p_{in}	- Chamber pressure	p_{atm}	- Ambient pressure
V_{in}	- Chamber volume	H_T	- heat transfer coefficient
m_{rob}	- Robot's main body mass	m_{leg}	- Robot's leg mass
d_{mem}	- Membrane diameter	t_m	- Membrane thickness
E_m	- Membrane Young's modulus	ρ	- Membrane Density
v_m	- Membrane Poisson's ratio	μ	- Gas viscosity



In[ ]:=

Listing the governing equations

In[ ]:=
```
ListOfAssumptions = {p[t] > 0, t > 0, R > 0, ρ[t] > 0, k > 0, V0 > 0, V > 0, Reals};
```

Flow equations

Perfect Gas equation

In[ ]:=
```
PerfectGasEq=p[t]==ρ[t]×R[t]×T[t];TextbookEq[PerfectGasEq]
```

$$p = R\,T\,\rho$$

Mass conservation equation

In[ ]:=
```
MassConservationEq=qOut[t]==qInC-D[ρ[t]×V[t],t];TextbookEq[MassConservationEq]
```

$$\mathrm{qOut} = \mathrm{qInC} - \frac{\partial \rho}{\partial t}\,V - \frac{\partial V}{\partial t}\,\rho$$

Hagen Poiseuille equation for gas exiting the chamber

In[ ]:=
```
qOutEq=qOut[t]== ρ[t] (π rEx^4)/(8 μ)(p[t]-pAtm)/lExit;TextbookEq[qOutEq]
```

$$\mathrm{qOut} = \frac{\pi\,\mathrm{rEx}^4\,(-\mathrm{pAtm} + p)\,\rho}{8\,\mathrm{lExit}\,\mu}$$

This equation is for an incompressible fluid. For a compressible fluid there is a different equation. The difference is significant only if the pressure within the chamber is O(Atm).

Elasticity

Elastic relation between chamber volume and pressure

In[ ]:=
```
ElasticReBen=w/rc==1/5(12(1-pr^2)q/ ( 64 Em h^3))(rc^2-r^2)^2;
wFunc[r_,q_,rc_,h_,Em_,pr_]=Flatten[Solve[ElasticReBen,w]][[1]][[2]];
VtoPEq=V[t]-V0==Integrate[wFunc[r,p[t]-pAtm,rc,h,Em,pr] 2 π r,{r,0,rc}];TextbookEq[VtoPEq]
```

$$-\mathrm{V0} + V = \frac{\pi\left(-1 + \mathrm{pr}^2\right)\mathrm{rc}^7\,(\mathrm{pAtm} - p)}{80\,\mathrm{Em}\,h^3}$$

Species concentrations

The number of molecules within a chamber does not change during combustion, so the specific gas constant can be calculated by.

In[ ]:=
```
GasConstantEq=R[t]==(1000(mO2[t]/32+mCH4[t]/160+mCO2[t]/44+mH2O[t]/18+mN2[t]/24)/(mN2[t]+mO2[t]
DensityEq=ρ[t]×V[t] ==mO2[t]+mN2[t]+mCH4[t]+mCO2[t]+mH2O[t];
mCH4ConservationEq=D[mCH4[t],t]==mCH4In-HeavisideTheta[qOut[t]]qOut[t] (mCH4[t]/(V[t]×ρ[t]))-16
mO2ConservationEq=D[mO2[t],t]==mO2In-HeavisideTheta[mO2[t]]HeavisideTheta[qOut[t]]qOut[t] (mO2[t
mCO2ConservationEq=D[mCO2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mCO2[t]/(V[t]×ρ[t]))+44 α IsI
mH2OConservationEq=D[mH2O[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mH2O[t]/(V[t]×ρ[t]))+H2OK 36 
mN2ConservationEq=D[mN2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mN2[t]/(V[t]×ρ[t]))-HeavisideThe
```

Upon combustion, one moll of CH4 vanishes, and along with 2 molls of O2. Two molls of H20 are created as well as 1 moll of CO2.
CH4+2O2->CO2+2H2O.

