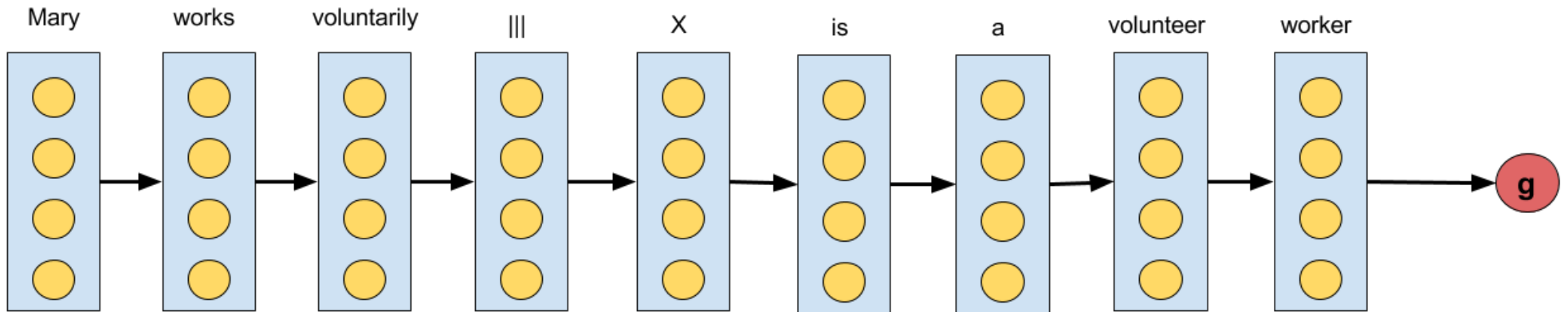


# Gated RNNs

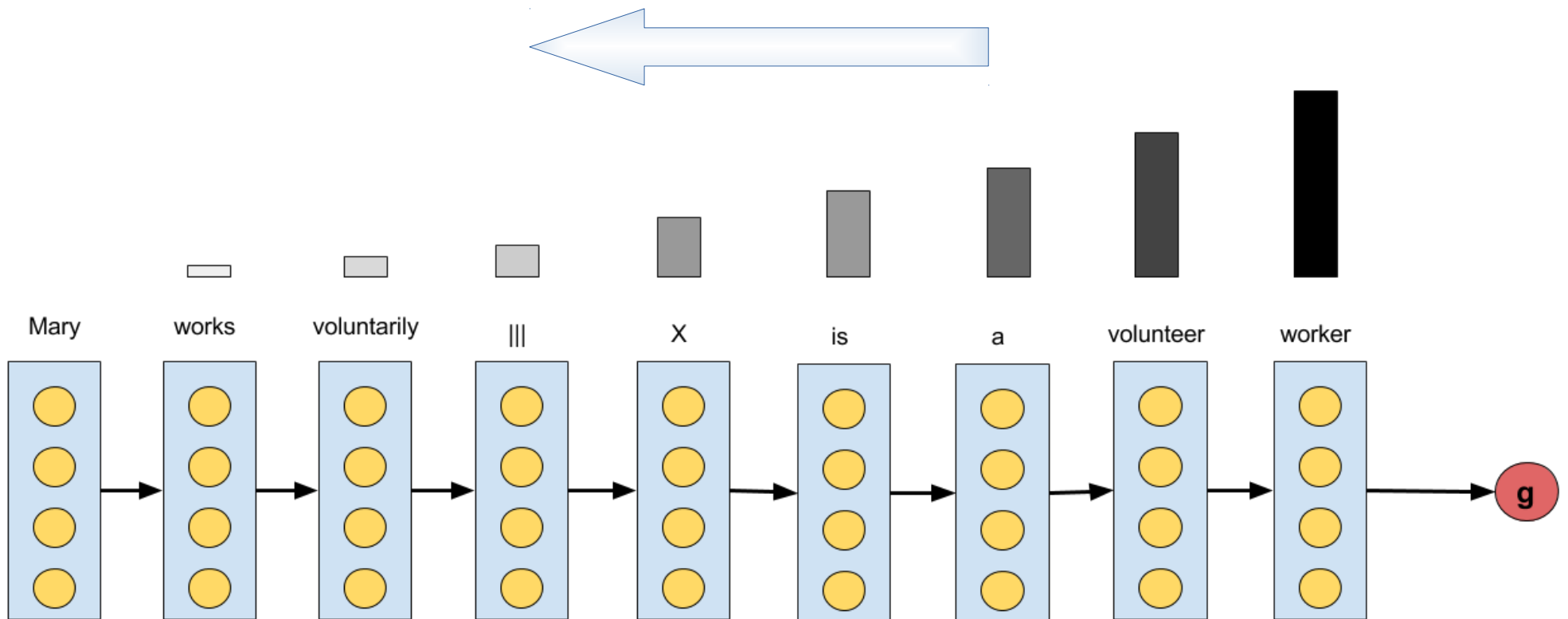
A Battle Against Vanishing Gradients

# RNN

$$\vec{h}_{t+1} = \tanh(W\vec{x}_{t+1} + U\vec{h}_t + b)$$



