In[228]:=

```
ClearAll["Global`*"]
```

A multigait microrobot driven by powerful, high - frequency combustion actuators

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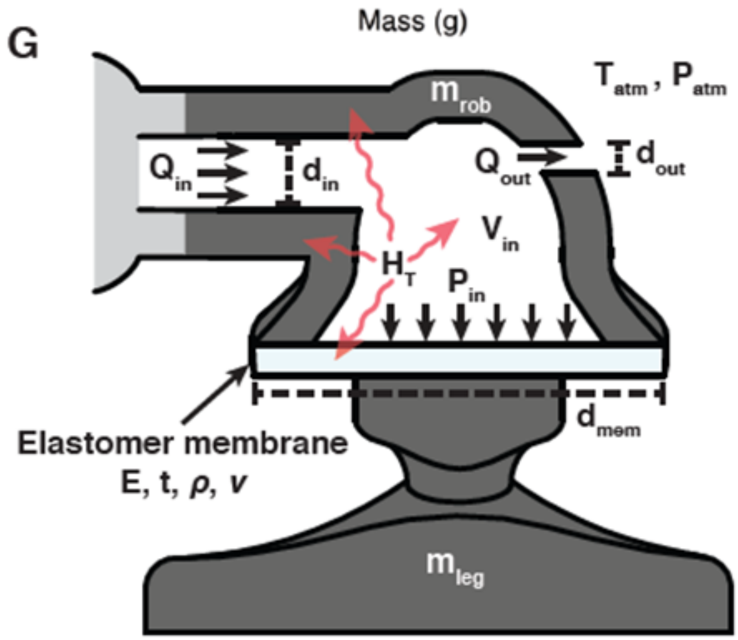
Auxiliary Functions

Overview

Nomenclature:

d_{in}	- Inlet diameter	d_{out}	- Outlet diameter
Q_{in}	- Inlet volumetric flux	Q_{out}	- Outlet volumetric flux
T_{in}	- Chamber temperature	T_{atm}	- Ambient temperature
P_{in}	- Chamber pressure	P_{atm}	- Ambient pressure
V_{in}	- Chamber volume	H_T	- heat transfer coefficient
m_{rob}	- Robot's main body mass	m_{leg}	- Robot's leg mass
d_{mem}	- Membrane diameter	t	- Membrane thickness
E	- Membrane Young's modulus	ρ	- Membrane Density
v	- Membrane Poisson's ratio	μ	- Gas viscosity

In[122]:=



In[236]:=

Listing the governing equations

In[237]:=

```
ListOfAssumptions = {p[t] > 0, t > 0, R > 0, ρ[t] > 0, k > 0, V0 > 0, V > 0, Reals};
```

Flow equations

Perfect Gas equation

In[238]:=

```
PerfectGasEq=p[t]==ρ[t]×R[t]×T[t];TextbookEq[PerfectGasEq]
```

$$p = R\,T\,\rho$$

Mass conservation equation

In[239]:=

```
MassConservationEq=qOut[t]==qInC-D[ρ[t]×V[t],t];TextbookEq[MassConservationEq]
```

$$\text{qOut} = \text{qInC} - \frac{\partial \rho}{\partial t} V - \frac{\partial V}{\partial t} \rho$$

Hagen Poiseuille equation for gas exiting the chamber

In[240]:=

```
qOutEq=qOut[t]== ρ[t] (π rEx^4)/(8 μ)(p[t]-pAtm)/lExit;TextbookEq[qOutEq]
```

$$\text{qOut} = \frac{\pi\,\text{rEx}^4\,(-\text{pAtm}+p)\,\rho}{8\,\text{lExit}\,\mu}$$

This equation is for an incompressible fluid. For a compressible fluid there is a different equation. The difference is significant only if the pressure within the chamber is O(Atm).

Elasticity

Elastic relation between chamber volume and pressure

In[241]:=

```
ElasticReBen=w/rc==1/5(12(1-pr^2)q/ ( 64 Em h^3))(rc^2-r^2)^2;
wFunc[r_,q_,rc_,h_,Em_,pr_]=Flatten[Solve[ElasticReBen,w]][[1]][[2]];
VtoPEq=V[t]-V0==Integrate[wFunc[r,p[t]-pAtm,rc,h,Em,pr] 2 π r,{r,0,rc}];TextbookEq[VtoPEq]
```

$$-\text{V0}+V = \frac{\pi\left(-1+\text{pr}^2\right)\text{rc}^7\,(\text{pAtm}-p)}{80\,\text{Em}\,h^3}$$

Species concentrations

The number of molecules within a chamber does not change during combustion, so the specific gas constant can be calculated by.

