## Stackelberg

Consider a scenario with 2 firms, which, in a sequential manner, define their level of production. Firm 1 is the leader and first chooses its strategy. Firm 2, being the follower, observes the choice of Firm 1 and then defines its strategy. The inverse demand is given by: P(Q) = 150 - 3Q. At the same time, the costs are identical for both firms, with  $C_{qi} = 18q_i$  for each firm. Determine quantities, the equilibrium price, and benefits.

## Solution

By backward induction, we set up the profit function for Firm 2:

$$\pi_{q_2} = (150 - 3q_2 - \frac{3q_1}{2})q_2 - 18q_2$$

We differentiate with respect to  $q_2$ :

$$150 - 6q_2 - \frac{3q_1}{2} - 18 = 0$$

$$132 - 6q_2 - \frac{3q_1}{2} = 0$$

$$q_2 = 22 - \frac{1}{2}q_1$$

We set up the profit function for Firm 1, substituting  $q_2$ :

$$\pi_{q_1} = (150 - 3(22 - \frac{1}{2}q_1) - 3q_1)q_1 - 18q_1$$

$$(150 - 66 + \frac{3}{2}q_1)q_1 - 18q_1 = (84 - \frac{3}{2}q_1)q_1 - 18q_1$$

We differentiate with respect to  $q_1$ :

$$84 - 3q_1 - 18 = 0$$

$$66 - 3q_1 = 0$$

$$q_1 = \frac{66}{3} = 22$$

We substitute  $q_1$  into  $q_2$  to find the quantities for the follower firm:

$$q_2 = 22 - \frac{1}{2} \times 22$$

$$q_2 = 11$$

We calculate the equilibrium price:

$$P = 150 - 11 \times 3 - 22 \times 3$$

$$P = 51$$

$$Q = 11 + 22 = 33$$

$$\pi_1 = 51 * 22 - 18 * 22 = 726$$

$$\pi_2 = 51 * 11 - 18 * 11 = 363$$