Differential equation with separable variables

Solve the following differential equation:

$$(1 + x^2)y^3 dx + (1 + y^2)dy = 0$$

Solution

We solve by the method of separable variables:

$$(1 + x^2)y^3 dx = -(1 + y^2)dy$$

$$(1 + x^2)dx = -\frac{(1 + y^2)}{y^3}dy$$

$$(1 + x^2)dx = (-y^{-3} - \frac{1}{y})dy$$

Integrate both sides:

$$x + \frac{x^3}{3} = \frac{y^{-2}}{2} - ln(y) + C$$

We assume that $y > 0$ to be within the natural logarithm.