Limit analysis

Calculate double, iterative, and radial limits, if possible. Draw conclusions.

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$$

Solution

Iterative limits:

$$L_1 = \lim_{y \to 0}[\lim_{x \to 0} \frac{xy^3}{x^2 + y^6}] = \lim_{y \to 0} \frac{0 \cdot y^3}{0^2 + y^6} = \lim_{y \to 0} \frac{0}{y^6} = \lim_{y \to 0} 0 = 0$$

$$L_2 = \lim_{x \to 0}[\lim_{y \to 0} \frac{xy^3}{x^2 + y^6}] = \lim_{x \to 0} \frac{x \cdot 0^3}{x^2 + 0^6} = \lim_{x \to 0} \frac{0}{x^2} = \lim_{x \to 0} 0 = 0$$

Radial limit:

$$y - y_0 = m(x - x_0)$$

$$y = mx$$

$$L_r = \lim_{x \to 0} \frac{xx^3 m^3}{x^2 + x^6 m^6} = \lim_{x \to 0} \frac{x^4 m^3}{x^2(1 + m^6 x^4)} = \lim_{x \to 0} \frac{x^2 m^3}{(1 + m^6 x^4)} = 0$$

The calculated limits cannot conclude the existence or non-existence of the limit