Removable Discontinuity

Find the domain, classify discontinuities into removable and essential. If removable, extend the function to make it continuous.

$$f(x) = \frac{3 - \sqrt{x+1}}{x - 8}$$

Solution

The domain is $(-1, 8) \cup (8, +\infty)$. The function is not continuous at $x = 8$. We take the limit:

$$\lim_{x \to 8} \frac{3 - \sqrt{x+1}}{x - 8} = \lim_{x \to 8} \frac{3 - \sqrt{x+1}}{x - 8} \frac{3 + \sqrt{x+1}}{3 + \sqrt{x+1}}$$

$$\lim_{x \to 8} \frac{9 - (x+1)}{(x-8)(3 + \sqrt{x+1})} = \lim_{x \to 8} \frac{8 - x}{(x-8)(3 + \sqrt{x+1})}$$

$$\lim_{x \to 8} -\frac{x - 8}{(x-8)(3 + \sqrt{x+1})} = \lim_{x \to 8} \frac{1}{3 + \sqrt{x+1}} = 1/6$$

For the function to be continuous at $x = 8$:

$$f(x) = \begin{cases} \dfrac{3 - \sqrt{x+1}}{x - 8} & \text{if } x \neq 8 \\ 1/6 & \text{if } x = 8 \end{cases}$$