Double limit

Given the following function:

$$f(x, y) = \frac{xy - 2y + x - 2}{(x^2 - 4)(y^3 + 1)}$$

Calculate, if it exists, the double limit at the following points: $P_1 = (1, 1)$ and $P_2 = (2, -1)$.

Solution

$$\lim_{x,y\to(1,1)} \frac{xy - 2y + x - 2}{(x^2 - 4)(y^3 + 1)} = \frac{1 - 2 + 1 - 2}{-3 \cdot 3} = \frac{2}{9}$$

$$\lim_{x,y\to(2,-1)} \frac{xy - 2y + x - 2}{(x^2 - 4)(y^3 + 1)} = \lim_{x,y\to(2,-1)} \frac{y(x - 2) + (x - 2)}{(x + 2)(x - 2)(y^3 + 1)} = \lim_{x,y\to(2,-1)} \frac{y + 1}{(x + 2)(y^3 + 1)}$$

Furthermore:

$$y^3 + 1 = (y + 1)(y^2 - y + 1)$$

Therefore:

$$\lim_{x,y\to(2,-1)} \frac{y + 1}{(x + 2)(y + 1)(y^2 - y + 1)} = \lim_{x,y\to(2,-1)} \frac{1}{(x + 2)(y^2 - y + 1)} = \frac{1}{12}$$