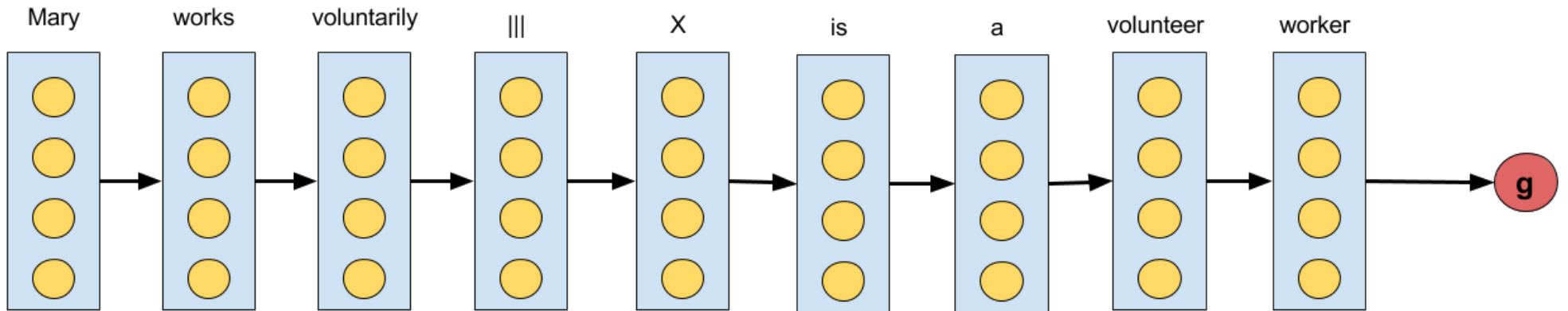
# Vanishing Gradient Problem



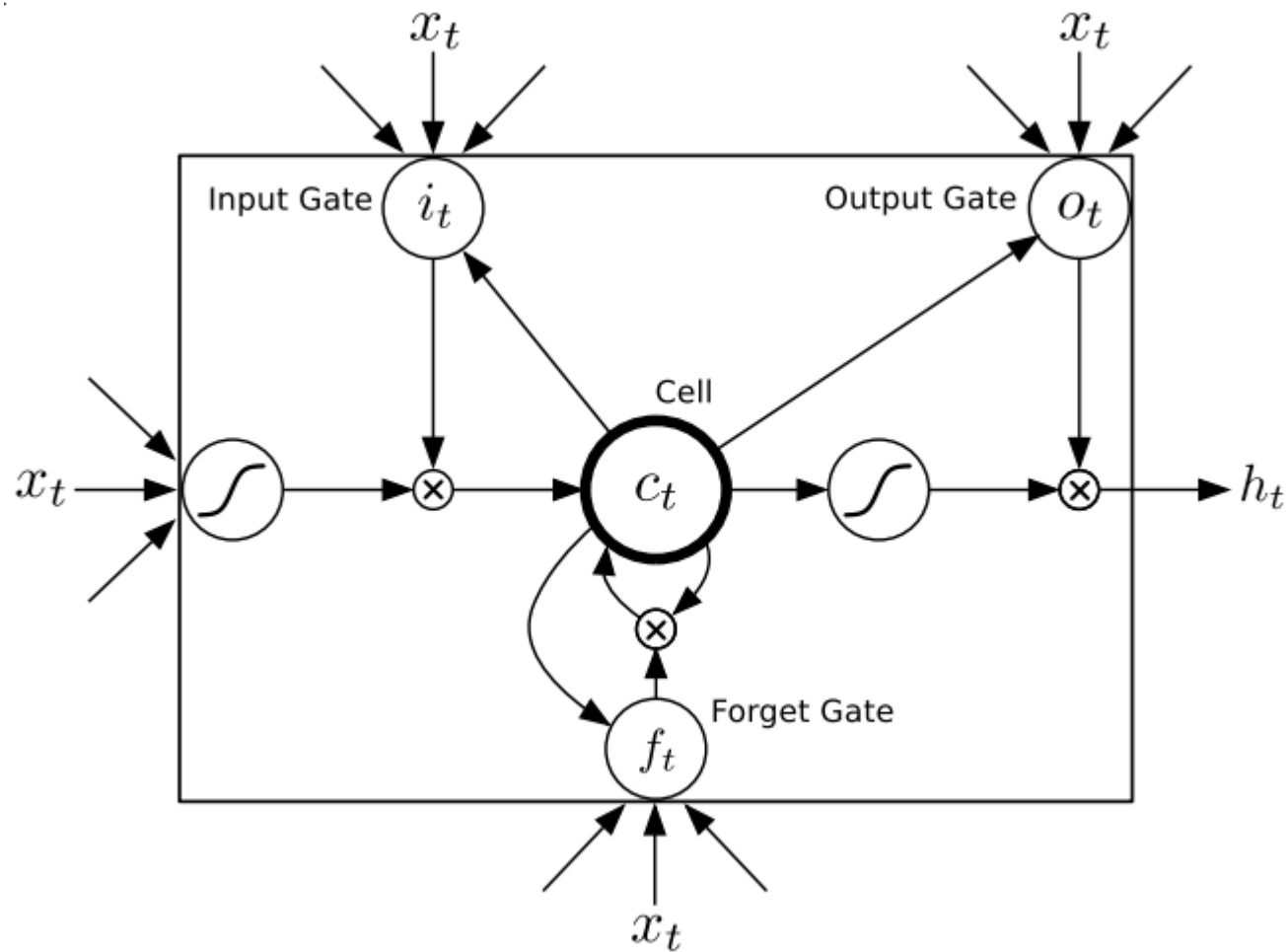
$$\nabla_{h^t} g = \frac{\partial_{h^{t+1}}}{\partial_{h^t}} \nabla_{h^{t+1}} g + \dots$$

# RNN

$$\vec{h}_{t+1} = \tanh(W\vec{x}_{t+1} + U\vec{h}_t + b)$$



# Long Short-Term Memory



[Hochreiter, Schmidhuber 1997; Graves 2013]

# Long Short-Term Memory

$$i_t = \sigma(W_{xi}x_t + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_i)$$

