

## Double Angle Formulas

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$= 2 \cos^2(x) - 1$$

$$= 1 - 2 \sin^2(x)$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

## Limits

$$\lim_{x \rightarrow n} f(x) = L$$

Means for any number  $\epsilon > 0$  there exists a number  $\delta > 0$  such that if  $0 < |x - n| < \delta$  then  $|f(x) - L| < \epsilon$ .

$$\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right) \text{ if } f(g(x)) \text{ is continuous at } x$$

Standard difference quotient:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Symmetric difference quotient:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$$

## Continuity and Differentiability

$f(x)$  is continuous at  $x$  if:

$$f(x) = \lim_{k \rightarrow x^+} f(k) = \lim_{k \rightarrow x^-} f(k)$$

$f(x)$  is differentiable at  $x$  if:

$$f'_+(x) = f'_-(x)$$

## Differentiation Rules

$$\begin{aligned}\frac{d}{dx}cf(x) &= cf'(x) \\ \frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\ \frac{d}{dx}(f(x) \pm g(x)) &= f'(x) \pm g'(x) \\ \frac{d}{dx}f(x)^n &= nf(x)^{n-1} \cdot f'(x) \\ \frac{d}{dx}\sqrt{f(x)} &= \frac{f'(x)}{2\sqrt{f(x)}} \\ \frac{d}{dx}(f(x)g(x)) &= f'(x)g(x) + f(x)g'(x) \\ \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\ \frac{d}{dx}|f(x)| &= \frac{|f(x)|}{f(x)} \cdot f'(x) \\ \frac{d}{dx}b^{f(x)} &= b^{f(x)} \ln b \cdot f'(x) \\ \frac{d}{dx}\log_b(f(x)) &= \frac{f'(x)}{f(x) \ln b}\end{aligned}$$

## Trigonometric Differentiation Rules

$$\begin{aligned}\frac{d}{dx}\sin(f(x)) &= \cos(f(x)) \cdot f'(x) \\ \frac{d}{dx}\cos(f(x)) &= -\sin(f(x)) \cdot f'(x) \\ \frac{d}{dx}\tan(f(x)) &= \sec^2(f(x)) \cdot f'(x) \\ \frac{d}{dx}\sec(f(x)) &= \sec(f(x)) \tan(f(x)) \cdot f'(x) \\ \frac{d}{dx}\csc(f(x)) &= -\csc(f(x)) \cot(f(x)) \cdot f'(x) \\ \frac{d}{dx}\cot(f(x)) &= -\csc^2(f(x)) \cdot f'(x)\end{aligned}$$

## Inverse Trigonometric Differentiation Rules

$$\begin{aligned}\frac{d}{dx}\sin^{-1}(f(x)) &= \frac{f'(x)}{\sqrt{1-f(x)^2}} \\ \frac{d}{dx}\cos^{-1}(f(x)) &= -\frac{f'(x)}{\sqrt{1-f(x)^2}} \\ \frac{d}{dx}\tan^{-1}(f(x)) &= \frac{f'(x)}{1+f(x)^2}\end{aligned}$$

## Mean Value Theorem

If  $f(x)$  is continuous on  $[a, b]$ , then there exists one value  $c$  on  $[a, b]$  such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

## Reimann Sums

$$\sum_{k=1}^n f(x_k) \Delta x$$

Assuming regular partition:

$$\begin{aligned} \text{LRAM} &= \sum_{k=1}^n f\left(a + \frac{(k-1)(b-a)}{n}\right) \frac{b-a}{n} \\ \text{MRAM} &= \sum_{k=1}^n f\left(a + \frac{(k-\frac{1}{2})(b-a)}{n}\right) \frac{b-a}{n} \\ \text{RRAM} &= \sum_{k=1}^n f\left(a + \frac{k(b-a)}{n}\right) \frac{b-a}{n} \end{aligned}$$

## Definite Integration

$$\begin{aligned} \int_a^b f(x) \, dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x \\ \int_a^a f(x) \, dx &= 0 \\ \int_a^b f(x) \, dx &= - \int_b^a f(x) \, dx \\ \int_a^b c f(x) \, dx &= c \int_a^b f(x) \, dx \\ \int_a^b (f(x) \pm g(x)) \, dx &= \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx \\ \int_a^p f(x) \, dx + \int_p^b f(x) \, dx &= \int_a^b f(x) \, dx \\ \int_a^b f(x) \, dx &> \int_a^b g(x) \, dx \text{ if } f(x) > g(x) \text{ on } [a, b] \end{aligned}$$

