Partial derivatives 3

Let

$$Z = \ln(x^2 + xy + y^2)$$

verify that $xz_x + yz_y = 2$

Solution

Given the partial derivatives of the function:

$$z_x = \frac{2x + y}{x^2 + xy + y^2}$$

and

$$z_y = \frac{x + 2y}{x^2 + xy + y^2}$$

Then, substituting into the given expression, it is verified:

$$x \cdot z_x + y \cdot z_y = 2$$

$$x \cdot \frac{2x + y}{x^2 + xy + y^2} + y \cdot \frac{x + 2y}{x^2 + xy + y^2} = 2$$

$$\frac{2x^2 + 2yx + 2y^2}{x^2 + xy + y^2} = 2$$

$$2 \cdot \frac{x^2 + xy + y^2}{x^2 + xy + y^2} = 2$$