Budget Constraint with a Free-Good Promotion

A consumer has an income of $m = \$100$. There are two goods: x and y. The price of good x is constant at $p_x = \$5$ per unit. The price of good y is normally $p_y = \$2$ per unit. However, there is a promotion on good y: if the consumer purchases enough y so that her total received quantity is at least 40 units, she receives an extra 10 units for free. (In other words, if the consumer's total consumption of y is at least 40 units, then she only pays for $y - 10$ units.)

Assume that the consumer's choice variable is the total number of units of y received. Write down the piecewise budget constraint and sketch the corresponding budget line on the (x, y) plane. Clearly indicate the key intercepts and the kink that arises from the promotion.

Solution

Step 1. Formulating the Budget Constraint

Since the promotion affects only good y, we treat the expenditure on y as follows:

$0 \leq y < 40$: When the consumer receives fewer than 40 units of y, she is not eligible for the promotion. Hence, she pays the full price:

$$\text{Expenditure on } y = 2y.$$

The expenditure on x is $5x$. Thus, the budget constraint in this region is:

$$5x + 2y = 100, \quad \text{for } 0 \leq y < 40.$$

$y \geq 40$: When the consumer receives 40 or more units of y, she obtains an extra 10 units free. That is, for every bundle with total $y \geq 40$ units, she pays only for $y - 10$ units. Therefore, her expenditure on y is:

$$\text{Expenditure on } y = 2(y - 10).$$

The budget constraint in this case becomes:

$$5x + 2(y - 10) = 100 \quad \Longrightarrow \quad 5x + 2y = 120, \quad \text{for } y \geq 40.$$

Step 2. Finding Key Intercepts

For $0 \leq y < 40$: The constraint is

$$5x + 2y = 100.$$

- x-intercept: Set $y = 0 \Rightarrow 5x = 100$ so $x = 20$.
- y-intercept (within this segment): Set $x = 0 \Rightarrow 2y = 100$ so $y = 50$. However, note that this segment is only valid for $y < 40$. Thus, we only use the portion for $0 \leq y < 40$. In particular, at $y = 40$ the corresponding x-value is:

$$5x + 2(40) = 100 \quad \Longrightarrow \quad 5x = 100 - 80 = 20, \quad x = 4.$$

For $y \geq 40$: The constraint is

$$5x + 2y = 120.$$

- At the threshold $y = 40$, we have:

$$5x + 2(40) = 120 \quad \Longrightarrow \quad 5x = 120 - 80 = 40, \quad x = 8.$$

- x-intercept: Set $y = 0 \Rightarrow 5x = 120$ so $x = 24$.
- y-intercept: Set $x = 0 \Rightarrow 2y = 120$ so $y = 60$.

Step 3. Sketching the Budget Line

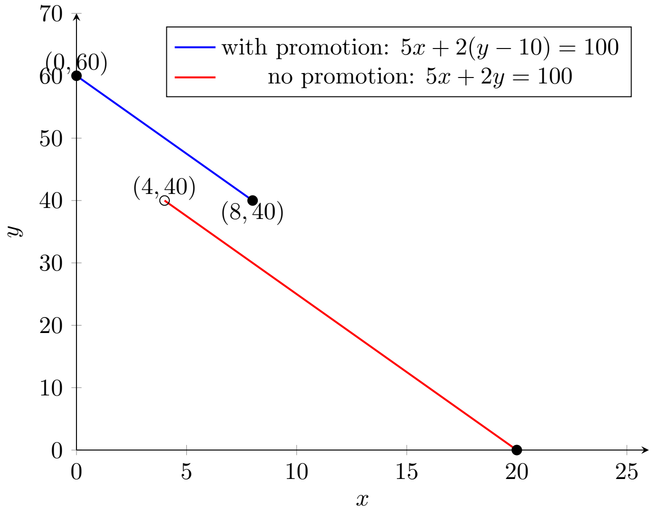
The budget line is piecewise linear with a kink at the promotion threshold:

$$\begin{aligned} \text{For } 0 \leq y < 40: &\quad 5x + 2y = 100 \quad \Longrightarrow \quad y = 50 - 2.5x, \\ \text{For } y \geq 40: &\quad 5x + 2y = 120 \quad \Longrightarrow \quad y = 60 - 2.5x. \end{aligned}$$

Thus, the two segments are:

- **Segment 1** (no promotion): from the x-intercept $(20, 0)$ to the point at $y = 40$, where $x = 4$.
- **Segment 2** (with promotion): from the point at $y = 40$, where $x = 8$, to the y-intercept $(0, 60)$.

Notice that there is a *jump* in the budget line at $y = 40$: if the consumer purchases less than 40 units of y (receiving no bonus), at $y = 40$ she can only afford $x = 4$; but if she qualifies for the promotion (receiving 10 free units), then at $y = 40$ she must pay for only 30 units, allowing her to afford $x = 8$.



Explanation: For $y < 40$, the consumer pays full price for every unit of y, so the budget line is given by $5x + 2y = 100$. However, if the consumer opts to receive at least 40 units of y (thereby qualifying for 10 extra free units), the cost for y becomes $2(y - 10)$ and the budget constraint shifts to $5x + 2y = 120$. The resulting budget line has a discontinuity (kink) at $y = 40$ where the affordable x-quantity jumps from $x = 4$ (without the promotion) to $x = 8$ (with the promotion).