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CENTRO DE DOCENCIA DE CIENCIAS BÁSICAS PARA INGENIERÍA

TUTORÍA 9+ Pauta

BAIN 036
Álgebra Lineal para Ingeniería
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1.
Solución:
(a)
(i) Sean $\vec{u}=(a,b,c)$ y $\vec{v}=(x,y,z)\in\mathbb{R}^3$. Luego:

$$\begin{aligned}
T(\vec{u}+\vec{v}) &= T((a,b,c)+(x,y,z))\\
&= T(a+x,b+y,c+z)\\
&= (a+x-(b+y),c+z-3(a+x))\\
&= (a-b,c-3a)+(x-y,z-3x)\\
&= T(a,b,c)+T(x,y,z)
\end{aligned}$$

(ii) Sea $\alpha\in\mathbb{R}$, $\vec{v}=(x,y,z)\in\mathbb{R}^3$. Luego:

$$\begin{aligned}
T(\alpha(x,y,z)) &= T(\alpha x,\alpha y,\alpha z)\\
&= (\alpha x-\alpha y,\alpha z-3\alpha x)\\
&= \alpha(x-y,z-3x)\\
&= \alpha T(x,y,z) \qquad \therefore\ Por\,(i)\,y\,(ii): T\ es\ transformación\ lineal.
\end{aligned}$$

(b)
(i) Sean $p(x)=a_1x^2+b_1x+c_1$ y $q(x)=a_2x^2+b_2x+c_2\in P_2(\mathbb{R})$. Luego:

$$\begin{aligned}
T(p(x)+q(x)) &= T(a_1x^2+b_1x+c_1+a_2x^2+b_2x+c_2)\\
&= T((a_1+a_2)x^2+(b_1+b_2)x+(c_1+c_2))\\
&= \begin{bmatrix} 2(a_1+a_2) & (a_1+a_2)-(b_1+b_2)\\ (a_1+a_2)+(b_1+b_2) & 3+(c_1+c_2)\end{bmatrix}\\
&= \begin{bmatrix} a_1 & a_1-b_1\\ a_1+b_1 & 3+c_1\end{bmatrix}+\begin{bmatrix} a_2 & a_2-b_2\\ a_2+b_2 & c_2\end{bmatrix}\\
&\neq T(a_1x^2+b_1x+c_1)+T(a_2x^2+b_2x+c_2) \qquad \therefore T\ no\ es\ transformación\ lineal.
\end{aligned}$$

(c)

(i) Sean $p(x) = a_1x + b$ y $q(x) = a_2x + b_2 \in P_1(\mathbb{R})$. Luego:

$$
\begin{aligned}
T(p(x)+q(x)) &= T(a_1x+b_1+a_2x+b_2) \\
&= T((a_1+a_2)x+(b_1+b_2)) \\
&= (2(a_1+a_2),0,(b_1+b_2)-(a_1+a_2)) \\
&= (2a_1,0,b_1-a_1)+(2a_2,0,b_2-a_2) \\
&= T(a_1x+b_1)+T(a_2x+b_2) \\
&= T(p(x))+ T(q(x))
\end{aligned}
$$

(ii) Sea $\alpha \in \mathbb{R}$, $p(x) = ax + b \in P_1(\mathbb{R})$. Luego:

$$
\begin{aligned}
T(\alpha(ax+b)) &= T(\alpha ax+\alpha b) \\
&= (2\alpha a,0,\alpha b-\alpha a) \\
&= \alpha(2a,0,b-a) \\
&= \alpha T(ax+b)
\end{aligned}
$$

$\therefore$ *Por (i) y (ii) : T es transformación lineal.*

2.

(a)

<u>Solución:</u>

$KerT = \{(x,y,z) \in \mathbb{R}^3 / T(x,y,z) = (0,0)\}$

$KerT = \{(x,y,z) \in \mathbb{R}^3 / (x+y+z, x-2y) = (0,0)\}$

$KerT = \{(x,y,z) \in \mathbb{R}^3 / x+y+z = 0 \wedge x-2y = 0\}$

$KerT = \{(x,y,z) \in \mathbb{R}^3 / -3y = z \wedge x = 2y\}$

$KerT = \langle (2,1,-3) \rangle$

$\therefore B = \{(2,1,-3)\}$ *es l.i y es base del KerT.*

(b)

Sea $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ base de $\mathbb{R}^3$. Luego:

$\operatorname{Im} T = \langle T(1,0,0), T(0,1,0), T(0,0,1) \rangle$

$\operatorname{Im} T = \langle (1,1),(1,-2),(1,0) \rangle$

Así: $B_1 = \{(1,1),(1,-2)\}$ es base de la $\operatorname{Im} T$ y la $\dim \operatorname{Im} T = 2$

3.
(a)
Solución:

$$KerT = \left\{ax^3 + bx^2 + cx + d \in P_3(\mathbb{R}) / T\left(ax^3 + bx^2 + cx + d\right) = 0x^3 + 0x^2 + 0x + 0\right\}$$

$$KerT = \left\{ax^3 + bx^2 + cx + d \in P_3(\mathbb{R}) / (a+b+d)x^3 + (2b-d)x^2 + (c-a)x + 2d = 0x^3 + 0x^2 + 0x + 0\right\}$$

$$KerT = \left\{ax^3 + bx^2 + cx + d \in P_3(\mathbb{R}) / (a+b+d) = 0 \wedge (2b-d) = 0 \wedge (c-a) = 0 \wedge 2d = 0\right\}$$

$$KerT = \left\{0x^3 + 0x^2 + 0x + 0\right\}$$

$\therefore \dim KerT = 0.$

(b)
Solución:

Sea $B = \left\{x^3, x^2, x, 1\right\}$ base de $P_3(\mathbb{R})$. Luego:

$$\operatorname{Im} T = \left\langle T\left(x^3\right), T\left(x^2\right), T(x), T(1)\right\rangle$$

$$\operatorname{Im} T = \left\langle x^3 - x, x^3 + 2x^2, x, x^3 - x^2 + 2\right\rangle$$

Así: $B_1 = \left\{x^3 - x, x^3 + 2x^2, x, x^3 - x^2 + 2\right\}$ es l.i. y es base de la $\operatorname{Im} T$ y la $\dim \operatorname{Im} T = 4$