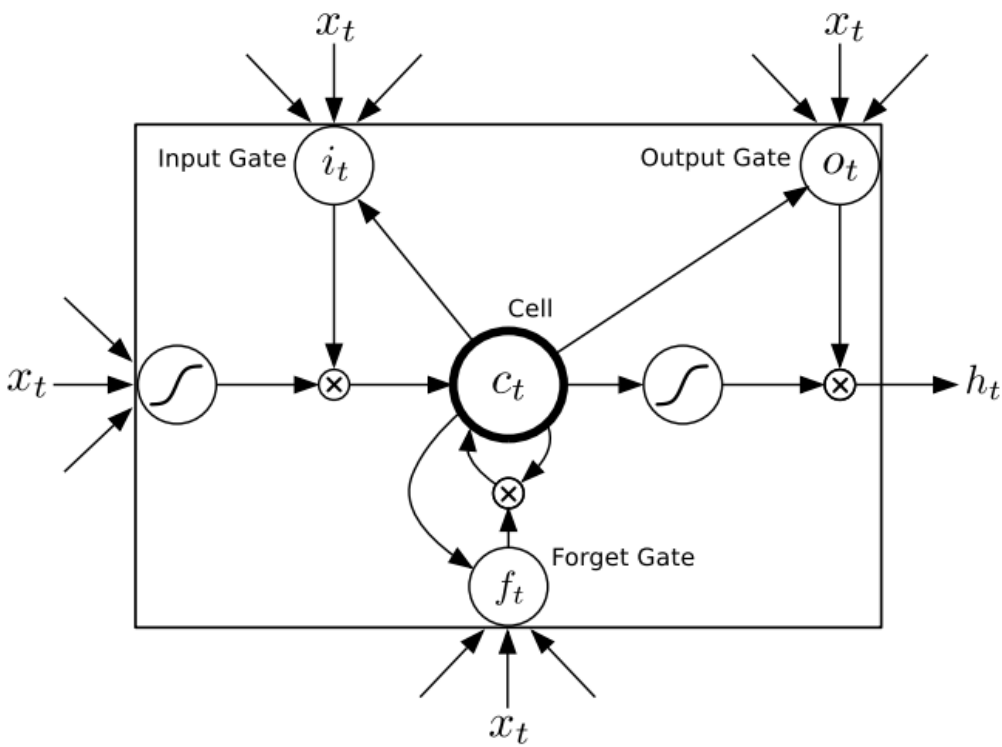
Input gate

$$f_t = \sigma(W_{xf}x_t + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_f)$$

Forget gate

$$c_t = f_t c_{t-1} + i_t \tanh(W_{xc}x_t + W_{hc}h_{t-1} + b_c)$$

$$o_t = \sigma(W_{xo}x_t + W_{ho}h_{t-1} + W_{co}c_t + b_o)$$



$$h_t = o_t \tanh(c_t)$$

# Long Short-Term Memory

$$i_t = \sigma(W_{xi}x_t + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_i)$$

**Input gate**

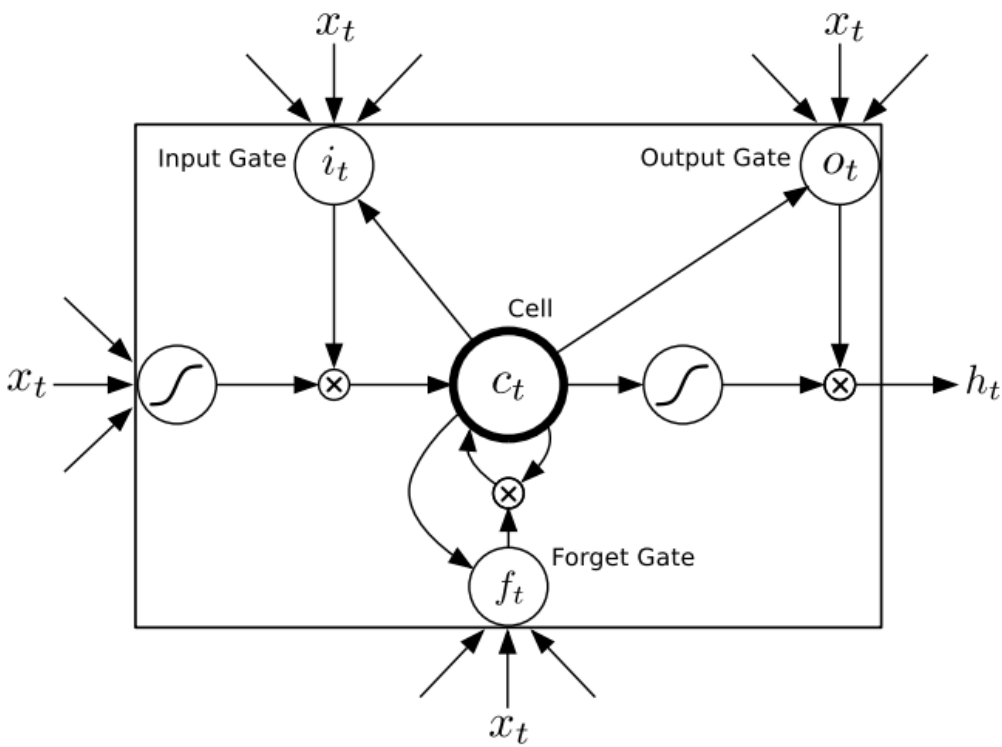
$$f_t = \sigma(W_{xf}x_t + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_f)$$

