CAAP Statistics - Lec17

Aug 2, 2022

Review

- Normality Condition
  - Large sample: by CLT
  - Population distribution is normal
  - T-distribution
- One sample mean/proportion
- Paired data
- Difference in two means/proportions
- Equivalence of Hypothesis Testing and Confidence Interval

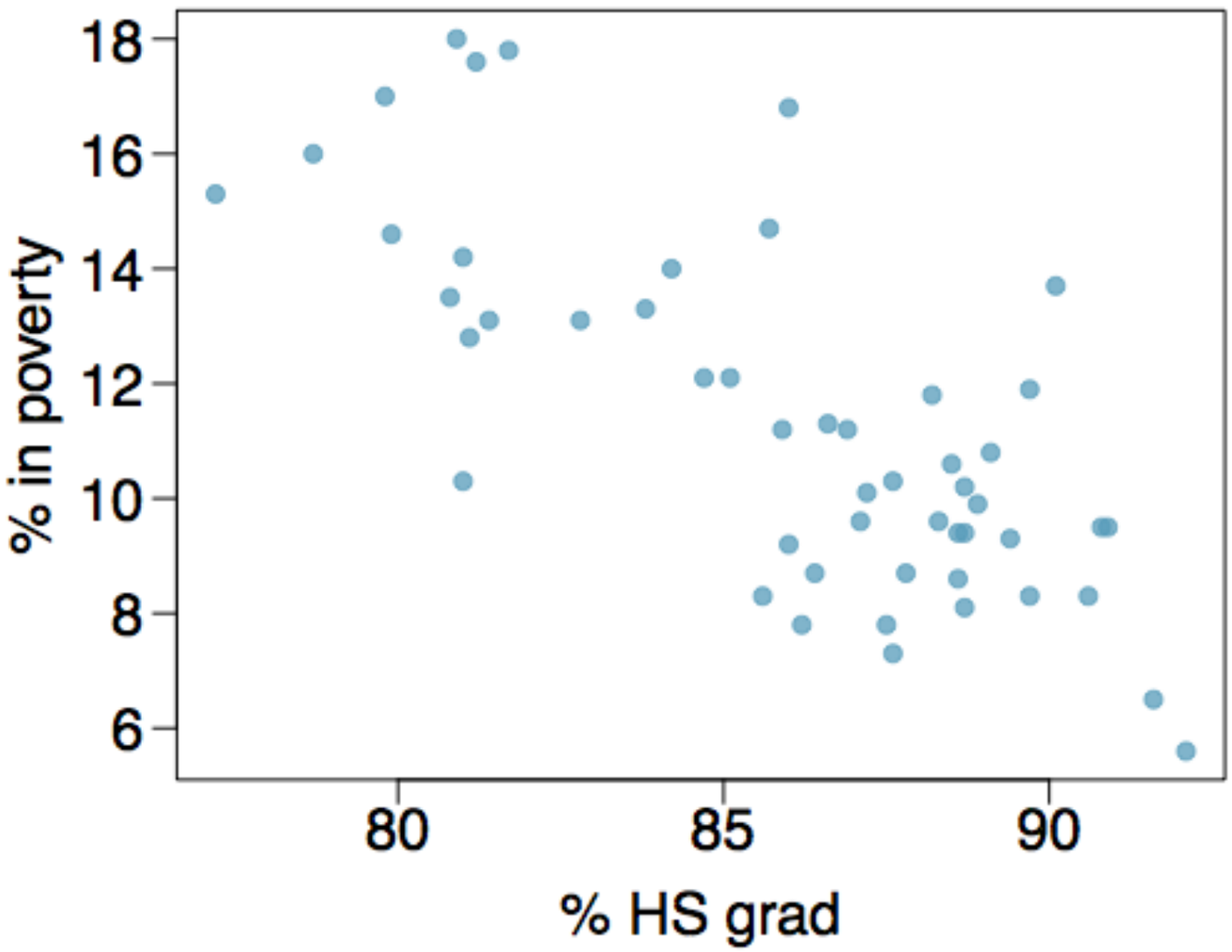
Learning Objectives

- Line Fitting, Residuals and Correlation
- Fitting a line by Least Squares Regression
- Types of outliers in Linear Regression
  - Outliers, High leverage point
  - Influential point
- Inference for Linear Regression
  - Hypothesis testing for the slope

Line Fitting, Residuals, and Correlation

Poverty vs. HS graduate rate

The *scatterplot* below shows the relationship between HS graduate rate in all 50 US states and DC and the percent of residents who live below the poverty line (income below $23,050 for a family of 4 in 2012).



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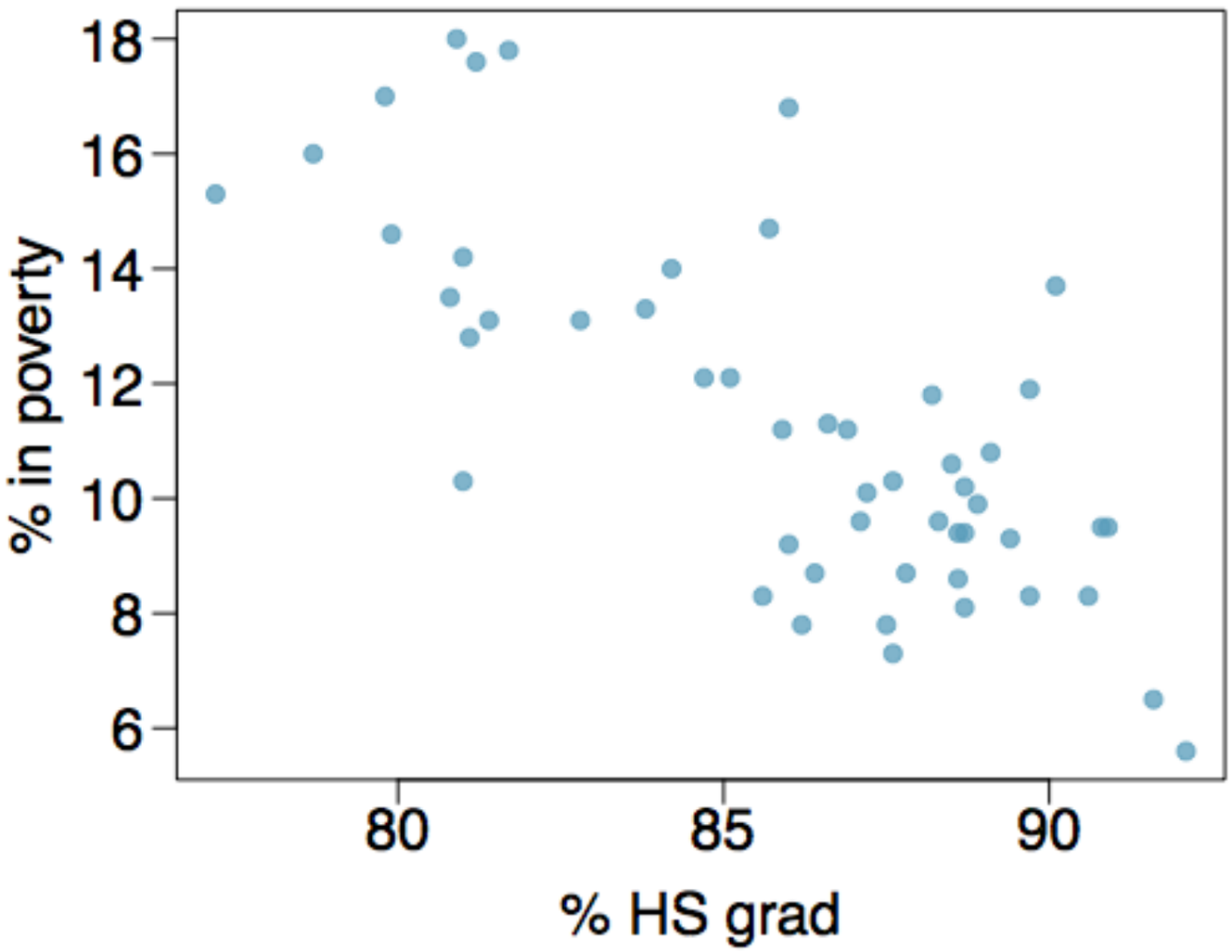

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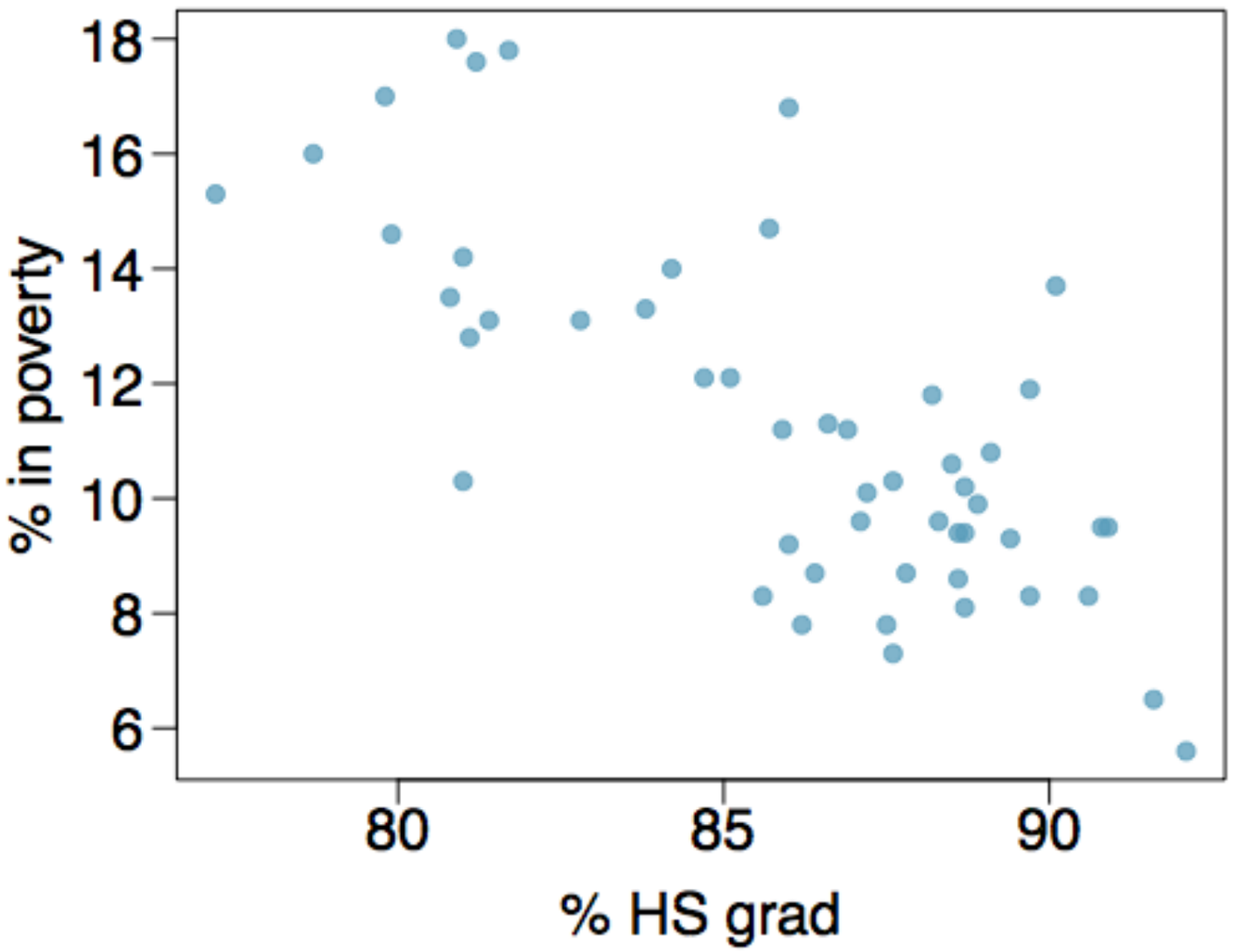

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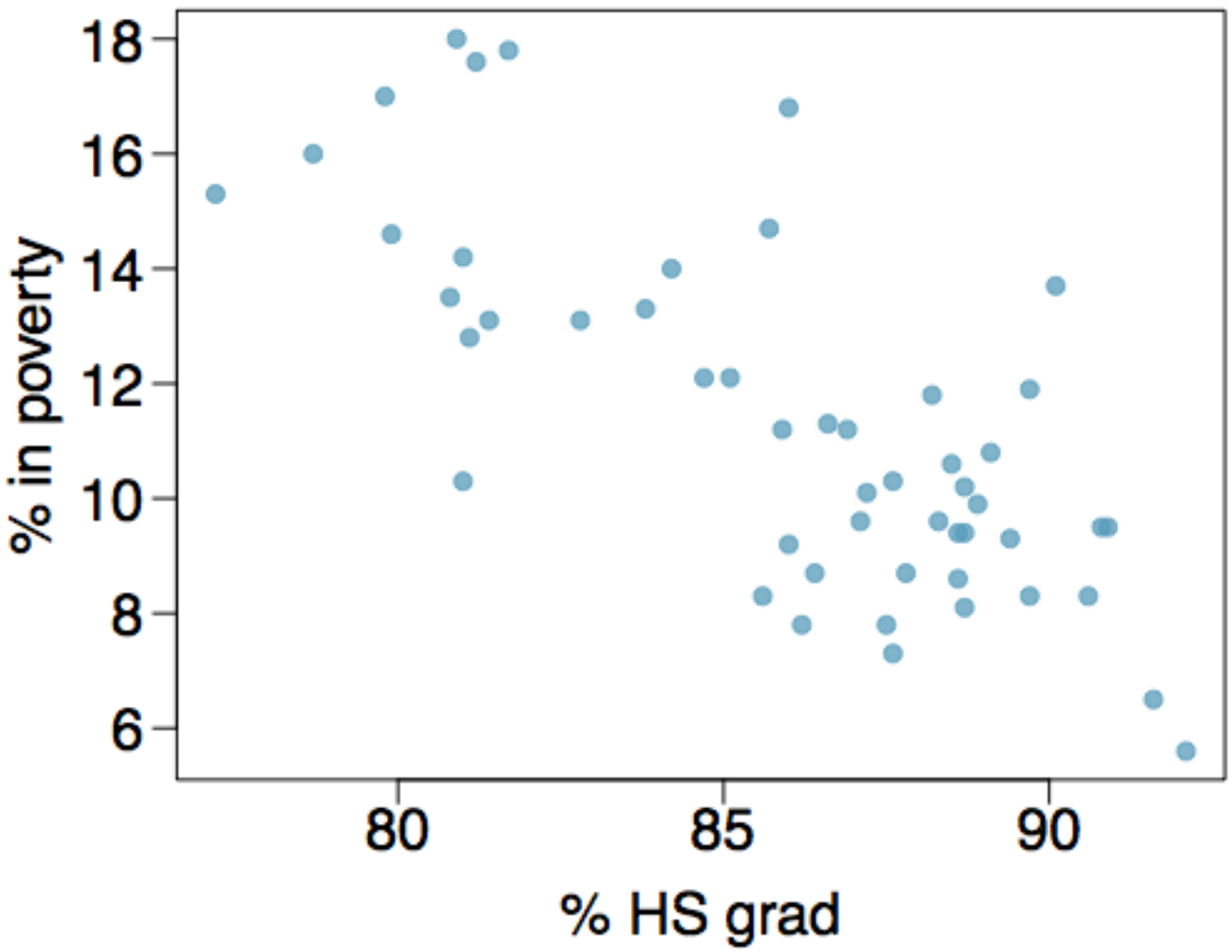

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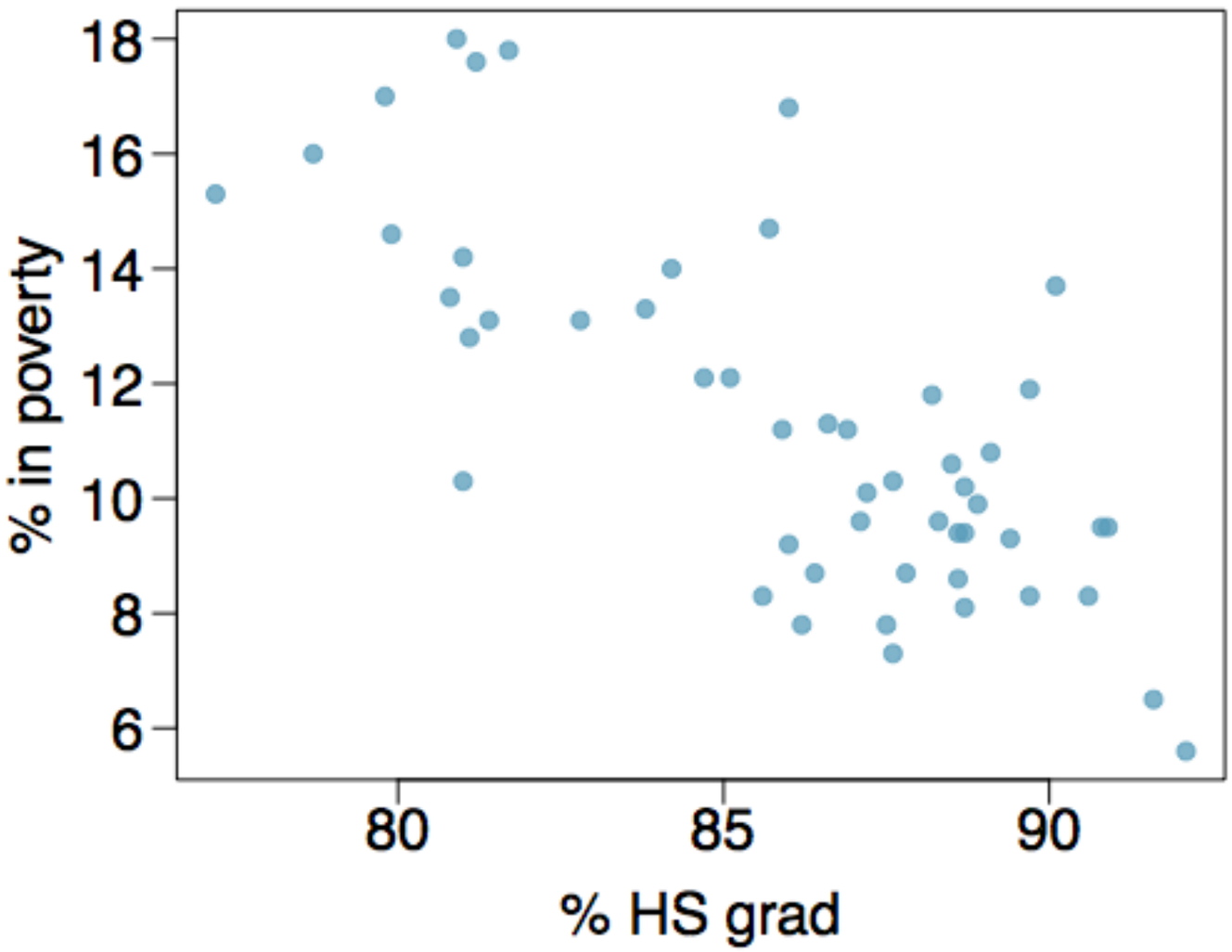

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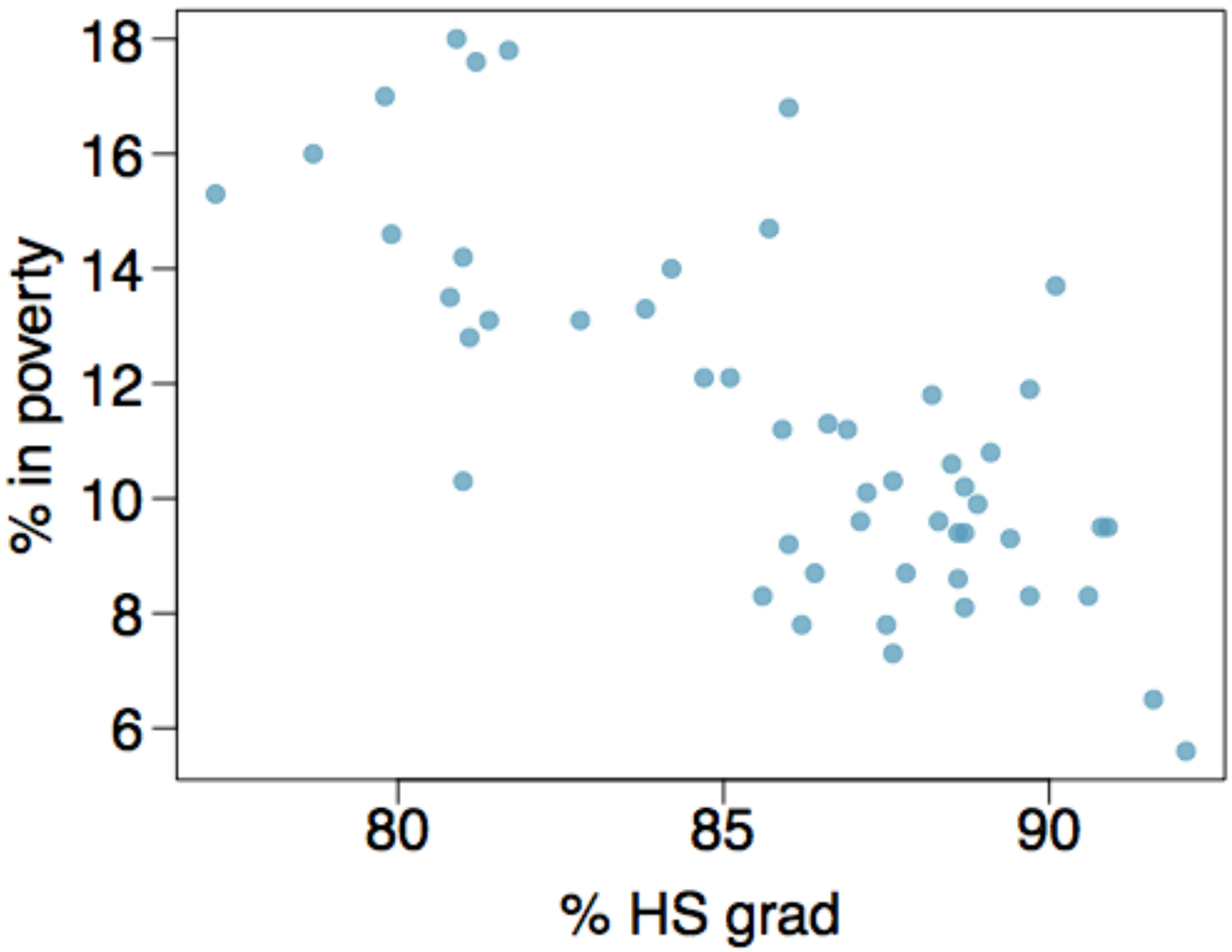

Response variable?

% in poverty

Explanatory variable?

% HS grad

Relationship?

linear, negative, moderately strong

Poverty vs. HS graduate rate

The linear model for predicting poverty from high school graduation rate in the US is

$$\hat{poverty} = 64.78 - 0.62 * HS_{grad}$$

The "hat" is used to signify that this is an estimate.

