Normal distribution percentile calculation

Monthly family expenses have a mean of \$700 and a standard deviation of \$80. Determine, for a period of 3 years (36 months), the minimum total consumption amount that will be exceeded with 85% probability.

We assume a normal distribution with:

- Monthly mean (μ): \$700
- Monthly standard deviation (σ): \$80

The total consumption over 36 months is the sum of 36 independent normal random variables. Therefore, the total consumption (X) also follows a normal distribution with:

- Total mean (μ_T):
$$\mu_T = n \times \mu = 36 \times \$700 = \$25{,}200$$
- Total standard deviation (σ_T):
$$\sigma_T = \sqrt{n} \times \sigma = \sqrt{36} \times \$80 = 6 \times \$80 = \$480$$

Hence,

$$X \sim N(\mu_T, \sigma_T^2) = N(\$25{,}200, (\$480)^2)$$

We want to find the value x such that the probability that the total consumption is less than or equal to x is 15% (because 85% will exceed it):

$$P(X \leq x) = 0.15$$

We standardize the variable:

$$Z = \frac{X - \mu_T}{\sigma_T}$$

Thus,

$$P\Big(Z \leq \frac{x - \mu_T}{\sigma_T}\Big) = 0.15$$

We look up in the standard normal distribution table (or use a suitable application) the value of z satisfying:

$$P(Z \leq z) = 0.15$$

The corresponding value is approximately:

$$z = -1.036$$

We solve for x from the standardization equation:

$$z = \frac{x - \mu_T}{\sigma_T} \quad \Rightarrow \quad x = \mu_T + z \cdot \sigma_T$$

Substituting the obtained values:

$$\begin{aligned} x &= \$25{,}200 + (-1.036) \times \$480 \\ &= \$25{,}200 - \$497.28 \\ &= \$24{,}702.72 \end{aligned}$$

The minimum total consumption that will be exceeded with 85% probability over a 36-month period is:

$$\boxed{\$24{,}702.72}$$