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18-660
HW 9

$$z = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned} L(z, U, V) &= f(z) + V_1 g_1(z) \\ &= x^2 + y^2 + V_1(x + y - 1) \end{aligned}$$

$$\begin{aligned} d(U, V) &= \inf_z L(z, U, V) \\ &= \inf_{x, y} [x^2 + y^2 + V_1(x + y - 1)] \\ &= \min_{x, y} x^2 + y^2 + V_1(x + y - 1) \end{aligned}$$

$$\frac{d}{dx} = 2x + V_1$$

$$\frac{d}{dy} = 2y + V_1$$

$$\text{min at } x, y = -\frac{V_1}{2}$$

$$\begin{aligned} d(U, V) &= \frac{V_1^2}{4} + \frac{V_1^2}{4} - \frac{V_1^2}{2} - \frac{V_1^2}{2} - V_1 \\ &= -\frac{V_1^2}{2} - V_1 \end{aligned}$$