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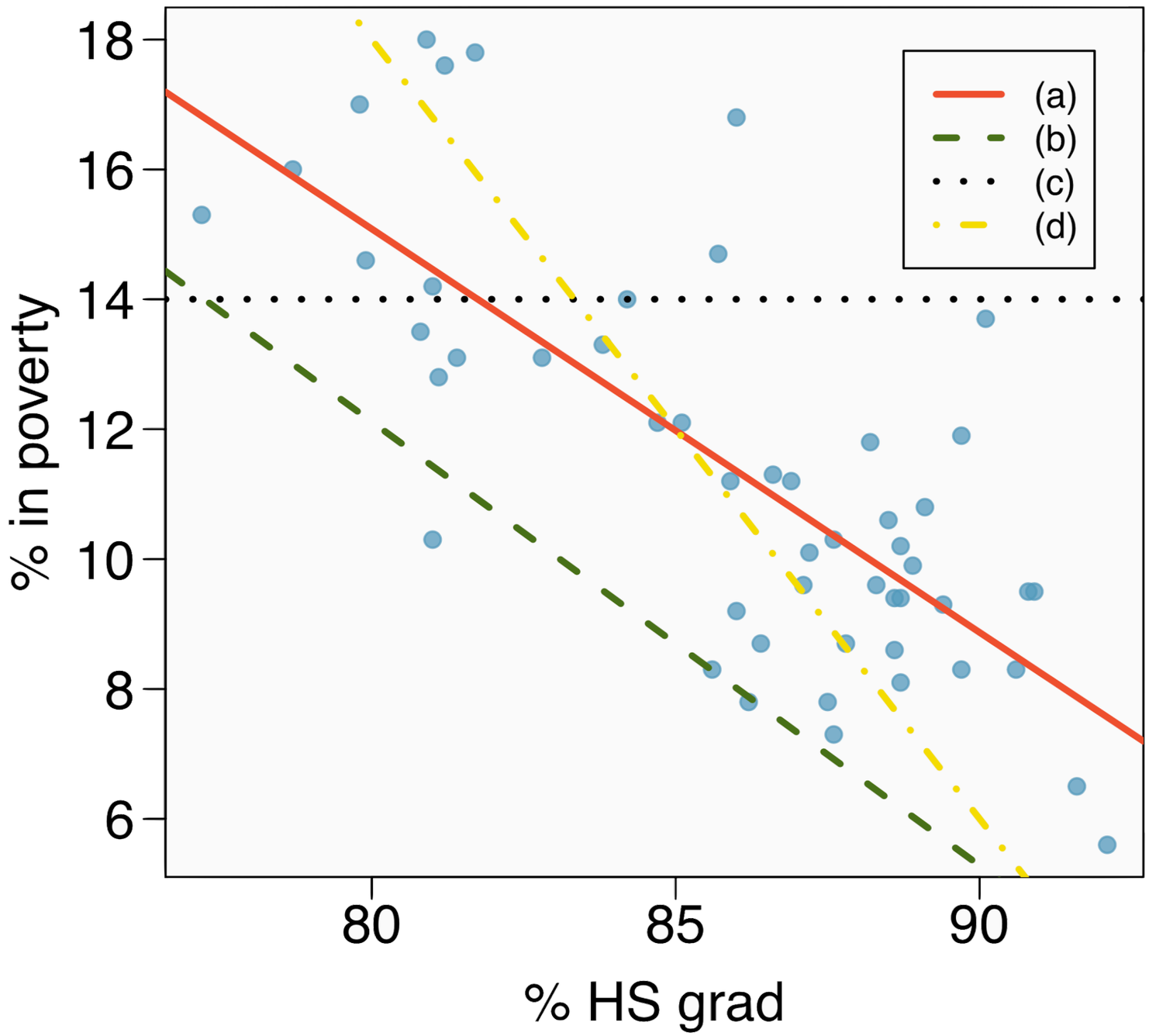
The high school graduate rate in Georgia is 85.1%. What poverty level does the model predict for this state?

$$64.78 - 0.62 \times 85.1 = 12.018$$

Eyeballing the line

Which of the following appears to be the line that best fits the linear relationship between % in poverty and % HS grad? Choose one.



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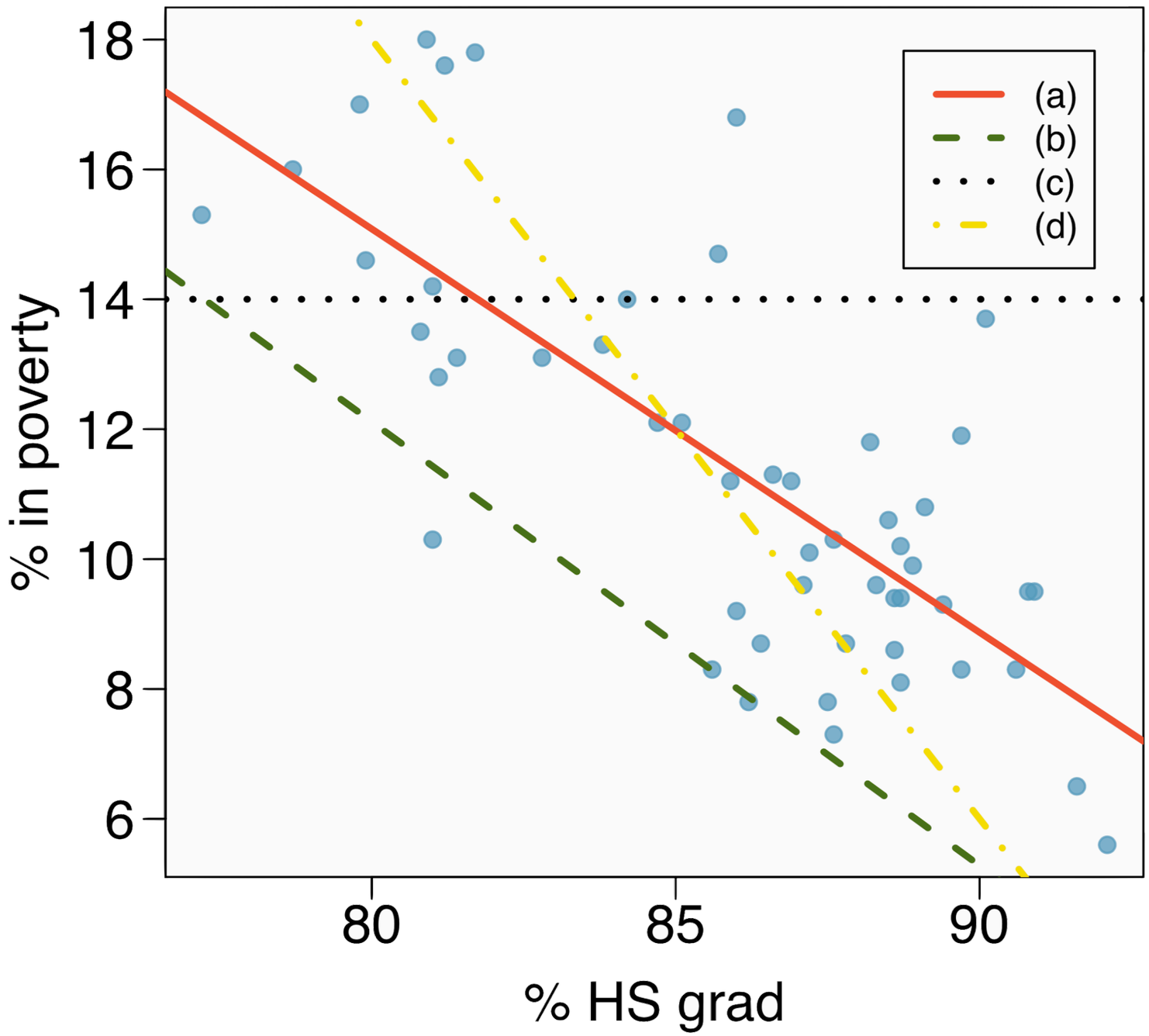
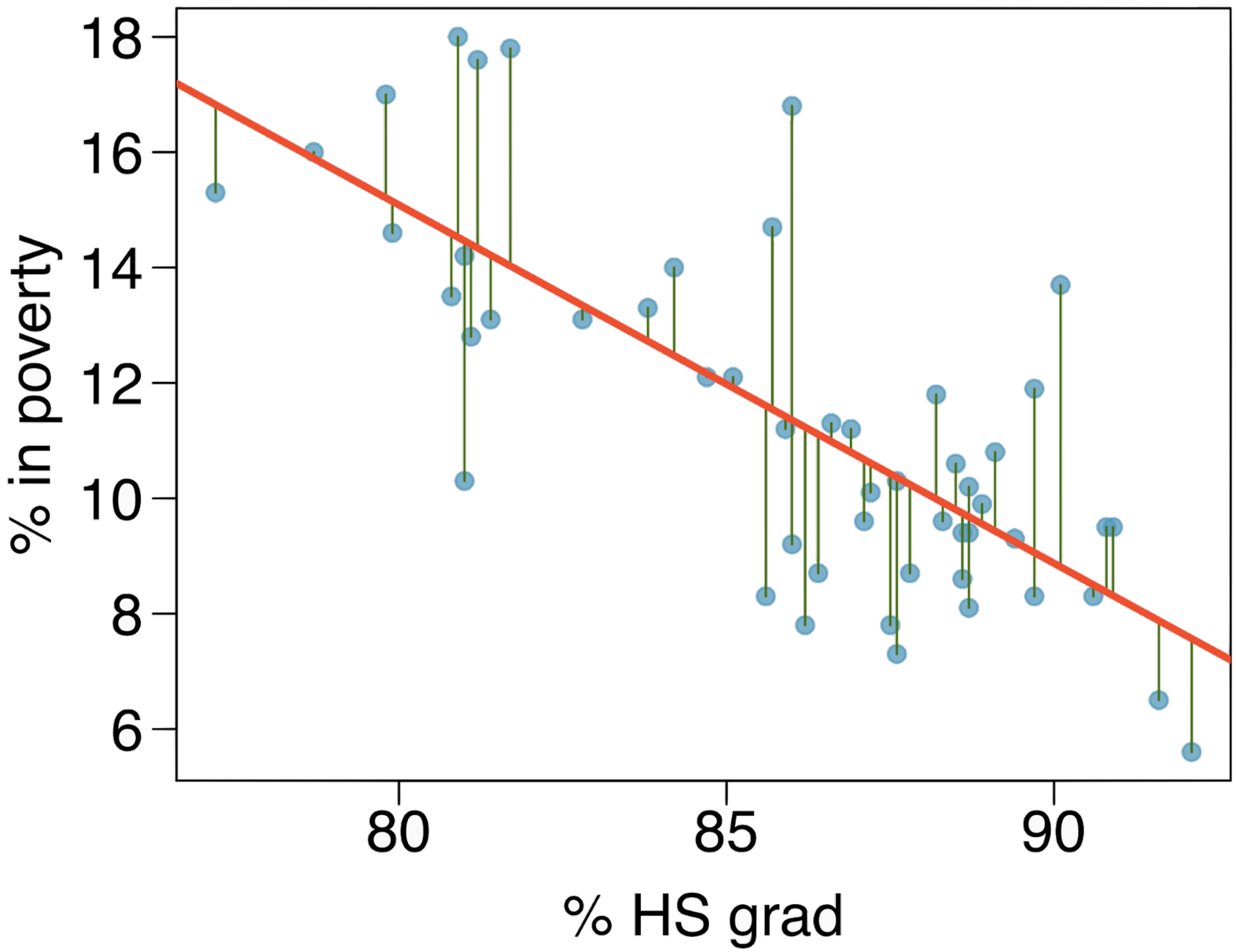


Residuals

Residuals are the leftovers from the model fit:

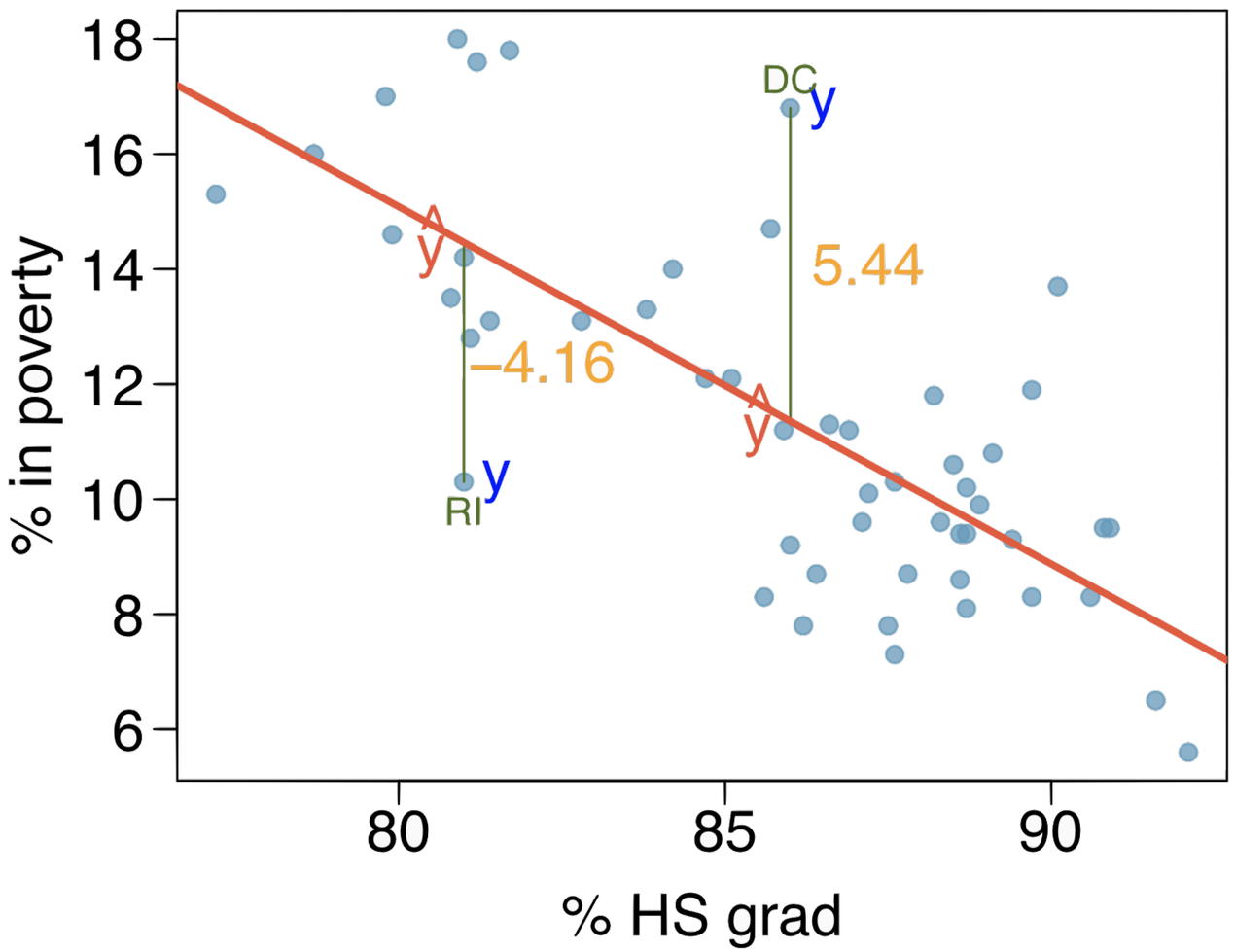
Data = Fit + Residual



Residuals (cont.)

Residual is the difference between the observed (y_i) and predicted $\hat{y}_i$.

$$e_i = y_i - \hat{y}_i$$



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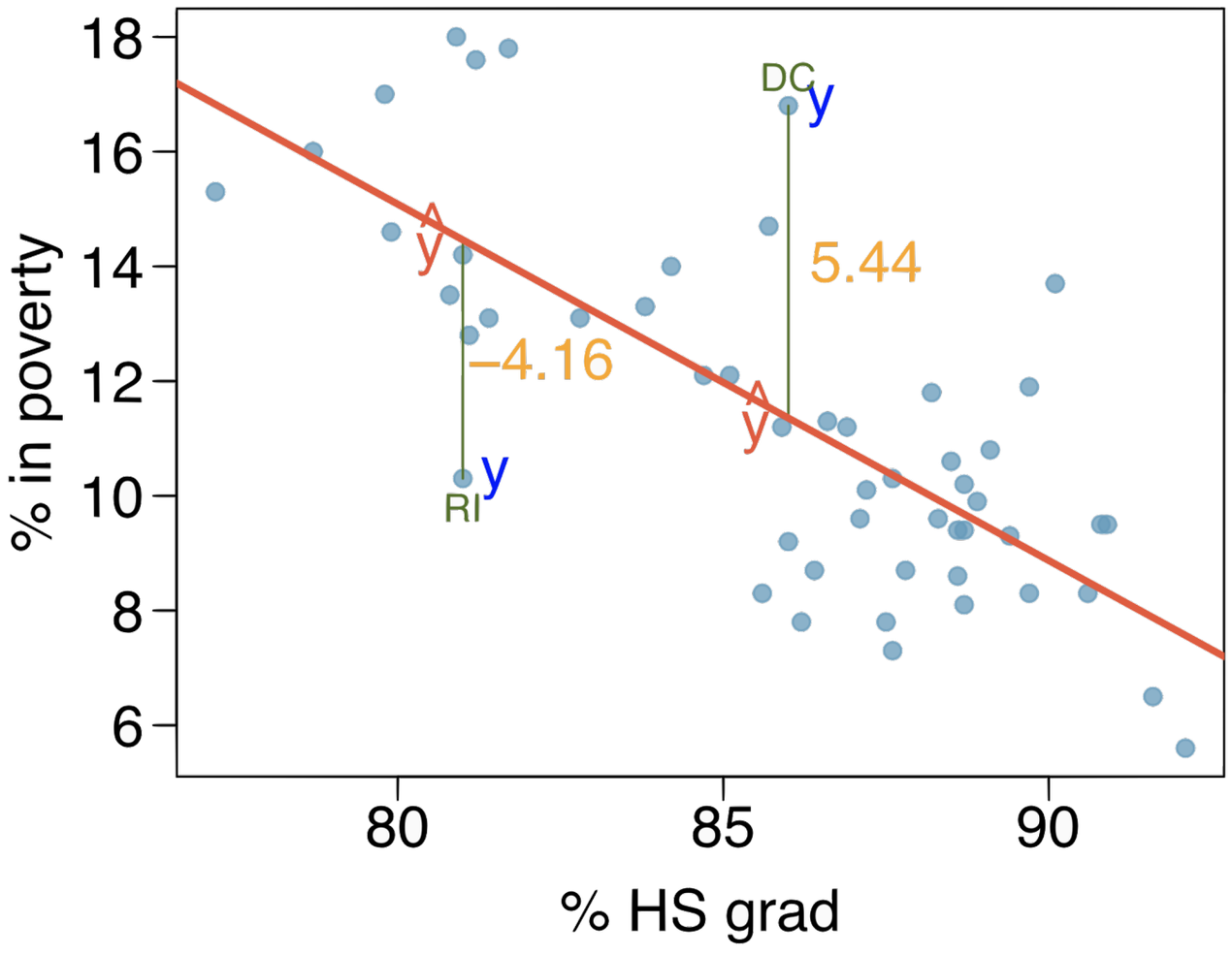

% living in poverty in DC is 5.44% more than predicted.

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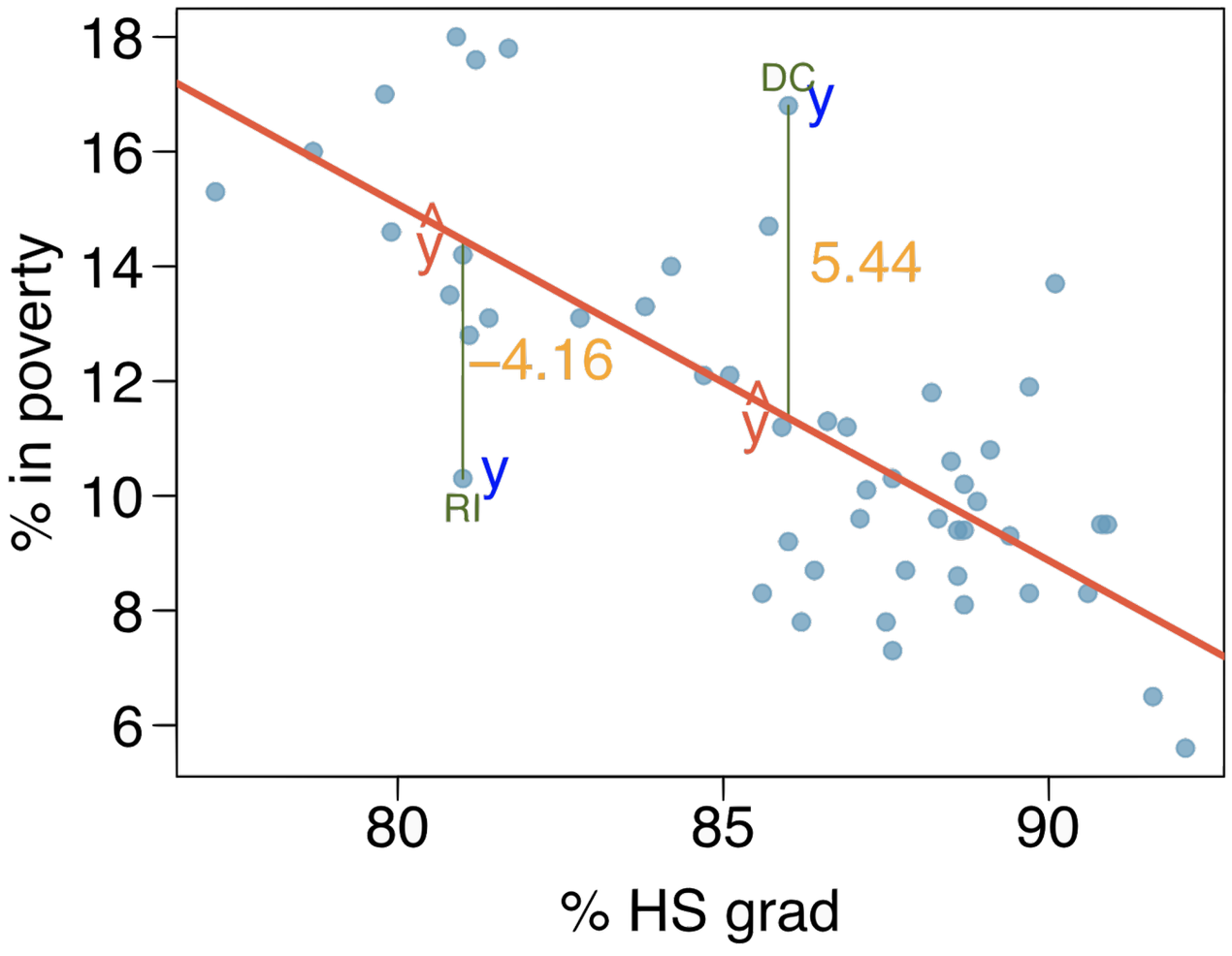

% living in poverty in DC is 5.44% more than predicted.

% living in poverty in RI is 4.16% less than predicted.