The mean value of  $f(x)$  over  $[a, b]$  is:

$$\frac{1}{b-a} \int_a^b f(x) \, dx$$

## Mean Value of a Function Theorem

If  $f(x)$  is continuous on  $[a, b]$ , then there exists one value  $c$  on  $[a, b]$  such that:

$$f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx$$

## Linear Approximation

The linearization of  $f(x)$  at  $a$ :

$$L(x) = f(a) + f'(a)(x - a)$$

## Fundamental Theorem of Calculus

### Part 1

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(t) \, dt = f(b(x))b'(x) - f(a(x))a'(x)$$

### Part 2

$$F'(x) = f(x)$$
$$\int_a^b f(x) \, dx = F(x) \Big|_a^b = F(b) - F(a)$$

## Solving Differential Equations Using A Definite Integral

$$\frac{dy}{dx} = f(x)$$
$$y = \int_a^x f(t) \, dt + C$$

The easiest solution occurs when  $a$  = the given  $x$  value and  $C$  = the given  $y$  value.

## Solving Differential Equations Using Antidifferentiation

$$\frac{dy}{dx} = f(x)$$
$$y = \int f(x) \, dx + C \text{ (general form)}$$

Specify largest domain of  $x$  that includes specified  $x$  and does not include any discontinuities.

## Euler's Method

Start at  $(x_0, y_0)$

$$\Delta y = \Delta x \cdot \left. \frac{dy}{dx} \right|_{(x_{n-1}, y_{n-1})}$$

$$(x_n, y_n) = (x_{n-1} + \Delta x, y_{n-1} + \Delta y)$$

## Antidifferentiation Rules

$$\int cf(x) dx = c \int f(x) dx$$

$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int b^x dx = \frac{b^x}{\ln b} + C$$

$$\int \ln x dx = x \ln x - x + C$$

## Trigonometric Antidifferentiation Rules

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \tan(x) dx = -\ln|\cos(x)| + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sin^2(x) dx = \frac{1}{2}(x - \sin(x) \cos(x)) + C$$

$$\int \cos^2(x) dx = \frac{1}{2}(x + \sin(x) \cos(x)) + C$$

$$\int \tan^2(x) dx = \tan(x) - x + C$$

$$\int \sec^2(x) dx = \tan(x) + C$$

$$\int \sec(x) \tan(x) dx = \sec(x) + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$$

## Inverse Trigonometric Antidifferentiation Rules

$$\int \sin^{-1}(x) = x \sin^{-1}(x) + \sqrt{1-x^2}$$

$$\int \tan^{-1}(x) = x \tan^{-1}(x) - \frac{1}{2} \ln(1+x^2) + C$$

## U-Substitution

To differentiate  $\int f(g(x)) dx$  where  $f(x)$  and  $g(x)$  are both antidifferentiable, set  $u = g(x)$ . Then solve for  $du$  and replace  $dx$  so that the equation  $A \int f(u) du$  results (where  $A$  is the necessary “fudge factor”). Antidifferentiate and replace  $u$  with  $g(x)$ .

## Integration by Parts

$$\int u dv = uv - \int v du + C$$

or

$$\begin{array}{rcl} + & u & dv \\ - & \frac{du}{dx} & \int dv dx \\ + & \frac{d^2u}{dx^2} & \int (\int dv dx) dx \\ - & \frac{d^3u}{dx^3} & \int (\int (\int dv dx) dx) dx \\ + & \dots & \dots \\ & 0 & \dots \end{array}$$

$$\int u dv = \left( u \cdot \int dv dx \right) - \left( \frac{du}{dx} \cdot \int (\int dv dx) dx \right) + \left( \frac{d^2u}{dx^2} \cdot \int (\int (\int dv dx) dx) dx \right) - \dots$$

## Partial Fraction Decomposition

Convert fractions of form  $\frac{\text{linear or constant}}{\text{factorable quadratic}}$  into its partial fraction.

$$\frac{h(x)}{f(x)g(x)} = \frac{A}{f(x)} + \frac{B}{g(x)} = \frac{Ag(x)}{f(x)g(x)} + \frac{Bf(x)}{f(x)g(x)} = \frac{Ag(x) + Bf(x)}{f(x)g(x)}$$

$$h(x) = Ag(x) + Bf(x)$$

Choose a value of  $x$  such that  $f(x) = 0$  and then  $g(x) = 0$  to solve for  $A$  and  $B$  respectively.

