Budget Constraint with Block Pricing and Discount

A consumer has an income of $m = \$100$. There are two goods, x and y. The price of good y is constant at $p_y = \$10$ per unit. Good x is sold under a block pricing schedule:

$$\text{Price of } x = \begin{cases} \$5 \text{ per unit}, & \text{if } x < 10, \\ \$3 \text{ per unit}, & \text{if } x \geq 10. \end{cases}$$

Assume that if the consumer purchases 10 or more units of x, the lower price applies to all units purchased.

(a) Write down the consumer's piecewise budget constraint.

(b) Sketch the corresponding budget line on the (x, y) plane. Clearly indicate the key intercepts and the discontinuity (kink) that occurs at $x = 10$.

(c) Solve the problem again, but this time assume that the discount applies only to units purchased beyond $x = 10$. Write down the new piecewise budget constraint and sketch the modified budget line on the (x, y) plane.

Solution

(a) *Formulation of the Piecewise Budget Constraint*

Since the expenditure on good x depends on the quantity purchased, we have two cases:

$x<10$: The price of x is \$5 per unit. Thus, the expenditure on x is $5x$ and on y is $10y$. The budget constraint is

$$5x + 10y = 100, \quad \text{for } 0 \leq x < 10.$$

$x \geq 10$: If the consumer buys 10 or more units, the price of x becomes \$3 per unit (for all units of x). Thus, the expenditure on x is $3x$ and the budget constraint becomes

$$3x + 10y = 100, \quad \text{for } x \geq 10.$$

(b) *Sketching the Budget Line*

$x<10$: Rewriting the constraint:

$$5x + 10y = 100 \quad \Longrightarrow \quad y = 10 - 0.5x.$$

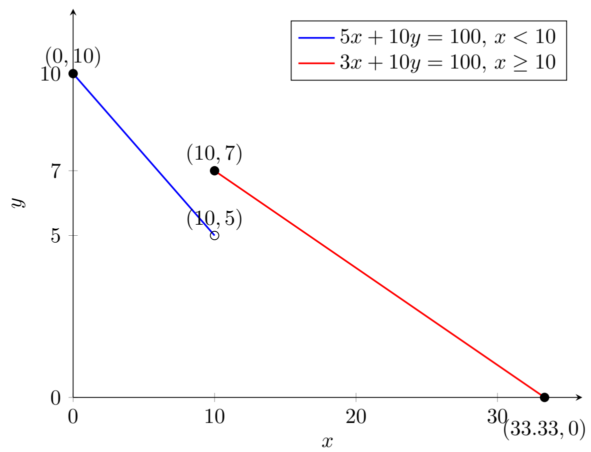
- When $x = 0$, $y = 10$ (the y-intercept).
- When $x = 10$, $y = 10 - 0.5(10) = 5$.

$x \geq 10$: Rewriting the discounted segment:

$$3x + 10y = 100 \quad \Longrightarrow \quad y = 10 - 0.3x.$$

- When $x = 10$, $y = 10 - 0.3(10) = 7$.
- When $y = 0$, $3x = 100$ so $x = \frac{100}{3} \approx 33.33$ (the x-intercept for this segment).

Graph: The budget line is piecewise linear with a jump at $x = 10$. Below is a sketch of the budget line.



Explanation: For $x < 10$, the consumer faces the higher price of \$5 per unit, so the budget line follows $y = 10 - 0.5x$ and passes through $(0, 10)$ and $(10, 5)$. However, if the consumer opts to purchase 10 or more units of x, they receive a discount, and the price falls to \$3 per unit for all units of x. In this case, the budget line becomes $y = 10 - 0.3x$ for $x \geq 10$ and passes through $(10, 7)$ and the x-intercept $(33.33, 0)$. Notice that at $x = 10$ there is a discontinuity: purchasing exactly 10 units under the higher price would allow only $y = 5$, whereas qualifying for the discount gives $y = 7$. This jump reflects the benefit of the discount when buying in bulk.

(c) *Formulation of the Piecewise Budget Constraint*

The expenditure on good x depends on the quantity purchased:

$$\text{Expenditure on } x = \begin{cases} 5x, & \text{if } 0 \leq x \leq 10, \\ 5(10) + 3(x - 10) = 50 + 3(x - 10), & \text{if } x > 10. \end{cases}$$

Thus, the consumer's total expenditure is the sum of spending on x and y.

$\underline{0 \leq x \leq 10}$: In this range, the expenditure on x is $5x$ and on y is $10y$. The budget constraint is:

$$5x + 10y = 100.$$

Solving for y:

$$y = 10 - 0.5x.$$

$\underline{x > 10}$: If $x > 10$, the first 10 units cost \$5 each (totaling \$50) and each extra unit costs \$3. Thus, the total cost for x is:

$$50 + 3(x - 10) = 3x + 20.$$

Adding the cost for y, the budget constraint becomes:

$$3x + 20 + 10y = 100 \quad \Longrightarrow \quad 3x + 10y = 80.$$

Solving for y:

$$y = 8 - 0.3x.$$

Sketching the Budget Line

For $0 \leq x \leq 10$:

$$y = 10 - 0.5x.$$

- y-intercept: When $x = 0$, $y = 10$.
- At $x = 10$, $y = 10 - 0.5(10) = 5$.

For $x > 10$:

$$y = 8 - 0.3x.$$

- At $x = 10$, $y = 8 - 0.3(10) = 8 - 3 = 5$ (ensuring the two segments connect at $x = 10$).
- x-intercept: Set $y = 0$, then $3x = 80$ so $x = \frac{80}{3} \approx 26.67$.

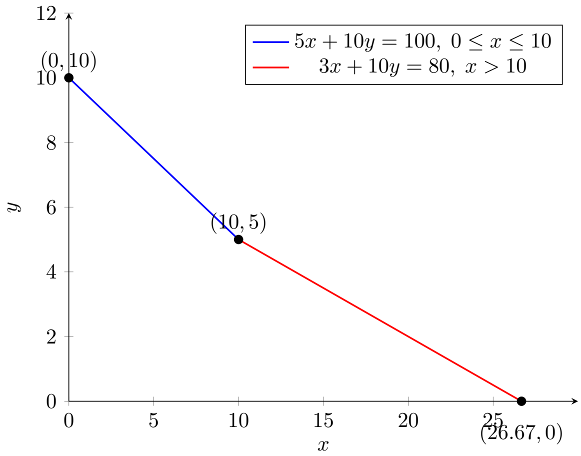
Thus, the budget line is given by:

$$\textbf{For } 0 \leq x \leq 10: \quad y = 10 - 0.5x,$$

$$\textbf{For } x > 10: \quad y = 8 - 0.3x.$$

There is a kink at $x = 10$, $y = 5$ where the pricing rule changes.

Graph:



12
10
8
6
4
2
0
0
5
10
15
20
25
x
y
$(0, 10)$
$(10, 5)$
$(26.67, 0)$
$5x + 10y = 100,\ 0 \leq x \leq 10$
$3x + 10y = 80,\ x > 10$