Calculation of radial limit in two variables

Calculate the radial limit of the following function:

$$\lim_{(x,y)\to(0,0)} (x+y)[\sin(1/x) + \sin(1/y)]$$

Solution

Calculate the radial limit:

$$y = mx$$

$$(x + mx)[\sin(1/x) + \sin(1/mx)]$$

$$x(1 + m)[\sin(1/x) + \sin(1/mx)]$$

$x(1 + m)$ tends to 0 while $[\sin(1/x) + \sin(1/mx)]$ is a bounded function between -2 and 2. Therefore, infinitesimal times bounded gives us 0.