## Exponential Growth/Decay

$$\frac{dQ}{dt} = kQ$$

$$Q = Q_0 e^{kt} = Q_0 b^{\frac{t}{p}}$$

$$k = -\frac{\ln(b)}{p}$$

### Half-life

$$k = -\frac{\ln(2)}{p}$$

## Logistic Differential Equation

$$\frac{dP}{dt} = kP(m - P) = kPm\left(1 - \frac{P}{m}\right)$$

where  $P$  is the population and  $M$  is the carrying capacity

$$P(t) = \frac{m}{1 + Ae^{-mkt}}$$

The maximum carrying capacity is reached when  $P(t) = m - 0.5$ , if  $P(0) < m$ , or right after  $P(t) = m + 0.5$ , if  $P(0) > m$ .

## Calculating Area of Regions Bounded by Two Functions

$$A = \int_a^b f(x) - g(x) \, dx$$

where  $f(x)$  is the upper function and  $g(x)$  is the lower function.

If the functions cross, multiple integrals must be taken with the appropriate upper and lower functions.

## Typical Cross-Sectional Areas

Square:  $A = b^2 = \frac{1}{2}d^2$

Rectangle:  $A = bh$

Equalateral triangle:  $A = \frac{\sqrt{3}}{4}b^2$

Right isoceles triangle:  $A = \frac{1}{2}l^2 = \frac{1}{4}h^2$

Hexagon:  $A = \frac{3\sqrt{3}}{2}s^2$

Circle:  $A = \pi r^2 = \frac{1}{4}\pi d^2$

Semicircle:  $A = \frac{1}{2}\pi r^2 = \frac{1}{8}\pi d^2$

Parabolic region:  $A = \frac{2}{3}bh$

## Calculating Volume Using the Disk/Washer Method for Solids of Rotation

$$V = \pi \int_a^b r_{\text{outer}}^2 - r_{\text{inner}}^2 dx$$

where  $dx$  is the differential thickness of the disk/washer.

## Calculating Volume Using the Cylindrical Shell Method for Solids of Rotation

$$V = 2\pi \int_a^b hr dx$$

where  $dx$  is the differential thickness of the shell.

## Calculating Arc Lengh

$$l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

## L'Hôpitals Rule

In order to evaluate a limit where

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \Rightarrow \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

evaluate

$$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$



In order to evaluate a limit where

$$\lim_{x \rightarrow a} f(x)^{g(x)} \Rightarrow "0^0" \text{ or } "1^\infty" \text{ or } "\infty^0"$$

set  $L$  equal to the original limit and immediately take the log of both sides

$$\begin{aligned} L &= \lim_{x \rightarrow a} f(x)^{g(x)} \\ \ln(L) &= \ln\left(\lim_{x \rightarrow a} f(x)^{g(x)}\right) \\ \ln(L) &= \lim_{x \rightarrow a} g(x) \ln f(x) \end{aligned}$$

Evaluate the resulting limit and then raise  $e$  to that result

## Relative Rates of Growth

To compare rates of  $f(x)$  and  $g(x)$  evaluate

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \begin{cases} \infty & \text{then } f(x) \text{ grows faster than } g \\ \langle 0, \infty \rangle & \text{then } f(x) \text{ and } g(x) \text{ grow at the same rate} \\ 0 & \text{then } f(x) \text{ grows slower than } g(x) \end{cases}$$

## Improper Integrals

### Type 1

$$\begin{aligned} \int_a^\infty f(x) \, dx &= \lim_{n \rightarrow \infty} \int_a^n f(x) \, dx \\ \int_{-\infty}^b f(x) \, dx &= \lim_{n \rightarrow \infty} \int_{-n}^b f(x) \, dx \\ \int_{-\infty}^\infty f(x) \, dx &= \lim_{n \rightarrow \infty} \int_{-n}^n f(x) \, dx \end{aligned}$$