Quantifying the relationship

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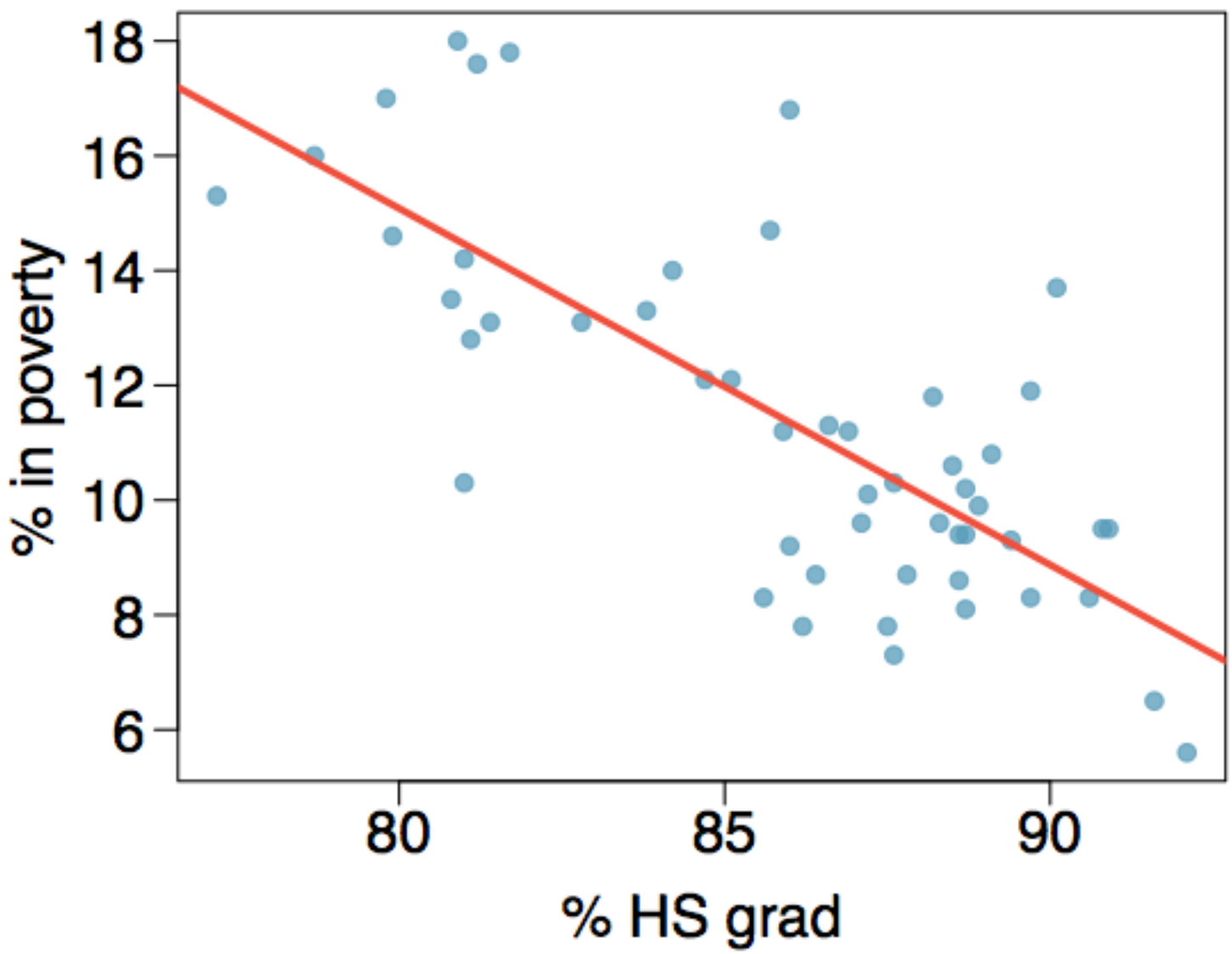
- A value of 0 indicates no linear association.

Guessing the correlation

Which of the following is the best guess for the correlation between percent in poverty and percent HS grad?

(a) 0.6
(b) -0.75
(c) -0.1
(d) 0.02
(e) -1.5



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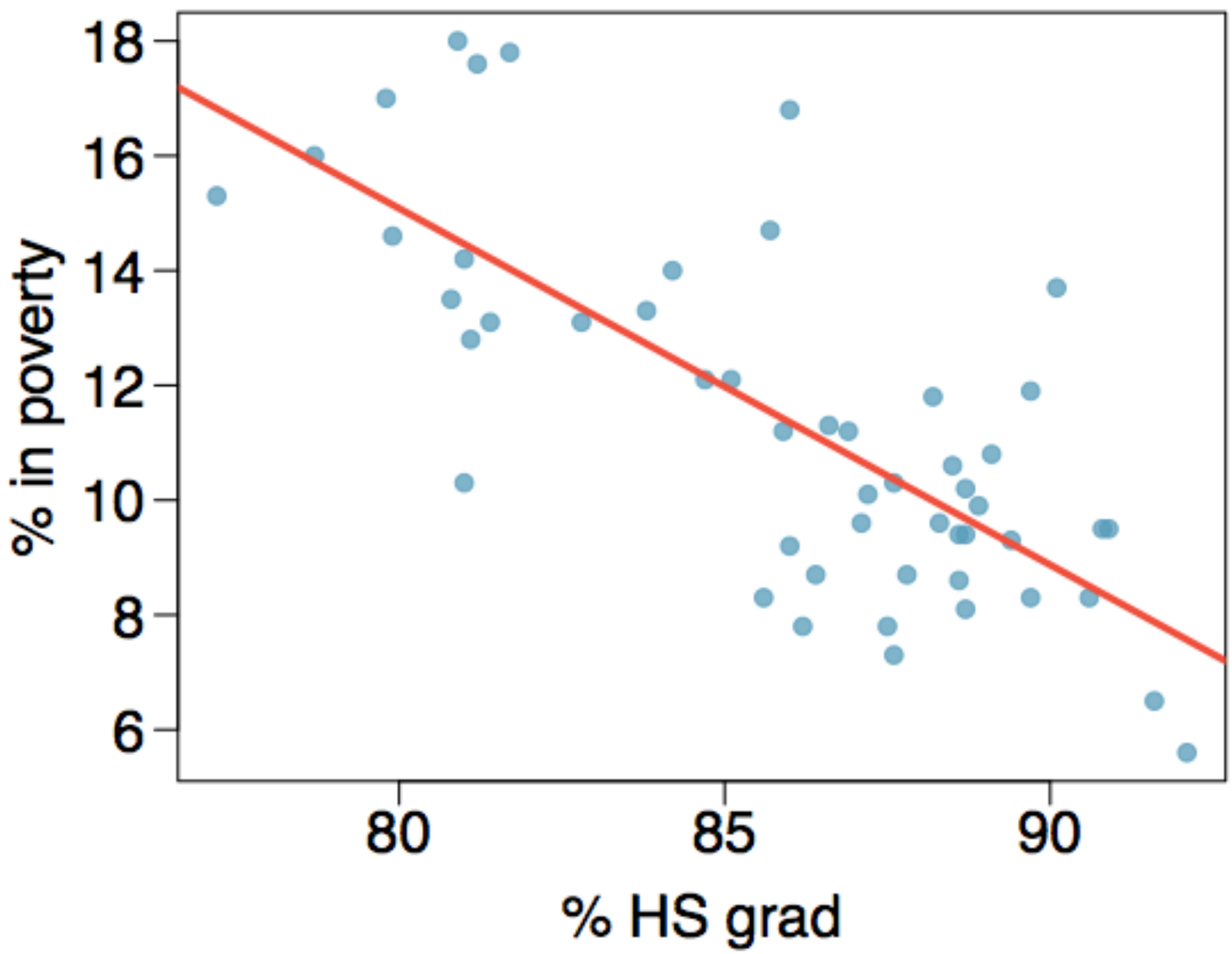

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Which of the following is the best guess for the correlation between percent in poverty and percent female householder?

(a) 0.1
(b) -0.6
(c) -0.4
(d) 0.9
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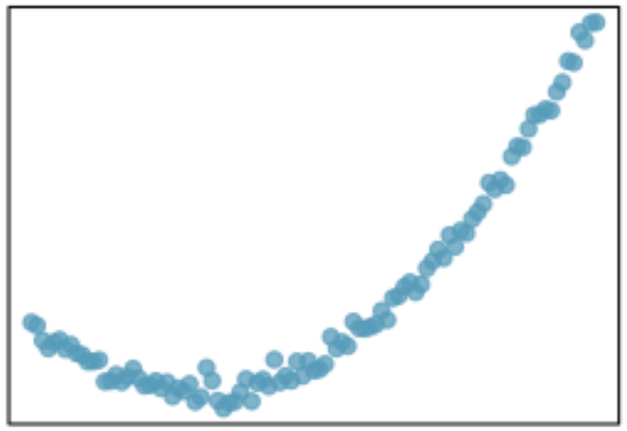
Assessing the correlation

Which of the following is has the strongest correlation, i.e. correlation coefficient closest to +1 or -1?



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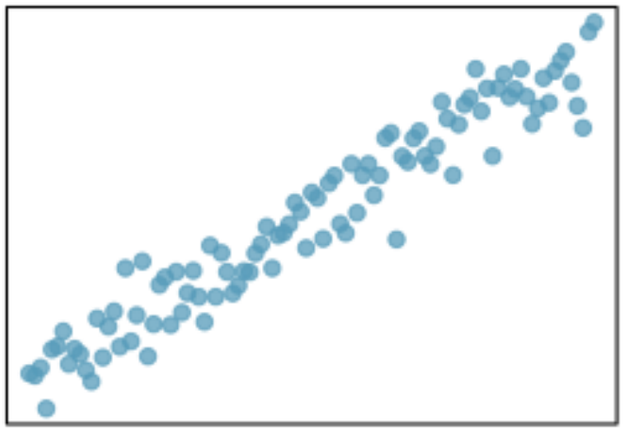

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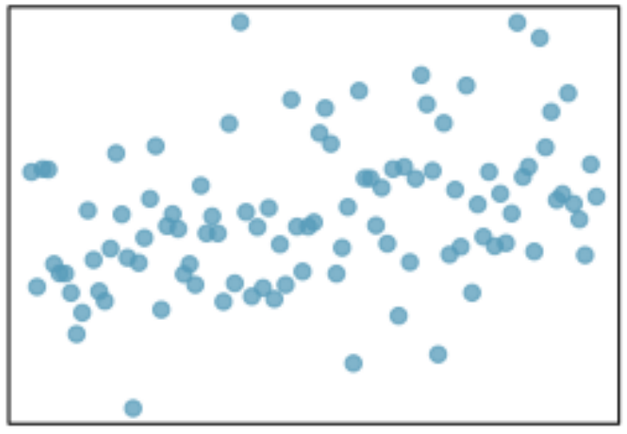

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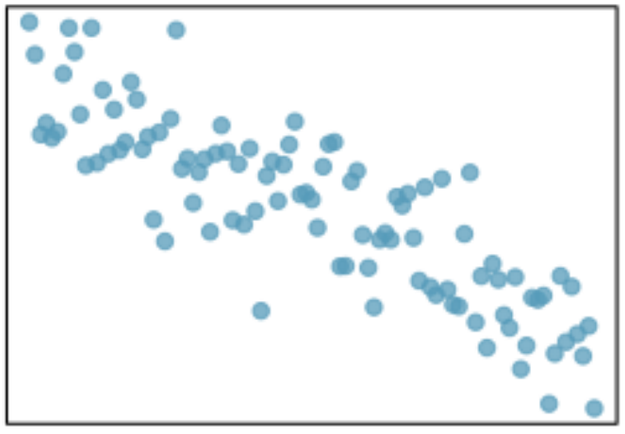

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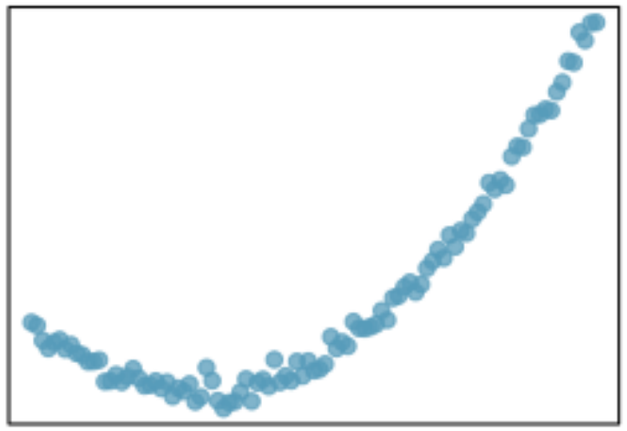

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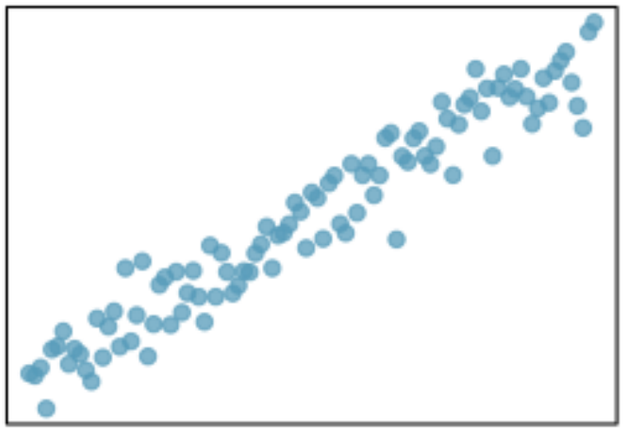

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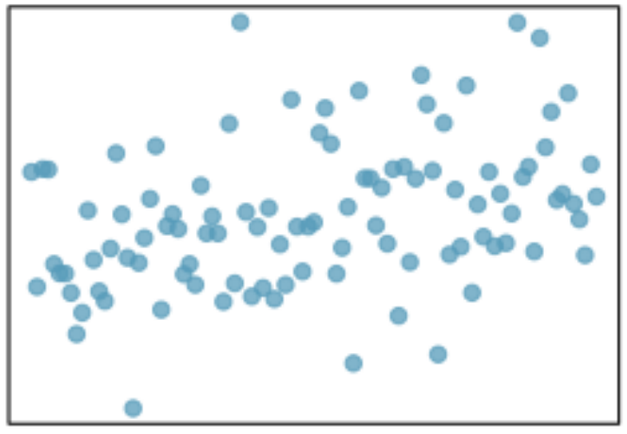

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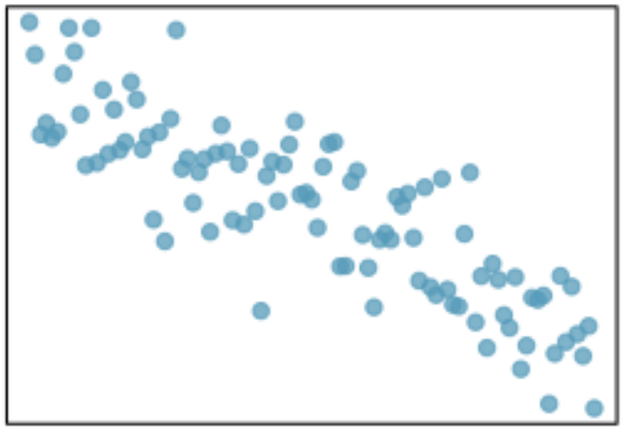

(d)

(b) → correlation means <u>linear</u> association

Fitting a line by least squares regression

A measure for the best line

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- Why least squares?
  1. Most commonly used
  2. Easier to compute by hand and using software
  3. In many applications, a residual twice as large as another is usually more than twice as bad

The least squares line

$$\hat{y} = \beta_0 + \beta_1 x$$

predicted y — *intercept* — *slope* — *explanatory variable*

Notation:

- Intercept:
  - Parameter: β_0
  - Point estimate: b_0

- Slope:
  - Parameter: β_1
  - Point estimate: b_1

Conditions for the least squares line

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2. Nearly normal residuals
3. Constant variability

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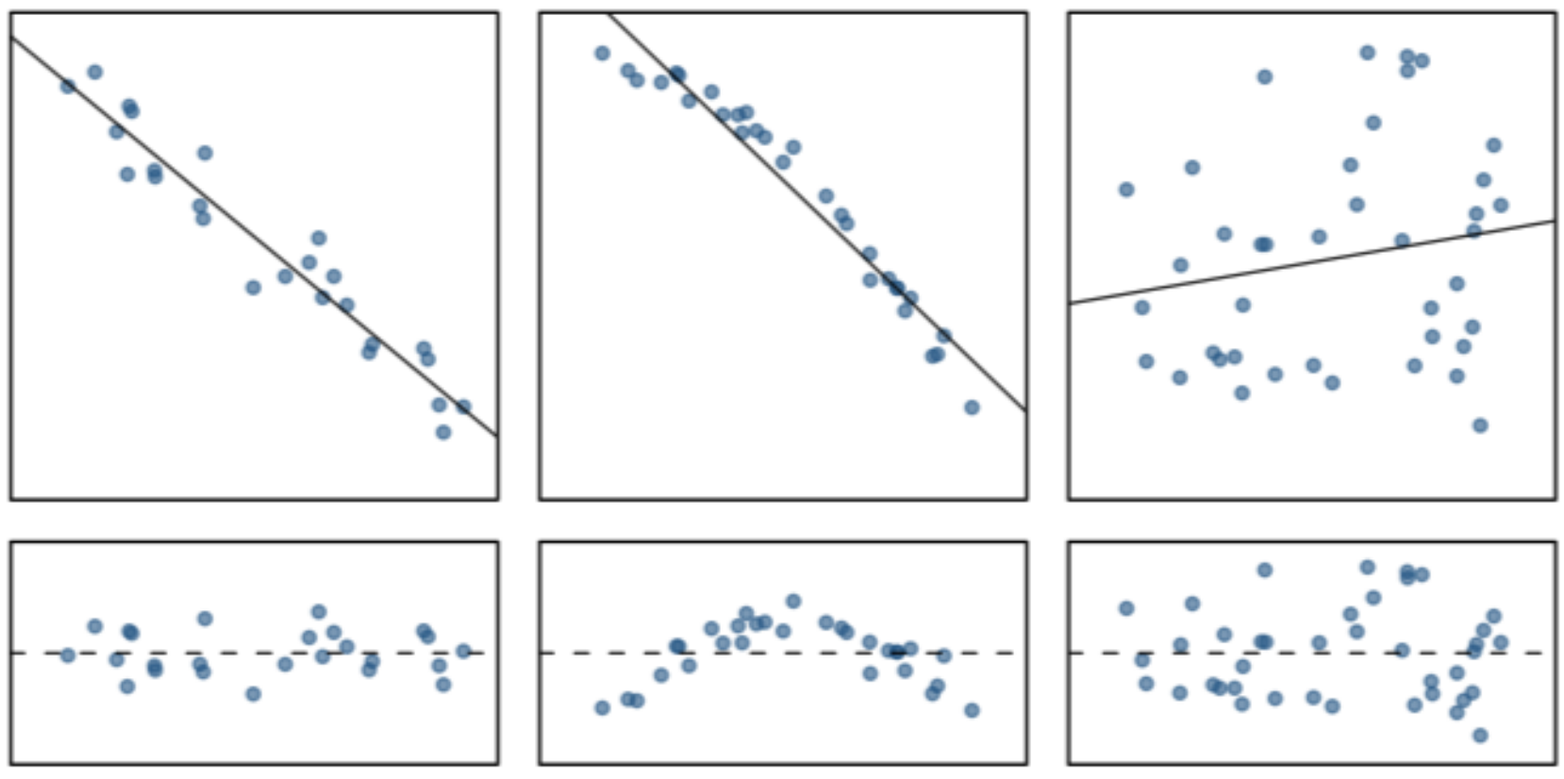
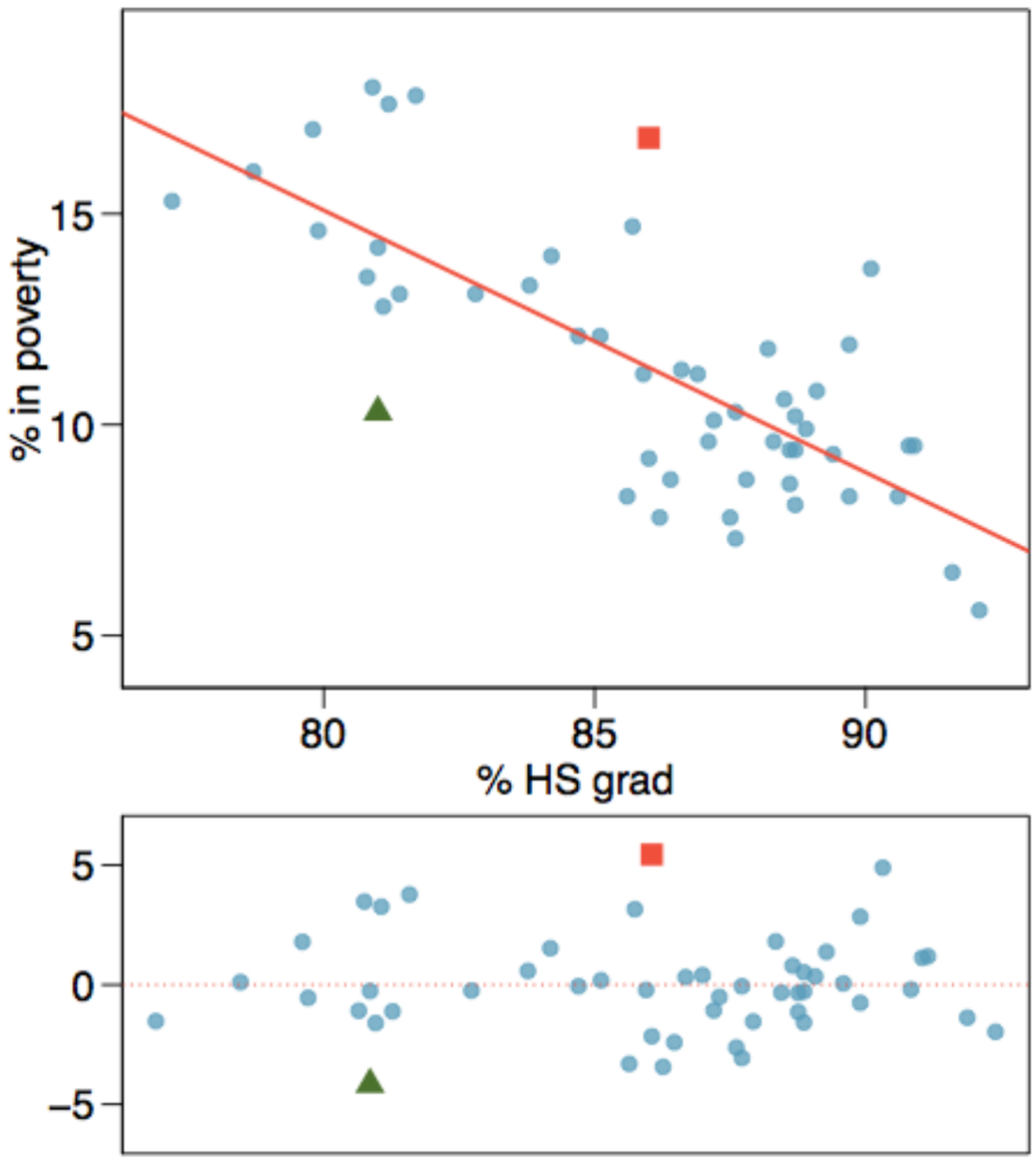
- Check using a scatterplot of the data, or a *residuals plot*.



