Exercises on Homogeneity and Elasticity

1. Find the value of a such that the following function is homogeneous and of degree 1: $f(x,y) = \sqrt[a]{x^3y^2 + 2x^2y^3}$
2. Prove that if $f(x,y)$ is a homogeneous function of degree n, then the function $g(x,y) = yx \cdot \frac{\partial f}{\partial x} + xy \cdot \frac{\partial f}{\partial y}$ is also homogeneous and of degree $n+1$.
3. Suppose that $f(x,y)$ is a homogeneous function of degree n. Determine if the following function is homogeneous and if so, determine its degree of homogeneity: $g(x,y) = f(x,y)y^2 - x^2y^n$.
4. Given a demand function $Q(p) = Ap^{-\alpha}$, where A and α are positive constants, calculate the price elasticity of demand. And show for what values of α the function is elastic.
5. Consider a supply function of the form $Q(p) = Ap^{\alpha}(1 + \beta p)$. Calculate the price elasticity of supply.

Solutions

1.
$$f(xt, yt) = \sqrt[a]{t^5x^3y^2 + t^5 2x^2y^3}$$
$$f(xt, yt) = t^{5/a}\sqrt[a]{x^3y^2 + 2x^2y^3}$$

Therefore if $a = 5$
$$f(xt, yt) = t^{5/5}\sqrt[a]{x^3y^2 + 2x^2y^3}$$
$$f(xt, yt) = t^1 f(x, y)$$

2.
$$g(xt, yt) = ytxt \cdot f_y(xt, ty) + xtyt \cdot f_x(tx, ty)$$
$$g(xt, yt) = xt^2t^{n-1} \cdot f_y(x, y) + t^2yt^{n-1} \cdot f_x(x, y)$$
$$g(xt, yt) = t^{n+1}x \cdot f_y(x, y) + yt^{n+1} \cdot f_x(x, y)$$
$$g(xt, yt) = t^{n+1}[x \cdot f_y(x, y) + y \cdot f_x(x, y)]$$
$$g(xt, yt) = t^{n+1}g(x, y)$$

3.
$$g(x, y) = f(x, y)y^2 - x^2y^n$$
$$g(xt, yt) = t^n f(x, y)t^2y^2 - t^{2+n}x^2y^n$$
$$g(xt, yt) = t^{n+2}g(x, y)$$

Therefore $g(x, y)$ is a homogeneous function of degree $n + 2$.

4.
$$\frac{\partial Q}{\partial p}\frac{p}{Q} = A(-\alpha)p^{-\alpha-1}\frac{p}{Ap^{-\alpha}} = -\alpha$$

5.
$$\frac{\partial Q}{\partial p}\frac{p}{Q} = [A\alpha p^{\alpha-1} + A\beta(\alpha+1)p^{\alpha}]\frac{p}{Ap^{\alpha} + A\beta p^{\alpha+1}}$$
$$\frac{\partial Q}{\partial p}\frac{p}{Q} = \frac{\alpha + \beta(\alpha+1)p}{A + \beta p}$$