Simultaneous 3-Player Game

50 pesos must be distributed among 3 players. Each player can choose $x_i \in [0, K]$ where x_i is an integer. Moreover, if the sum of what each player chooses exceeds 50, then the 50 pesos go to player 3 and the other players get 0. But if the sum is less than or equal to 50, each player gets what they chose. In other words, the payouts would be:

$$(0;0;50) \text{ if } x_1 + x_2 + x_3 > 50$$

$$(x_1, x_2, x_3) \text{ if } x_1 + x_2 + x_3 \leq 50$$

1. If $K = 40$, determine if the following cases are NE, and in the case that there are incentives for deviation, indicate which player has such deviation incentive and to what strategy.
   (a) $(40, 40, 40)$
   (b) $(0, 0, 40)$
   (c) $(20, 20, 40)$
   (d) $(25, 25, 25)$
2. Find all the NE if $K = 60$

Answers

1. (a) It is a Nash equilibrium since even if player 1 or 2 change their choice, the sum will always be greater than 50 and they will have a payout of 0. On the other hand, player 3 has the maximum payout (50) and therefore has no incentive to deviate.

   (b) It is not a Nash equilibrium because both player 1 and player 2 can choose a number between 1 and 10 and receive a payout greater than 0, since the sum would not be greater than 50.

   (c) It is a Nash equilibrium, just like in the first case, player 1 and player 2 cannot get a payout greater than 0 no matter what they do, the sum is always greater than 50 and therefore player 3 gets everything. And player 3 has no incentives to lower their choice since they are getting the maximum payout.

   (d) It is a Nash equilibrium, although player 1 or player 2 choose a lower number, the payout they get is equal to 0. While player 3 cannot get a higher payout

2. If $K = 60$, then the Nash equilibria are as follows:

   - (x_1, x_2, x_3) where $x_3 \in [50, 60]$ and $x_1, x_2 \in [0, 60]$. For example $(0, 0, 50)$, $(10, 4, 60)$, $(2, 1, 55)$
   - (x_1, x_2, x_3) where $x_3 \in [0, 49]$, $x_1 + x_3 \geq 50$ and $x_2 + x_3 \geq 50$. For example: $(20, 15, 45)$, $(2, 2, 49)$, $(10, 5, 46)$, $(1, 1, 49)$, $(4, 4, 46)$