In[ ]:=
```
ClearAll["Global`*"]
```

A multigait microrobot driven by powerful, high - frequency combustion actuators

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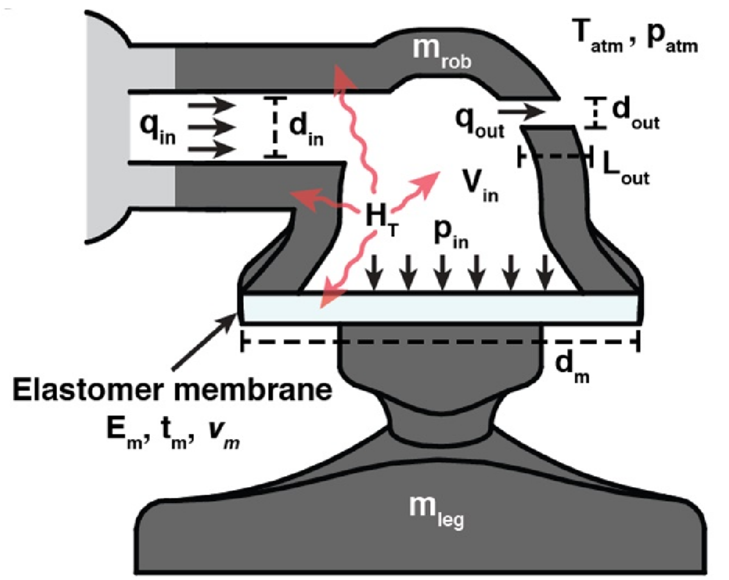
Auxiliary Functions

Overview

Nomenclature:

d_{in}	- Inlet diameter	d_{out}	- Outlet diameter
q_{in}	- Inlet volumetric flux	q_{out}	- Outlet volumetric flux
T_{in}	- Chamber temperature	T_{atm}	- Ambient temperature
p_{in}	- Chamber pressure	p_{atm}	- Ambient pressure
V_{in}	- Chamber volume	H_T	- heat transfer coefficient
m_{rob}	- Robot’s main body mass	m_{leg}	- Robot’s leg mass
d_{mem}	- Membrane diameter	t_m	- Membrane thickness
E_m	- Membrane Young’s modulus	ρ	- Membrane Density
v_m	- Membrane Poisson’s ratio	μ	- Gas viscosity



In[ ]:=

Listing the governing equations

In[ ]:=
```
ListOfAssumptions = {p[t] > 0, t > 0, R > 0, ρ[t] > 0, k > 0, V0 > 0, V > 0, Reals};
```

Flow equations

Perfect Gas equation

In[ ]:=
```
PerfectGasEq=p[t]==ρ[t]×R[t]×T[t];TextbookEq[PerfectGasEq]
```

$$p = R\,T\,\rho$$

Mass conservation equation

In[ ]:=
```
MassConservationEq=qOut[t]==qInC-D[ρ[t]×V[t],t];TextbookEq[MassConservationEq]
```

$$\mathrm{qOut} = \mathrm{qInC} - \frac{\partial \rho}{\partial t} V - \frac{\partial V}{\partial t} \rho$$

Hagen Poiseuille equation for gas exiting the chamber

In[ ]:=
```
qOutEq=qOut[t]== ρ[t] (π rEx^4)/(8 μ) (p[t]-pAtm)/lExit;TextbookEq[qOutEq]
```

$$\mathrm{qOut} = \frac{\pi\,\mathrm{rEx}^4\,(-\mathrm{pAtm} + p)\,\rho}{8\,\mathrm{lExit}\,\mu}$$

This equation is for an incompressible fluid. For a compressible fluid there is a different equation. The difference is significant only if the pressure within the chamber is O(Atm).

Elasticity

Elastic relation between chamber volume and pressure

In[ ]:=
```
ElasticReBen=w/rc==1/5(12(1-pr^2)q/ ( 64 Em h^3))(rc^2-r^2)^2;
wFunc[r_,q_,rc_,h_,Em_,pr_]=Flatten[Solve[ElasticReBen,w]][[1]][[2]];
VtoPEq=V[t]-V0==Integrate[wFunc[r,p[t]-pAtm,rc,h,Em,pr] 2 π r,{r,0,rc}];TextbookEq[VtoPEq]
```

$$-\mathrm{V0}+V=\frac{\pi\left(-1+\mathrm{pr}^2\right)\mathrm{rc}^7\,(\mathrm{pAtm}-p)}{80\,\mathrm{Em}\,h^3}$$

Species concentrations

The number of molecules within a chamber does not change during combustion, so the specific gas constant can be calculated by.