**Forget gate**

$$c_t = f_t c_{t-1} + i_t \tanh(W_{xc}x_t + W_{hc}h_{t-1} + b_c)$$

**Memory cell**

$$o_t = \sigma(W_{xo}x_t + W_{ho}h_{t-1} + W_{co}c_t + b_o)$$



$$h_t = o_t \tanh(c_t)$$

# Long Short-Term Memory

$$i_t = \sigma(W_{xi}x_t + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_i)$$

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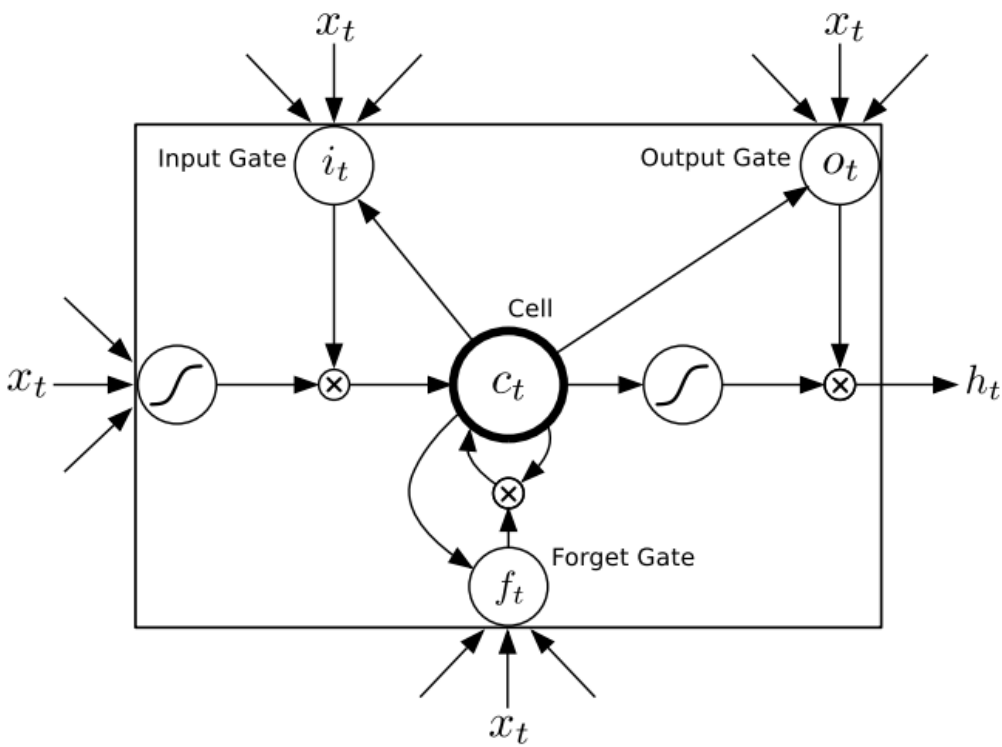
**Forget gate**

