

In[228]:=

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A multigait microrobot driven by powerful, high - frequency combustion actuators

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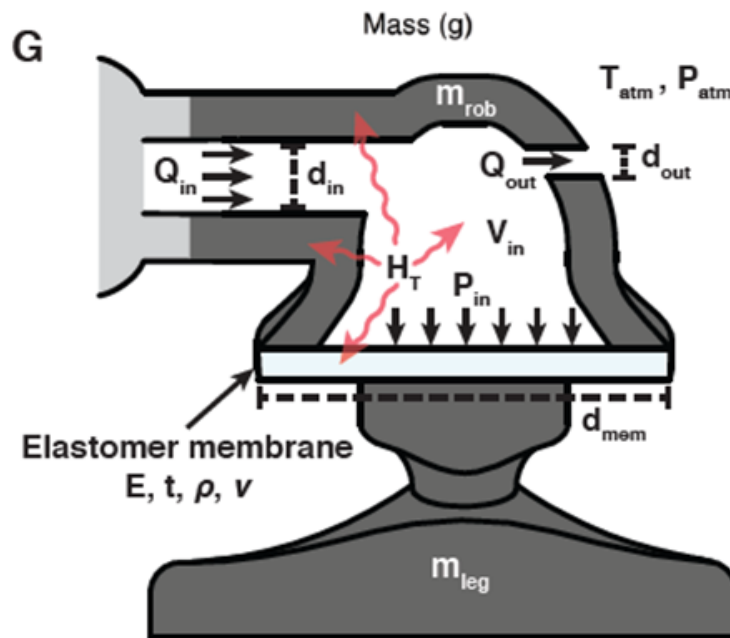
Auxiliary Functions

Overview

Nomenclature:

d_{in}	- Inlet diameter	d_{out}	- Outlet diameter
Q_{in}	- Inlet volumetric flux	Q_{out}	- Outlet volumetric flux
T_{in}	- Chamber temperature	T_{atm}	- Ambient temperature
P_{in}	- Chamber pressure	P_{atm}	- Ambient pressure
V_{in}	- Chamber volume	H_T	- heat transfer coefficient
m_{rob}	- Robot's main body mass	m_{leg}	- Robot's leg mass
d_{mem}	- Membrane diameter	t	- Membrane thickness
E	- Membrane Young's modulus	ρ	- Membrane Density
ν	- Membrane Poisson's ratio	μ	- Gas viscosity

In[122]:=



In[236]:=

Listing the governing equations

In[237]:=

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ListOfAssumptions = {p[t] > 0, t > 0, R > 0, rho[t] > 0, k > 0, V0 > 0, V > 0, Reals};
```

Flow equations

Perfect Gas equation

In[238]:=

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PerfectGasEq=p[t]==rho[t]*R[t]*T[t];TextbookEq[PerfectGasEq]
```

$$p = R T \rho$$

Mass conservation equation

In[239]:=

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MassConservationEq=qOut[t]==qInC-D[rho[t]*V[t],t];TextbookEq[MassConservationEq]
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$$q_{\text{Out}} = q_{\text{InC}} - \frac{\partial \rho}{\partial t} V - \frac{\partial V}{\partial t} \rho$$

Hagen Poiseuille equation for gas exiting the chamber

In[240]:=

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qOutEq=qOut[t]==ρ[t] (π rEx^4)/(8 μ) (p[t]-pAtm)/lExit;TextbookEq[qOutEq]
```

$$q_{\text{Out}} = \frac{\pi r_{\text{Ex}}^4 (-p_{\text{Atm}} + p) \rho}{8 l_{\text{Exit}} \mu}$$

This equation is for an incompressible fluid. For a compressible fluid there is a different equation. The difference is significant only if the pressure within the chamber is O(Atm).

Elasticity

Elastic relation between chamber volume and pressure

In[241]:=

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ElasticReBen=w/rc==1/5(12(1-pr^2)q/(64 Em h^3))(rc^2-r^2)^2;
wFunc[r_,q_,rc_,h_,Em_,pr_]=Flatten[Solve[ElasticReBen,w]][[1]][[2]];
VtoPEq=V[t]-V0==Integrate[wFunc[r,p[t]-pAtm,rc,h,Em,pr] 2 π r,{r,0,rc}];TextbookEq[VtoPEq]
```

$$-V_0 + V = \frac{\pi(-1 + pr^2)rc^7(p_{\text{Atm}} - p)}{80 Em h^3}$$

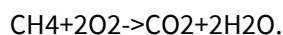
Species concentrations

The number of molecules within a chamber does not change during combustion, so the specific gas constant can be calculated by.