### Type 2

$$\int_a^b f(x)$$

such that there is at least one vertical-asymptotic discontinuity on  $[a, b]$ .

$$\begin{aligned} \int_a^b f(x) \, dx \text{ (where discontinuity at } a) &= \lim_{n \rightarrow a^+} \int_n^b f(x) \, dx \\ \int_a^b f(x) \, dx \text{ (where discontinuity at } b) &= \lim_{n \rightarrow b^-} \int_a^n f(x) \, dx \\ \int_a^b f(x) \, dx \text{ (where discontinuity } d \text{ on } \langle a, b \rangle) &= \lim_{n \rightarrow d^-} \int_a^n f(x) \, dx + \lim_{n \rightarrow d^+} \int_n^b f(x) \, dx \end{aligned}$$

## P-Integrals

An integral that fits the pattern:

$$\int_1^\infty \frac{1}{x^p} dx \text{ where } p \text{ is positive}$$

$$p > 1 \Rightarrow \text{the integral converges to } \frac{1}{p-1}.$$

$$0 < p \leq 1 \Rightarrow \text{the integral diverges.}$$

## Comparison Tests

For positive type 1 improper integrals ( $a = \infty$  and/or  $b = \infty$ ) where the integrand is not easily antidifferentiable:

$$\int_a^b f(x) dx$$

Establish bound functions which can be antidifferentiated:

$$l(x) \leq f(x) \leq u(x)$$

$$\text{Convergence if } \int_a^b f(x) dx \leq \int_a^b u(x) dx \text{ and } \int_a^b u(x) dx \text{ converges}$$

$$\text{Divergence if } \int_a^b f(x) dx \geq \int_a^b l(x) dx \text{ and } \int_a^b l(x) dx \text{ diverges}$$

## Sequences

### Explicit Definition

$$x_k = f(k)$$

where  $k \in \{1, 2, 3, \dots\}$ .

### Recursive Definition

$$x_k = m(x_{k-1})$$

where  $x_1$  is defined and  $k \in \{2, 3, 4, \dots\}$ .

### Arithmetic

$$x_k = d(k-1) + x_1$$

$$x_k = t_{k-1} + d$$

## Geometric

$$x_k = x_1 r^{k-1}$$

$$x_k = x_{k-1} r$$

## Convergence/Divergence

A sequence converges if the limit  $\lim_{n \rightarrow \infty} x_n$  is a finite number else it diverges.

## Infinite Series

$$\sum_{k=0}^{\infty} x_k$$

### Convergence

Consider the sequence of “partial sums” ( $s$ ) where:

$$s_n = \sum_{k=0}^n x_k$$

If  $\{s_n\}$  converges to  $S$  then the infinite series converges to  $S$ .

### Power Series

$$\sum_{k=0}^{\infty} c_k (x - a)^k$$

### Taylor/Maclaurin Series

Given an infinitely differentiable function  $f(x)$ , the Taylor series centered around  $a$  is equal to the power series with a specific  $c_k$ :

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x - a)^k$$

A Maclaurin series is a Taylor series centered around 0 ( $a = 0$ ).

The  $n$ -order polynomial function  $P_n(x)$  that best fits  $f(x)$  is equal to the Taylor series evaluated with upper limit  $n$ :

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x - a)^k$$

## Common Maclaurin Series

$$\frac{a}{1-x} = \sum_{k=0}^{\infty} ax^k \quad x \in \langle -1, 1 \rangle$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \quad x \in \mathbb{R}$$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} \quad x \in \mathbb{R}$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} \quad x \in \mathbb{R}$$

$$\tan^{-1}(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \quad x \in [-1, 1]$$

$$\ln(x+1) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} \quad x \in \langle -1, 1 \rangle$$

## Calculating Error for a Taylor Polynomial

$$P_{\infty}(x) = P_n(x) + R_n(x)$$

where  $P_{\infty}$  is the entire infinite Taylor series and  $R_n$  is the remainder (note that the remainder series starts with  $k = n + 1$ ).

Lagrange Error:

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1} \quad \text{for an unknown } c \in \langle a, x \rangle.$$

$$|R_n(x)| = \left| \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1} \right|$$

$$|R_n(x)| = \frac{|f^{(n+1)}(c)|}{(n+1)!} |x-a|^{n+1}$$

To bound the error:

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$$

Where  $M$  is a positive number greater than or equal to every possible value  $f^{(n+1)}(t)$  where  $t$  is in the interval of study.

