Implicit system of equations

The demand for good 1 (D_1) is defined implicitly through a system of equations involving D_2, p_1, and p_2. Find $\dfrac{\partial D_1}{\partial p_1}$ and classify the good based on the sign of the derivative.

$$D_1^2 p_1^3 p_2 - 10 + 4D_2 = 0$$
$$D_1 + 3p_2 + p_1^3 + 20 - 10D_2^2 = 0$$

Solution

$$2D_1p_1^3p_2dD_1 + 3D_1^2p_1^2p_2dp_1 + D_1^2p_1^3dp_2 + 4dD_2 = 0$$
$$dD_1 + 3dp_2 + 3p_1^2dp_1 - 20D_2dD_2 = 0$$

Separate dependent and independent variables:

$$2D_1p_1^3p_2dD_1 + 4dD_2 = -3D_1^2p_1^2p_2dp_1 - D_1^2p_1^3dp_2$$
$$dD_1 - 20D_2dD_2 = -3p_1^2dp_1 - 3dp_2$$

Set $dp_2 = 0$:

$$2D_1p_1^3p_2dD_1 + 4dD_2 = -3D_1^2p_1^2p_2dp_1$$
$$dD_1 - 20D_2dD_2 = -3p_1^2dp_1$$

Divide by dp_1:

$$2D_1p_1^3p_2\frac{dD_1}{dp_1} + 4\frac{dD_2}{dp_1} = -3D_1^2p_1^2p_2$$
$$\frac{dD_1}{dp_1} - 20D_2\frac{dD_2}{dp_1} = -3p_1^2$$

Write in matrix form:

$$\begin{bmatrix} 2D_1p_1^3p_2 & 4 \\ 1 & -20D_2 \end{bmatrix} \begin{bmatrix} D_1'p_1 \\ D_2'p_1 \end{bmatrix} = \begin{bmatrix} -3D_1^2p_1^2p_2 \\ -3p_1^2 \end{bmatrix}$$

The determinant of the Jacobian:

$$|J| = -402D_1p_1^3p_2D_2 - 4 = -(402D_1p_1^3p_2D_2 + 4)$$

The final result then:

$$D_1'p_1 = \frac{\begin{vmatrix} -3D_1^2p_1^2p_2 & 4 \\ -3p_1^2 & -20D_2 \end{vmatrix}}{8D_1p_1^3p_2 + 20D_2} = \frac{60D_1^2p_1^2p_2D_2 + 12p_1^2}{-(402D_1p_1^3p_2D_2 + 4)} = -\frac{60D_1^2p_1^2p_2D_2 + 12p_1^2}{402D_1p_1^3p_2D_2 + 4} < 0$$

The good is a typical good as when the price increases, the demand decreases.