In[244]:=

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GasConstantEq=R[t]==(1000(mO2[t]/32+mCH4[t]/16+mCO2[t]/44+mH2O[t]/18+mN2[t]/24)/(mN2[t]+mO2[t]
DensityEq=ρ[t]×V[t]==mO2[t]+mN2[t]+mCH4[t]+mCO2[t]+mH2O[t];
mCH4ConservationEq=D[mCH4[t],t]==mCH4In-HeavisideTheta[qOut[t]]qOut[t] (mCH4[t]/(V[t]×ρ[t]))-16
mO2ConservationEq=D[mO2[t],t]==mO2In-HeavisideTheta[mO2[t]]HeavisideTheta[qOut[t]]qOut[t] (mO2[t]
mCO2ConservationEq=D[mCO2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mCO2[t]/(V[t]×ρ[t]))+44 α IsI
mH2OConservationEq=D[mH2O[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mH2O[t]/(V[t]×ρ[t]))+H2OK 36 α
mN2ConservationEq=D[mN2[t],t]==-HeavisideTheta[qOut[t]]qOut[t] (mN2[t]/(V[t]×ρ[t]))-HeavisideThe
```

Upon combustion, one mola of CH₄ vanishes, and along with 2 mols of O₂. Two mols of H₂O are created as well as 1 mola of CO₂.



Ignition and energy equations

First we model the time in which ignition occurs

In[251]:=

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HeatEquation = D[ρ[t] V[t], t] Cp T[t], t] == -hT π rc rc (T[t] - Tatm) + 16 HeatGen α IsIgnitionFunc
TextbookEq[HeatEquation]
```

$$C_p \frac{\partial \rho}{\partial t} T V + C_p \frac{\partial V}{\partial t} T \rho + C_p \frac{\partial T}{\partial t} V \rho =$$

$$16 \text{ HeatGen } \alpha \theta(\text{mCH4}) \text{ IsIgnitionFunc} - \frac{\partial V}{\partial t} p - hT \pi r c^2 (-T_{\text{atm}} + T) + C_p (q_{\text{InC}} T_{\text{atm}} - q_{\text{Out}} T)$$

Analysis

Lets define a list with all of our equations

In[253]:=

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EquationList = {PerfectGasEq, MassConservationEq, qOutEq, VtoPEq, GasConstantEq, DensityEq, mCH4Conservat:
TextbookEq[EquationList]}
```

$$T \rho$$

$$= q_{\text{InC}} - \frac{\partial \rho}{\partial t} V - \frac{\partial V}{\partial t} \rho$$

$$= \frac{\pi r_{\text{Exit}}^4 (-p_{\text{atm}} + p) \rho}{8 l_{\text{Exit}} \mu}$$

$$- V = \frac{\pi (-1 + p r^2) r c^7 (p_{\text{atm}} - p)}{80 E m h^3}$$

$$314. \left(\frac{m_{\text{CH4}}}{160} + \frac{m_{\text{CO2}}}{44} + \frac{m_{\text{H2O}}}{18} + \frac{m_{\text{N2}}}{24} + \frac{m_{\text{O2}}}{32} \right)$$

$$\frac{m_{\text{CH4}} + m_{\text{CO2}} + m_{\text{H2O}} + m_{\text{N2}} + m_{\text{O2}}}{m_{\text{CH4}} + m_{\text{CO2}} + m_{\text{H2O}} + m_{\text{N2}} + m_{\text{O2}}}$$

$$m_{\text{CH4}} + m_{\text{CO2}} + m_{\text{H2O}} + m_{\text{N2}} + m_{\text{O2}}$$

$$= m_{\text{CH4}} \text{In} - 16 \alpha \theta(m_{\text{CH4}}) \text{ IsIgnitionFunc} - \frac{\theta(q_{\text{Out}}) m_{\text{CH4}} q_{\text{Out}}}{V \rho}$$

$$= m_{\text{O2}} \text{In} - 64 \alpha \theta(m_{\text{CH4}}) \text{ IsIgnitionFunc} - 0.21 \theta(-q_{\text{Out}}) q_{\text{Out}} - \frac{\theta(m_{\text{O2}}) \theta(q_{\text{Out}}) m_{\text{O2}} q_{\text{Out}}}{V \rho}$$

$$= 44 \alpha \theta(m_{\text{CH4}}) \text{ IsIgnitionFunc} - \frac{\theta(q_{\text{Out}}) m_{\text{CO2}} q_{\text{Out}}}{V \rho}$$

$$= 36 \text{ H2OK } \alpha \theta(m_{\text{CH4}}) \text{ IsIgnitionFunc} - \frac{\theta(q_{\text{Out}}) m_{\text{H2O}} q_{\text{Out}}}{V \rho}$$

$$= -0.79 \theta(-q_{\text{Out}}) q_{\text{Out}} - \frac{\theta(q_{\text{Out}}) m_{\text{N2}} q_{\text{Out}}}{V \rho}$$

$$T V + C_p \frac{\partial V}{\partial t} T \rho + C_p \frac{\partial T}{\partial t} V \rho = 16 \text{ HeatGen } \alpha \theta(m_{\text{CH4}}) \text{ IsIgnitionFunc} - \frac{\partial V}{\partial t} p - hT \pi r c^2 (-T_{\text{atm}} + T) + C_p (q_{\text{InC}} T_{\text{atm}} - q_{\text{Out}} T)$$

In[255]:=

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NameList = {"PerfectGasEq", "MassConservationEq", "qOutEq", "VtoPEq", "GasConstantEq", "DensityEq", "DensityEq"}
```

