Tutorial 8: Graph Algorithms I

CAB301 - Algorithms and Complexity

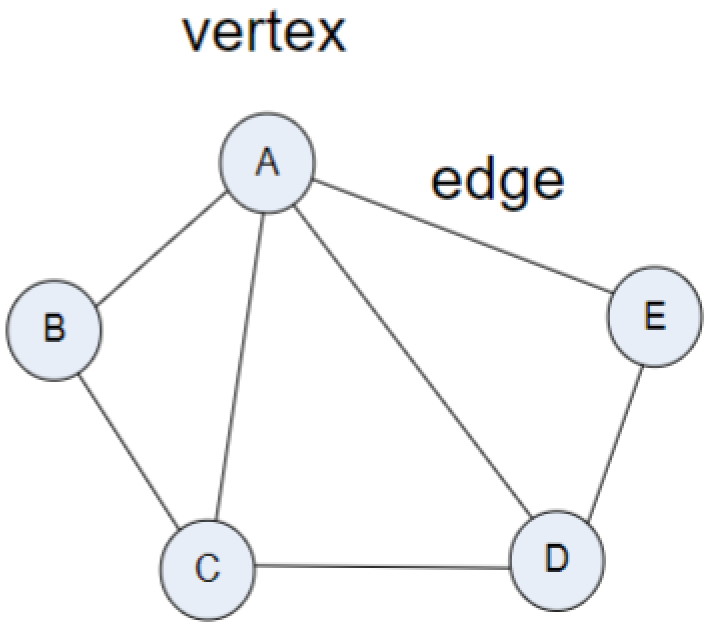
School of Computer Science, Faculty of Science

Agenda

1. **Lecture Recap**: Graph Algorithms I
    - Graphs
    - Graph Representations
    - Graph Traversals
    - Topological Sort
    - Spanning Tree
2. **Tutorial Questions** + **Q&A**

Graphs

A collection of nodes (**vertices**) and **edges** connecting them.



G = (V, E), where V = {A, B, C, D, E} and
E = {(A, B), (A, C), (A, D), (A, E), (B, C), (C, D), (D, E)}.

Directionality and Weight

Graphs can be **directed**, where edges have a direction, or **undirected**.

Graphs can be **weighted**, where edges have a weight, or **unweighted**.

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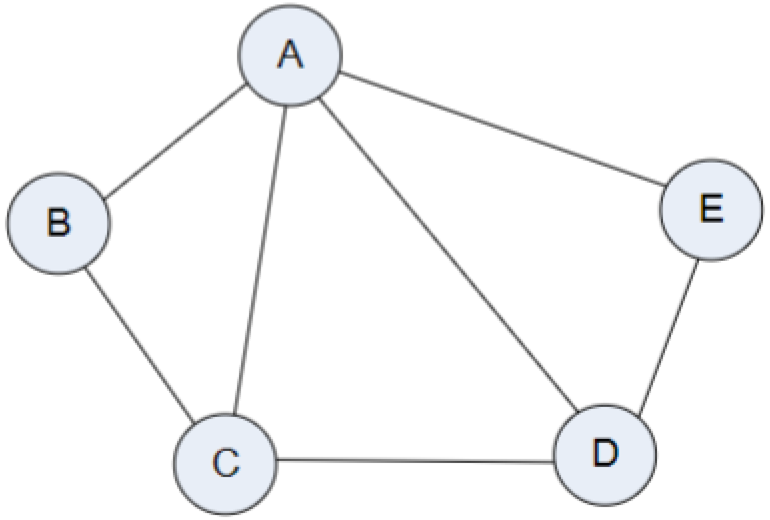
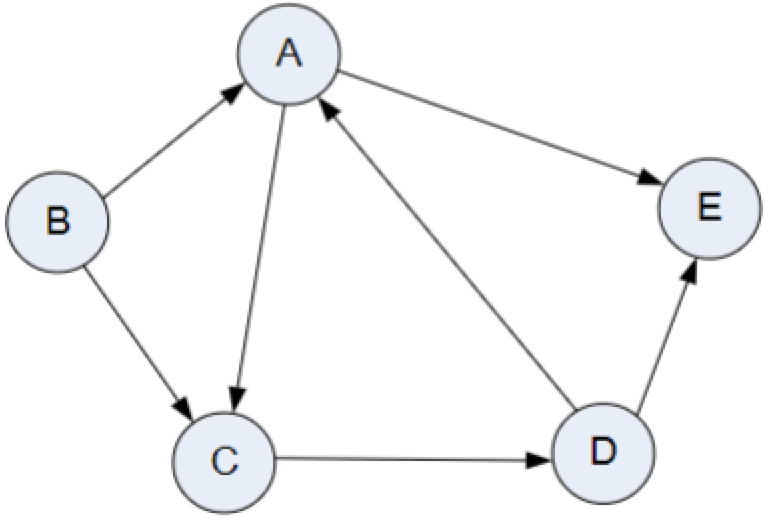


