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18-660
Hw2

1.(1) $f(x) = e^x$ $\frac{dy}{dx} = e^x$ $\frac{d^2y}{dx^2} = e^x$ $e^x > 0$ $\forall x$ so e^x is convec

(2) $f(x_1, x_2) = x_1 x_2$ $H(x_1, x_2) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $H(x_1, x_2)$ is not positive definite because:

$$z^T H(x_1, x_2) z = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b & a \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = ab + ba = 2ab$$

if exactly one of a or b is negative, then $2ab$ is negative
Therefore, $f(x_1, x_2) = x_1 x_2$ is not convex.

(3) $\{x \mid \log(x) \leq 0\}$, if $\log(x)$ is convex, the α sublevel set is convex

$\frac{df}{dx} = \frac{1}{x}$ $\frac{d^2f}{dx^2} = -\frac{1}{x^2}$ $\frac{-1}{x^2}$ is always negative, so

the set $\{x \mid \log(x) \leq 0\}$ is not convex

(4) $\sqrt{x_1^2+x_2^2}$, $|x_1|$, $|x_2|$ all must be convex. $|x_1|$ and $|x_2|$ are convex when ≤ 1, so that is fine. Now

$$H\left(\sqrt{x_1^2+x_2^2}\right) = \begin{bmatrix} \frac{x_2^2}{(x_1^2+x_2)^{3/2}} & -\frac{x_1 x_2}{(x_1^2+x_2^2)^{3/2}} \\ -\frac{x_1 x_2}{(x_1^2+x_2^2)^{3/2}} & \frac{x^2}{(x_1^2+x_2^2)^{3/2}} \end{bmatrix}$$

determinent $\left[H\left(\sqrt{x_1^2+x_2^2}\right)\right] = 0$, so $\sqrt{x_1^2+x_2^2}$ is convex, and thus, the set is convex.

(5) Here we are trying to maximize a concave function. We can write this as such:

$$\min_x \; -\log(x)$$

$$\text{S.T.} \quad x \leq 2$$

Since it's the minimization of a ~~concave~~ convex function, and all constraints are convex and upper-bounded, this optimization is convex.

(6) We do the same as above, so:

$$\min_x \; -x$$

$$\text{S.T.} \quad |x| \geq 1$$

We would think that this is a convex optimization, except the constraint is lower bounded. This makes this not a convex optimization problem.

2. (7) QR decomposition:

$r_{11} = \|A_1\|_2 = 2, \quad Q_1 = \frac{A_1}{r_{12}} = [1, 0, 0]$

$r_{12} = 3 \quad\quad Q_2 = \frac{A_2 - Q_1 r_{12}}{r_{22}} = [0, 0, 1]$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$r_{22} = 1 \quad r_{23} = 1 \quad r_{33} = 1 \quad Q_3 = [0, 1, 0]$