Anatomy of a residuals plot



▲ *RI:*

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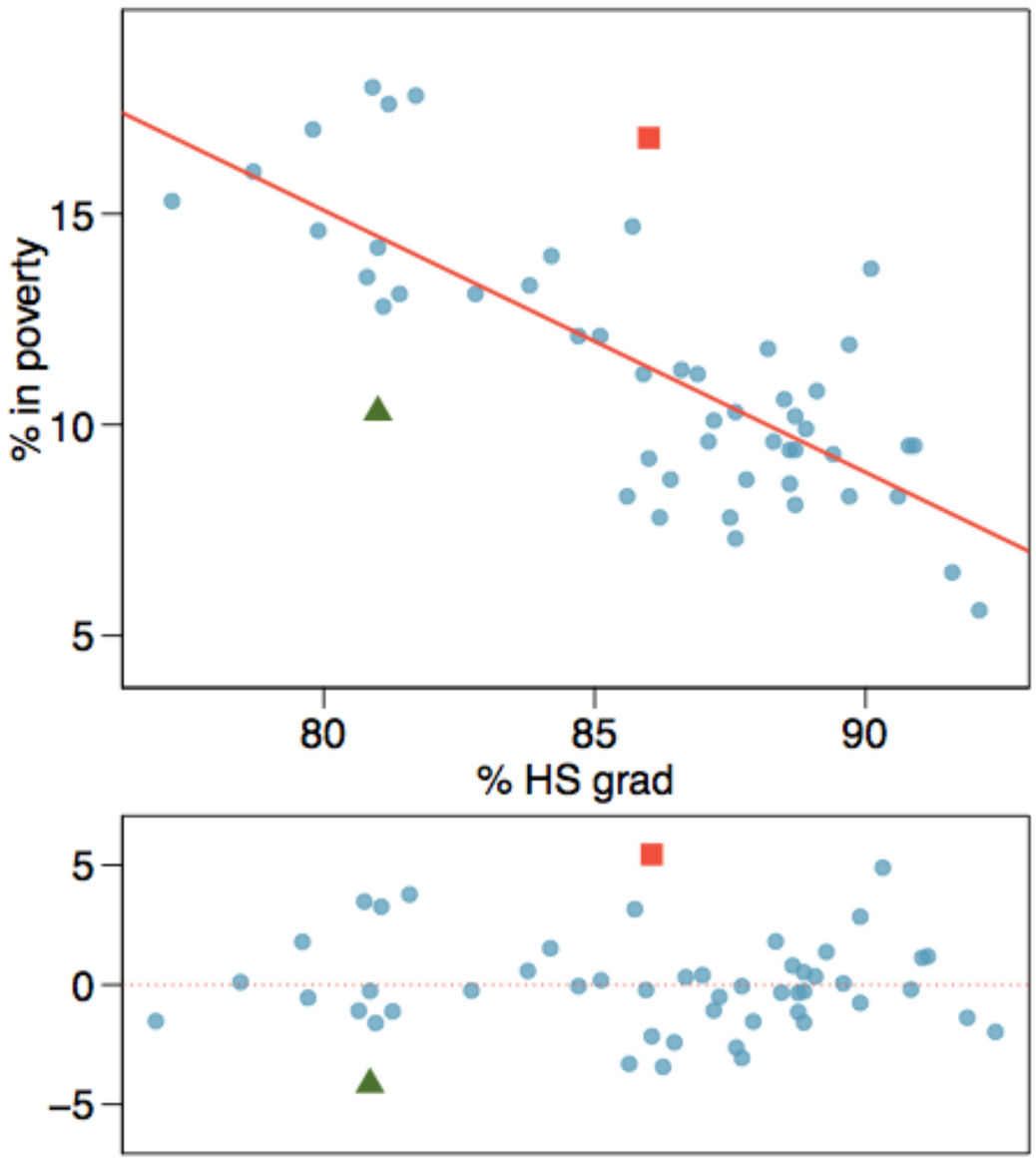

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■ *DC:*

$$\%\ HS\ grad = 86 \qquad \%\ in\ poverty = 16.8$$

$$\widehat{\%\ in\ poverty} = 64.68 - 0.62 * 86 = 11.36$$

$$e = \%\ in\ poverty - \widehat{\%\ in\ poverty}$$

$$= 16.8 - 11.36 = 5.44$$

Conditions: (2) Nearly normal residuals

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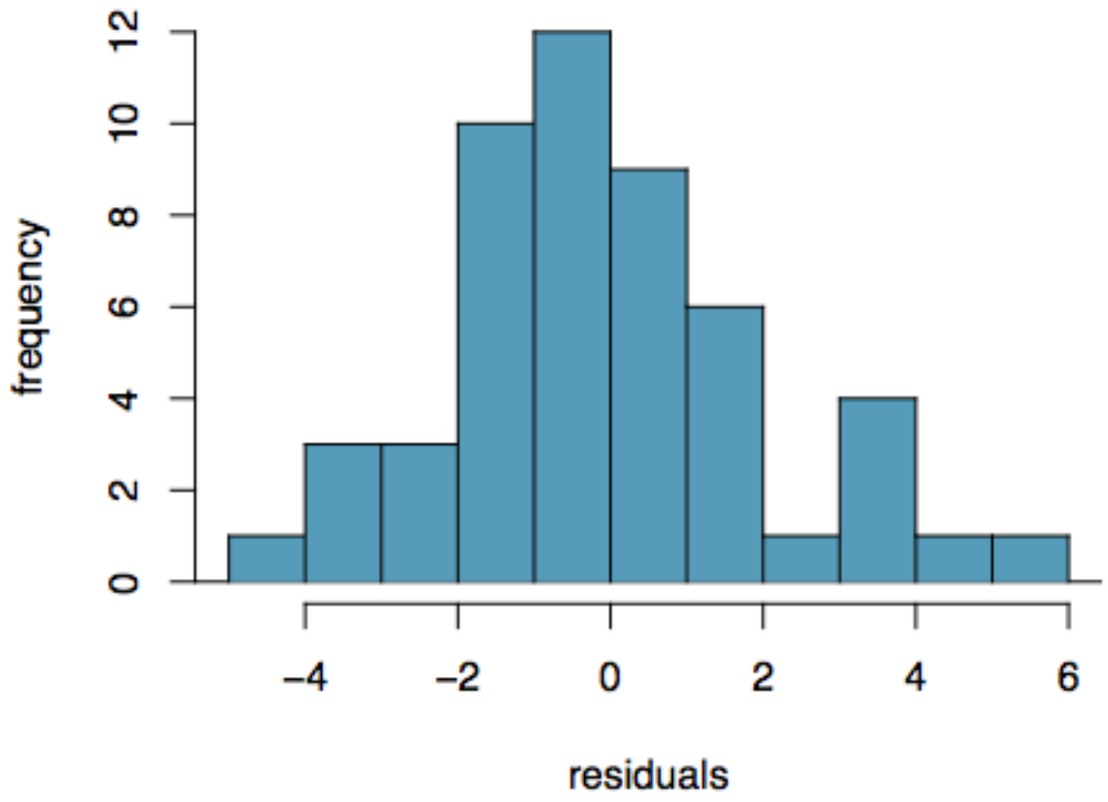
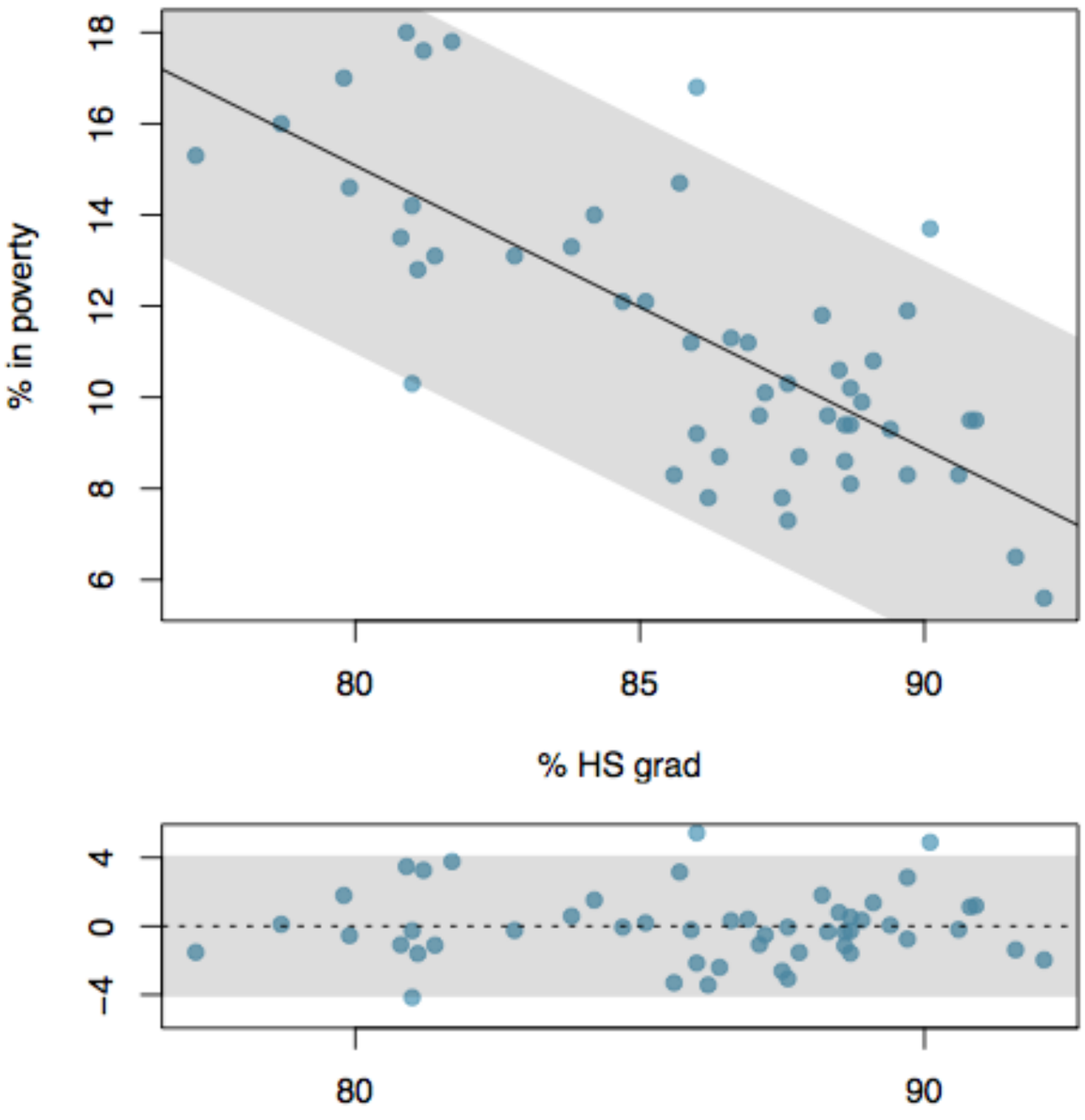
- Check using a histogram or normal probability plot of residuals.



Conditions: (3) Constant variability



- The variability of points around the least squares line should be roughly constant.

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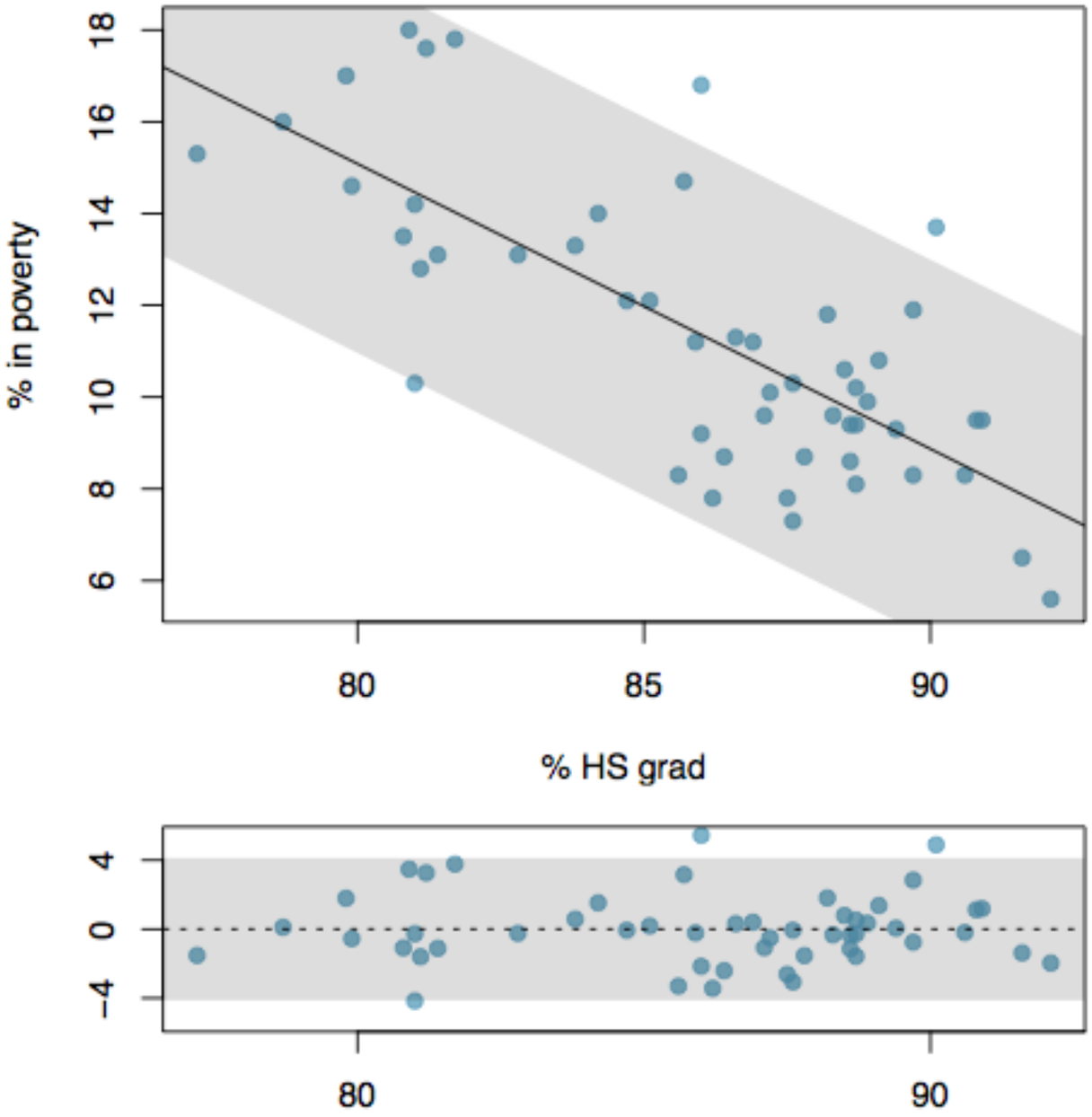

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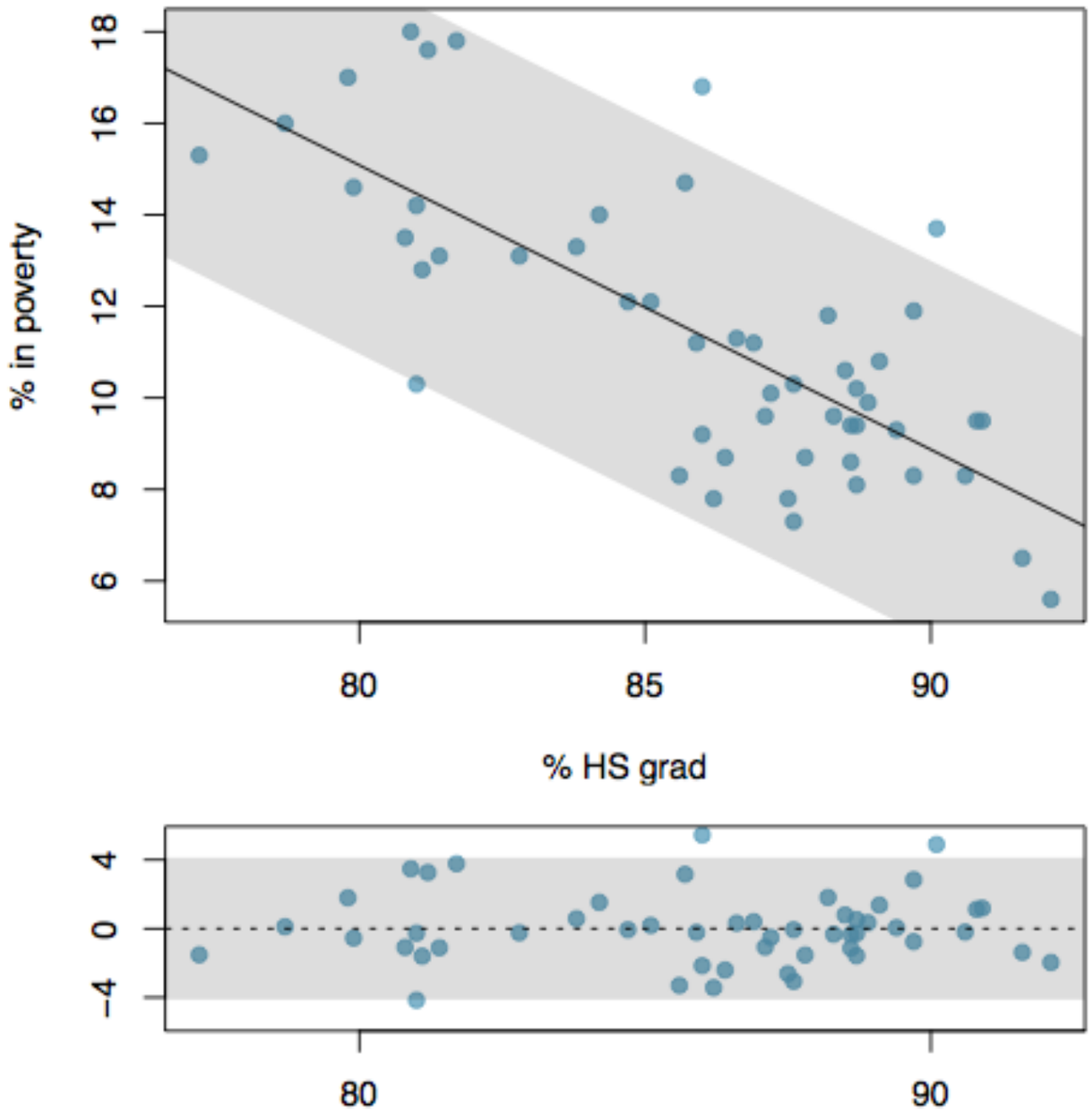

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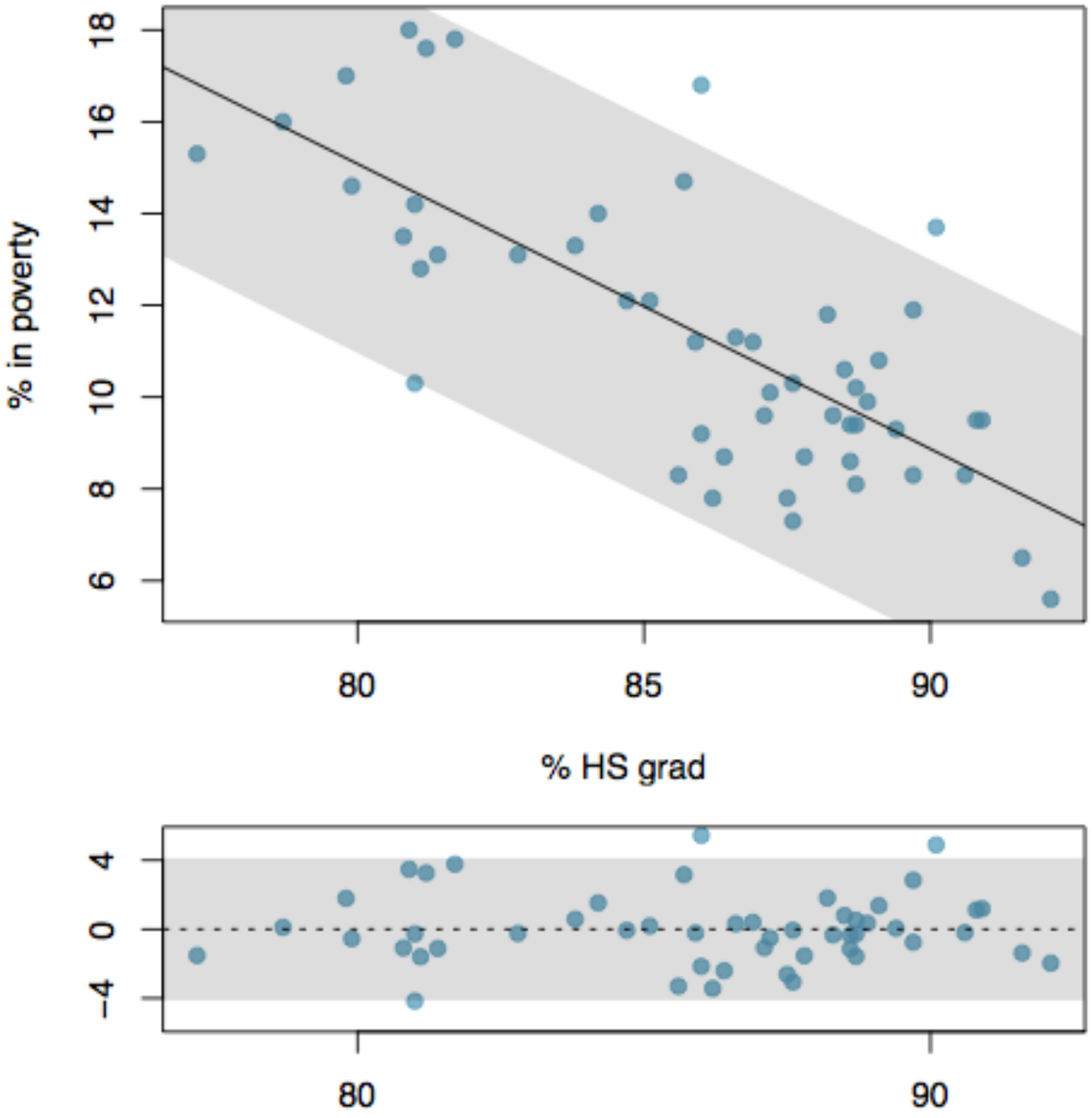