The series then converges if:

$$\lim_{n \rightarrow \infty} |R_n(x)| = 0$$

# Infinite Series Tests

## **n<sup>th</sup>-Term Test**

If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , the series  $\sum_{n=0}^{\infty} a_n$  diverges.

## **Comparison Test**

To conclude whether a series  $S$  converges or diverges establish a known series  $T$ .

Convergence if  $S \leq T$  and  $T$  converges

Divergence if  $S \geq T$  and  $T$  diverges

## **Alternating Test**

Any alternating series that passes the  $n^{\text{th}}$ -term test where the magnitude of successive terms decreases will converge.

A series which depends on alternation has conditional convergence. A series which converges whether all terms are positive or negative has absolute convergence.

Additionally the error is less than the next term in the sequence for an alternating sequence.

## **P-Series Test**

If the series is in the form  $a_n = \frac{1}{n^p}$ , then if  $p > 1$  the series converges, else it diverges.

## **Telescoping Series Test**

For a series where subsequent terms cancel out previous terms such as:

$$\sum_{k=1}^{\infty} \frac{1}{(k)(k+1)}$$

Separate into partial fractions:

$$\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+1} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$$

## **Ratio Test**

For a series:

$$\sum_{k=0}^{\infty} t_k \text{ where } t_k \geq 0$$

consider

$$\lim_{k \rightarrow \infty} \frac{t_{k+1}}{t_k} = \begin{cases} [0, 1) & \text{the series converges} \\ 1 & \text{the test is inconclusive} \\ \langle 1, \infty \rangle \text{ or } \infty & \text{the series is divergent} \end{cases}$$

To evaluate for a power series take the absolute value of the ratio:

$$\lim_{k \rightarrow \infty} \left| \frac{c_{k+1}(x-a)^{k+1}}{c_k(x-a)^k} \right| = \lim_{k \rightarrow \infty} \left| \frac{c_{k+1}}{c_k} \right| |x-a|$$

Then to find radius of convergence:

$$l = \lim_{k \rightarrow \infty} \left| \frac{c_{k+1}}{c_k} \right| = \begin{cases} 0 & \text{the series converges for } x \in \mathbb{R} \\ \langle 0, \infty \rangle & \text{the series converges for } |x-a| < R = \frac{1}{l} \\ \infty & \text{the series converges only for } x = a \end{cases}$$

## Integral Test

Find positive, continuous, and decreasing function where  $f(n) = a_n$ . If  $\int_0^{\infty} f(x) dx$  converges the series will converge; if it diverges the series will diverge.

## Parametric Functions

Both  $x$  and  $y$  defined in terms of third variable:

$$x = f(t)$$

$$y = g(t)$$

## Eliminating the Parameter

Solve for  $t$  in one of the equations and substitute or find another way to combine the two equations to eliminate  $t$ ;

## Taking Derivatives

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

## Calculating Arc Length

$$\int_{t_1}^{t_2} \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt$$

## Polar Functions

Points are identified by radius  $r$  and angle  $\theta$ :  $(r, \theta)$ . There exist infinitely many polar representations for the same point.

Equations are usually  $r$  in terms of  $\theta$ :  $r = f(\theta)$ .

### Relationships With Cartesian Points

$$r = \sqrt{x^2 + y^2}$$

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

### Calculating Instantaneous Slope

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}(r \sin(\theta))}{\frac{d}{d\theta}(r \cos(\theta))} = \frac{\frac{dr}{d\theta} \sin(\theta) + r \cos(\theta)}{\frac{dr}{d\theta} \cos(\theta) - r \sin(\theta)}$$

### Calculating Area Bounded by Curve

$$\frac{1}{2} \int_{\alpha}^{\beta} (r(\theta))^2 d\theta$$

## Vector Functions

$$\vec{s}(t) = \langle f(t), g(t) \rangle$$

$$\vec{s}'(t) = \vec{v}(t) = \langle f'(t), g'(t) \rangle$$

$$\vec{s}''(t) = \vec{v}'(t) = \vec{a}(t) = \langle f''(t), g''(t) \rangle$$

$$\text{speed} = |\vec{v}| = \sqrt{(f'(t))^2 + (g'(t))^2}$$