

Derivative of a composite function

Calculate the partial derivatives of the following composite function:

$$z = \ln(3u^2 - uv + v^3)$$

where

$$u = \sin x \quad \text{and} \quad v = xy.$$

Solution

$$\begin{aligned} z_x &= z'_u u'_x + z'_v v'_x \\ z_x &= \frac{6u - v}{3u^2 - uv + v^3} \cdot \cos x + \frac{-u + 3v^2}{3u^2 - uv + v^3} \cdot y \end{aligned}$$

$$\begin{aligned} z_y &= z'_u u'_y + z'_v v'_y \\ z_y &= \frac{-u + 3v^2}{3u^2 - uv + v^3} \cdot x \end{aligned}$$