An undirected graph



A directed graph

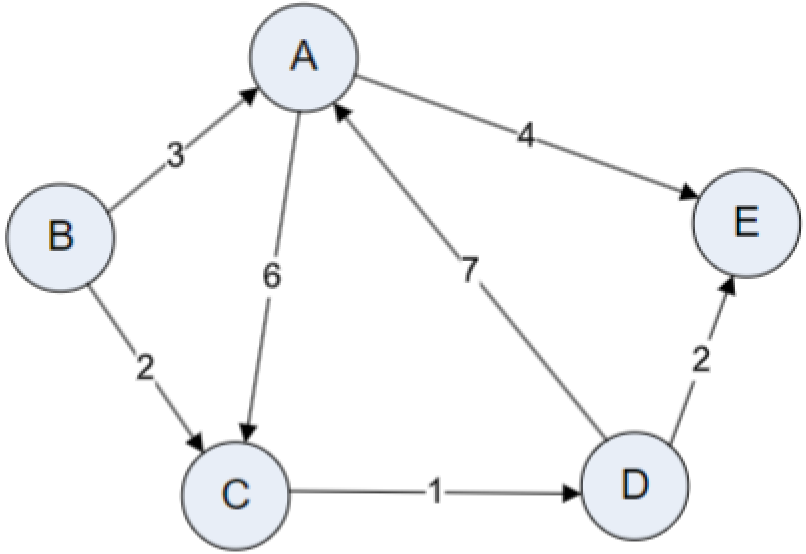
Terminology

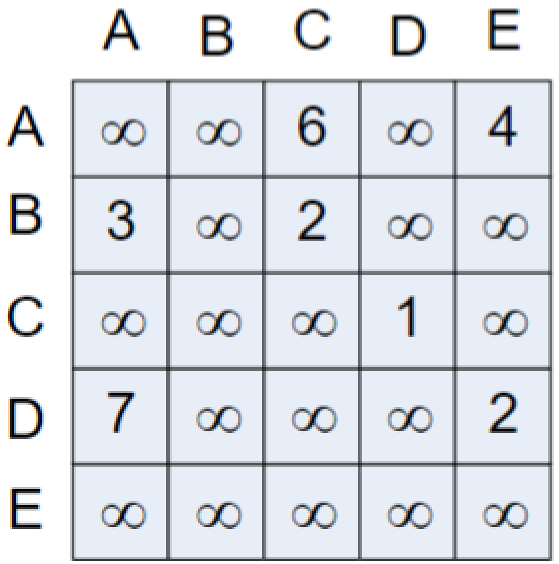
- **Path**: A sequence of vertices connected by edges.
- **Cycle**: A path that starts and ends at the **same vertex**.
- **Connected**: A graph where there is a path between every pair of vertices.
- **Subgraph**: A graph whose vertices and edges are a subset of another graph.

Graph Representations - Adjacency Matrix

2D array, A, where $A[i][j] = w$ if there is an edge from i to j with weight w, or ∞ if there is no edge.

If unweighted, $A[i][j] = 1$ if there is an edge, or 0 if there is no edge.