Ignition and energy equations

First we model the time in which ignition occurs

```
In[◦]:= HeatEquation = D[ρ[t] 15V[t] Cp T[t], t] == -hT π rc rc (T[t] - Tatm) + 16 HeatGen α IsIgnitionFu
TextbookEq[HeatEquation]
```

$$15\,\mathrm{Cp}\,\frac{\partial\rho}{\partial t}\,T\,V + 15\,\mathrm{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho + 15\,\mathrm{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho = 16\,\mathrm{HeatGen}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - \frac{\partial V}{\partial t}\,p - \mathrm{hT}\,\pi\,\mathrm{rc}^2\,(-\mathrm{Tatm}+T) + \mathrm{Cp}\,(\mathrm{qInC}\,\mathrm{TAtm} - \mathrm{qOut}\,T)$$

Analysis

Lets define a list with all of our equations

```
In[◦]:= EquationList={PerfectGasEq,MassConservationEq,qOutEq,VtoPEq,GasConstantEq,DensityEq,mCH4Conservat:
TextbookEq[EquationList]
```

$$\mathrm{ıC} - \frac{\partial\rho}{\partial t}\,V - \frac{\partial V}{\partial t}\,\rho$$

$$\frac{\mathrm{Ex}^4\,(-\mathrm{pAtm}+p)\,\rho}{8\,\mathrm{lExit}\,\mu}$$

$$= \frac{\pi\left(-1+\mathrm{pr}^2\right)\mathrm{rc}^7\,(\mathrm{pAtm}-p)}{80\,\mathrm{Em}\,h^3}$$

$$\frac{\frac{\mathrm{ıCH4}}{160}+\frac{\mathrm{mCO2}}{44}+\frac{\mathrm{mH2O}}{18}+\frac{\mathrm{mN2}}{24}+\frac{\mathrm{mO2}}{32}\Big)}{+\mathrm{mCO2}+\mathrm{mH2O}+\mathrm{mN2}+\mathrm{mO2}}$$

$$\mathrm{H4} + \mathrm{mCO2} + \mathrm{mH2O} + \mathrm{mN2} + \mathrm{mO2}$$

$$\mathrm{ıCH4In} - 16\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - \frac{\theta(\mathrm{qOut})\,\mathrm{mCH4}\,\mathrm{qOut}}{V\,\rho}$$

$$\mathrm{O2In} - 64\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - 0.21\,\theta(-\mathrm{qOut})\,\mathrm{qOut} - \frac{\theta(\mathrm{mO2})\,\theta(\mathrm{qOut})\,\mathrm{mO2}\,\mathrm{qOut}}{V\,\rho}$$

$$4\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - \frac{\theta(\mathrm{qOut})\,\mathrm{mCO2}\,\mathrm{qOut}}{V\,\rho}$$

$$6\,\mathrm{H2OK}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - \frac{\theta(\mathrm{qOut})\,\mathrm{mH2O}\,\mathrm{qOut}}{V\,\rho}$$

$$.79\,\theta(-\mathrm{qOut})\,\mathrm{qOut} - \frac{\theta(\mathrm{qOut})\,\mathrm{mN2}\,\mathrm{qOut}}{V\,\rho}$$

$$V + 15\,\mathrm{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho + 15\,\mathrm{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho = 16\,\mathrm{HeatGen}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc} - \frac{\partial V}{\partial t}\,p - \mathrm{hT}\,\pi\,\mathrm{rc}^2\,(-\mathrm{Tatm}+T) + \mathrm{Cp}\,(\mathrm{qInC}\,\mathrm{TAtm}$$

```
In[◦]:= NameList = {"PerfectGasEq", "MassConservationEq", "qOutEq", "VtoPEq", "GasConstantEq", "DensityEq'
```

In[ ]:=
```
qOutFunc[t_]=Flatten[Solve[EquationList[[3]],qOut[t]]][[1]][[2]];
EquationList1=Simplify[EquationList//.qOut→Function[t,qOutFunc[t]]];
ρFunc[t_]=Flatten[Solve[EquationList1[[6]],ρ[t]]][[1]][[2]];
EquationList2=Simplify[EquationList1//.ρ→Function[t,ρFunc[t]]];
VFunc[t_]=Flatten[Solve[EquationList2[[4]],V[t]]][[1]][[2]];
EquationList3=Simplify[EquationList2//.V→Function[t,VFunc[t]],ListOfAssumptions];
TFunc[t_]= Flatten[Solve[EquationList3[[1]],T[t]]][[1]][[2]];
EquationList4=Simplify[EquationList3//.T→Function[t,TFunc[t]],ListOfAssumptions];
RFunc[t_]=Solve[EquationList4[[5]],R[t]][[1]][[1]][[2]];
EquationList5=EquationList4//.R→Function[t,RFunc[t]];
```

