Marshallian Demand and Indirect Utility for a Quasilinear Utility Function

Consider a consumer with the utility function

$$u(q_1, q_2) = q_1 + 2q_2^2 - 10,$$

subject to the budget constraint

$$p_1 q_1 + p_2 q_2 = m, \quad q_1, q_2 \geq 0,$$

where $p_1, p_2 > 0$ and $m > 0$.

Derive the Marshallian (uncompensated) demand functions for q_1 and q_2.

Derive the corresponding indirect utility function.

Solution

Since the utility function is separable, linear in q_1, and quadratic in q_2, we substitute the budget constraint into the utility function:

$$q_1 = \frac{m - p_2 q_2}{p_1}.$$

Rewriting the utility function in terms of q_2,

$$u(q_2) = \frac{m}{p_1} - \frac{p_2}{p_1} q_2 + 2q_2^2 - 10.$$

Since the term $2q_2^2$ is convex in q_2, and the objective function consists of a linear term decreasing in q_2 and a convex term, the overall function is convex. Therefore, the maximum occurs at one of the endpoints of the feasible set, where $q_2 \leq \frac{m}{p_2}$.

Corner 1: Specialization in Good 1 ($q_2 = 0$)
If $q_2 = 0$, then

$$q_1 = \frac{m}{p_1}.$$

The resulting utility is:

$$u(0) = \frac{m}{p_1} - 10.$$

Corner 2: Specialization in Good 2 ($q_1 = 0$)
If $q_1 = 0$, then from the budget constraint:

$$q_2 = \frac{m}{p_2}.$$

The corresponding utility is:

$$u\left(0, \frac{m}{p_2}\right) = 2\left(\frac{m}{p_2}\right)^2 - 10 = \frac{2m^2}{p_2^2} - 10.$$

Optimal Choice
To determine which corner solution is chosen, compare:

$$\frac{m}{p_1} - 10 \quad \text{and} \quad \frac{2m^2}{p_2^2} - 10.$$

Setting them equal to find the threshold value of m,

$$\frac{m}{p_1} = \frac{2m^2}{p_2^2}.$$

Solving for m,

$$m = \frac{p_2^2}{2p_1}.$$

Thus, the Marshallian demands are:

$$q_1^*(p_1, p_2, m) = \begin{cases} \frac{m}{p_1}, & \text{if } m \leq \frac{p_2^2}{2p_1}, \\ 0, & \text{if } m > \frac{p_2^2}{2p_1}. \end{cases}$$

$$q_2^*(p_1, p_2, m) = \begin{cases} 0, & \text{if } m \leq \frac{p_2^2}{2p_1}, \\ \frac{m}{p_2}, & \text{if } m > \frac{p_2^2}{2p_1}. \end{cases}$$

Indirect Utility Function

Since the indirect utility function, $V(m, p_1, p_2)$, represents the maximum utility attainable given m and prices:

$$V(m, p_1, p_2) = \begin{cases} \frac{m}{p_1} - 10, & \text{if } m \leq \frac{p_2^2}{2p_1}, \\ \frac{2m^2}{p_2^2} - 10, & \text{if } m > \frac{p_2^2}{2p_1}. \end{cases}$$