# Marshallian Demands for Quasi-Linear Preferences

Given the utility function:

$$U = x_1 + \ln(x_2)$$

with prices $p_1$, $p_2$ and income $m$. Calculate:

1. Marshallian demands
2. Indirect utility function

# Solution

1. We set up the Lagrangian:
$$L = x_1 + \ln(x_2) + \lambda(m - p_1x_1 - p_2x_2)$$
The first-order conditions are:
$$\frac{\partial L}{\partial x_1} = 1 - \lambda p_1 = 0 \quad \frac{\partial L}{\partial x_2} = \frac{1}{x_2} - \lambda p_2 = 0 \quad \frac{\partial L}{\partial \lambda} = m - p_1x_1 - p_2x_2 = 0$$
From the first two:
$$\frac{1}{p_1} = \lambda \quad \frac{1}{x_2p_2} = \lambda$$
Equating $\lambda$:
$$\frac{1}{p_1} = \frac{1}{x_2p_2} \quad \Longrightarrow \quad x_2 = \frac{p_1}{p_2}$$
Using the budget constraint:
$$m - p_1x_1 - p_2x_2 = 0 \quad \Longrightarrow \quad m - p_1x_1 - p_2\left(\frac{p_1}{p_2}\right) = 0 \quad \Longrightarrow \quad m - p_1x_1 - p_1 = 0 \quad \Longrightarrow \quad x_1 = \frac{m - p_1}{p_1}$$
If $m - p_1 < 0$, we must consider a corner solution $x_1 = 0$ and $x_2 = \frac{m}{p_2}$. Thus, the Marshallian demands are piecewise:
$$\mathbf{x_1^m} = \begin{cases} \frac{m-p_1}{p_1} & \text{if } m \geq p_1 \\ 0 & \text{if } m < p_1 \end{cases}$$
$$\mathbf{x_2^m} = \begin{cases} \frac{p_1}{p_2} & \text{if } m \geq p_1 \\ \frac{m}{p_2} & \text{if } m < p_1 \end{cases}$$

2. The indirect utility function is also piecewise:
$$\mathbf{V(p_1, p_2, m)} = \begin{cases} \dfrac{m - p_1}{p_1} + \ln\left(\dfrac{p_1}{p_2}\right) & \text{if } m \geq p_1 \\ \ln\left(\dfrac{m}{p_2}\right) & \text{if } m < p_1 \end{cases}$$