PHYS 3211 – Assignment 1

Due: 09/09/2015

Pencil & Paper problems

1. Many problems in physics that are solved computationally require the use of dimensional analysis. Consider the Planck length, which is the size at which quantum gravitational effects create a "foam" of spacetime. Knowing that this size can be made up of only fundamental, universal constants, use dimensional analysis to find the Planck length.

2. The radial equation for a planetary orbit is given by
$$-\frac{GMm}{r^2} + \frac{L^2}{mr^3} = m\frac{d^2r}{dt^2} ,$$
where L is the (constant) angular momentum.

   (a) Choose a new system of units for time and for distance to eliminate the variables G, M, m and possibly L. That is, set
$$r = \alpha\rho, \qquad t = \beta\tau$$
and determine α, β in terms of those variables. [Hint: First divide both sides of the equation by GMm to make the first term $-1/r^2$, then make the above substitutions.]

   (b) In SI units, what are the values of α and β, numerically (note these will be in units of meters and seconds, respectively), and what is their interpretation? [Use the estimate $L = mvR$ to determine the Earth's average angular momentum.]

   (c) In this system, would the values you would put into a computer for ρ and τ be "reasonable," that is, not too large or not too small? [That is, would you be in danger of running into overflow or underflow errors?]

3. Suppose you are standing on Eddy's Parade at Fordham University, which is at a latitude of 40 5/6°, and that you are 2.0 m tall.

   (a) Analytically find the speed of your feet, v, due to the rotation of the Earth. (Assume that the Earth is a perfect sphere, which is not, of radius $R = 6378$ km and that a day is exactly 24 hours long.)

   (b) Using your result from (a), determine the centripetal acceleration of your feet.

   (c) Repeat parts (a) and (b) for your head and compute the difference between the acceleration at your head and feet. Show how round-off can corrupt your calculation and offer a way to fix the problem.

Computer problems

4. In mathematics, there is a famous sequence of numbers called the Fibonacci numbers, after the thirteenth-century Italian mathematician Leonardo Fibonacci. The first two numbers are defined as 0 and 1. Every subsequent term is the sum of the preceding two. Thus, the numbers continue as $(0 + 1) = 1, (1 + 1) = 2, (2 + 1) = 3$, etc. Write a program that displays the Fibonacci numbers for the number of terms specified by the user.

5. Write a C++ program that determines the absolute round off error in the simple calculation $|10 + h| - 10$ for 21 repetitions in which the value of h is changed in each repetition to:

   (a) $h/10$,

   (b) $h/2$.

   That is, for part (a), set $h = 1$, then for the first repetition, set $h = 1/10$, then $h = 1/100$, and so forth.

   Obviously you are calculating h using this method (essentially). What happens in the two cases, and why are they so different?

6. In class we wrote a code to calculate e^x using the Taylor series formulation. Write two codes that calculate the sine and cosine using the Taylor series representations

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} , \qquad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

   Given that $\sin(x) = \cos(x - \pi/2)$, how do these codes differ when calculating $\sin(1.0)$ or $\cos(1 - \pi/2)$? How about $\sin(0.001)$ or $\cos(0.001 - \pi/2)$? Go ahead and hard-code π as

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double PI = 3.141592653589793
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7. Write a C++ program that calculates π using both of the following series representations:

$$\pi = 4 \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \right] ,$$

$$\pi^2 = 6 \sum_{i=1}^{\infty} \frac{1}{i^2} .$$

   Allow for user-specified number of iterations and give the error between each method and the accepted value of π.

   How many iterations are needed to get π to seven decimal places using each representation? Determine this number by comparing to π as given above, and for each series calculate $\delta = |\pi - \pi_{\text{series}}|$. How does δ compare for the two series?