In[244]:=

```
GasConstantEq=R[t]==(1000(mO2[t]/32+mCH4[t]/160+mCO2[t]/44+mH2O[t]/18+mN2[t]/24)/(mN2[t]+mO2[t]
DensityEq=ρ[t]×V[t] ==mO2[t]+mN2[t]+mCH4[t]+mCO2[t]+mH2O[t];
mCH4ConservationEq=D[mCH4[t],t]==mCH4In-HeavisideTheta[qOut[t]]qOut[t] (mCH4[t]/(V[t]×ρ[t]))-16
mO2ConservationEq=D[mO2[t],t]==mO2In-HeavisideTheta[mO2[t]]HeavisideTheta[qOut[t]]qOut[t] (mO2[t
mCO2ConservationEq=D[mCO2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mCO2[t]/(V[t]×ρ[t]))+44 α IsI
mH2OConservationEq=D[mH2O[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mH2O[t]/(V[t]×ρ[t]))+H2OK 36 α
mN2ConservationEq=D[mN2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mN2[t]/(V[t]×ρ[t]))-HeavisideThe
```

Upon combustion, one moll of CH4 vanishes, and along with 2 molls of O2. Two molls of H20 are created as well as 1 moll of CO2.
CH4+2O2->CO2+2H2O.

Ignition and energy equations

First we model the time in which ignition occurs

In[251]:=

```
HeatEquation = D[ρ[t]×V[t] Cp T[t], t] == -hT π rc rc (T[t] - Tatm) + 16 HeatGen α IsIgnitionFun
TextbookEq[HeatEquation]
```

$$\mathrm{Cp}\,\frac{\partial \rho}{\partial t}\,T\,V+\mathrm{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho+\mathrm{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho=$$
$$16\,\mathrm{HeatGen}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\partial V}{\partial t}\,p-\mathrm{hT}\,\pi\,\mathrm{rc}^2\,(-\mathrm{Tatm}+T)+\mathrm{Cp}\,(\mathrm{qInC}\,\mathrm{TAtm}-\mathrm{qOut}\,T)$$

Analysis

Lets define a list with all of our equations

In[253]:=

```
EquationList={PerfectGasEq,MassConservationEq,qOutEq,VtoPEq,GasConstantEq,DensityEq,mCH4Conservat
TextbookEq[EquationList]
```

$T\,\rho$

$= \mathrm{qInC}-\frac{\partial\rho}{\partial t}\,V-\frac{\partial V}{\partial t}\,\rho$

$= \frac{\pi\,\mathrm{rEx}^4\,(-\mathrm{pAtm}+p)\,\rho}{8\,\mathrm{lExit}\,\mu}$

$V=\frac{\pi\left(-1+\mathrm{pr}^2\right)\mathrm{rc}^7\,(\mathrm{pAtm}-p)}{80\,\mathrm{Em}\,h^3}$

$\frac{314.\left(\frac{\mathrm{mCH4}}{160}+\frac{\mathrm{mCO2}}{44}+\frac{\mathrm{mH2O}}{18}+\frac{\mathrm{mN2}}{24}+\frac{\mathrm{mO2}}{32}\right)}{\mathrm{mCH4}+\mathrm{mCO2}+\mathrm{mH2O}+\mathrm{mN2}+\mathrm{mO2}}$

$\mathrm{mCH4}+\mathrm{mCO2}+\mathrm{mH2O}+\mathrm{mN2}+\mathrm{mO2}$

$= \mathrm{mCH4In}-16\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\theta(\mathrm{qOut})\,\mathrm{mCH4}\,\mathrm{qOut}}{V\,\rho}$

$= \mathrm{mO2In}-64\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-0.21\,\theta(-\mathrm{qOut})\,\mathrm{qOut}-\frac{\theta(\mathrm{mO2})\,\theta(\mathrm{qOut})\,\mathrm{mO2}\,\mathrm{qOut}}{V\,\rho}$

$= 44\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\theta(\mathrm{qOut})\,\mathrm{mCO2}\,\mathrm{qOut}}{V\,\rho}$

$= 36\,\mathrm{H2OK}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\theta(\mathrm{qOut})\,\mathrm{mH2O}\,\mathrm{qOut}}{V\,\rho}$

$= -0.79\,\theta(-\mathrm{qOut})\,\mathrm{qOut}-\frac{\theta(\mathrm{qOut})\,\mathrm{mN2}\,\mathrm{qOut}}{V\,\rho}$

