Bounded Diameter Minimum Spanning Trees

Literature Survey and Related Problems

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Table of Contents

Introduction

Background and Motivation

- Consider a set of points of size n
- We want to find the minimum spanning tree with a twist. The twist is we don't want to minimise the weight, we want to minimise the diameter.
- Recall, the diameter of a spanning tree is maximum distance between any two points in that tree.
- Now let's assign weights to the edges. The problems changes slightly. That is if we find different trees with different weight but same diameter we have to return min weight tree.

Frame content [1].

Problem Definitions

Given a set of points $P = \{p_i\}_{i=1}^{n}$. The euclidean graph G induced by P is a weighted complete graph $G = (V, E)$, where cost(or weight) of each edge is distance between the two points.

MDST Problem

- Given graph $G = (V, E)$ and a cost function $W(e) \in \mathbb{Z}^+ \forall e \in E$
- Find a spanning tree for G such that $max_{simplepathp \in T} \Sigma_{e \in p} W(e)$ is minimised.

In [1] GMDST is discussed. The graph is a euclidean graph as discussed above

BDBCST Problem

- Given graph $G = (V, E)$ and a cost function $W(e) \in \mathbb{Z}^+ \forall e \in E$ and two positive integers C and D.
- Find a spanning tree T for G such that $\Sigma_{e \in p} W(e) \leq C$ and $\Sigma_{e \in p} W(e) \leq D$ for all simple paths p in T.

BRBCST Problem

- Given graph $G = (V, E)$ and a cost function $W(e) \in \mathbb{Z}^{+} \forall e \in E$ and two positive integers C and R and also a distinguished vertex $v \in V$ as root.
- Find a spanning tree T for G such that $\Sigma_{e \in p} W(e) \leq C$ and $\Sigma_{e \in p} W(e) \leq R$ for all simple paths p in T starting from v.

Steiner MDST Problem

- Given graph $G = (V, E)$ a subset of S of V, and a cost function $W(e) \in \mathbb{Z}^{+} \forall e \in E$
- Find a steiner tree $T = (V_T, E_T)$ where $S \subseteq V_T \subseteq V$ of G such that $max_{simplepathp \in T} \Sigma_{e \in P} W(e)$ is minimised.

GMDST Problem and Analysis

Algorithm

Recall Geometric Minimum Diameter Spanning Tree Problem. Authors of the paper [1] tried to a $\mathcal{O}(n^3)$ algorithm for solving the problem. Let's first see two definitions,

- A point P in set S is **monopole** if all the the remaining points are connected to it.
- A spanning tree of n point set is **monopolar** if there exists a monopole.
- A spanning tree of n point set is **dipolar** if there exist two points such that all the remaining points are directly connected to either of two points

Algorithm Continued...

ALGORITHM FUNCTION GMDST(set of n Points S):

- **find-min-MPST(S)**: returns the monopolar spanning tree of set S with minimum diameter and the corresponding diameter M
- **find-min-DPST(S)**: returns the dipolar spanning tree of the set S of a minimum diameter and the corresponding diameter D
- if $M \leq D$ then $GMDST = M$ else $GMDST = D$

Algorithm Analysis

- **FIND-MIN-MPST(S)**: for every point $p \in S$
  - construct the monopolar spanning tree by association of an edge with the points in the set $S - \{p\}$.
  - compute the furthest distance by adding the distances between furthest point and second furthest point from p. Let's denote the distance by d_p

  return the minimum of all d_p where $p \in S$

Algorithm Analysis Continued...

- **FIND-MIN-DPST(S)**: for every two points p_i and $p_j \in S$, where $i \neq j$ as possible centers of dipolar MDST and select that pair that give minimum diameter

Algorithm Analysis Continued...

- **FIND-MIN-MPST(S)**: This one runs for $\mathcal{O}(n^2)$
- **FIND-MIN-DPST(S)**: This one runs for $\mathcal{O}(n^3)$
- The last if condition checking is a constant time operation.

Note: finding minimum MPST can be done in $\mathcal{O}(nlogn)$ time using furthest voronoi diagram.

Conclusions

Conclusions

- Here we have presented an $\theta(n^3)$ algorithm for finding minimum diameter spanning tree of set of n point in the plane.
- This result is applicable to any graphs whose edge satisfies triangle inequality.
- It is also proven that finding minimum diameter min cost spanning tree is NP Hard.But bounded diameter bounded cost spanning tree is NP complete.
- The proof BDBCST problem is in NP comes from the fact that the most heavy weighted simple path must be a path connecting two leaf nodes, we may randomly guess n-1 edges and check if the tree satisfies the constraints in polynomial time.

References I

J.-M. Ho, D. Lee, C.-H. Chang, and C. Wong, "Minimum diameter spanning trees and related problems," *SIAM J. Comput.*, vol. 20, pp. 987–997, 10 1991.

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