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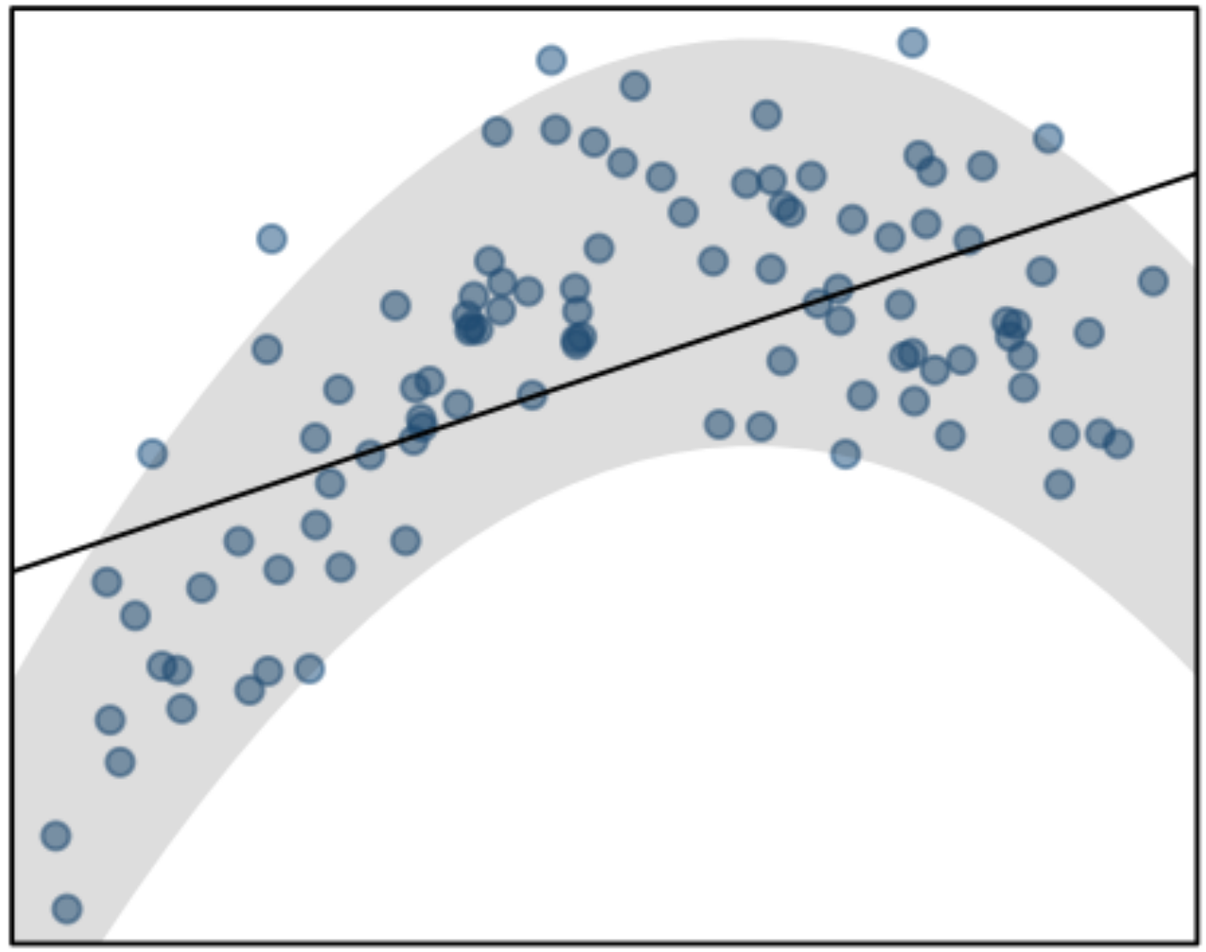
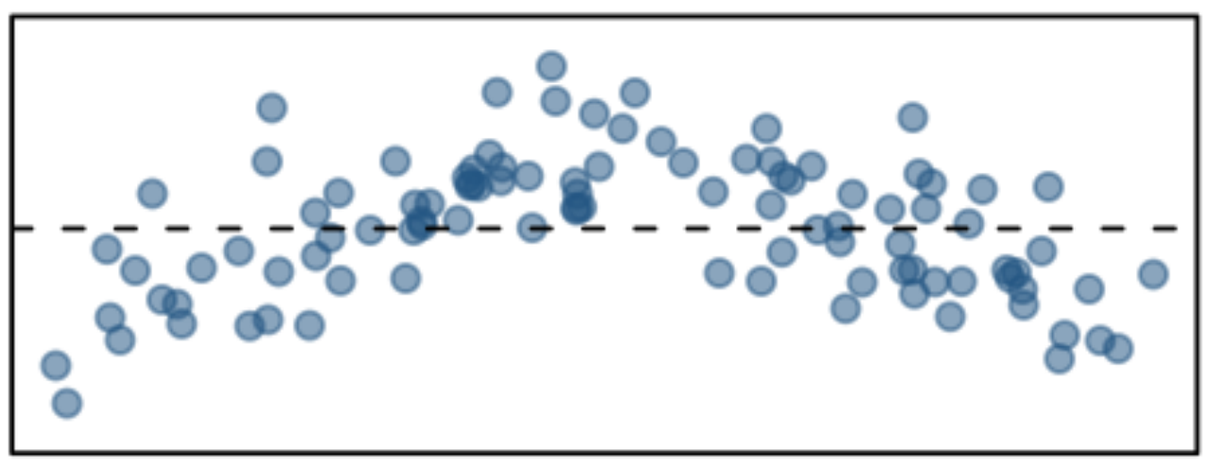
- This implies that the variability of residuals around the 0 line should be roughly constant as well.
- Also called *homoscedasticity*.
- Check using a histogram or normal probability plot of residuals.

Checking conditions

What condition is this linear model obviously violating?

(a) Constant variability
(b) Linear relationship
(c) Normal residuals
(d) No extreme outliers





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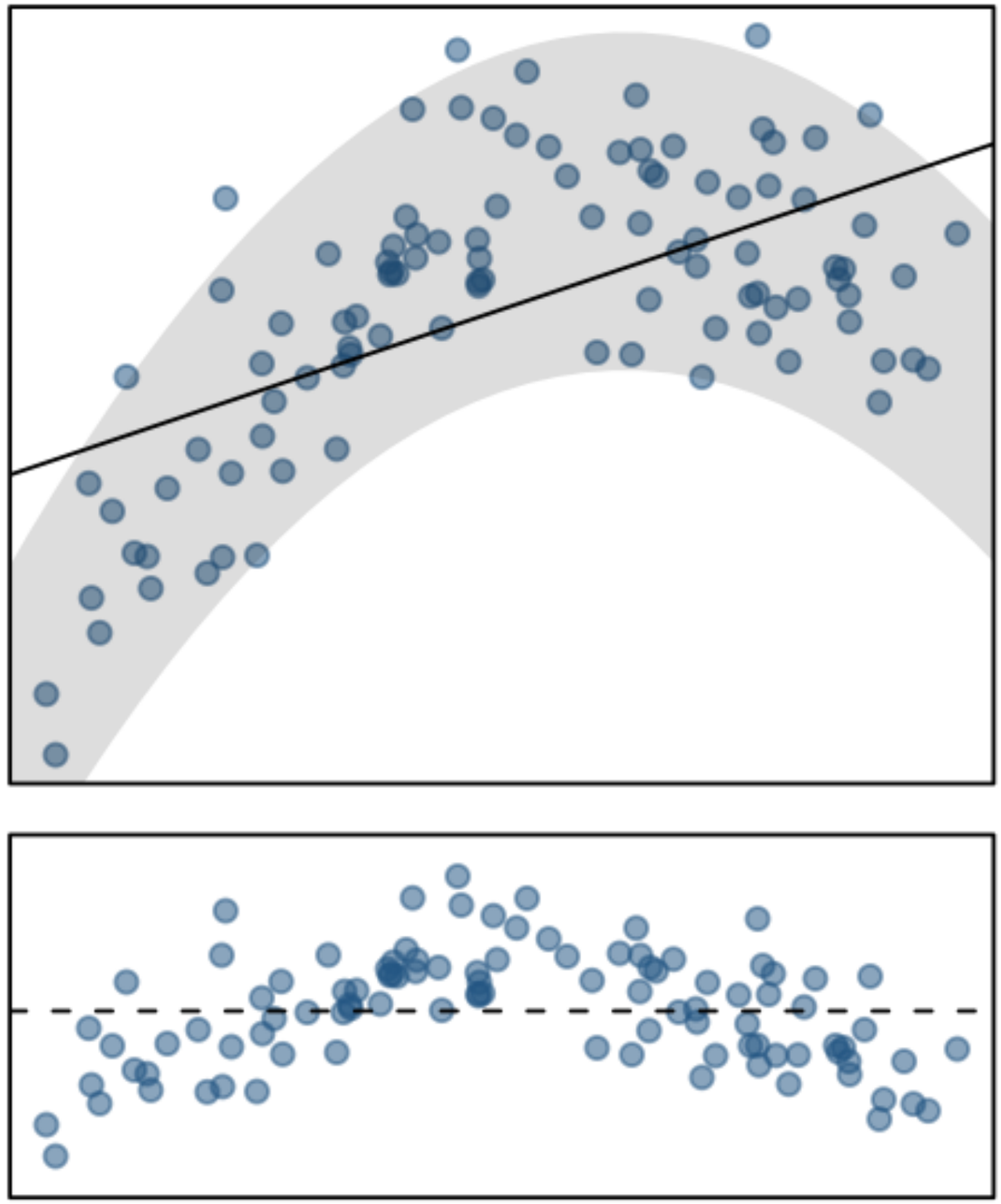

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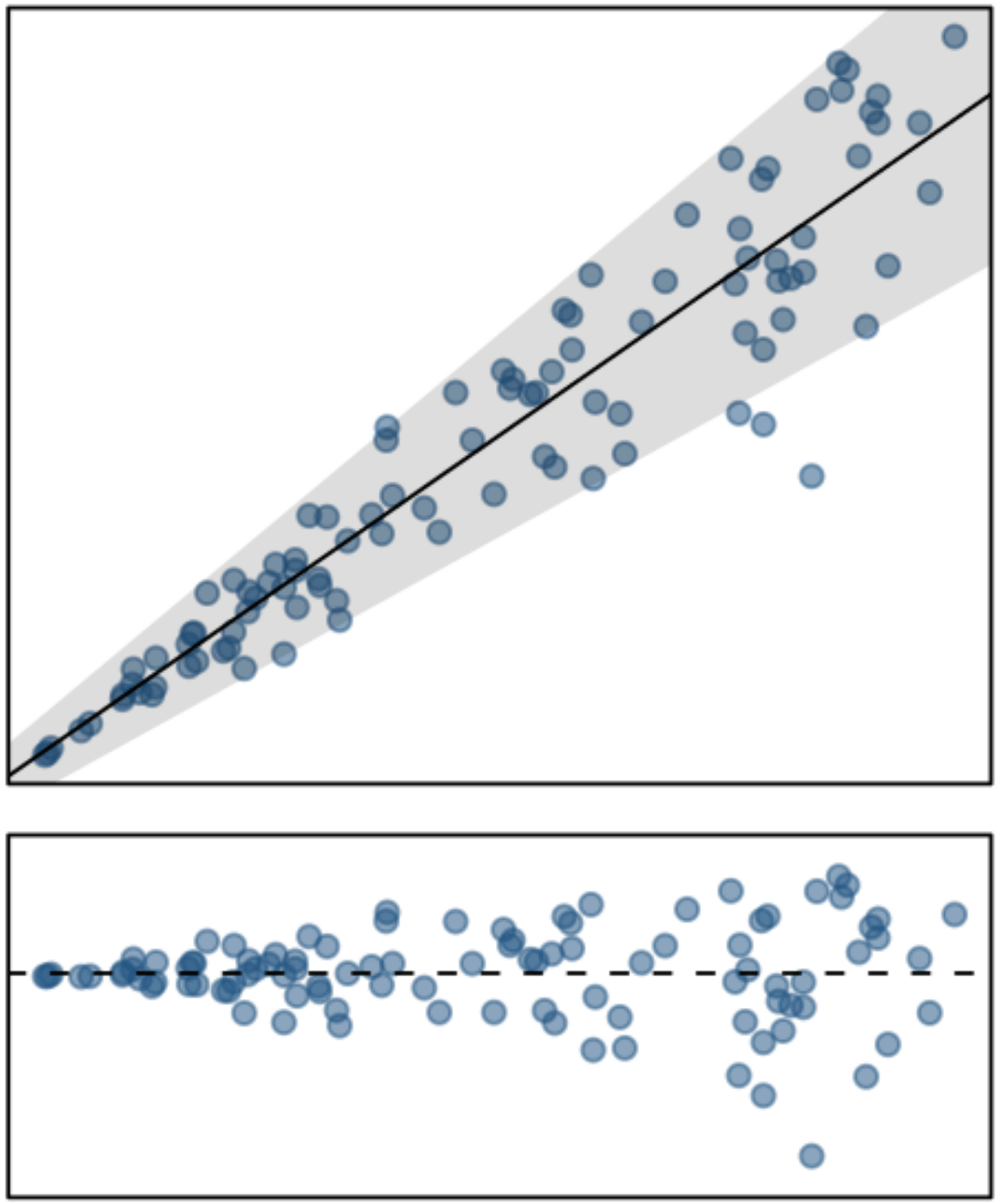

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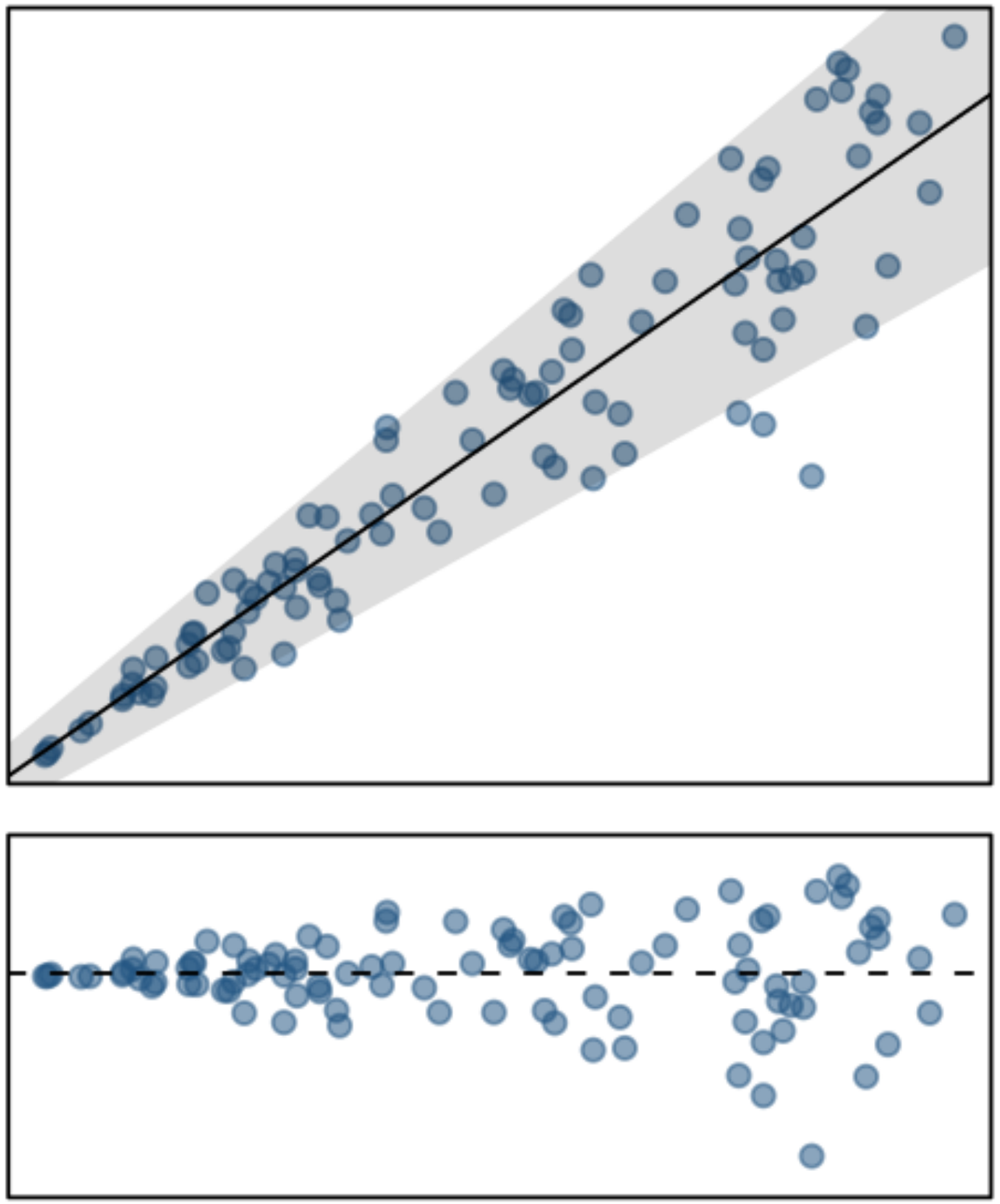
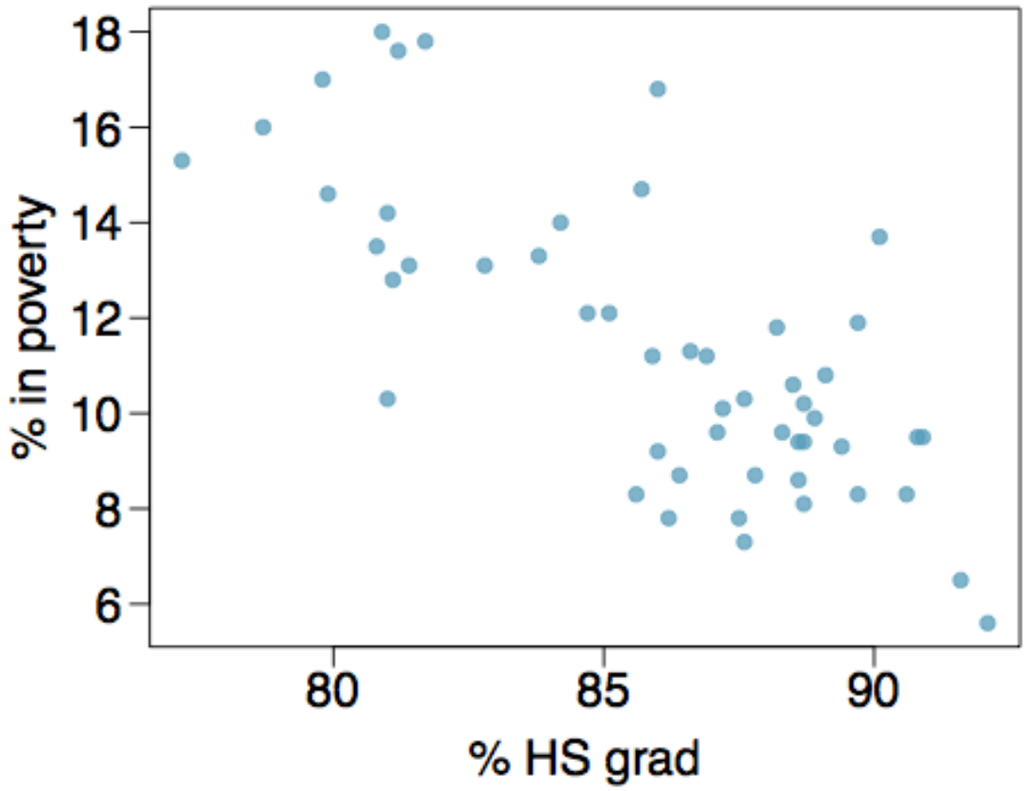


Given...



	% HS grad (x)	% in poverty (y)
mean	$\bar{x} = 86.01$	$\bar{y} = 11.35$
sd	$s_x = 3.73$	$s_y = 3.1$
	correlation	$R = -0.75$

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Interpretation

For each additional % point in HS graduate rate, we would expect the % living in poverty to be lower on average by 0.62% points.

Intercept

The intercept is where the regression line intersects the y-axis. The calculation of the intercept uses the fact the a regression line always passes through ($\bar{x}$, $\bar{y}$).

$$b_0 = \bar{y} - b_1 \bar{x}$$

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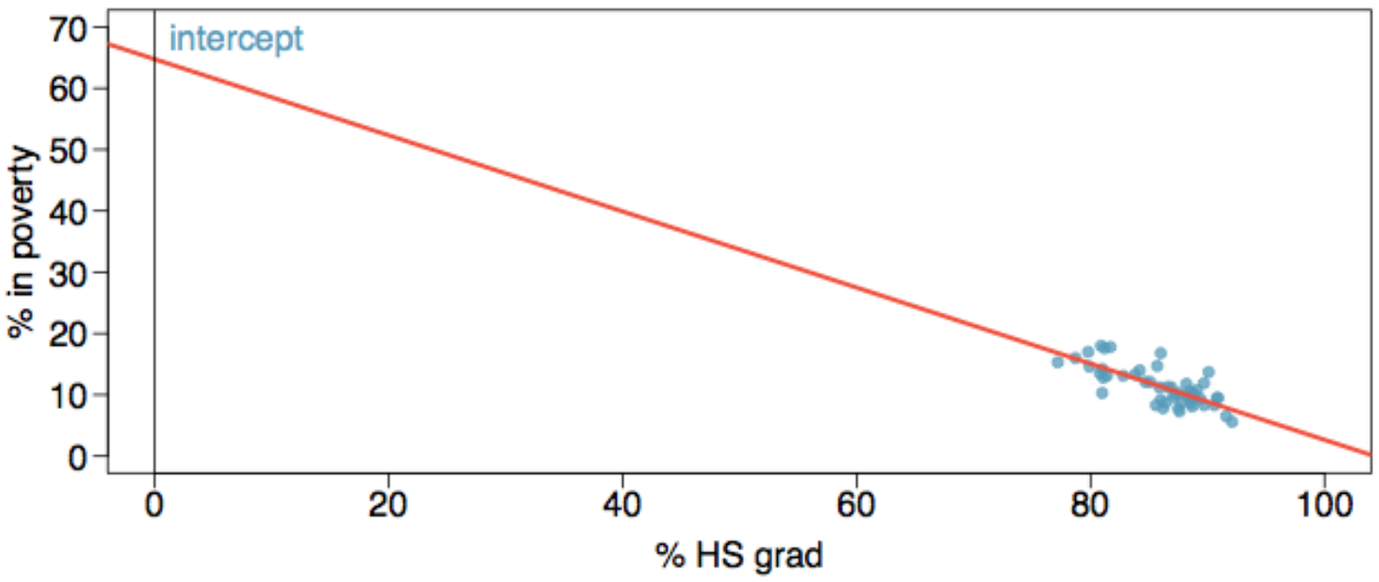

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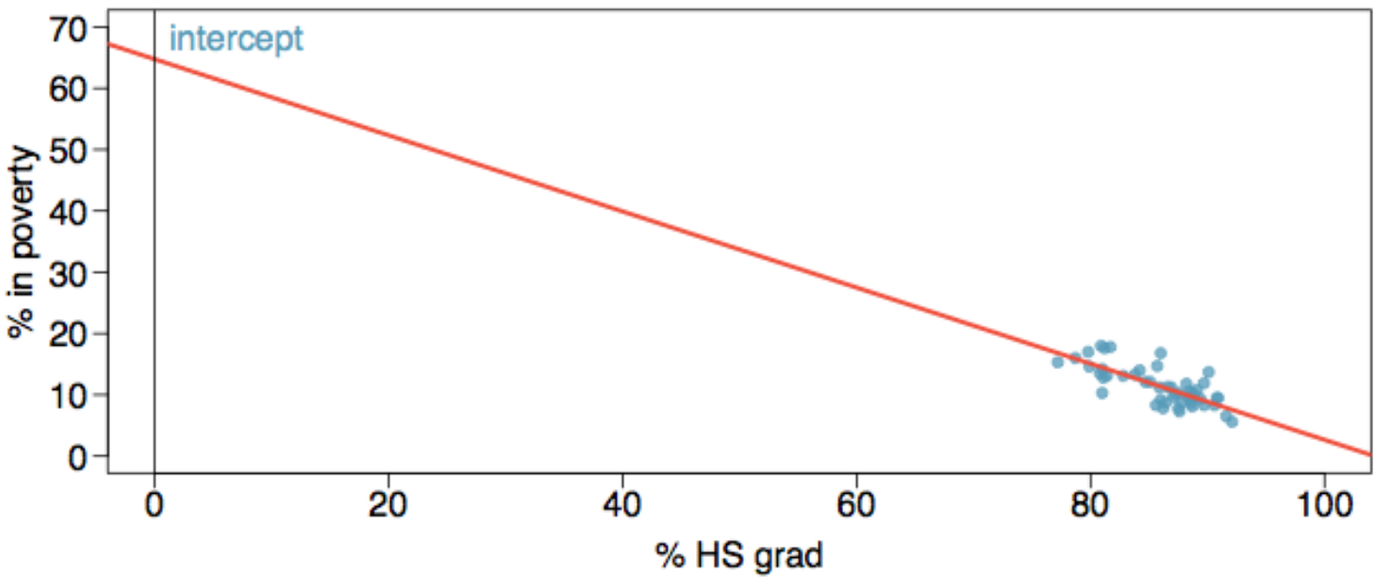

b_0 = 11.35 - (-0.62) x 86.01

= 64.68

Which of the following is the correct interpretation of the intercept?

