

Aggregation of Linear Consumer Demands

Consider a market for a good x with price p . There are three consumers in the market with individual linear demand functions given by:

$$x_A(p) = \max\{20 - 2p, 0\},$$

$$x_B(p) = \max\{15 - p, 0\},$$

$$x_C(p) = \max\{10 - 0.5p, 0\}.$$

Derive the aggregate market demand $Q(p)$ by summing the individual demands. Express the aggregate demand as a piecewise function of the price p .

Solution

Identify Demand Thresholds

Each consumer's demand becomes zero when the expression inside the $\max\{\cdot\}$ is non-positive:

$$\begin{aligned} 20 - 2p \leq 0 &\implies p \geq 10, \\ 15 - p \leq 0 &\implies p \geq 15, \\ 10 - 0.5p \leq 0 &\implies p \geq 20. \end{aligned}$$

Aggregate Demand by Price Intervals

(i) **For $p < 10$:** All consumers are active. The aggregate demand is:

$$\begin{aligned} Q(p) &= x_A(p) + x_B(p) + x_C(p) \\ &= (20 - 2p) + (15 - p) + (10 - 0.5p) \\ &= 45 - 3.5p. \end{aligned}$$

(ii) **For $10 \leq p < 15$:** Consumer A drops out ($x_A(p) = 0$), while B and C remain active:

$$\begin{aligned} Q(p) &= 0 + (15 - p) + (10 - 0.5p) \\ &= 25 - 1.5p. \end{aligned}$$

(iii) **For $15 \leq p < 20$:** Consumers A and B are inactive, leaving only Consumer C :

$$\begin{aligned} Q(p) &= 0 + 0 + (10 - 0.5p) \\ &= 10 - 0.5p. \end{aligned}$$

(iv) **For $p \geq 20$:** All consumers have zero demand:

$$Q(p) = 0.$$

Final Aggregate Demand Function

The aggregate market demand function $Q(p)$ is given by:

$$Q(p) = \begin{cases} 45 - 3.5p, & \text{if } p < 10, \\ 25 - 1.5p, & \text{if } 10 \leq p < 15, \\ 10 - 0.5p, & \text{if } 15 \leq p < 20, \\ 0, & \text{if } p \geq 20. \end{cases}$$