# Maximizing profit

Consider a market participant firm that operates under perfect competition. This firm sells its output at a constant market price $p$ greater than zero, and incurs a cost $w$ greater than zero for each unit of the single input it utilizes. The firm's production function is denoted by $f(x)$, with $x$ being the amount of the variable input used. We aim to determine the input level $x$ that will maximize the firm's profit. The production function is given by $f(x) = x^2$.

1. Write the profit function and explain if it is concave.

2. Find the critical points (if there are any).

## Solution

1. The profit $\Pi$ of a competitive firm as a function of input $x$ can be modeled by the expression

$$\Pi(x) = px^2 - wx$$

   This quadratic function does not display concavity as its second derivative, $2p$ is positive for all values of $x$. The profit function is convex.

2. For profit maximization, if we set the derivative of the profit function to zero, yielding $\Pi'(x) = 2px - w = 0$, which implies $x = \frac{w}{2p}$. Evaluating the profit function at this point gives us $p\left(\frac{w}{2p}\right)^2 - w\left(\frac{w}{2p}\right)$, resulting in a negative value.

   Given the convex nature of the profit function, it is evident that with an increase in $x$, the profit function escalates without bound, signifying that it does not attain a maximal value.