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18-660
Hw 8

1. $\Delta x^{(k)} = x^{(k+1)} - x^{(k)}$ $\qquad \frac{df}{dx}\Big|_{x^{(k+1)}} = 0$

$$\frac{df}{dx}\Big|_{x^{(k+1)}} \approx \frac{df}{dx}\Big|_{x^{(k)}} + \frac{d^2f}{dx^2}\Big|_{x^{(k)}} \cdot \left[x^{(k+1)} - x^{(k)}\right]$$

$$x^{(0)} = 1 \qquad \frac{df}{dx}\Big|_{x^{(0)}} = 4 \qquad \frac{d^2f}{dx^2}\Big|_{x^{(0)}} = 12$$

$$\frac{df}{dx}\Big|_{x^{(1)}} \approx 4 + 12 \cdot \left[x^{(1)} - 1\right] = 0$$

$$\Rightarrow x^{(1)} = \frac{2}{3} \qquad \frac{df}{dx}\Big|_{x^{(1)}} = \frac{32}{27} \qquad \frac{d^2f}{dx^2}\Big|_{x^{(1)}} = \frac{48}{9}$$

$$0 = \frac{32}{27} + \frac{48}{9} \cdot \left[x^{(2)} - \frac{2}{3}\right] = \frac{32}{27} + \frac{48}{9} x^{(2)} - \frac{96}{27} \Rightarrow$$

$$\Rightarrow x^{(2)} = \frac{64}{27} \cdot \frac{9}{48} = \frac{64}{144} = \frac{32}{72} = \frac{4}{9} \qquad \frac{df}{dx}\Big|_{x^{(2)}} = 4 \cdot \left(\frac{64}{729}\right) = \frac{256}{729} \qquad \frac{d^2f}{dx^2} = 12 \cdot \left(\frac{16}{81}\right) = 2.37$$

$$0 = 0.35 + 2.37\left[x^{(3)} - 0.44\right] = 2.37\, x^{(3)} - 0.69$$

$$\Rightarrow x^{(3)} = 0.29$$

2. $\min_{x} \quad x^4 + y^4 \qquad x^{(0)}=1,\ y^{(0)}=1$

S.T. $x+y=1$

$\min \ f(z) \qquad z^{(0)} = \begin{bmatrix}1\\1\end{bmatrix}$

S.T. $A_x = B$

$A = \begin{bmatrix}1 & 1\end{bmatrix}$

$B = 1$

$$\nabla^2 f(z) = \begin{bmatrix}12x^2 & 0\\ 0 & 12y^2\end{bmatrix} \qquad \nabla f(z) = \begin{bmatrix}4x^3\\4y^3\end{bmatrix}$$

$$\begin{bmatrix}12 & 0 & 1\\ 0 & 12 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\\ v\end{bmatrix} = \begin{bmatrix}-4\\-4\\-1\end{bmatrix} \Rightarrow \begin{array}{l}\Delta x = -1/2\\ \Delta y = -1/2\end{array}$$

$$z^{(1)} = \begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}-1/2\\-1/2\end{bmatrix} = \begin{bmatrix}1/2\\1/2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3 & 0 & 1\\ 0 & 3 & 1\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\\ v\end{bmatrix} = \begin{bmatrix}-1/2\\-1/2\\0\end{bmatrix}$$

$$z^{(2)} = \begin{bmatrix}1/2\\1/2\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}1/2\\1/2\end{bmatrix}$$

Since $\Delta x = 0$ and $\Delta y = 0$, we have reached our solution, so $z^{(3)} = z^{(2)} = \begin{bmatrix}1/2\\1/2\end{bmatrix}$. So $x^{(3)} = 1/2$, $y^{(3)} = 1/2$.