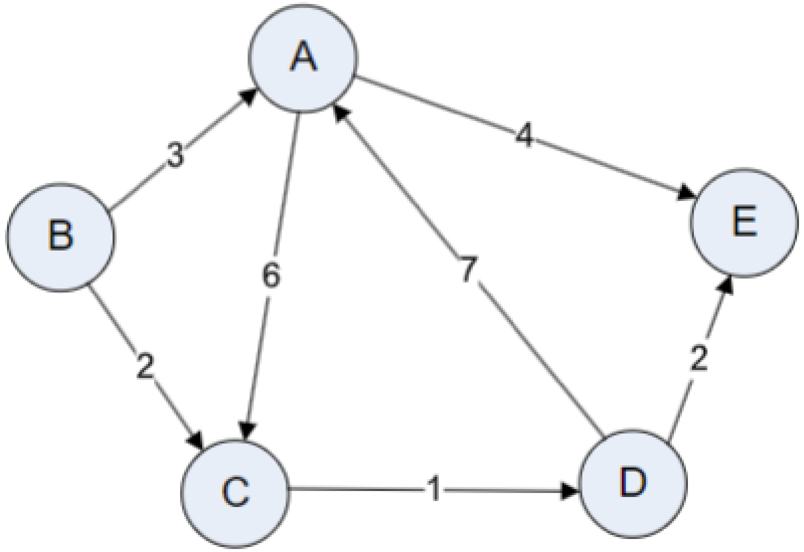




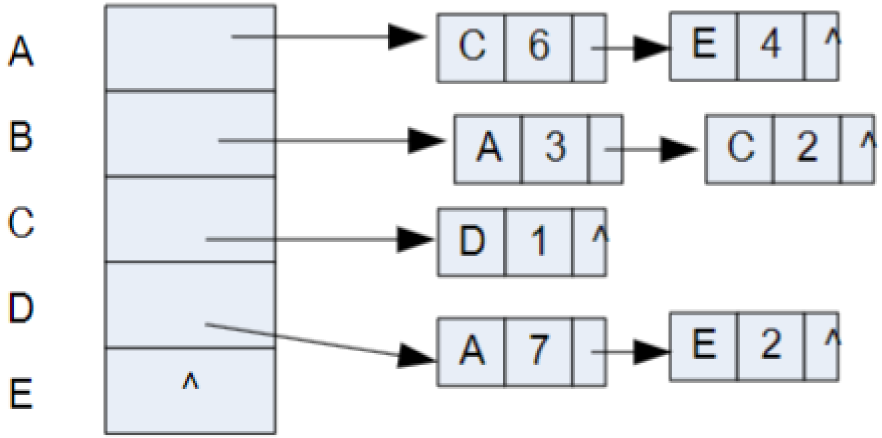
	A	B	C	D	E
A	∞	∞	6	∞	4
B	3	∞	2	∞	∞
C	∞	∞	∞	1	∞
D	7	∞	∞	∞	2
E	∞	∞	∞	∞	∞

Graph Representations - Adjacency List

For each vertex, i, store a list of vertices that i is connected to.



(a) A weighted directed graph



(b) The adjacency matrix representation of (a)

Graph Traversals

Depth-First Search (DFS): Explore **as far as possible** along each branch before backtracking.

- Uses either a **stack** or **recursion**.

Breadth-First Search (BFS): Explore **all neighbours of a vertex before moving** to the next level.

- Uses a **queue**.

Very similar to tree traversals, but need to keep track of visited vertices (and not visit them again).

Topological Sort

Given a **directed acyclic graph (DAG)**, order the vertices such that for every edge (u, v), u comes before v. Steps:

1. Find a vertex with **no incoming edges** (in-degree = 0).
2. Add it to the **topological order**.
3. **Remove** the vertex and its outgoing edges.
4. **Repeat** until all vertices are ordered.

Spanning Tree

A **subgraph** of a graph that is a **tree** and connects all vertices.

Weighted graphs can have a **minimum spanning tree (MST)**, which is a spanning tree with the **minimum total weight**.