In[ ]:=
```
GasConstantEq=R[t]==(1000(mO2[t]/32+mCH4[t]/160+mCO2[t]/44+mH2O[t]/18+mN2[t]/24)/(mN2[t]+mO2[t]
DensityEq=ρ[t]×V[t]  ==mO2[t]+mN2[t]+mCH4[t]+mCO2[t]+mH2O[t];
mCH4ConservationEq=D[mCH4[t],t]==mCH4In-HeavisideTheta[qOut[t]]qOut[t]  (mCH4[t]/(V[t]×ρ[t]))-16
mO2ConservationEq=D[mO2[t],t]==mO2In-HeavisideTheta[mO2[t]]HeavisideTheta[qOut[t]]qOut[t]  (mO2[t
mCO2ConservationEq=D[mCO2[t],t]==-HeavisideTheta[qOut[t]]qOut[t]  (mCO2[t]/(V[t]×ρ[t]))+44 α IsI
mH2OConservationEq=D[mH2O[t],t]==-HeavisideTheta[qOut[t]]qOut[t]  (mH2O[t]/(V[t]×ρ[t]))+H2OK 36 α
mN2ConservationEq=D[mN2[t],t]==-HeavisideTheta[qOut[t]]qOut[t]  (mN2[t]/(V[t]×ρ[t]))-HeavisideThe
```

Upon combustion, one moll of CH4 vanishes, and along with 2 molls of O2. Two molls of H20 are created as well as 1 moll of CO2.
 CH4+2O2->CO2+2H2O.

Ignition and energy equations

First we model the time in which ignition occurs

In[ ]:=
```
HeatEquation = D[ρ[t]×V[t] Cp T[t], t] == -hT π rc rc (T[t] - Tatm) + 16 HeatGen α IsIgnitionFun
TextbookEq[HeatEquation]
```

$$\mathrm{Cp}\,\frac{\partial\rho}{\partial t}\,T\,V+\mathrm{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho+\mathrm{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho=$$
$$16\,\mathrm{HeatGen}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\partial V}{\partial t}\,p-\mathrm{hT}\,\pi\,\mathrm{rc}^2\,(-\mathrm{Tatm}+T)+\mathrm{Cp}\,(\mathrm{qInC}\,\mathrm{TAtm}-\mathrm{qOut}\,T)$$

Analysis

Lets define a list with all of our equations

In[ ]:=
```
EquationList={PerfectGasEq,MassConservationEq,qOutEq,VtoPEq,GasConstantEq,DensityEq,mCH4Conservat
TextbookEq[EquationList]
```

$T\,\rho$

$= \text{qInC} - \frac{\partial\rho}{\partial t}\,V - \frac{\partial V}{\partial t}\,\rho$

$= \frac{\pi\,\text{rEx}^4\,(-\text{pAtm}+p)\,\rho}{8\,\text{lExit}\,\mu}$

$V = \frac{\pi\left(-1+\text{pr}^2\right)\text{rc}^7\,(\text{pAtm}-p)}{80\,\text{Em}\,h^3}$

$\frac{314.\left(\frac{\text{mCH4}}{160}+\frac{\text{mCO2}}{44}+\frac{\text{mH2O}}{18}+\frac{\text{mN2}}{24}+\frac{\text{mO2}}{32}\right)}{\text{mCH4}+\text{mCO2}+\text{mH2O}+\text{mN2}+\text{mO2}}$

$\text{mCH4} + \text{mCO2} + \text{mH2O} + \text{mN2} + \text{mO2}$

$= \text{mCH4In} - 16\,\alpha\,\theta(\text{mCH4})\,\text{IsIgnitionFunc} - \frac{\theta(\text{qOut})\,\text{mCH4}\,\text{qOut}}{V\,\rho}$

$= \text{mO2In} - 64\,\alpha\,\theta(\text{mCH4})\,\text{IsIgnitionFunc} - 0.21\,\theta(-\text{qOut})\,\text{qOut} - \frac{\theta(\text{mO2})\,\theta(\text{qOut})\,\text{mO2}\,\text{qOut}}{V\,\rho}$

$= 44\,\alpha\,\theta(\text{mCH4})\,\text{IsIgnitionFunc} - \frac{\theta(\text{qOut})\,\text{mCO2}\,\text{qOut}}{V\,\rho}$

$= 36\,\text{H2OK}\,\alpha\,\theta(\text{mCH4})\,\text{IsIgnitionFunc} - \frac{\theta(\text{qOut})\,\text{mH2O}\,\text{qOut}}{V\,\rho}$