(a) For each % point increase in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
(b) For each % point decrease in HS graduate rate, % living in poverty is expected to increase on average by 64.68%.
(c) Having no HS graduates leads to 64.68% of residents living below the poverty line.
(d) States with no HS graduates are expected on average to have 64.68% of residents living below the poverty line.
(e) In states with no HS graduates % living in poverty is expected to increase on average by 64.68%.

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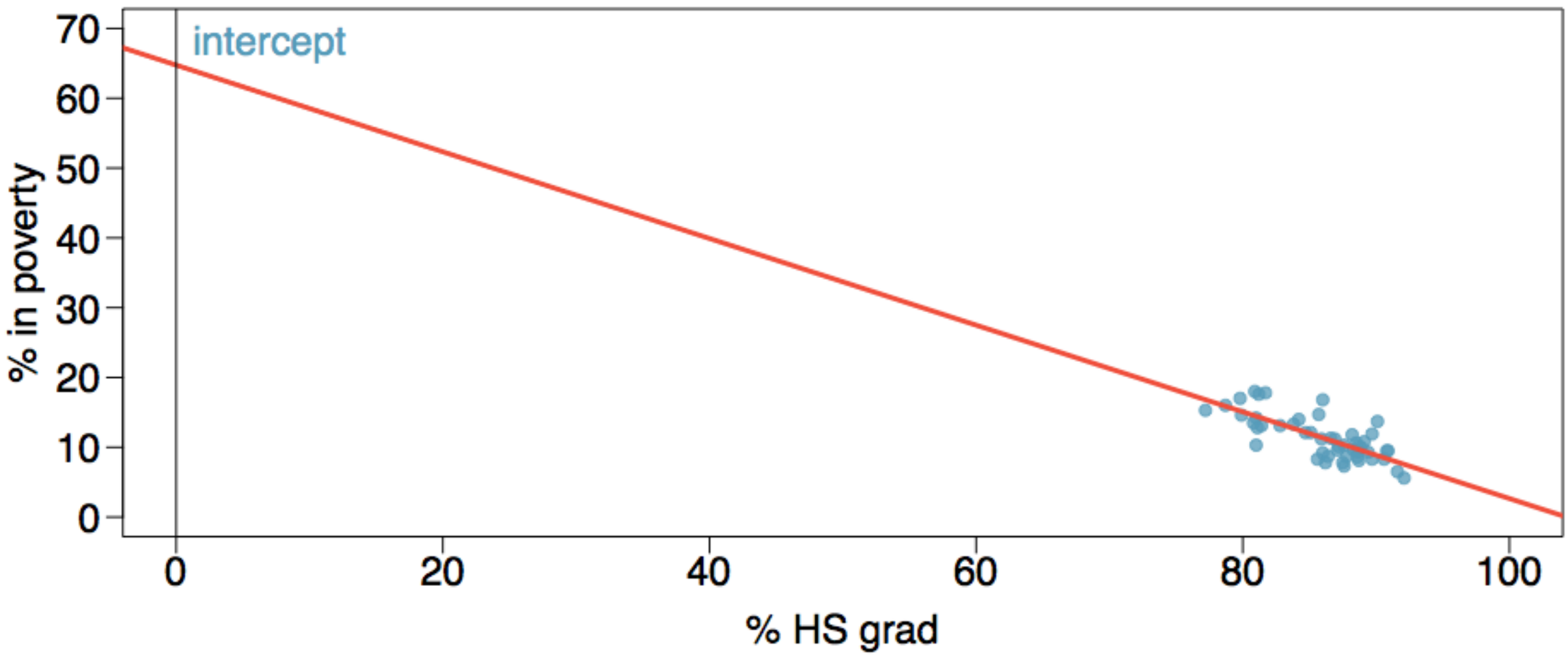
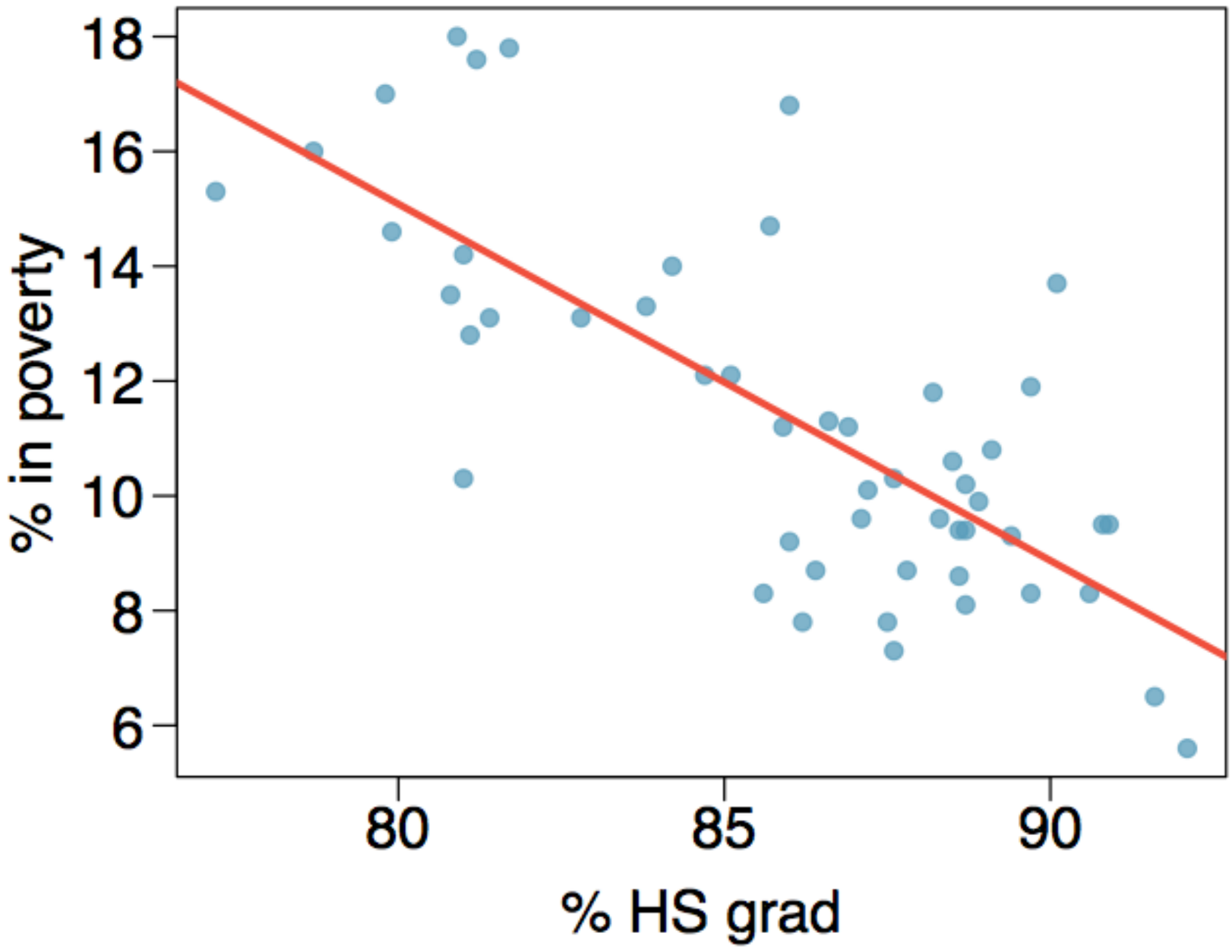
More on the intercept

Since there are no states in the dataset with no HS graduates, the intercept is of no interest, not very useful, and also not reliable since the predicted value of the intercept is so far from the bulk of the data.



Regression line

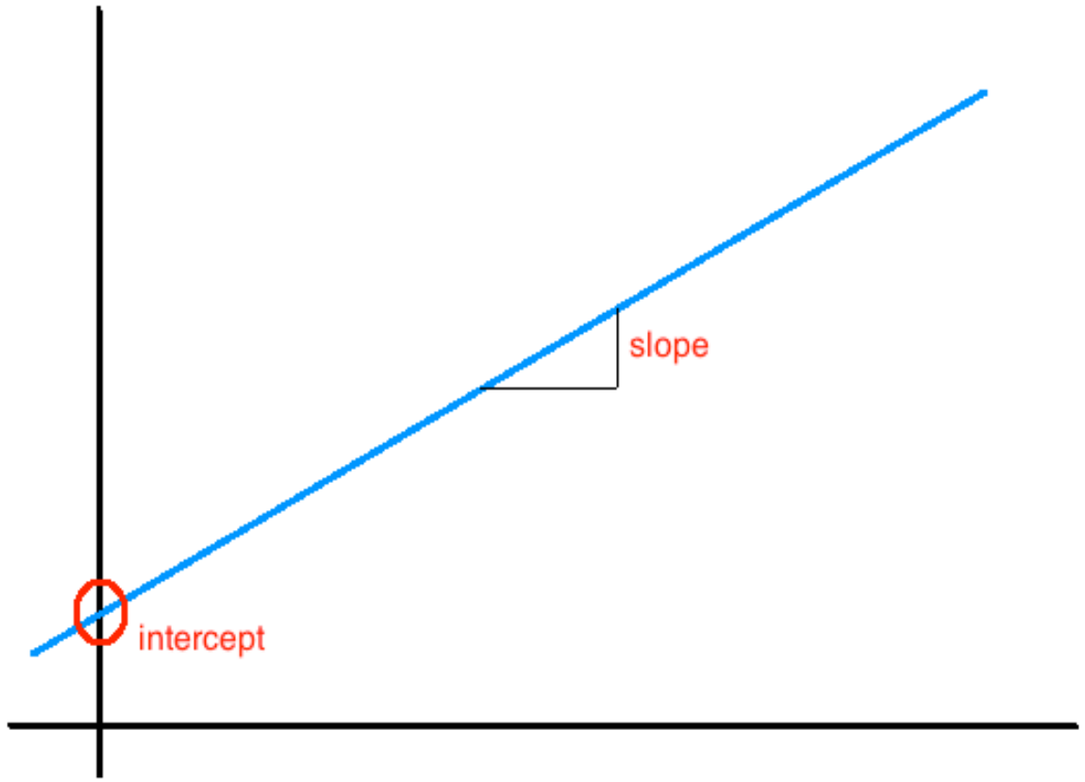
$$\widehat{\%\ in\ poverty} = 64.68 - 0.62\ \%\ HS\ grad$$



Interpretation of slope and intercept

- *Intercept*: When $x = 0$, y is expected to equal the intercept.

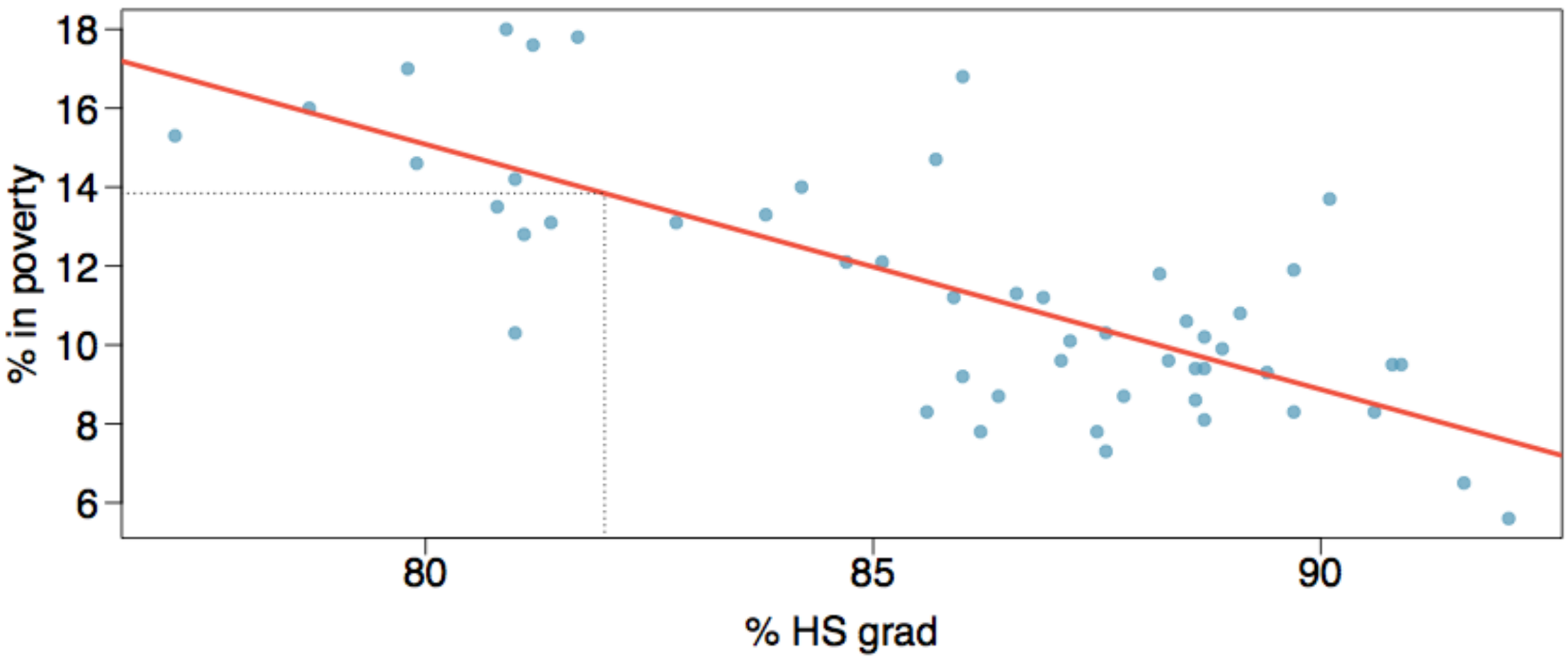
- *Slope*: For each unit in x, y is expected to increase / decrease on average by the slope.



Note: These statements are not causal, unless the study is a randomized controlled experiment.

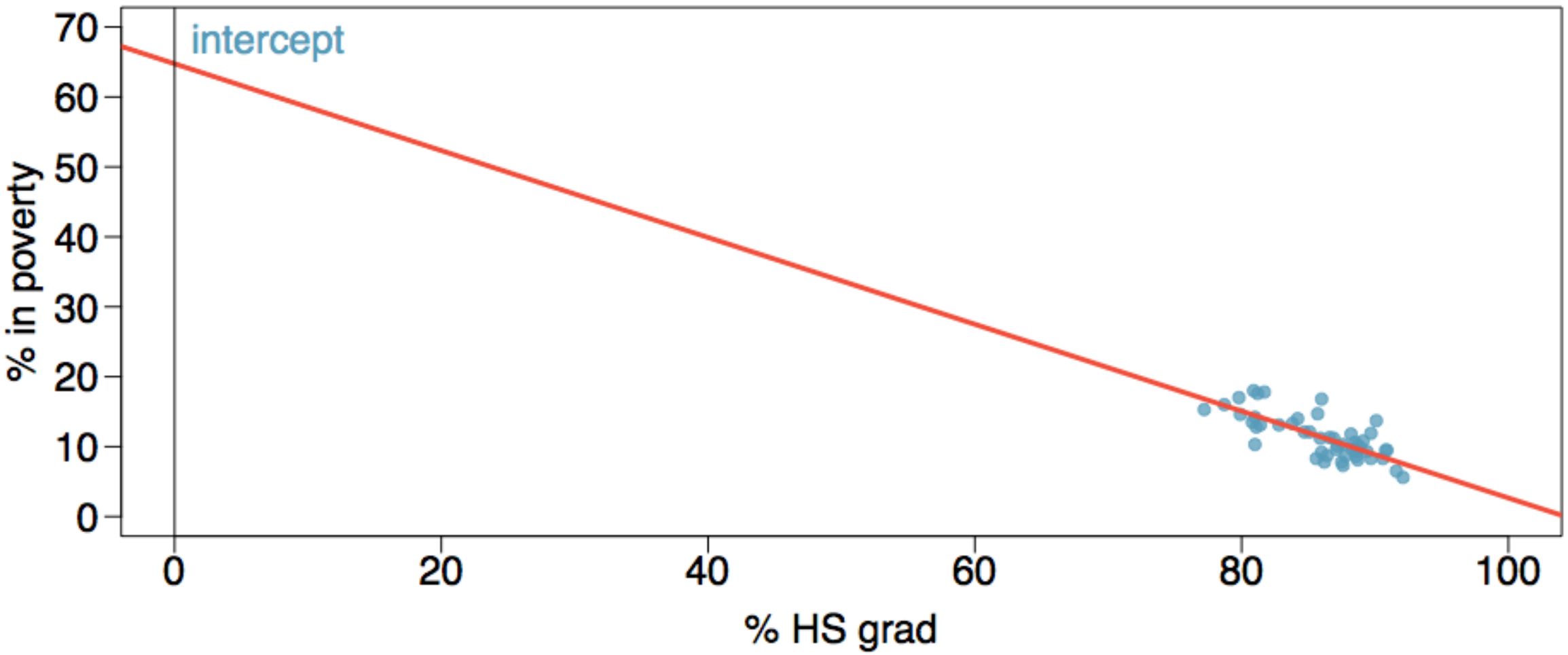
Prediction

- Using the linear model to predict the value of the response variable for a given value of the explanatory variable is called *prediction*, simply by plugging in the value of x in the linear model equation.
- There will be some uncertainty associated with the predicted value.



Extrapolation

- Applying a model estimate to values outside of the realm of the original data is called *extrapolation*.
- Sometimes the intercept might be an extrapolation.



Examples of extrapolation

Momentous sprint at the 2156 Olympics?

Women sprinters are closing the gap on men and may one day overtake them.

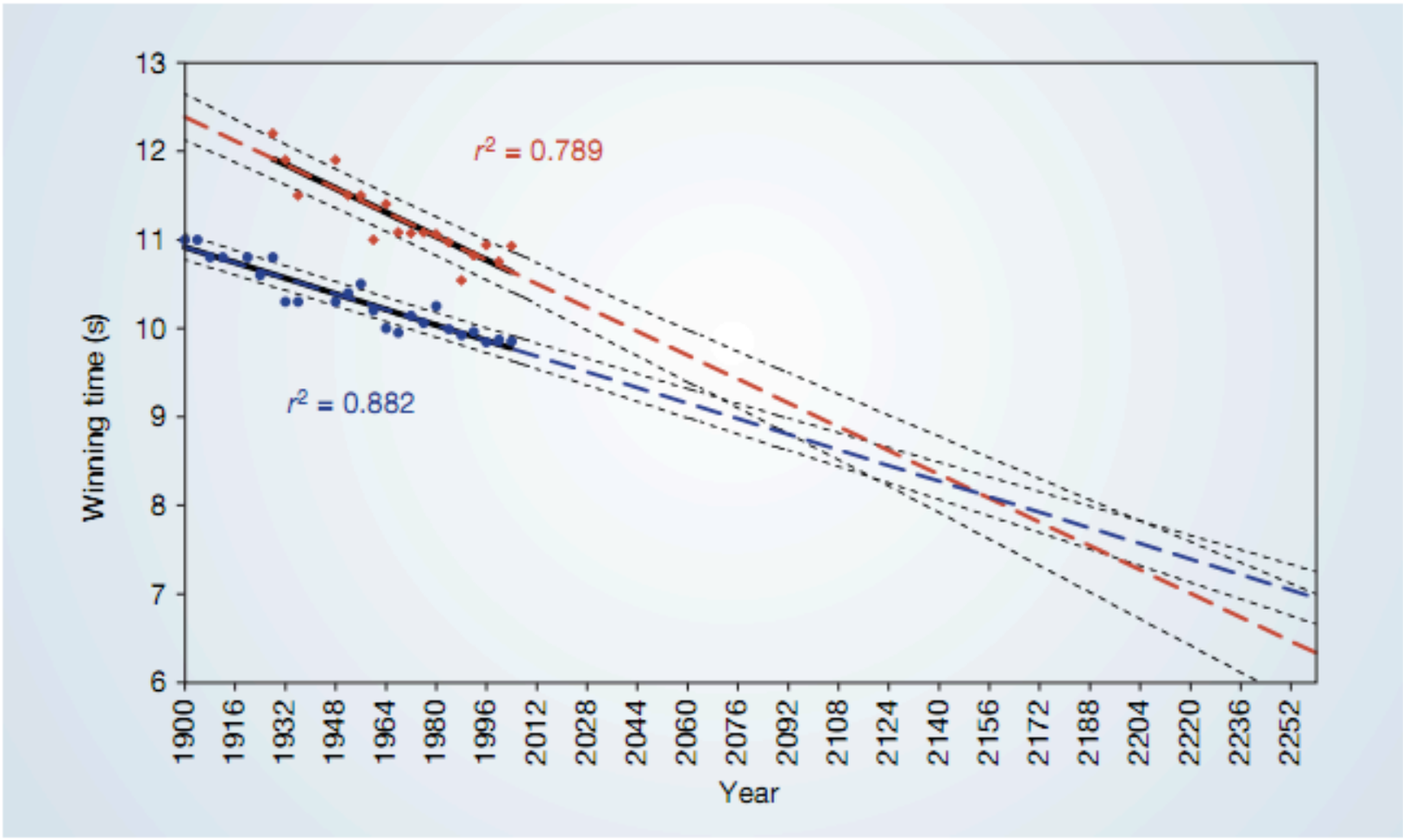


Figure 1 The winning Olympic 100-metre sprint times for men (blue points) and women (red points), with superimposed best-fit linear regression lines (solid black lines) and coefficients of determination. The regression lines are extrapolated (broken blue and red lines for men and women, respectively) and 95% confidence intervals (dotted black lines) based on the available points are superimposed. The projections intersect just before the 2156 Olympics, when the winning women's 100-metre sprint time of 8.079 s will be faster than the men's at 8.098 s.

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- The remainder of the variability is explained by variables not included in the model or by inherent randomness in the data.
- For the model we've been working with, $R^2 = -0.62^2 = 0.38$.

Interpretation of R^2

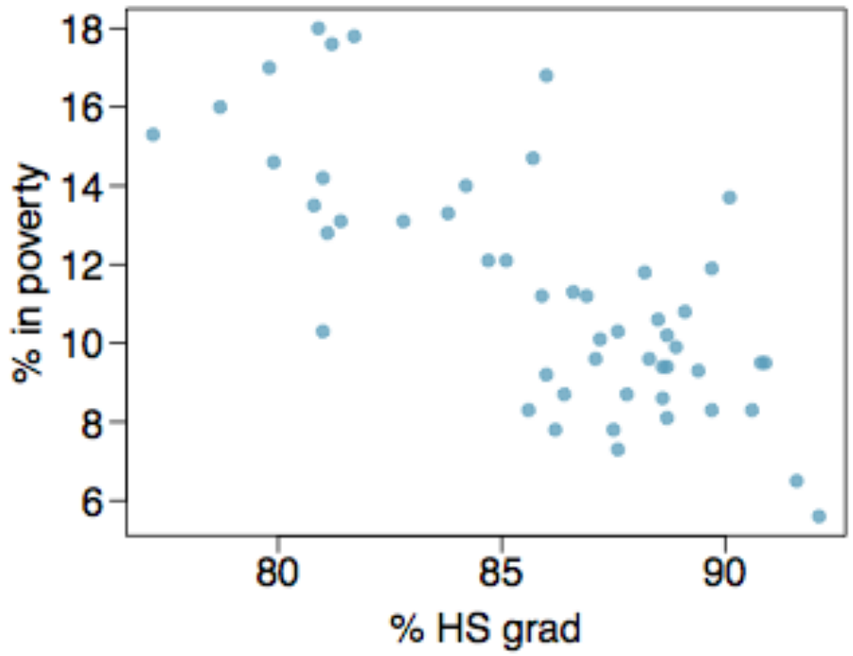
Which of the below is the correct interpretation of $R = -0.62$, $R^2 = 0.38$?