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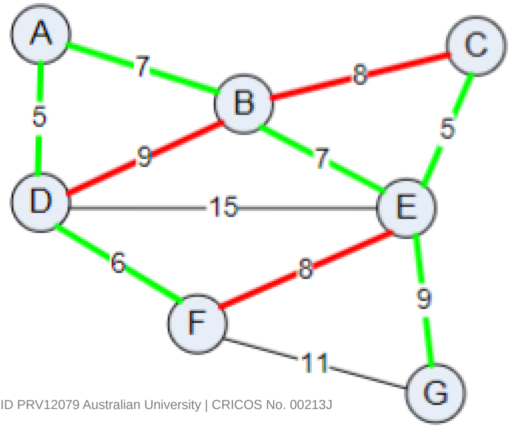

Kruskal's Algorithm

1. **Sort** edges by weight in **non-decreasing order**.
2. **Iterate** through edges:
   - **Add** edge to MST if it **does not create a cycle**.
   - **Repeat** until $|V| - 1$ edges are added.
3. **Output** the MST.

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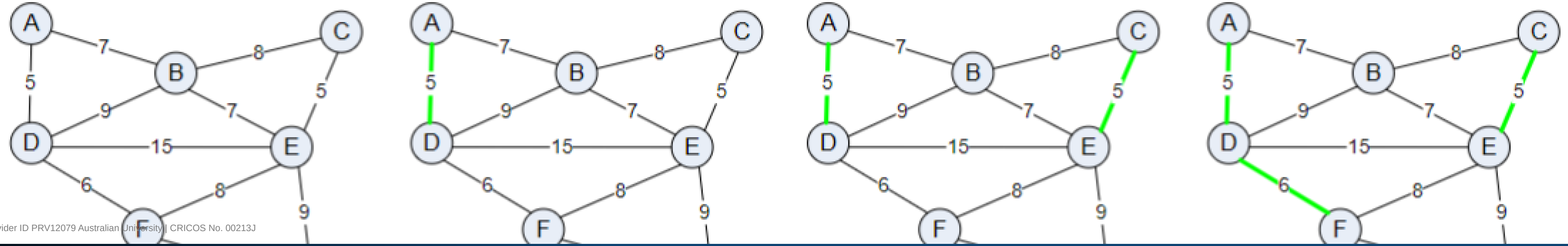

Prim's Algorithm

Keep track a set of vertices in the MST, V_T and a set of edges in the MST, E_T. Starting with $V_T = \{v_0\}$ and $E_T = \emptyset$, repeat for $|V| - 1$ times:

1. Find a **minimum weight edge** $e^* = (v^*, u^*)$ among edges connecting V_T to the rest of the graph.
2. **Add** u^* to V_T.
3. **Add** e^* to E_T.

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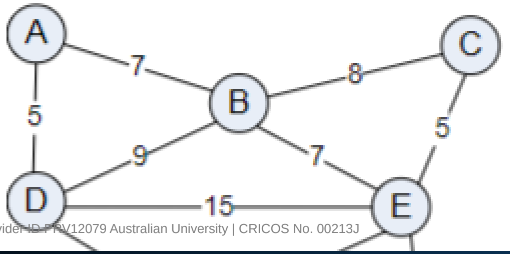
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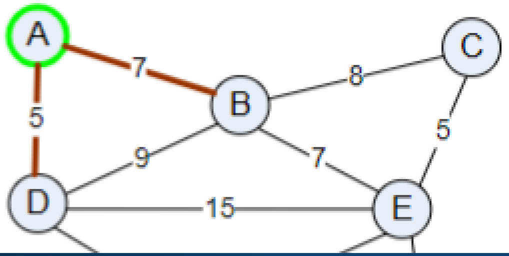
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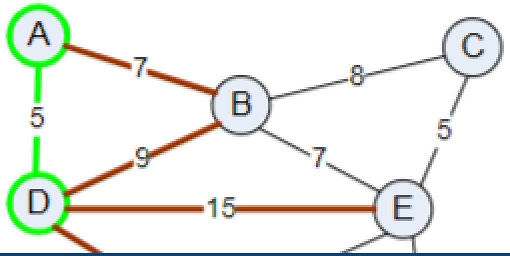
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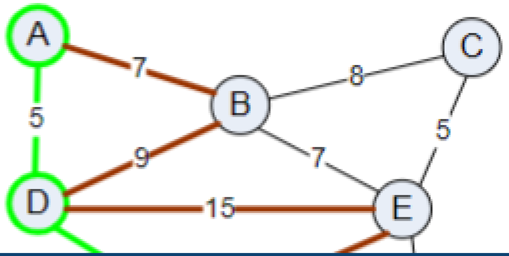
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