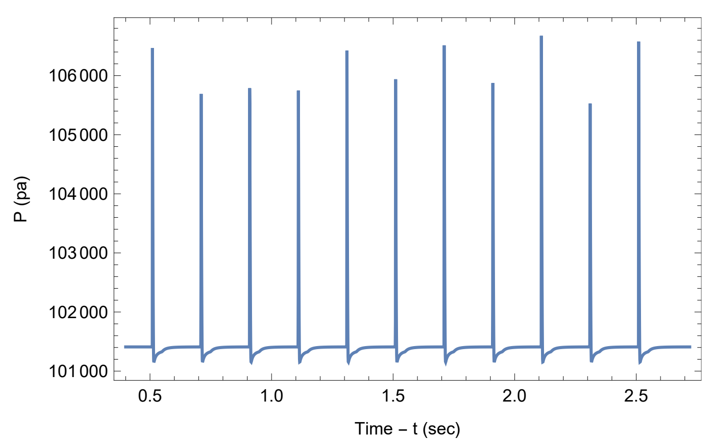
In[ ]:=
```
QinO2=100;(*ml/min*)
ρO2=1.331;(*kg/m^3*)
ρNH4=0.668;(*kg/m^3*)
mInO2=QinO2/60 ρO2/10^6;
QinNH4val=12/60 ρNH4/10^6;
```

In[ ]:=
```
PRV={rEx→.2/1000,pAtm→101325.0,TAtm→320,V0→(π 0.003×0.003×0.0035),lExit→3.0/1000,hT→1000,Cp→8*:
 mO2In→mInO2,
 mCH4In→ QinNH4val,
 qInC→mInO2+QinNH4val,
 fr→5,dt→2*10^-4,MaxStepSizeVal→0.0151/100,α→5*5*10^-10,H2OK→1,NCycles→10,Tmax→0.52+11/5,Rtot→268,Cp

IsIgnitionFunc[t_,fr_,dt_]=Sum[PDF[NormalDistribution[i/fr+0.51,dt],t],{i,0,10,1}];
EquationList6=EquationList5//.PRV;
{mO20,mCO20,mH2O0,mCH40,mN20}={4.537×10^-8,0,0,0,0};
s=NDSolve[{EquationList6[[7]],EquationList6[[8]],EquationList6[[9]],EquationList6[[10]],EquationList6[[11]]
```

