Expected value and probabilities in a joint distribution

The following table shows the joint probabilities of the number of goals scored by team X and team Y when they face each other.

$Y \backslash X$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	0.1	0.1	0.04	0.02
$Y = 1$	0.1	0.15	0.05	0.03
$Y = 2$	0.09	0.13	0.02	0.01
$Y = 3$	0.01	0.1	0.02	$\dots$

Solution

The sum of all probabilities is 0.97. We calculate the missing value:

$$p = 1 - S = 1 - 0.97 = 0.03$$

We update the complete table:

$Y \backslash X$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	0.10	0.10	0.04	0.02
$Y = 1$	0.10	0.15	0.05	0.03
$Y = 2$	0.09	0.13	0.02	0.01
$Y = 3$	0.01	0.10	0.02	**0.03**

The total number of goals is $T = X + Y$.
The expectation of T is:

$$E[T] = E[X] + E[Y]$$

Marginal probabilities for X:

$$\begin{aligned}
P(X = 0) &= 0.10 + 0.10 + 0.09 + 0.01 = 0.30 \\
P(X = 1) &= 0.10 + 0.15 + 0.13 + 0.10 = 0.48 \\
P(X = 2) &= 0.04 + 0.05 + 0.02 + 0.02 = 0.13 \\
P(X = 3) &= 0.02 + 0.03 + 0.01 + 0.03 = 0.09
\end{aligned}$$

Marginal probabilities for Y:

$$\begin{aligned}
P(Y = 0) &= 0.10 + 0.10 + 0.04 + 0.02 = 0.26 \\
P(Y = 1) &= 0.10 + 0.15 + 0.05 + 0.03 = 0.33 \\
P(Y = 2) &= 0.09 + 0.13 + 0.02 + 0.01 = 0.25 \\
P(Y = 3) &= 0.01 + 0.10 + 0.02 + 0.03 = 0.16
\end{aligned}$$

Calculation of $E[X]$:

$$\begin{aligned}
E[X] &= \sum_{x=0}^{3} x \cdot P(X = x) \\
&= 0 \cdot 0.30 + 1 \cdot 0.48 + 2 \cdot 0.13 + 3 \cdot 0.09 \\
&= 0 + 0.48 + 0.26 + 0.27 \\
&= 1.01
\end{aligned}$$

Calculation of $E[Y]$:

$$\begin{aligned}
E[Y] &= \sum_{y=0}^{3} y \cdot P(Y = y) \\
&= 0 \cdot 0.26 + 1 \cdot 0.33 + 2 \cdot 0.25 + 3 \cdot 0.16 \\
&= 0 + 0.33 + 0.50 + 0.48 \\
&= 1.31
\end{aligned}$$

Calculate $E[T]$:

$$E[T] = E[X] + E[Y] = 1.01 + 1.31 = 2.32$$

Team X wins when $X > Y$.
Identify the combinations where $X > Y$:

X	Y	$P(X, Y)$
1	0	0.10
2	0	0.04
2	1	0.05
3	0	0.02
3	1	0.03
3	2	0.01

Sum the probabilities of these combinations:

$$\begin{aligned} P(X > Y) &= P(1,0) + P(2,0) + P(2,1) + P(3,0) + P(3,1) + P(3,2) \\ &= 0.10 + 0.04 + 0.05 + 0.02 + 0.03 + 0.01 \\ &= 0.25 \end{aligned}$$

The expectation of the total number of goals is:

$$E[T] = 2.32$$

On average, 2.32 goals are expected in the match.
The probability that team X wins is:

$$P(X > Y) = 0.25$$