Determining the domain in $\mathbb{R}^2$

1. $$f(x,y) = 5xy + 3y$$

2. $$f(x,y) = \sqrt{9 - x^2 - y^2}$$

3. $$f(x,y) = \sqrt{x^2/4 + y^2/25 - 1}$$

4. $$f(x,y) = ln(-x^2 + 4 - y)$$

5. $$f(x,y) = \frac{\sqrt{25 - x^2 - y^2}}{xy}$$

6. $$f(x,y) = \frac{1}{ln(x + y - 5)}$$

Solution

1.
$$D(f) = \mathbb{R}$$

2.
$$9 - x^2 - y^2 \geq 0$$
$$9 \geq x^2 + y^2$$

$$D(f) = \{(x, y) \in \mathbb{R}^2 \mid 9 \geq x^2 + y^2\}$$

3.
$$x^2/4 + y^2/25 - 1 \geq 0$$
$$x^2/4 + y^2/25 \geq 1$$
$$D(f) = \{(x, y) \in \mathbb{R}^2 \mid x^2/4 + y^2/25 \geq 1\}$$

4.
$$-x^2 + 4 - y > 0$$
$$4 > y + x^2$$
$$D(f) = \{(x, y) \in \mathbb{R}^2 \mid 4 > y + x^2\}$$

5.
$$x \neq 0$$
$$y \neq 0$$
$$25 - x^2 - y^2 \geq 0$$

$$25 \geq x^2 + y^2$$
$$D(f) = \{(x, y) \in \mathbb{R}^2 \mid (25 \geq x^2 + y^2) \wedge (x \neq 0 \wedge \neq 0)\}$$

6.
$$x + y - 5 > 0$$
$$x + y > 5$$

$$x + y - 5 \neq 1$$

$$D(f) = \{(x, y) \in \mathbb{R}^2 \mid (x + y > 5) \wedge (x + y - 5 \neq 1)\}$$