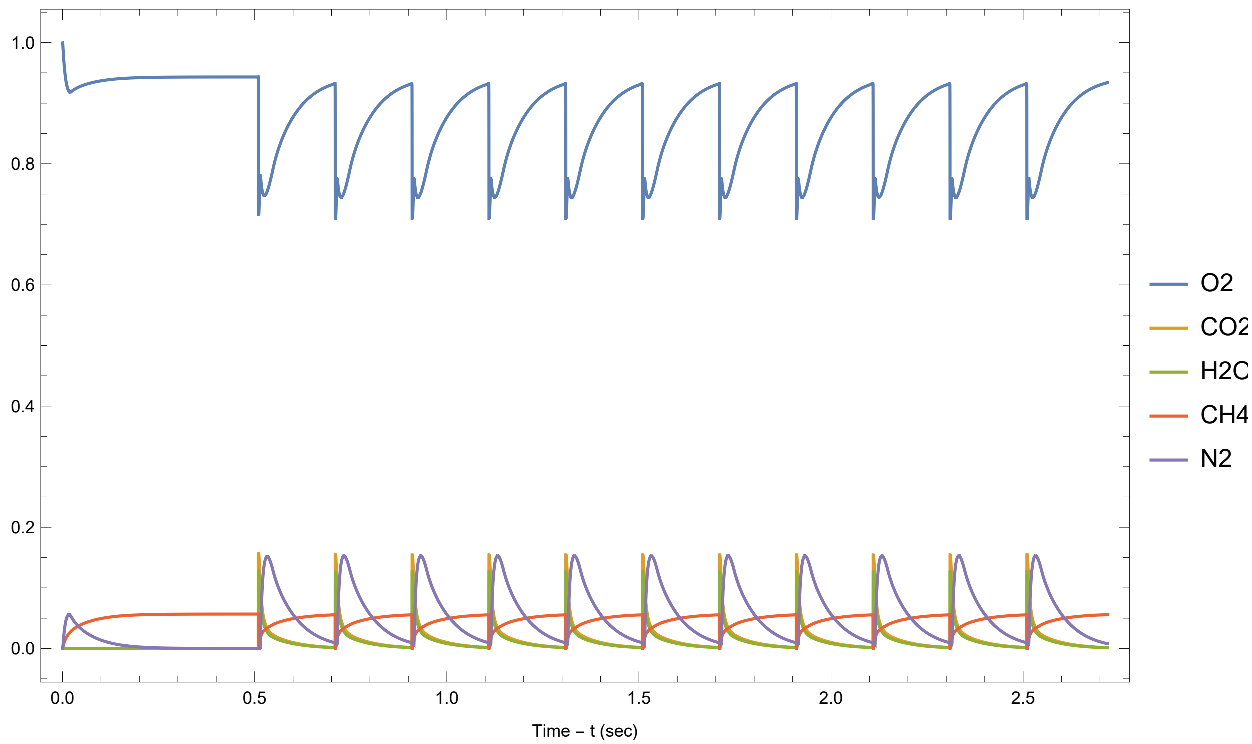
In[ ]:=
```
sol=First[p/.s];
Plot[sol[t],{t,0.4,Tmax/.PRV},PlotRange→All,PlotRange→All,Frame→True,FrameLabel→{"Time - t (sec)
```

Out[ ]=



In[∘]:=
```
m[t_] = (Evaluate[mO2[t] /. s] + Evaluate[mCO2[t] /. s] + Evaluate[mH2O[t] /. s] + Evaluate[mCH
PanelA = Plot[{(mO2[t] /. s)/m[t], (mCO2[t] /. s)/m[t], (mH2O[t] /. s)/m[t], (mCH4[t] /. s)/m[t
```

Out[∘]=



Thermal analysis

The flame temperature is taken from the model presented above

In[∘]:=
```
TModel=TFunc[t]/.R→Function[t,RFunc[t]]/.s/.PRV;
ti=(Tmax/.PRV)-10/(fr/.PRV)-0.5/(fr/.PRV);
tf=(Tmax/.PRV)-0.5/(fr/.PRV);
TdataModel=Table[temp/.{t→tt},{t,ti,tf,(tf-ti)/1000}];
```

The amount of flux exits trough the exhaust and the amount of heat generated within the chamber are given by

In[∘]:=
```
QoutModel=qOutFunc[t]/.(ρ[t]→ρFunc[t])/.(V[t]→VFunc[t])/.s/.PRV;
HeatGeneretedModel=ρ[t]×V[t] Cp TFunc[t]/.(ρ[t]→ρFunc[t])/.(V[t]→VFunc[t])/.R→Function[t,RFunc
Qdata=Table[{t-ti,First[HeatGeneretedModel]},{t,ti,tf,(tf-ti)/1000}];
Qfunc=Interpolation[Qdata,t];
```

The internal wall temperature is therefore given by a transient heat equation (see Supplementary Information for more details):

In[ ]:=

```
ncycles=200;
Qdata2=Flatten[Table[Table[{j (tf-ti+ tf-ti/1000 )+t,If[j (tf-ti+ tf-ti/1000 )+t<120,Qfunc,0]},{t,0,tf-ti, tf-t/100
QfuncContinous=Interpolation[Qdata2,t];
Tend=260;
Tinternal=First[T[t]/.NDSolve[{(CpActuator/.PRV)[[1]]T'[t]==QfuncContinous- (T[t]-(TAtm/.PRV))/(Rtot/.PRV) ,T[0
Plot[Tinternal,{t,0,Tend},Frame→True,FrameLabel→{"Time - t (sec)","Internal Wall Temperature (K)
```

Out[ ]=