18-660: Numerical Methods for Engineering Design and Optimization

Homework 7

Issued: October 10
Due: October 17 (midnight, Pittsburgh time)

Please submit the PDF file of your solution to the course web site before midnight on the due day.

Problem 1: Nonlinear Equation Solver

Apply Newton-Raphson method to solve the following nonlinear equations. Show your equations to calculate the solution x for the first three iterations.

$$x^3 = 0 \quad where \quad x^{(0)} = 1 \tag{1}$$

$$e^x = 1 \quad where \quad x^{(0)} = 1 \tag{2}$$

$x^{(0)}$ represents the initial starting point.

(1): $F[x^{(0)}] = 1$, $F'[x^{(0)}] = 3$

$F[x^{(1)}] \approx 1 + 3 \cdot (x^{(1)} - 1) = 0$

$\Rightarrow x^{(1)} = 2/3$, $F[x^{(1)}] = \frac{8}{27}$, $F'[x^{(1)}] = \frac{12}{9} = \frac{4}{3}$

$F[x^{(2)}] = \frac{8}{27} + \frac{4}{3}\left(x^{(2)} - \frac{2}{3}\right) = 0$

~~$\Rightarrow x^{(2)} = 0$, $F[x^{(2)}] = 0$, $F'[x^{(2)}] = 0$~~

~~$F[x^{(3)}] = 0 + 0 \cdot (x^{(3)})$~~

$\Rightarrow x^{(2)} = \frac{4}{9}$, $F[x^{(2)}] = \frac{64}{729}$, $F'[x^{(2)}] = \frac{16}{27}$

$F[x^{(3)}] = \frac{64}{729} + \frac{16}{27}\left(x^{(3)} - \frac{4}{9}\right) = 0 = \frac{16}{27}x^{(3)} - \frac{124}{729}$

$\Rightarrow x^{(3)} = \frac{3456}{11664} = 0.296$, $F[x^{(3)}] = 0.026$, $F'[x^{(3)}] = 0.263$

(2): $F[x^{(0)}] = e$, $F'[x^{(0)}] = e$

$F[x^{(1)}] \approx e + e(x^{(1)} - 1) = 0$

$\Rightarrow x^{(1)} = 0$ $F[x^{(1)}] = 1$, $F'[x^{(1)}] = 1$

$F[x^{(2)}] = 1 + 1(x^{(2)} - 0) = 0$

$\Rightarrow x^{(2)} = -1$ $F[x^{(2)}] = \frac{1}{e}$ $F'[x^{(2)}] = \frac{1}{e}$

$F[x^{(3)}] = \frac{1}{e} + \frac{1}{e}(x^{(3)} + 1) = 0$

$\Rightarrow x^{(3)} = -2$, $F[x^{(3)}] = \frac{1}{e^2}$, $F'[x^{(3)}] = \frac{1}{e^2}$