Calculate approximate value of a Function using differential

Given the function of the exercise $z = x^2 + y^2$, calculate the approximate value of $f(5.12, 6.85)$ applying the differential.

Solution

First, we propose a nearby point $(5, 7)$. Now we calculate the difference between the point we have and the original point: $(5.12, 6.85) - (5, 7) = (0.12, -0.15)$. Additionally, we calculate the value of the function at the proposed point $f(5, 7) = 25 + 49 = 74$. Finally, we calculate the differential of the function:

$$dz = 2x dx + 2y dy$$

We insert the differences between our point and the original point into dx and dy, while we evaluate the derivatives at the point we chose:

$$2 * 5(0.12) + 2 * 7(-0.15) = 1.2 - 2.1 = -0.9$$

And with this function, we subtract the variation which is -0.9.

$$f(5.12, 6.85) \approx 74 - 0.9 = 73.1$$