Elasticity and market equilibrium

Consider a market with 800 consumers. Each one has a Marshallian demand for good y which is given by m/p_y.

1. Calculate the aggregate demand y^A
2. Calculate the price elasticity of aggregate demand. And show that this elasticity is equal to $\frac{\partial ln(y^A)}{\partial ln(p_y)}$, (this is another way to calculate elasticity).
3. Assume there is only one company in the market that has the following long-term cost function: $C(y) = 150 + y^2$. However, this company is owned by the State and acts as if it were in a perfectly competitive market. Find the equilibrium quantity and price, as well as the company's profits. (Hint: obtain the company's supply curve and equate aggregate demand and supply.)

Solution

1. If each individual has the following demand m/p_y. Multiplying by 800 we get the aggregate demand:

$$y^A = \frac{800}{p_y}$$

2. We calculate elasticity with the formula:

$$\frac{Ey^A}{Ep_y} = \frac{\partial y^A}{\partial p_y}\frac{p_y}{y^A} = -\frac{m}{p_y^2}\frac{p_y}{m/p_y} = -1$$

   Using the other form $ln(y^A) = ln(800) - ln(p_y)$:

$$\frac{\partial ln(y^A)}{\partial ln(p_y)} = -1$$

3. We get the supply curve which is equal to the marginal cost:

$$C'_y = 2y$$

   We equate with the inverse demand:

$$800/y = 2y$$
$$400 = y^2$$
$$y = 20$$

   We get the price:

$$p_y = 800/20 = 40$$

   And we get the company's profits:

$$B = 20 * 40 - (150 + 20^2) = 250$$