Marshallian Demands for an n-Good Perfect-Complements Utility

Consider a consumer whose utility function is given by

$$u(x_1, x_2, \ldots, x_n) \;=\; \min\{\, x_1, x_2, \ldots, x_n\}$$

with prices $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and income m. We want to find the Marshallian (ordinary) demands $x_i(p, m)$ and the indirect utility function $v(p, m)$.

Solution

1. **Utility maximization problem:**

$$\max_{x_1,\ldots,x_n} \quad \min\{\,x_1, x_2, \ldots, x_n\} \quad \text{subject to} \quad \sum_{i=1}^{n} p_i x_i \;=\; m, \quad x_i \ge 0.$$

2. **Perfect-complements condition:** Since all coefficients in the $\min\{\cdot\}$ function are equal (each good appears with a coefficient of 1), the consumer maximizes utility by setting

$$x_1 \;=\; x_2 \;=\; \cdots \;=\; x_n \;=\; x.$$

3. **Budget constraint:** Substituting $x_i = x$ into the budget constraint, we get

$$p_1\,x \;+\; p_2\,x \;+\; \cdots \;+\; p_n\,x \;=\; \Bigl(\sum_{i=1}^{n} p_i\Bigr)\,x \;=\; m.$$

   Solving for x:

$$x \;=\; \frac{m}{\sum_{i=1}^{n} p_i}\,.$$

4. **Marshallian demands:** Because $x_1 = x_2 = \cdots = x_n = x$, the demand for each good i is:

$$x_i(p,m) \;=\; \frac{m}{\sum_{j=1}^{n} p_j}\,.$$

5. **Indirect utility function:** Substitute these demands into the utility function:

$$u(x_1,\ldots,x_n) \;=\; \min\{x_1,\ldots,x_n\} \;=\; x \;=\; \frac{m}{\sum_{i=1}^{n} p_i}\,.$$

   Hence the indirect utility is:

$$v(p,m) \;=\; \frac{m}{\sum_{i=1}^{n} p_i}\,.$$