(a) 38% of the variability in the % of HG graduates among the 51 states is explained by the model.

(b) 38% of the variability in the % of residents living in poverty among the 51 states is explained by the model.

(c) 38% of the time % HS graduates predict % living in poverty correctly.

(d) 62% of the variability in the % of residents living in poverty among the 51 states is explained by the model.



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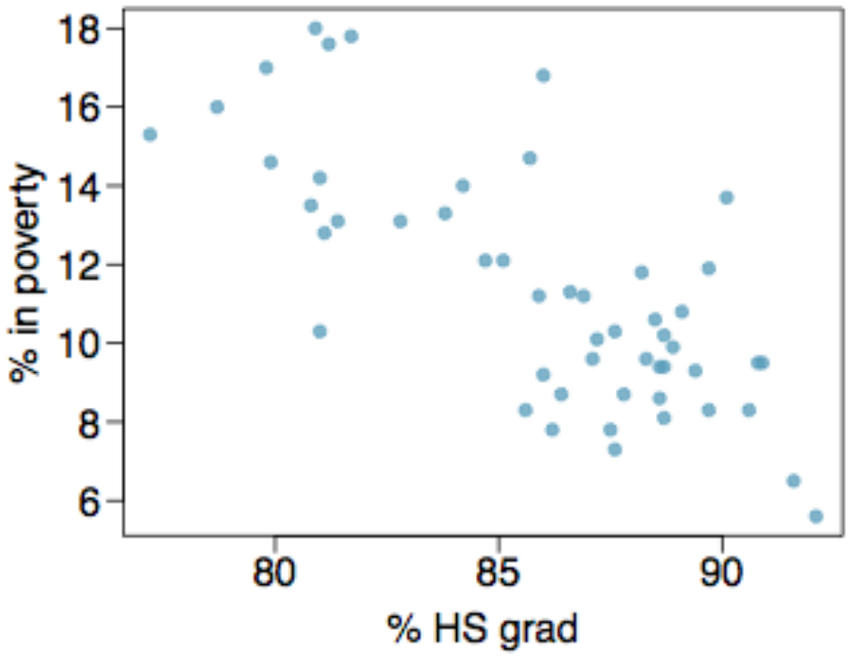

Poverty vs. region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, *reference level*: east

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Poverty vs. region (northeast, midwest, west, south)

Which region (northeast, midwest, west, or south) is the reference level?

	Estimate	Std. Error	t value	Pr(>|t|)
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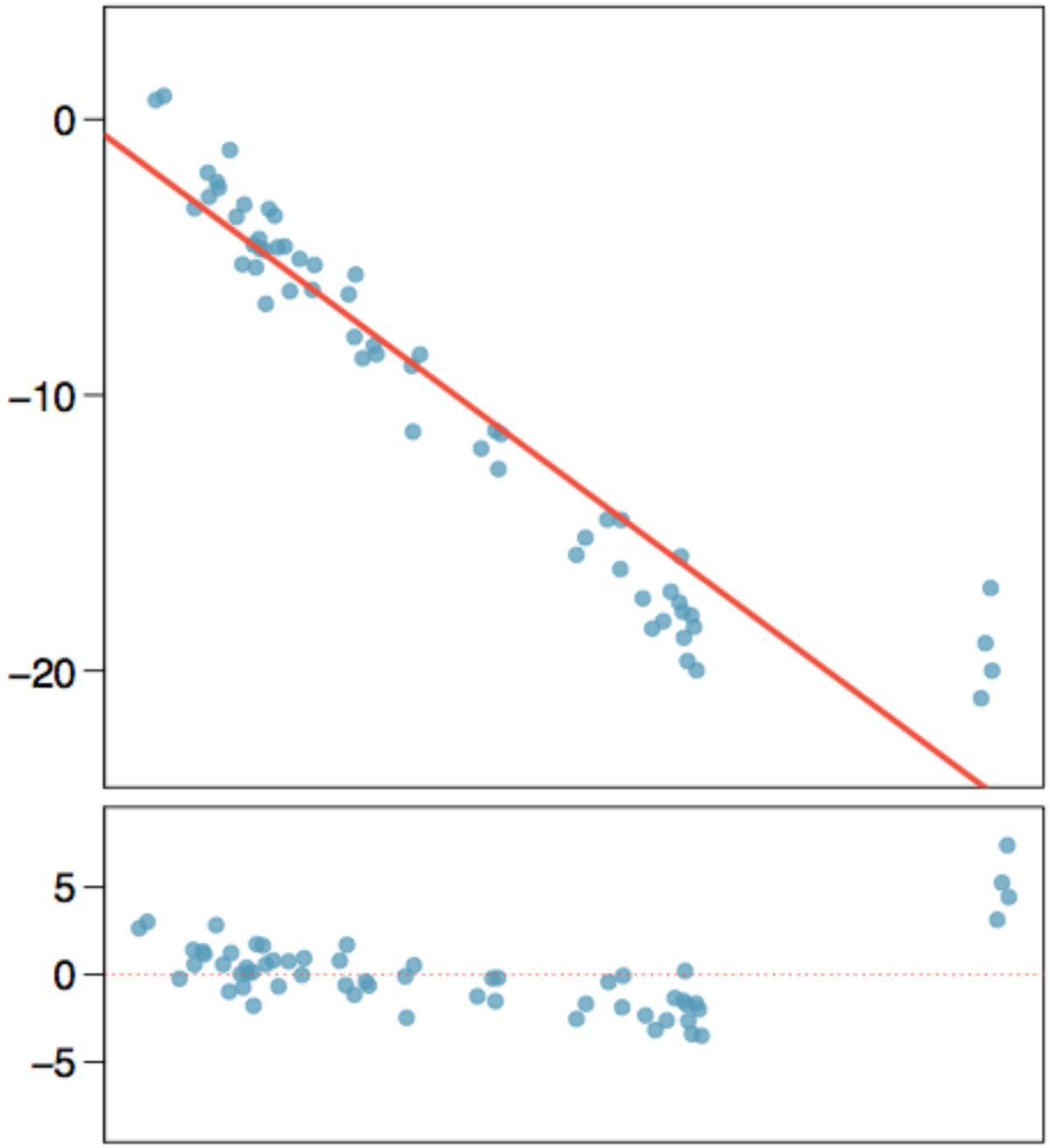
Types of outliers in linear regression

Types of outliers

How do outliers influence the least squares line in this plot?

To answer this question think of where the regression line would be with and without the outlier(s). Without the outliers the regression line would be steeper, and lie closer to the larger group of observations. With the outliers the line is pulled up and away from some of the observations in the larger group.



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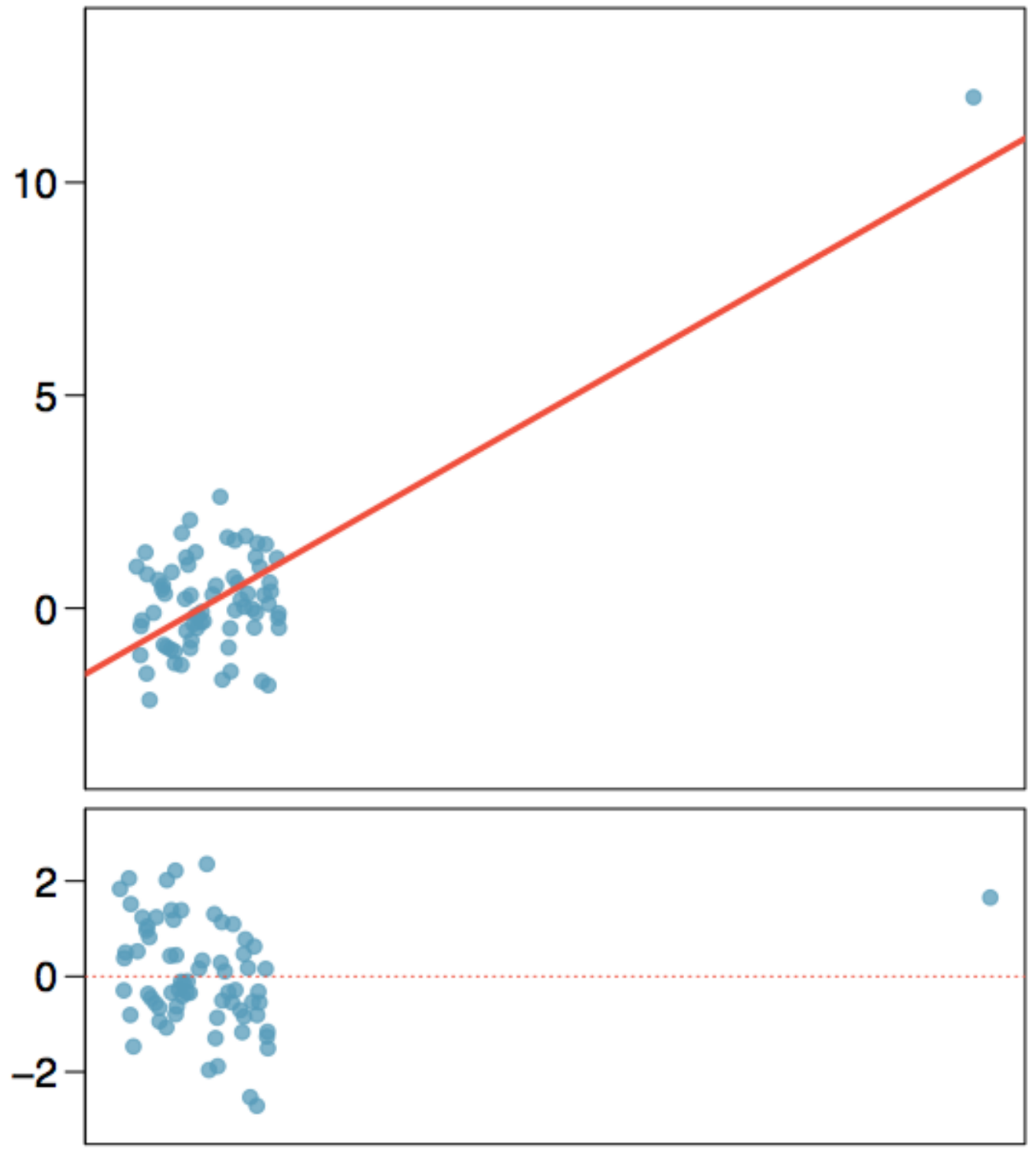

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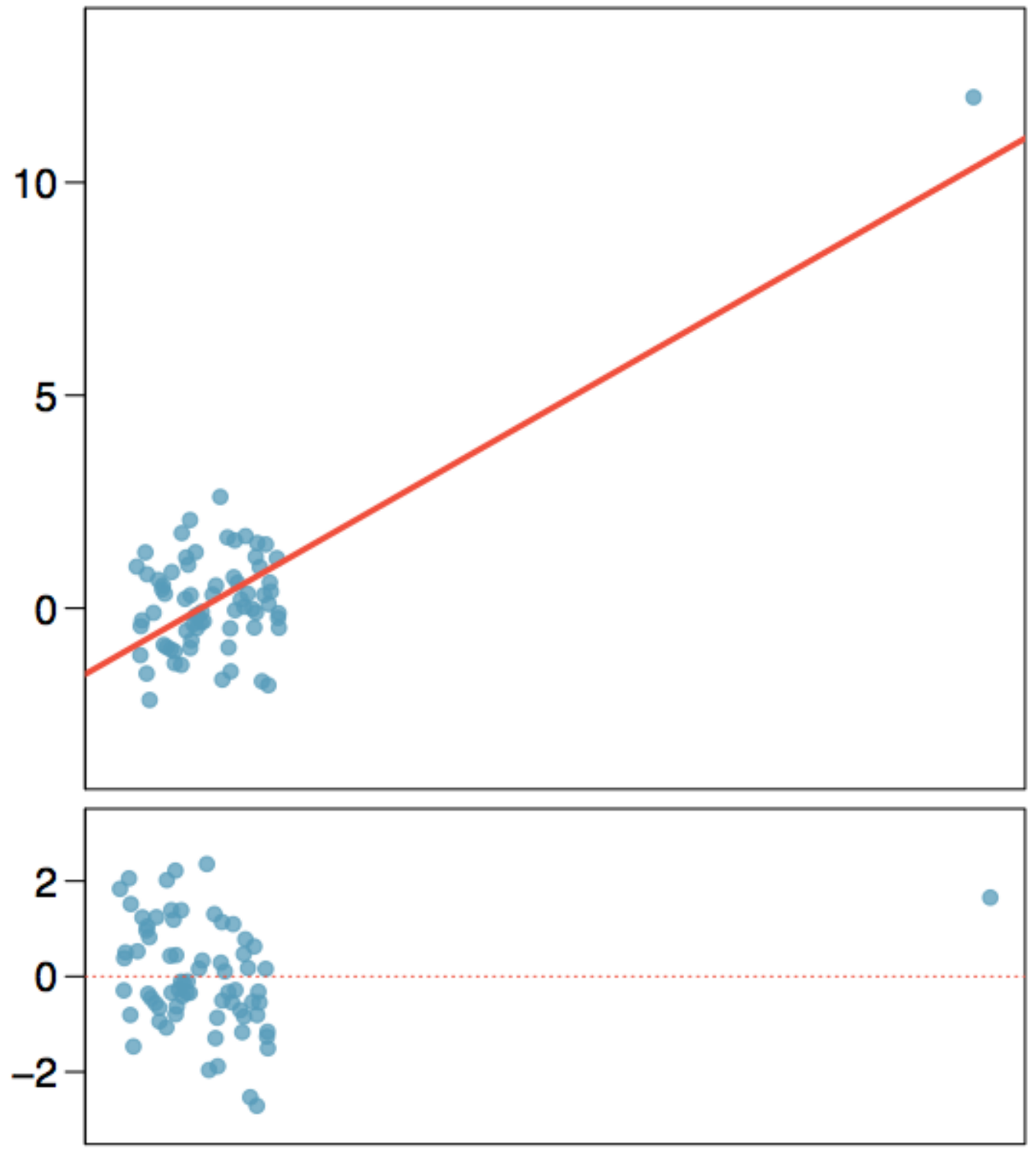
Without the outlier there is no evident relationship between *x* and *y*.



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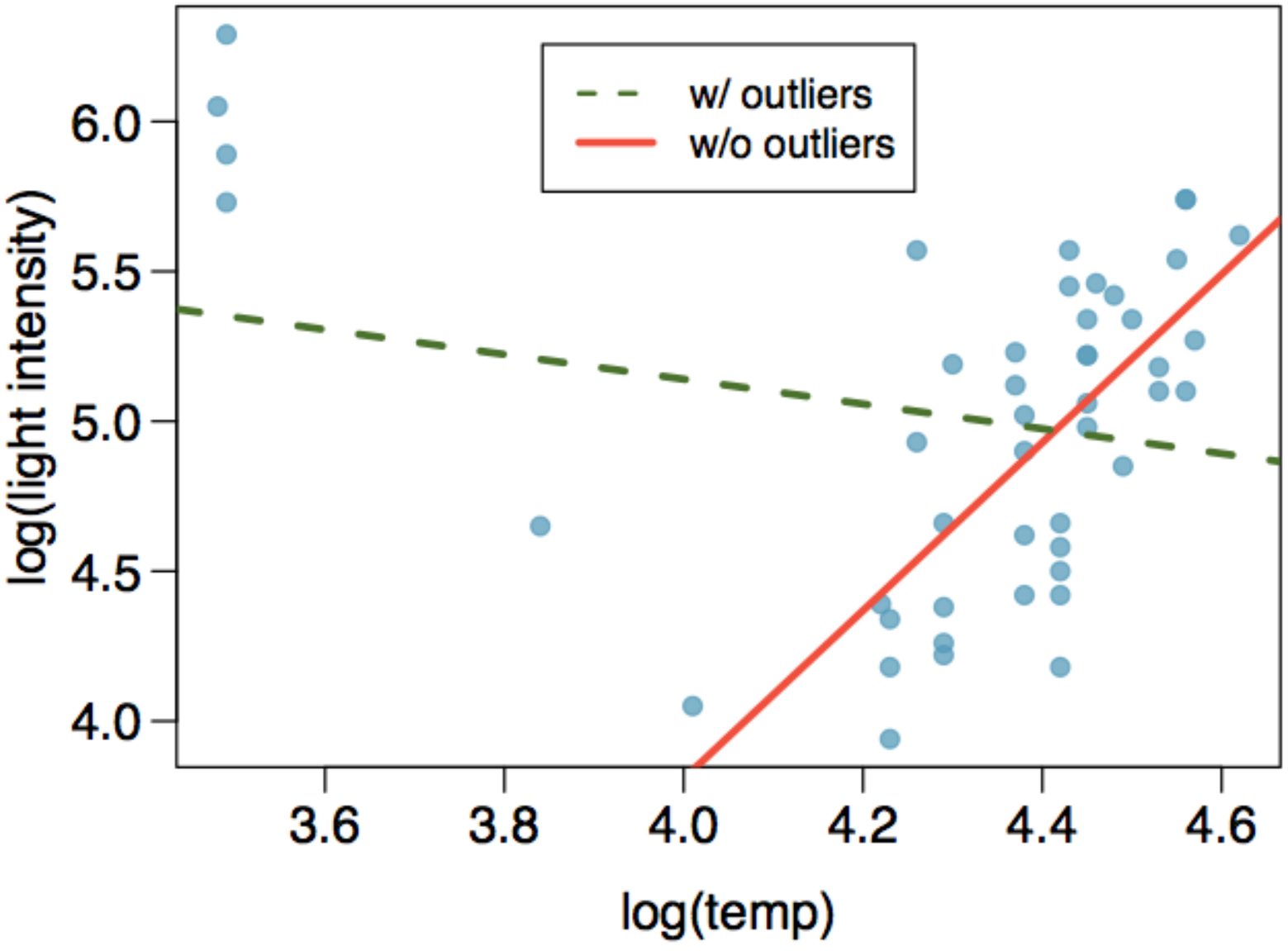
- Outliers that lie horizontally away from the center of the cloud are called *high leverage* points.
- High leverage points that actually influence the slope of the regression line are called *influential* points.
- In order to determine if a point is influential, visualize the regression line with and without the point. Does the slope of the line change considerably? If so, then the point is influential. If not, then it's not an influential point.

Influential points

Data are available on the log of the surface temperature and the log of the light intensity of 47 stars in the star cluster CYG OB1.



Types of outliers

Which of the below best describes the outlier?

(a) influential
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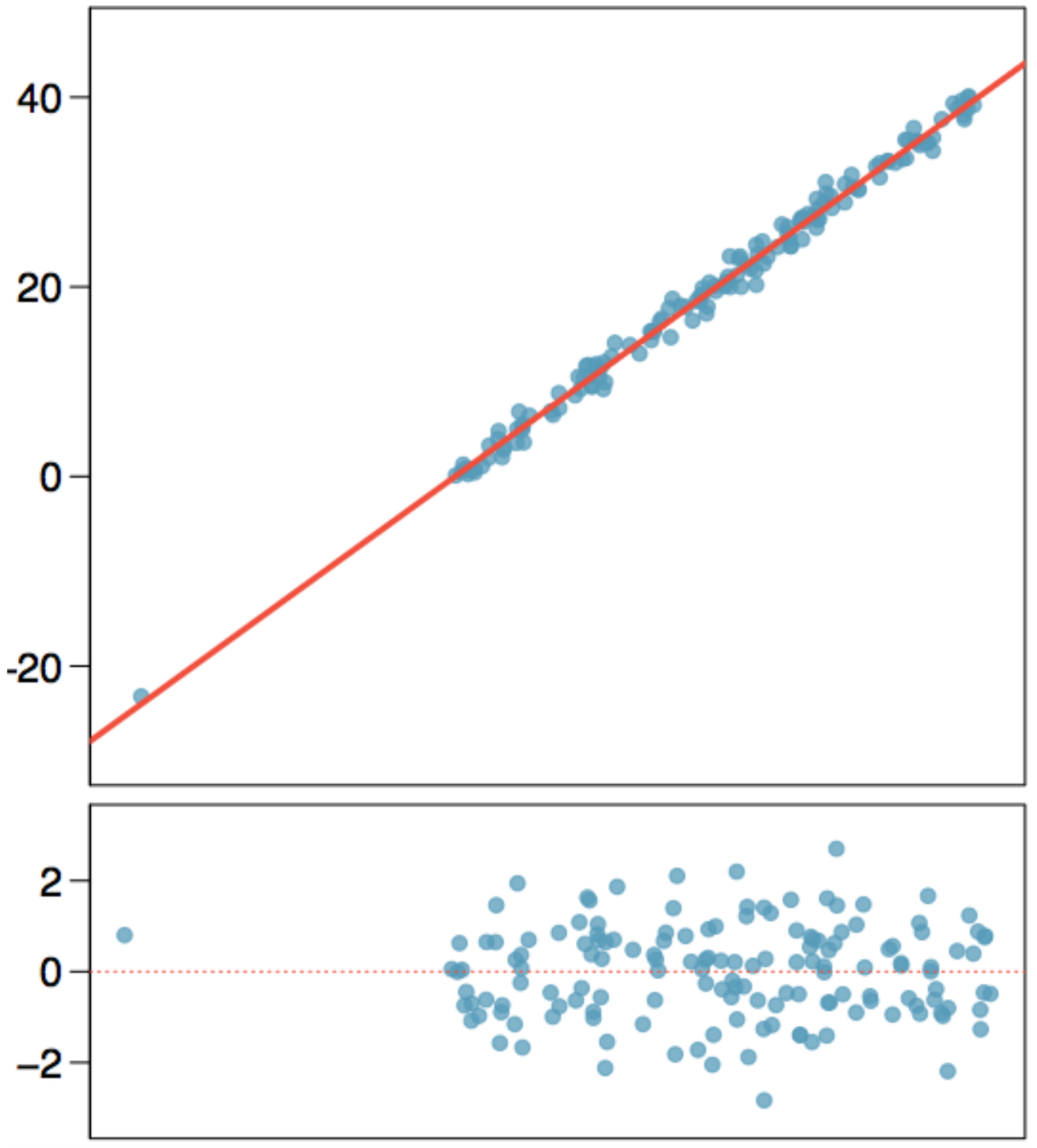

