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18-660
Hw 12

1. $f(x) = x_1^2 + x_2^2 \qquad \nabla f(x) = \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\nabla f^{(0)}(x) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + u^{(0)} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$R^{(0)} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad S^{(0)} = \text{span}\{R^{(0)}\} = \text{span}\left\{\begin{bmatrix} 2 \\ 2 \end{bmatrix}\right\}$

$D^{(0)} = R^{(0)} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$u^{(0)} = \frac{8}{16} = \frac{1}{2}$

$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\nabla f^{(1)}(x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x^{(2)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + u^{(1)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$ no matter what $u^{(k)}$ is. Convergence is reached

2. $f(x) = x_1^2 + x_1 x_2 + 2x_2^2 \qquad \nabla f(x) = \begin{bmatrix} 2x_1 + x_2 \\ 4x_2 + x_1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$

$\nabla f^{(0)}(x) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$x^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - u^{(0)} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$R^{(0)} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad S^{(0)} = \text{span}\left\{\begin{bmatrix} 3 \\ 5 \end{bmatrix}\right\}$

$D^{(0)} = R^{(0)} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad u^{(0)} = \frac{34}{148} = \frac{17}{74}$

$x^{(1)} = \begin{bmatrix} 23/74 \\ -11/74 \end{bmatrix}$

$$\nabla f^{(1)}(x) = \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} 23/74 \\ -11/74 \end{bmatrix} - \mu^{(1)} D^{(1)}$$

$$R^{(1)} = \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix} \qquad S^{(1)} = \text{span}\left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix} \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix} \right\}$$

$$D^{(1)} = R^{(1)} + \beta_{10} D^{(0)} \qquad D^{(0)T} A D^{(1)} = 0$$

$$= \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix} + \beta_{10} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\beta_{10} = - \frac{\begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix}}{\begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}} = - \left(\frac{-1.32}{148} \right) = 0.0089$$

$$D^{(1)} = \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix} + .0089 \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{matrix} 0.0263 \\ 0.0445 \end{matrix} \begin{bmatrix} 0.4998 \\ -0.2393 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} 23/74 \\ -11/74 \end{bmatrix} - \mu^{(1)} \begin{bmatrix} 0.4998 \\ -0.2393 \end{bmatrix}$$

$$\mu^{(1)} = \frac{\begin{bmatrix} 0.4998, & -0.2393 \end{bmatrix} \begin{bmatrix} 35/74 \\ -21/74 \end{bmatrix}}{\begin{bmatrix} 0.4998, & -0.2393 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 0.4998 \\ -0.2393 \end{bmatrix}} = \frac{0.3043}{0.4895} = 0.6217$$

$$x^{(2)} = \begin{bmatrix} 23/74 \\ -11/74 \end{bmatrix} - (0.6217) \begin{bmatrix} 0.4998 \\ -0.2393 \end{bmatrix}$$

$$x^{(2)} = \begin{bmatrix} 0.3108 \\ -0.1486 \end{bmatrix} - \begin{bmatrix} 0.3107 \\ -0.1488 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ Convergence is reached