1) $$\frac{cscx}{secx(cscx-1)}=\frac{\frac{1}{senx}}{\frac{1}{senxcosx}-\frac{1}{cosx}}=\frac{\frac{1}{senx}}{\frac{1-senx}{senxcosx}}=\frac{1}{senx}\cdot\frac{senxcosx}{1-senx}=\frac{cosx}{1-senx}\cdot\frac{1+senx}{1+senx}=$$

$$\frac{cosx(1+senx)}{cos^2x}=\frac{1+senx}{cosx}=\frac{1}{cosx}+\frac{senx}{cosx}=secx+tanx$$

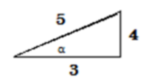
2) Calcule

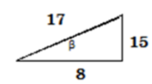
$sen\left[2arcsen\left(\frac{4}{5}\right)+\frac{1}{2}arccos\left(\frac{8}{17}\right)\right]$=

Si $\alpha=arcsen\left(\frac{4}{5}\right)\ y\ \beta=arccos\left(\frac{8}{17}\right)$

$sen\,\alpha=\frac{4}{5}, cos\beta=\frac{8}{17}$

$cos\alpha=\frac{3}{5}, sen\beta=\frac{15}{17}$





$$sen\left[2arcsen\left(\frac{4}{5}\right)+\frac{1}{2}arccos\left(\frac{8}{17}\right)\right]=sen\left[2\alpha+\frac{1}{2}\beta\right]$$
$$=sen2\alpha cos\frac{\beta}{2}+sen\frac{\beta}{2}cos2\alpha$$
$$=2sen\alpha cos\alpha\sqrt{\frac{1+cos\beta}{2}}+(cos^2\alpha-sen^2\alpha)\sqrt{\frac{1-cos\beta}{2}}$$
$$=2\cdot\frac{4}{5}\cdot\frac{3}{5}\cdot\sqrt{\frac{1+\frac{8}{17}}{2}}+\left(\frac{9}{25}-\frac{16}{25}\right)\cdot\sqrt{\frac{1-\frac{8}{17}}{2}}=$$
$$=\frac{24}{25}\cdot\sqrt{\frac{25}{34}}+\left(-\frac{7}{25}\right)\cdot\sqrt{\frac{9}{34}}=$$
$$=\frac{120}{25\sqrt{34}}-\frac{21}{25\sqrt{34}}=\frac{99}{25\sqrt{34}}$$

3) a) Sea π: plano determinado por $\overrightarrow{AB}\wedge\overrightarrow{AC}$, donde $\overrightarrow{AB}=(3,-4,8),\overrightarrow{AC}=(5,-6,0)$

$\vec{N}: vector\ normal\ de\ \pi$, se cumple que $\vec{N}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & -4 & 8\\ 5 & -6 & 10\end{vmatrix}=(8,10,2)$

Como $A(1,9,-2)\in\pi$, se tiene que $\begin{matrix}\pi: 8(x-1)+10(y-9)+2(z+2)=0\\ \pi: 4x+5y+z-47=0\end{matrix}$ es la ecuación del plano pedido.

b) Area del $\Delta ABC=\frac{\|\overrightarrow{AB}\times\overrightarrow{AC}\|}{2}=\frac{\sqrt{168}}{2}=\sqrt{42}u^2$

c) $\mathrm{P}(1,\mathrm{n},5)\in\pi$, luego debe satisfacer la ecuación $4+5n+5=47\Rightarrow n=\frac{38}{5}$

d) Sea $\vec{u}: vector\ normal\ de\ \pi, \vec{u}\ unitario\Longrightarrow\vec{u}=\frac{\vec{N}}{\|\vec{N}\|}=\frac{(8,10,2)}{2\sqrt{42}}=\left(\frac{4}{\sqrt{42}},\frac{5}{\sqrt{42}},\frac{1}{\sqrt{42}}\right)$