$$c_t = f_t c_{t-1} + i_t \tanh(W_{xc}x_t + W_{hc}h_{t-1} + b_c)$$

**Memory cell**

$$o_t = \sigma(W_{xo}x_t + W_{ho}h_{t-1} + W_{co}c_t + b_o)$$

**Output gate**

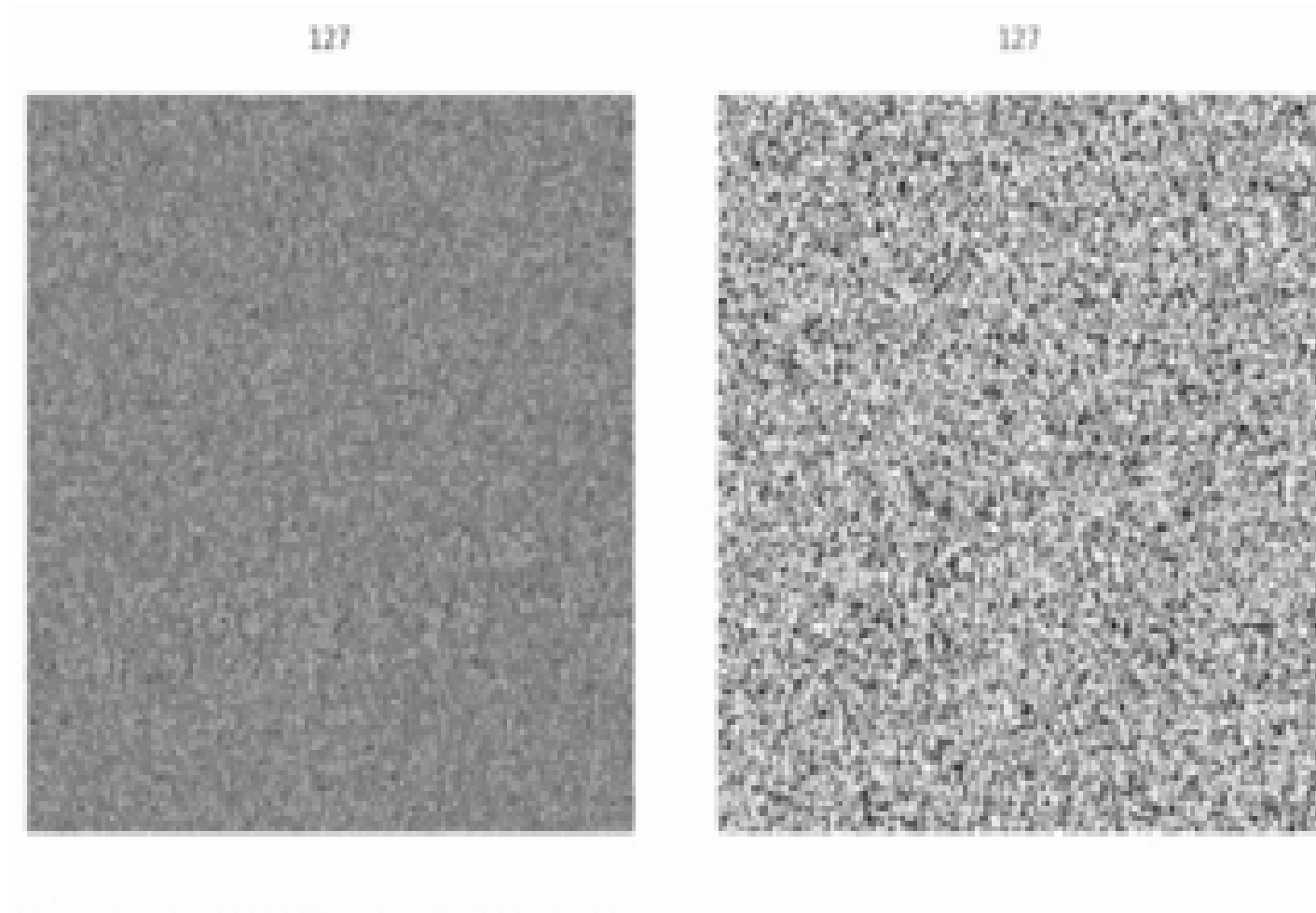


$$h_t = o_t \tanh(c_t)$$

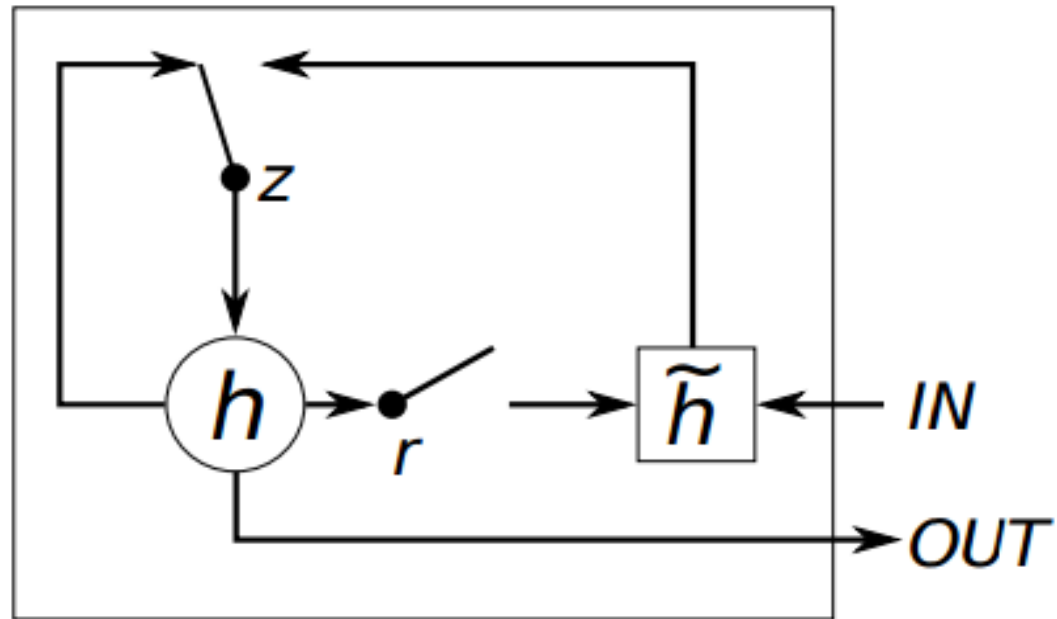


# Vanishing Gradients

## RNN vs LSTM



# Gated Recurrent Unit



[Chung 2014; Cho 2014]

