John was consuming 120 units of X and 60 units of Y. The price of X rose from 3 to 5. The price of Y remained at 6.

(a) How much would John's income have to rise so that he can still exactly afford 120 units of X and 60 units of Y?

(b) If John's income did not increase and remained at the initial leel, how many units of X could he afford if he still purchased 60 units of Y?

(c) If John decided to consume 100 units of X instead of 120 units after the price change, how much income would he save compared to the required income to maintain the original consumption?

(d) If the price of Y also increased from 6 to 8, how much would John's income have to rise to still afford 120 units of X and 60 units of Y?

(e) If John's income increased to 1,000 instead of the exact amount required to maintain the initial consumption, what is the remaining income after buying 120 units of X and 60 units of Y at the new prices?

(f) If John decided to spend all his income only on X at the new price, how many units of X could he purchase with the initial income?

Solution

(a) Let us calculate the initial income required by John to afford 120 units of X and 60 units of Y at the original prices.

$$\text{Initial income} = (\text{Price of } X) \times (\text{Quantity of } X) + (\text{Price of } Y) \times (\text{Quantity of } Y)$$
$$\text{Initial income} = 3 \times 120 + 6 \times 60$$
$$\text{Initial income} = 360 + 360 = 720$$

After the price of X increases to 5, the new income required to purchase the same quantities is:

$$\text{New income} = (\text{New price of } X) \times (\text{Quantity of } X) + (\text{Price of } Y) \times (\text{Quantity of } Y)$$
$$\text{New income} = 5 \times 120 + 6 \times 60$$
$$\text{New income} = 600 + 360 = 960$$

The required increase in John's income is the difference between the new income and the initial income:

$$\text{Income increase} = \text{New income} - \text{Initial income}$$
$$\text{Income increase} = 960 - 720 = 240$$

Thus, John's income would have to rise by $240.

(b) If John's income did not increase and remained at the initial level, how many units of X could he afford if he still purchased 60 units of Y?

Let us calculate the remaining income after purchasing 60 units of Y at the price of 6:

$$\text{Remaining income} = \text{Initial income} - (\text{Price of } Y) \times (\text{Quantity of } Y)$$
$$\text{Remaining income} = 720 - 6 \times 60$$
$$\text{Remaining income} = 720 - 360 = 360$$

Now, calculate how many units of X can be purchased at the new price of 5:

$$\text{Quantity of } X = \frac{\text{Remaining income}}{\text{New price of } X}$$
$$\text{Quantity of } X = \frac{360}{5} = 72$$

Thus, John could afford 72 units of X.

(c) If John decided to consume 100 units of X instead of 120 units after the price change, how much income would he save compared to the required income to maintain the original consumption?

The new income required for 100 units of X and 60 units of Y is:

$$\text{New income (100 units)} = (\text{New price of } X) \times (\text{Quantity of } X) + (\text{Price of } Y) \times (\text{Quantity of } Y)$$
$$\text{New income (100 units)} = 5 \times 100 + 6 \times 60$$
$$\text{New income (100 units)} = 500 + 360 = 860$$

The savings compared to the new income for 120 units of X and 60 units of Y are:

$$\text{Savings} = \text{New income (120 units)} - \text{New income (100 units)}$$
$$\text{Savings} = 960 - 860 = 100$$

Thus, John would save $100.

(d) If the price of Y also increased from 6 to 8, how much would John's income have to rise to still afford 120 units of X and 60 units of Y?

The new income required under the new prices is:

$$\text{New income} = (\text{New price of } X) \times (\text{Quantity of } X) + (\text{New price of } Y) \times (\text{Quantity of } Y)$$

$$\text{New income} = 5 \times 120 + 8 \times 60$$

$$\text{New income} = 600 + 480 = 1080$$

The required increase in John's income is:

$$\text{Income increase} = \text{New income} - \text{Initial income}$$

$$\text{Income increase} = 1080 - 720 = 360$$

Thus, John's income would have to rise by $360.

(e) If John's income increased to 1,000 instead of the exact amount required to maintain the initial consumption, what is the remaining income after buying 120 units of X and 60 units of Y at the new prices?

The total cost of 120 units of X and 60 units of Y at the new prices is:

$$\text{Total cost} = (\text{New price of } X) \times (\text{Quantity of } X) + (\text{Price of } Y) \times (\text{Quantity of } Y)$$

$$\text{Total cost} = 5 \times 120 + 6 \times 60$$

$$\text{Total cost} = 600 + 360 = 960$$

The remaining income is:

$$\text{Remaining income} = \text{New income} - \text{Total cost}$$

$$\text{Remaining income} = 1000 - 960 = 40$$

Thus, John would have $40 remaining.

(f) If John decided to spend all his income only on X at the new price, how many units of X could he purchase with the initial income?

The total number of units of X John can purchase with his initial income is:

$$\text{Quantity of } X = \frac{\text{Initial income}}{\text{New price of } X}$$

$$\text{Quantity of } X = \frac{720}{5} = 144$$

Thus, John could afford 144 units of X.