Sum of partial elasticities and degree of homogeneity

Let $q = f(k, l)$ be a homogeneous production function of degree n, where k and l are the quantities of the production inputs. Prove that the sum of the partial elasticities with respect to both inputs is equal to n.

Solution

We start by using Euler's theorem for homogeneous functions:

$$\frac{\partial f}{\partial k}k + \frac{\partial f}{\partial l}l = nf(k,l)$$

we divide everything by f:

$$\frac{\partial f}{\partial k}\frac{\partial k}{\partial f} + \frac{\partial f}{\partial l}\frac{l}{f} = \frac{nf(k,l)}{f}$$

Let's remember that partial elasticities have the following formulas:

$$\frac{Ef}{Ek} = f'k\frac{k}{f}$$

$$\frac{Ef}{El} = f'l\frac{l}{f}$$

Therefore:

$$\frac{Ef}{Ek} + \frac{Ef}{El} = n$$