$= -0.79\,\theta(-\text{qOut})\,\text{qOut} - \frac{\theta(\text{qOut})\,\text{mN2}\,\text{qOut}}{V\,\rho}$

$T\,V + \text{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho + \text{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho = 16\,\text{HeatGen}\,\alpha\,\theta(\text{mCH4})\,\text{IsIgnitionFunc} - \frac{\partial V}{\partial t}\,p - \text{hT}\,\pi\,\text{rc}^2\,(-\text{Tatm}+T) + \text{Cp}\,(\text{qInC TAtm} - \text{qO}$

In[ ]:=
```
NameList = {"PerfectGasEq", "MassConservationEq", "qOutEq", "VtoPEq", "GasConstantEq", "DensityEq"
```

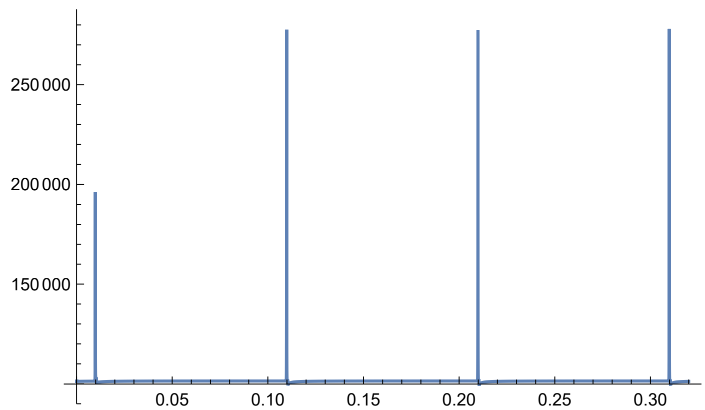
In[ ]:=
```
qOutFunc[t_]=Flatten[Solve[EquationList〚3〛,qOut[t]]]〚1〛〚2〛;
EquationList1=Simplify[EquationList//.qOut→Function[t,qOutFunc[t]]];
ρFunc[t_]=Flatten[Solve[EquationList1〚6〛,ρ[t]]]〚1〛〚2〛;
EquationList2=Simplify[EquationList1//.ρ→Function[t,ρFunc[t]]];
VFunc[t_]=Flatten[Solve[EquationList2〚4〛,V[t]]]〚1〛〚2〛;
EquationList3=Simplify[EquationList2//.V→Function[t,VFunc[t]],ListOfAssumptions];
TFunc[t_]= Flatten[Solve[EquationList3〚1〛,T[t]]]〚1〛〚2〛;
EquationList4=Simplify[EquationList3//.T→Function[t,TFunc[t]],ListOfAssumptions];
RFunc[t_]=Solve[EquationList4〚5〛,R[t]]〚1〛〚1〛〚2〛;
EquationList5=EquationList4//.R→Function[t,RFunc[t]];
```

In[ ]:=
```
QinO2=100;(*ml/min*)
ρO2=1.331;(*kg/m^3*)
ρNH4=0.668;(*kg/m^3*)
mInO2=QinO2/60 ρO2/10^6;
QinNH4val=18/60 ρNH4/10^6;
```

In[ ]:=
```
PRV={rEx→.2/1000,pAtm→101325.0,TAtm→300,V0→(π 0.003×0.003×0.0035),lExit→3.0/1000,hT→0.05*1000,C
 mO2In→mInO2,
 mCH4In→ QinNH4val,
 qInC→mInO2+QinNH4val,
  fr→10,dt→10^-4,MaxStepSizeVal→0.0151/100,α→5*5*10^-10,H2OK→1,NCycles→10,Tmax→0.02+3/10};
IsIgnitionFunc[t_,fr_,dt_]=Sum[PDF[NormalDistribution[i/fr+0.01,dt],t],{i,0,3,1}];
EquationList6=EquationList5//.PRV;
{mO20,mCO20,mH2O0,mCH40,mN20}={4.537×10^-8,0,0,0,0};
s=NDSolve[{EquationList6[[7]],EquationList6[[8]],EquationList6[[9]],EquationList6[[10]],EquationList6[[11]
```

In[ ]:=
```
sol=First[p/.s];
Plot[sol[t],{t,0,Tmax/.PRV},PlotRange→All,PlotPoints→10000]
```

Out[ ]=



In[ ]:=

```
m[t_] = (Evaluate[mO2[t] /. s] + Evaluate[mCO2[t] /. s] + Evaluate[mH2O[t] /. s] + Evaluate[mCH4
PanelA = Labeled[Plot[{(mO2[t] /. s)/m[t], (mCO2[t] /. s)/m[t], (mH2O[t] /. s)/m[t], (mCH4[t] /
```

Out[ ]=