$T\,V+\mathrm{Cp}\,\frac{\partial V}{\partial t}\,T\,\rho+\mathrm{Cp}\,\frac{\partial T}{\partial t}\,V\,\rho=16\,\mathrm{HeatGen}\,\alpha\,\theta(\mathrm{mCH4})\,\mathrm{IsIgnitionFunc}-\frac{\partial V}{\partial t}\,p-\mathrm{hT}\,\pi\,\mathrm{rc}^2\,(-\mathrm{Tatm}+T)+\mathrm{Cp}\,(\mathrm{qInC}\,\mathrm{TAtm}-\mathrm{qO}$

In[255]:=

```
NameList = {"PerfectGasEq", "MassConservationEq", "qOutEq", "VtoPEq", "GasConstantEq", "DensityEq"
```

In[256]:=

```
qOutFunc[t_]=Flatten[Solve[EquationList⟦3⟧,qOut[t]]]⟦1⟧⟦2⟧;
EquationList1=Simplify[EquationList//.qOut→Function[t,qOutFunc[t]]];
ρFunc[t_]=Flatten[Solve[EquationList1⟦6⟧,ρ[t]]]⟦1⟧⟦2⟧;
EquationList2=Simplify[EquationList1//.ρ→Function[t,ρFunc[t]]];
VFunc[t_]=Flatten[Solve[EquationList2⟦4⟧,V[t]]]⟦1⟧⟦2⟧;
EquationList3=Simplify[EquationList2//.V→Function[t,VFunc[t]],ListOfAssumptions];
TFunc[t_]= Flatten[Solve[EquationList3⟦1⟧,T[t]]]⟦1⟧⟦2⟧;
EquationList4=Simplify[EquationList3//.T→Function[t,TFunc[t]],ListOfAssumptions];
RFunc[t_]=Solve[EquationList4⟦5⟧,R[t]]⟦1⟧⟦1⟧⟦2⟧;
EquationList5=EquationList4//.R→Function[t,RFunc[t]];
```

In[266]:=

```
QinO2=100;(*ml/min*)
ρO2=1.331;(*kg/m^3*)
ρNH4=0.668;(*kg/m^3*)
```

$$mInO2=\frac{QinO2}{60}\ \frac{\rho O2}{10^6};$$

$$QinNH4val=\frac{18}{60}\ \frac{\rho NH4}{10^6};$$

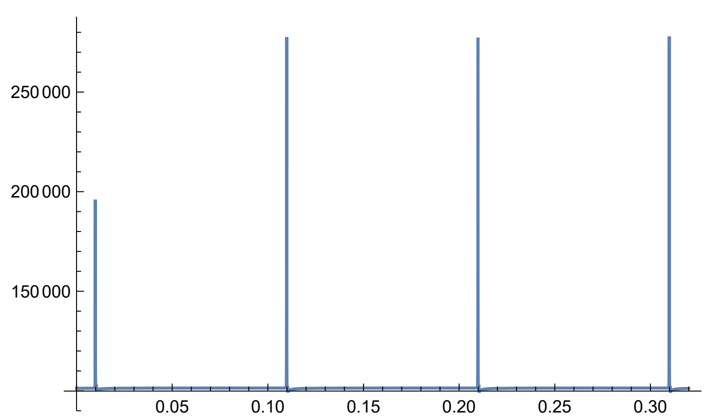
In[271]:=

```
PRV={rEx→.2/1000,pAtm→101325.0,TAtm→300,V0→(π 0.003×0.003×0.0035),lExit→3.0/1000,hT→0.05*1000,C
mO2In→mInO2,
mCH4In→ QinNH4val,
qInC→mInO2+QinNH4val,
fr→10,dt→10^-4,MaxStepSizeVal→0.0151/100,α→5*5*10^-10,H2OK→1,NCycles→10,Tmax→0.02+3/10};
IsIgnitionFunc[t_,fr_,dt_]=Sum[PDF[NormalDistribution[i/fr+0.01,dt],t],{i,0,3,1}];
EquationList6=EquationList5//.PRV;
{mO20,mCO20,mH2O0,mCH40,mN20}={4.537×10^-8,0,0,0,0};
s=NDSolve[{EquationList6⟦7⟧,EquationList6⟦8⟧,EquationList6⟦9⟧,EquationList6⟦10⟧,EquationList6⟦11⟧
```

In[276]:=

```
sol=First[p/.s];
Plot[sol[t],{t,0,Tmax/.PRV},PlotRange→All,PlotPoints→10000]
```

Out[277]=



In[278]:=

```
m[t_] = (Evaluate[mO2[t] /. s] + Evaluate[mCO2[t] /. s] + Evaluate[mH2O[t] /. s] + Evaluate[mCH4
PanelA = Labeled[Plot[{(mO2[t] /. s)/m[t], (mCO2[t] /. s)/m[t], (mH2O[t] /. s)/m[t], (mCH4[t] /
```

Out[279]=