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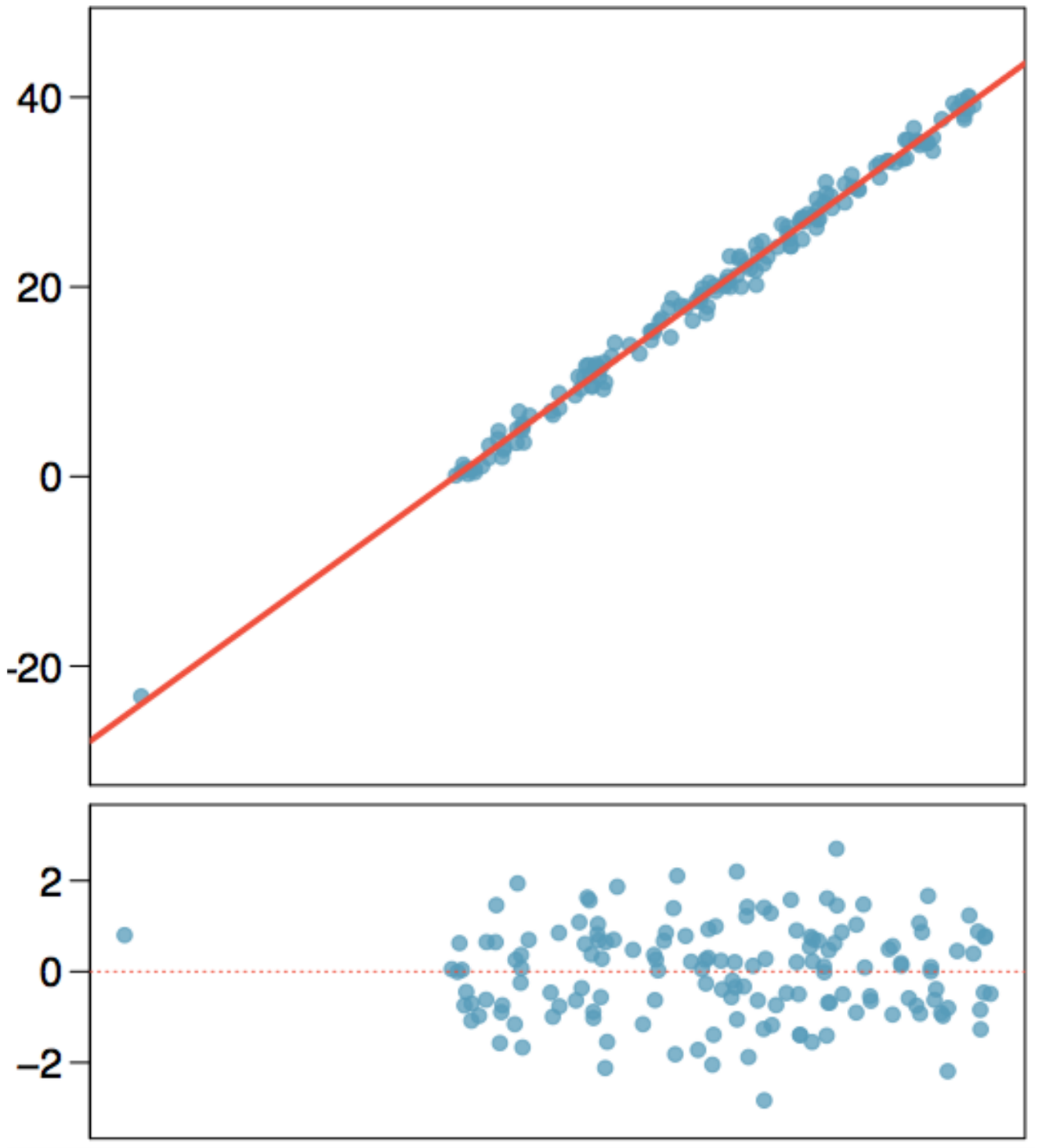
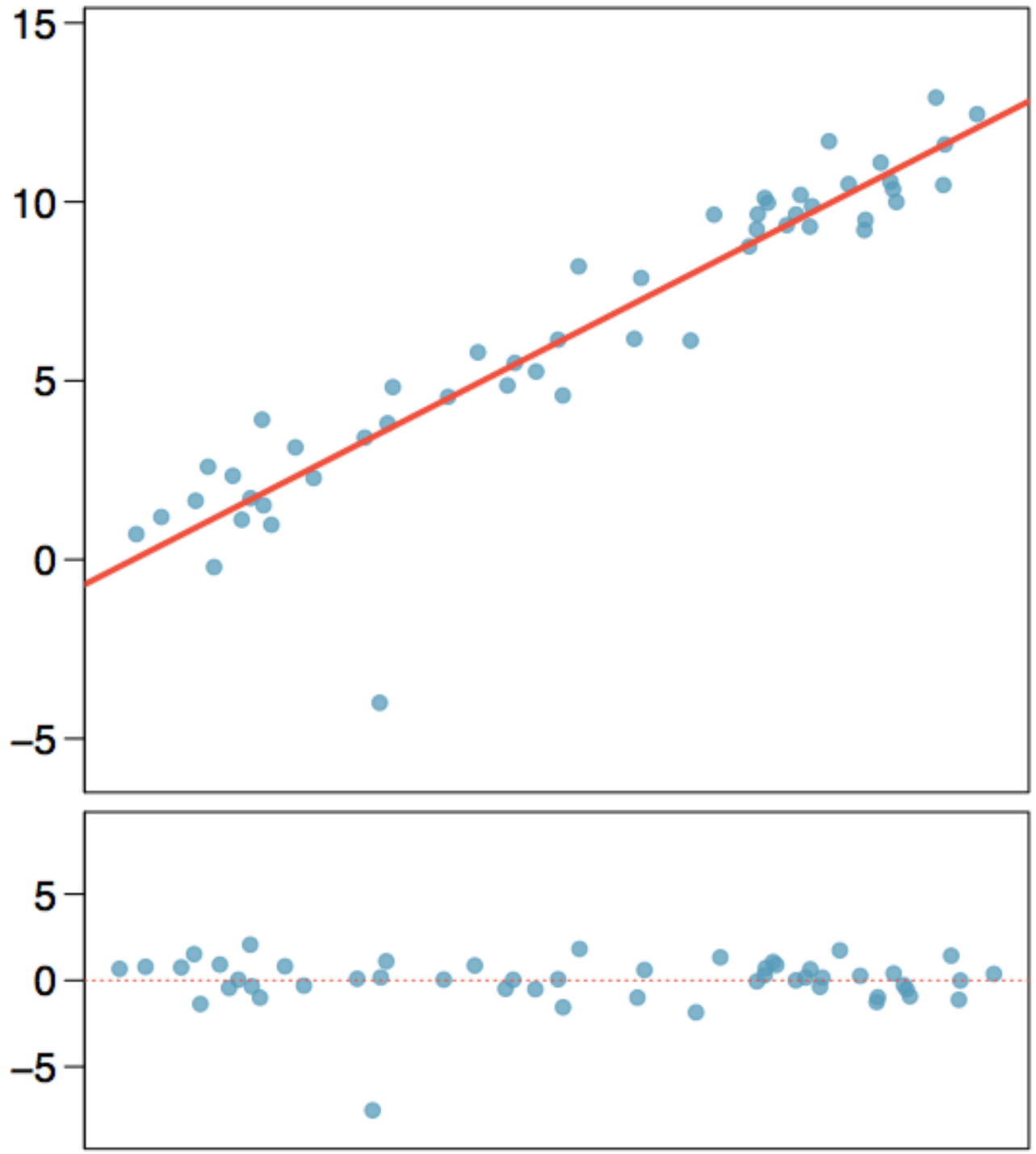


Types of outliers

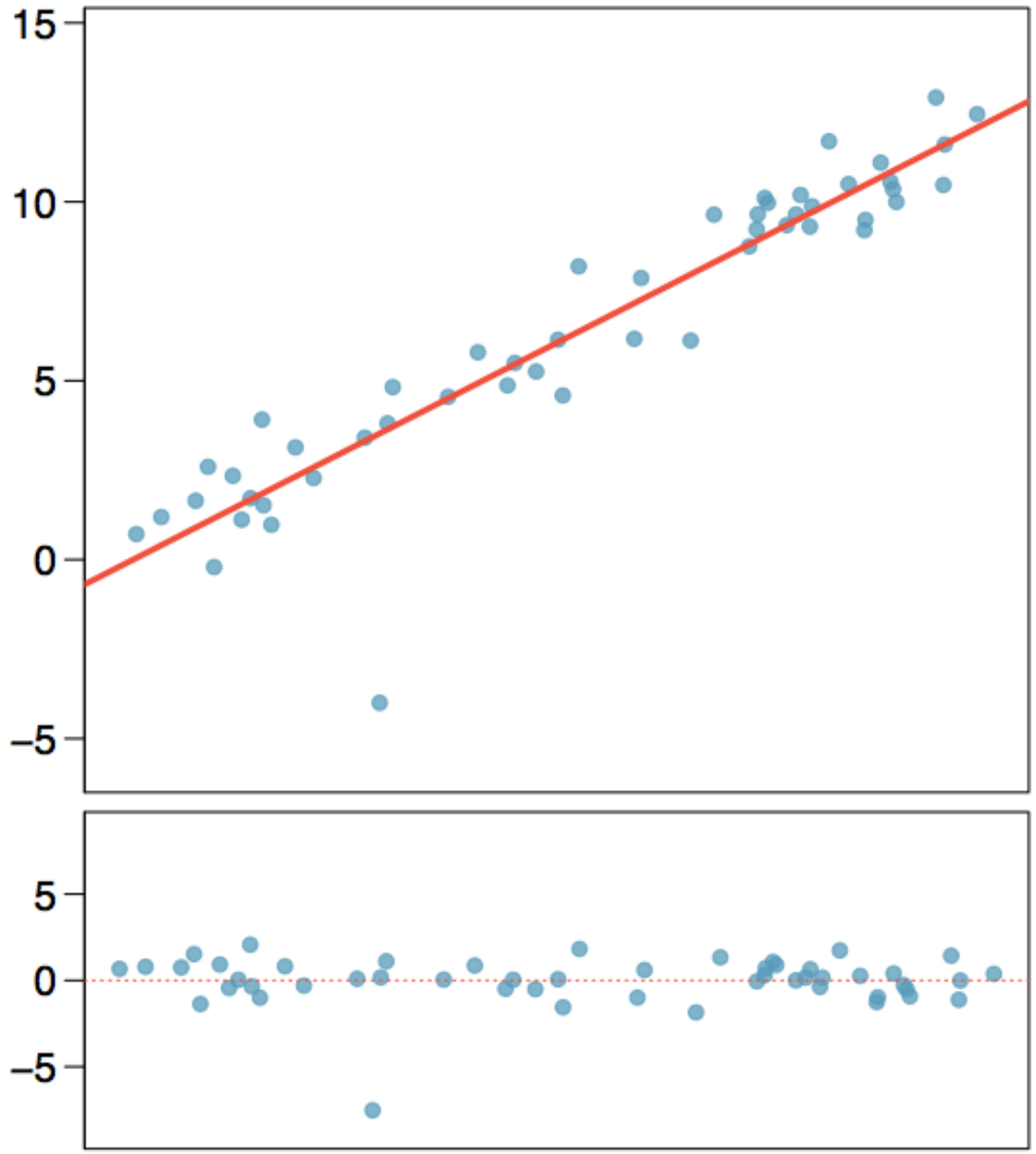
Does this outlier influence the slope of the regression line?



Types of outliers

Does this outlier influence the slope of the regression line?

Not much...



Recap

Which of following is true?

(a) Influential points always change the intercept of the regression line.
(b) Influential points always reduce R^2.
(c) It is much more likely for a low leverage point to be influential, than a high leverage point.
(d) When the data set includes an influential point, the relationship between the explanatory variable and the response variable is always nonlinear.
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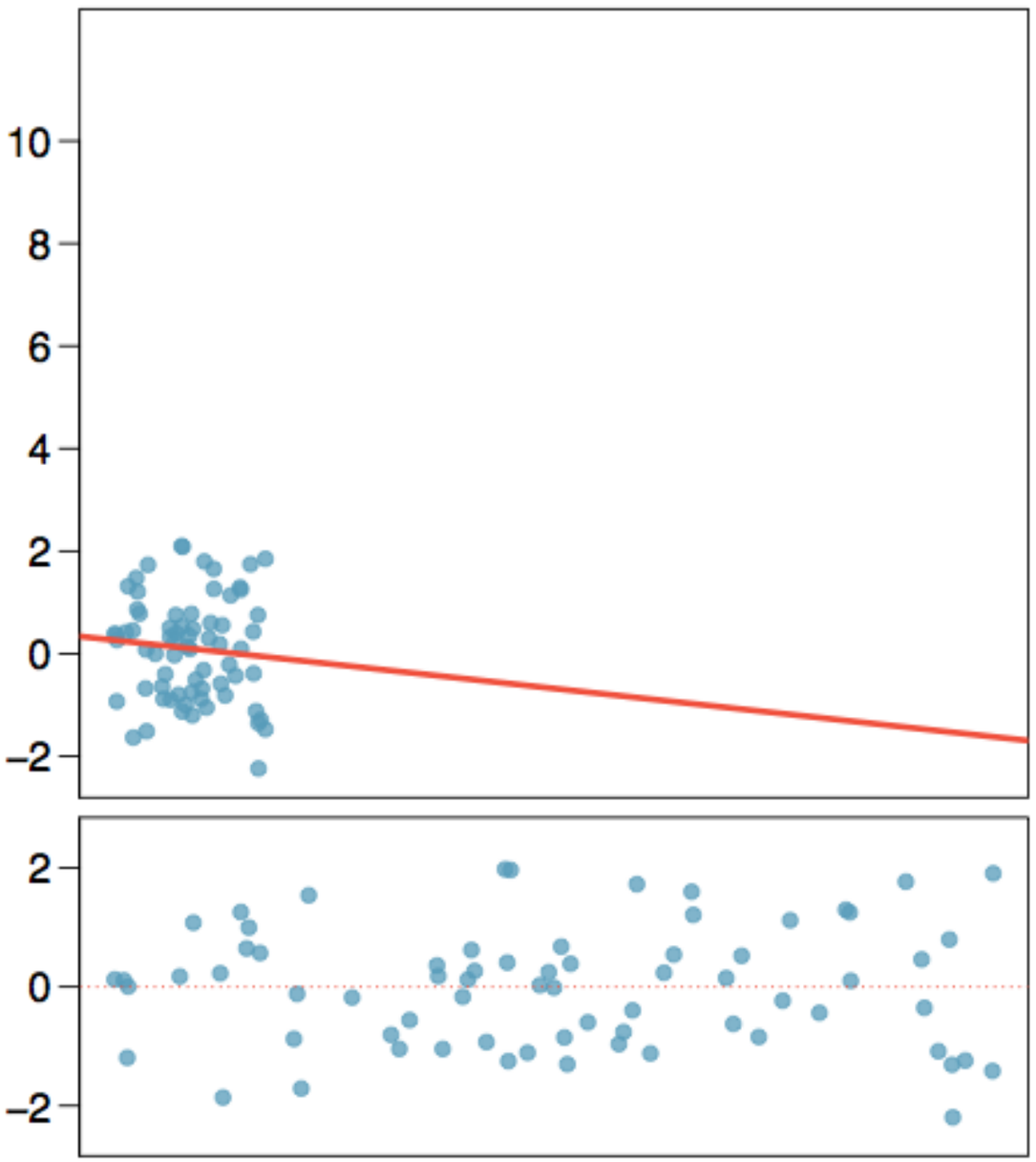
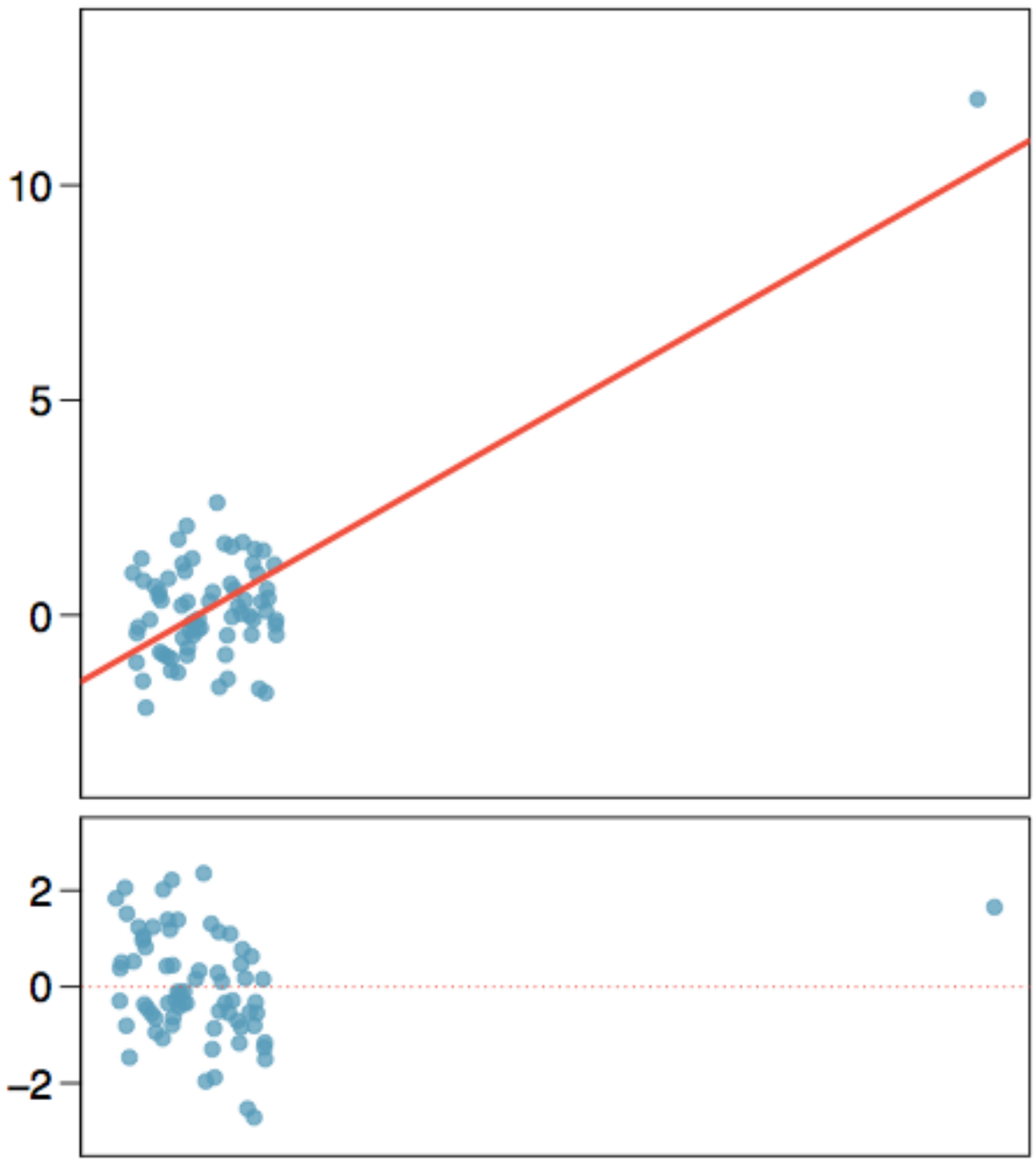
Recap (cont.)

$$R = 0.08, R^2 = 0.0064$$



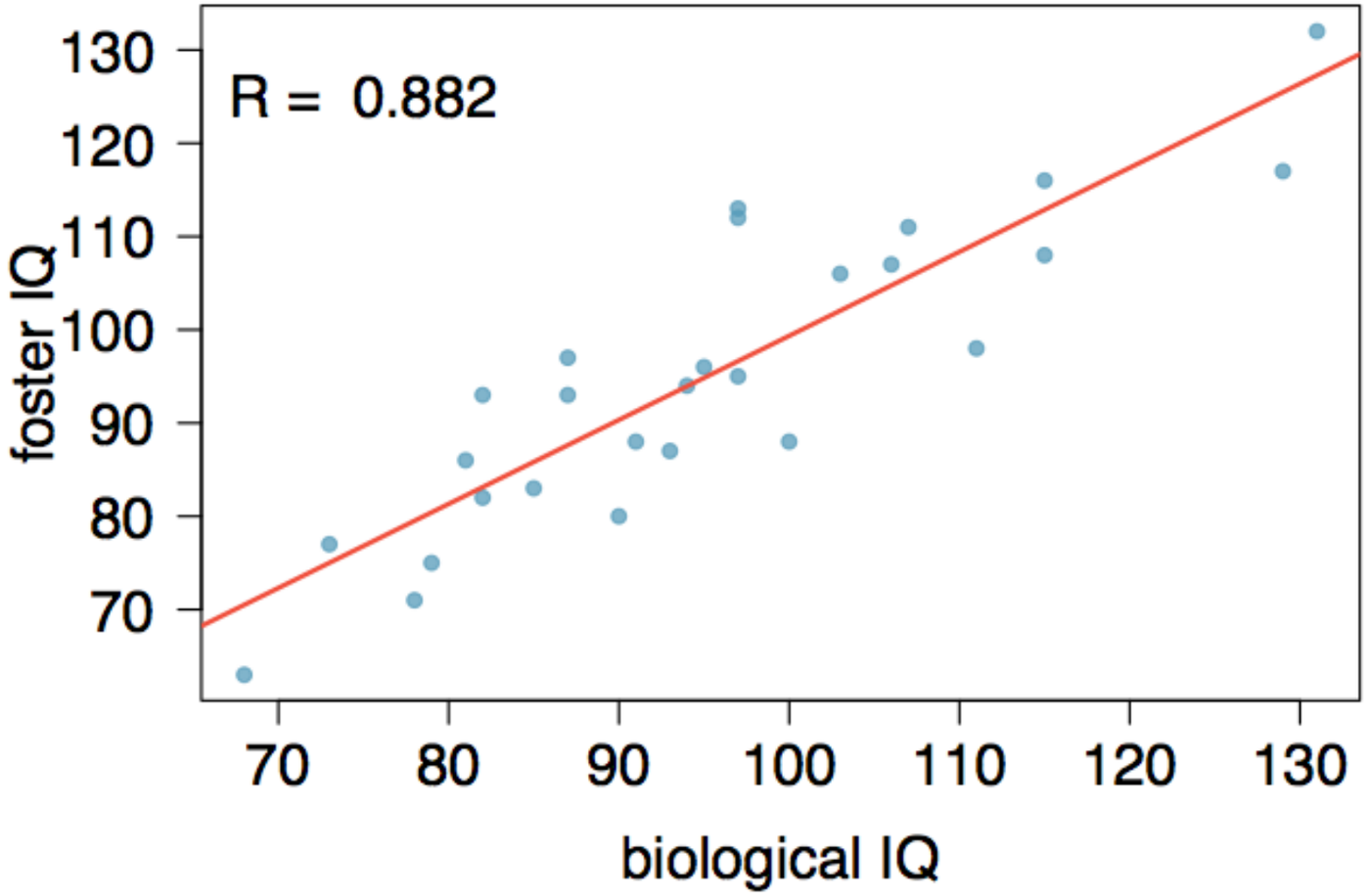
$$R = 0.79, R^2 = 0.6241$$



Inference for Linear Regression

Nature or nurture?

In 1966 Cyril Burt published a paper called "The genetic determination of differences in intelligence: A study of monozygotic twins reared apart?" The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Practice

Which of the following is false?

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   9.20760    9.29990   0.990    0.332
bioIQ         0.90144    0.09633   9.358  1.2e-09

Residual standard error: 7.729 on 25 degrees of freedom
Multiple R-squared: 0.7779,  Adjusted R-squared: 0.769
F-statistic: 87.56 on 1 and 25 DF,  p-value: 1.204e-09
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(a) Additional 10 points in the biological twin's IQ is associated with additional 9 points in the foster twin's IQ, on average.

(b) Roughly 78% of the foster twins' IQs can be accurately predicted by the model.

(c) The linear model is $\widehat{fosterIQ} = 9.2 + 0.9 \times bioIQ$

(d) Foster twins with IQs higher than average IQs tend to have biological twins with higher than average IQs as well.

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Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

(a) H_0: $b_0 = 0$; H_A: $b_0 \neq 0$

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	Estimate	Std. Error	t value	Pr(>|t|)
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Remember: we lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, β_0 and β_1.

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(0.7, 1.1)

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- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

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- The ultimate goal is to have independent observations.

Thursday is R Session!

- The first half of the lecture will be R session
- The second half of the lecture will be the time for the project discussion
  - Objective: Hypothesis Testing
    - Try to formally test your question/ hypotheses from exploratory data analysis(EDA)
    - Interpret your conclusion and present your finding in an appealing way