$$\dfrac{\dfrac{}{— \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{— \vdash x : A}\,(\mathrm{V})}{— \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}\,(\Pi\mathrm{E})$$

$$\dfrac{— \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}{—, x : A \vdash \mathtt{fst}\,(r\,x) : B}\,(\Sigma\mathrm{EL}) \qquad \dfrac{—, x : A \vdash \mathtt{fst}\,(r\,x) : B}{— \vdash \lambda x.\,\mathtt{fst}\,(r\,x) : A \to B}\,(\to\mathrm{I})$$

$$\dfrac{\dfrac{\dfrac{}{— \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{— \vdash x' : A}\,(\mathrm{V})}{— \vdash r\,x' : \Sigma[y{:}B]\ R\,x'\,y}\,(\Pi\mathrm{E})}{— \vdash \mathtt{snd}\,(r\,x') : R\,x'\,(\mathtt{fst}\,(r\,x'))}\,(\Sigma\mathrm{ER})$$

$$\dfrac{\dfrac{\dfrac{}{— \vdash R : A \to B \to \mathcal{U}}\,(\mathrm{V}) \qquad \dfrac{}{— \vdash x' : A}\,(\mathrm{V})}{— \vdash R\,x' : B \to \mathcal{U}}\,(\to\mathrm{I})}{— \vdash R\,x' = R\,x' \in B \to \mathcal{U}}\,(\mathrm{refl})$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{— \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{— \vdash x : A}\,(\mathrm{V})}{— \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}\,(\Pi\mathrm{E})}{—, x : A \vdash \mathtt{fst}\,(r\,x) : B}\,(\Sigma\mathrm{EL}) \qquad \dfrac{}{— \vdash x' : A}\,(\mathrm{V})}{— \vdash (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' = \mathtt{fst}\,(r\,x') \in B}\,(\to\mathrm{C})$$

$$\dfrac{— \vdash (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' = \mathtt{fst}\,(r\,x') \in B}{— \vdash \mathtt{fst}\,(r\,x') = (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' \in B}\,(\mathrm{sym})$$

$$\dfrac{— \vdash R\,x' = R\,x' \in B \to \mathcal{U} \qquad — \vdash \mathtt{fst}\,(r\,x') = (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' \in B}{— \vdash R\,x'\,(\mathtt{fst}\,(r\,x')) = R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x') \in \mathcal{U}}\,(\mathrm{cong})$$

$$\dfrac{— \vdash \mathtt{snd}\,(r\,x') : R\,x'\,(\mathtt{fst}\,(r\,x')) \qquad — \vdash R\,x'\,(\mathtt{fst}\,(r\,x')) = R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x') \in \mathcal{U}}{—, x' : A \vdash \mathtt{snd}\,(r\,x') \in R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}\,(\mathrm{conv})$$

$$\dfrac{—, x' : A \vdash \mathtt{snd}\,(r\,x') \in R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}{— \vdash \lambda x'.\,\mathtt{snd}\,(r\,x') : \Pi[x'{:}A]\ R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}\,(\Pi\mathrm{I})$$

$$\dfrac{— \vdash \lambda x.\,\mathtt{fst}\,(r\,x) : A \to B \qquad — \vdash \lambda x'.\,\mathtt{snd}\,(r\,x') : \Pi[x'{:}A]\ R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}{—, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash ((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}\,(\Sigma\mathrm{I})$$

$$\dfrac{—, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash ((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U} \vdash \lambda r.\,((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : (\Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y) \to \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}\,(\to\mathrm{I})$$

$$\dfrac{\dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x : A \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x : A \vdash x : A}\,(\mathrm{V})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x : A \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}\,(\Pi\mathrm{E})$$

$$\dfrac{\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x : A \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x : A \vdash \mathtt{fst}\,(r\,x) : B}\,(\Sigma\mathrm{EL})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash \lambda x.\,\mathtt{fst}\,(r\,x) : A \to B}\,(\to\mathrm{I})$$

$$\dfrac{\dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash x' : A}\,(\mathrm{V})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash r\,x' : \Sigma[y{:}B]\ R\,x'\,y}\,(\Pi\mathrm{E})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash r\,x' : \Sigma[y{:}B]\ R\,x'\,y}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{snd}\,(r\,x') : R\,x'\,(\mathtt{fst}\,(r\,x'))}\,(\Sigma\mathrm{ER})$$

$$\dfrac{\dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R : A \to B \to \mathcal{U}}\,(\mathrm{V}) \qquad \dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash x' : A}\,(\mathrm{V})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x' : B \to \mathcal{U}}\,(\to\mathrm{I})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x' : B \to \mathcal{U}}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x' = R\,x' \in B \to \mathcal{U}}\,(\mathrm{refl})$$

$$\dfrac{\dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y}\,(\mathrm{V}) \qquad \dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash x : A}\,(\mathrm{V})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}\,(\Pi\mathrm{E})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash r\,x : \Sigma[y{:}B]\ R\,x\,y}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash \mathtt{fst}\,(r\,x) : B}\,(\Sigma\mathrm{EL})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A, x : A \vdash \mathtt{fst}\,(r\,x) : B \qquad \dfrac{}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash x' : A}\,(\mathrm{V})}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' = \mathtt{fst}\,(r\,x') \in B}\,(\to\mathrm{C})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' = \mathtt{fst}\,(r\,x') \in B}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{fst}\,(r\,x') = (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' \in B}\,(\mathrm{sym})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x' = R\,x' \in B \to \mathcal{U} \qquad A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{fst}\,(r\,x') = (\lambda x.\,\mathtt{fst}\,(r\,x))\,x' \in B}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x'\,(\mathtt{fst}\,(r\,x')) = R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x') \in \mathcal{U}}\,(\mathrm{cong})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{snd}\,(r\,x') : R\,x'\,(\mathtt{fst}\,(r\,x')) \qquad A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash R\,x'\,(\mathtt{fst}\,(r\,x')) = R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x') \in \mathcal{U}}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{snd}\,(r\,x') \in R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}\,(\mathrm{conv})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y, x' : A \vdash \mathtt{snd}\,(r\,x') \in R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash \lambda x'.\,\mathtt{snd}\,(r\,x') : \Pi[x'{:}A]\ R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}\,(\Pi\mathrm{I})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash \lambda x.\,\mathtt{fst}\,(r\,x) : A \to B \qquad A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash \lambda x'.\,\mathtt{snd}\,(r\,x') : \Pi[x'{:}A]\ R\,x'\,((\lambda x.\,\mathtt{fst}\,(r\,x))\,x')}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash ((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}\,(\Sigma\mathrm{I})$$

$$\dfrac{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U}, r : \Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y \vdash ((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}{A : \mathcal{U}, B : \mathcal{U}, R : A \to B \to \mathcal{U} \vdash \lambda r.\,((\lambda x.\,\mathtt{fst}\,(r\,x)), (\lambda x'.\,\mathtt{snd}\,(r\,x'))) : (\Pi[x{:}A]\ \Sigma[y{:}B]\ R\,x\,y) \to \Sigma[f{:}A \to B]\ \Pi[x'{:}A]\ R\,x'\,(f\,x')}\,(\to\mathrm{I})$$