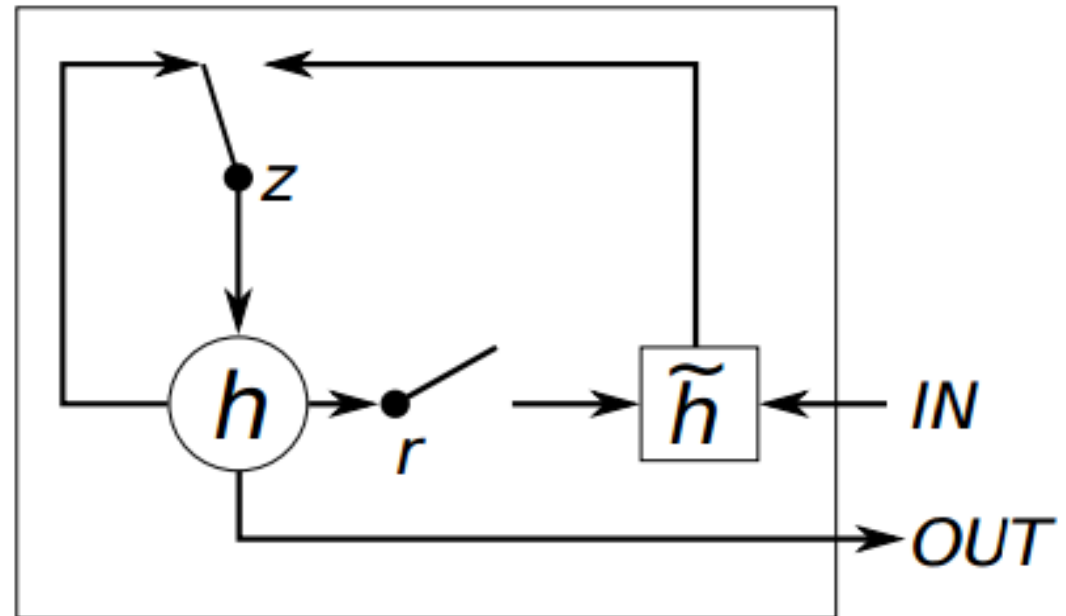
# Gated Recurrent Unit

**Update gate**

$$z_t = \sigma(W_z x_t + U_z h_{t-1} + b_z)$$

**Reset gate**

$$r_t = \sigma(W_r x_t + U_r h_{t-1} + b_r)$$



$$\tilde{h}_t = \tanh(W x_t + U(r_t \odot h_{t-1}))$$

$$h_t = (1 - z_t)h_{t-1} + z_t\tilde{h}_t$$