Homogeneity and Euler's theorem

For the following function, check if it is homogeneous; if affirmative, indicate the degree and verify Euler's theorem:

$$f(x, y) = \sqrt{2x^2 + y^2}$$

Solution

$$f(hx, hy) = \sqrt{2x^2h^2 + y^2h^2}$$
$$f(hx, hy) = \sqrt{h^2(2x^2 + y^2)}$$
$$f(hx, hy) = h\sqrt{2x^2 + y^2}$$
$$f(hx, hy) = hf(x, y)$$

The function is homogeneous of degree 1. We calculate the derivatives to verify Euler's theorem:

$$f'_x = \frac{1}{2}(2x^2 + y^2)^{-1/2}4x$$

$$f'_y = \frac{1}{2}(2x^2 + y^2)^{-1/2}2y$$

We state Euler's theorem:

$$xf'_x + yf'_y = nf(x, y)$$

Where n is the homogeneity degree, and in this case it is 1.

$$x\frac{1}{2}(2x^2 + y^2)^{-1/2}4x + y\frac{1}{2}(2x^2 + y^2)^{-1/2}2y = \sqrt{2x^2 + y^2}$$

$$\frac{1}{2}(2x^2 + y^2)^{-1/2}4x^2 + \frac{1}{2}(2x^2 + y^2)^{-1/2}2y^2 = \sqrt{2x^2 + y^2}$$

$$(2x^2 + y^2)^{-1/2}2x^2 + (2x^2 + y^2)^{-1/2}y^2 = \sqrt{2x^2 + y^2}$$

Taking out a common factor:

$$(2x^2 + y^2)^{-1/2}(2x^2 + y^2) = \sqrt{2x^2 + y^2}$$
$$(2x^2 + y^2)^{1/2} = \sqrt{2x^2 + y^2}$$