In[256]:=

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qOutFunc[t_]=Flatten[Solve[EquationList[[3]],qOut[t]]][[1]][[2]];
EquationList1=Simplify[EquationList[/.qOut->Function[t,qOutFunc[t]]];
ρFunc[t_]=Flatten[Solve[EquationList1[[6]],ρ[t]]][[1]][[2]];
EquationList2=Simplify[EquationList1[/.ρ->Function[t,ρFunc[t]]];
VFunc[t_]=Flatten[Solve[EquationList2[[4]],V[t]]][[1]][[2]];
EquationList3=Simplify[EquationList2[/.V->Function[t,VFunc[t]],ListOfAssumptions];
TFunc[t_]=Flatten[Solve[EquationList3[[1]],T[t]]][[1]][[2]];
EquationList4=Simplify[EquationList3[/.T->Function[t,TFunc[t]],ListOfAssumptions];
RFunc[t_]=Solve[EquationList4[[5]],R[t]][[1]][[1]][[2]];
EquationList5=EquationList4[/.R->Function[t,RFunc[t]]];

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In[266]:=

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QinO2=100; (*ml/min*)
ρO2=1.331; (*kg/m^3*)
ρNH4=0.668; (*kg/m^3*)
mInO2= $\frac{QinO2}{60} \frac{\rho O2}{10^6}$ ;
QinNH4val= $\frac{18}{60} \frac{\rho NH4}{10^6}$ ;

```

In[271]:=

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PRV={rEx->.2/1000,pAtm->101325.0,TAtm->300,V0->(π 0.003*0.003*0.0035),lExit->3.0/1000,hT->0.05*1000,C
mO2In->mInO2,
mCH4In->QinNH4val,
qInC->mInO2+QinNH4val,

fr->10,dt->10-4,MaxStepSizeVal->0.0151/100,α->5*5*10-10,H2OK->1,NCycles->10,Tmax->0.02+ $\frac{3}{10}$ };

IsIgnitionFunc[t_,fr_,dt_]=Sum[PDF[NormalDistribution[i/fr+0.01,dt],t],{i,0,3,1}];
EquationList6=EquationList5[/.PRV];
{mO20,mCO20,mH2O0,mCH40,mN20}={4.537*10-8,0,0,0,0};
s=NDSolve[{EquationList6[[7]],EquationList6[[8]],EquationList6[[9]],EquationList6[[10]],EquationList6[[11]]

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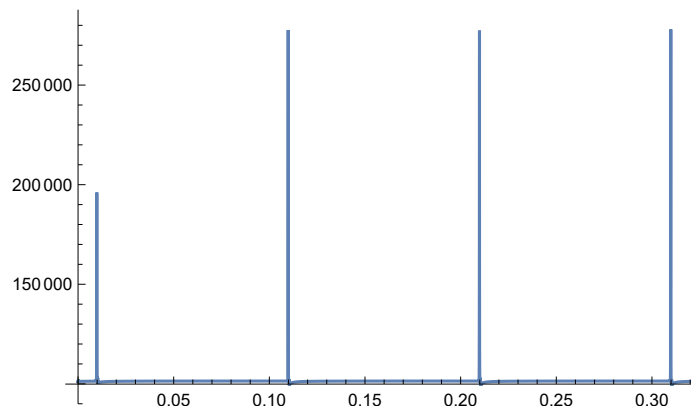
In[276]:=

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sol=First[p/.s];
Plot[sol[t],{t,0,Tmax/.PRV},PlotRange->All,PlotPoints->10000]

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Out[277]=



In[278]:=

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m[t_] = (Evaluate[mO2[t] /. s] + Evaluate[mCO2[t] /. s] + Evaluate[mH2O[t] /. s] + Evaluate[mCH4[t] /. s]) /. s
PanelA = Labeled[Plot[{(mO2[t] /. s)/m[t], (mCO2[t] /. s)/m[t], (mH2O[t] /. s)/m[t], (mCH4[t] /. s)/m[t], (mN2[t] /. s)/m[t]}, {t, 0, 0.35}], "Panel A"]

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Out[279]=

