Tangent Plane

Find the equation of the tangent plane to the surface:

1.
$$z = 2x^2y + y^2 - x + 1 \quad \text{at} \quad P_0 = (1, 3, 15)$$

2.
$$z = x^2 + y^2 \quad \text{at} \quad P_0 = (2, -1)$$

Solution

Definition of the tangent plane:

Given a surface:

$$z = f(x, y)$$

$$\Delta z = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$$

$$dz = f'_x(x_0, y_0)\Delta x + f'_y(x_0, y_0)\Delta y$$

The equation of the tangent plane to the surface at the point (x_0, y_0):

$$z = f(x_0, y_0) + f'_x(x_0, y_0)(x - x_0) + f'_y(x_0, y_0)(y - y_0)$$

1.

$$z = 15 + (4 \cdot 3 - 1)(x - 1) + (2 + 6)(y - 3)$$
$$z = 15 + 11x - 11 + 8y - 24$$
$$z = 11x + 8y - 20$$

2.

$$\frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial y} = 2y$$

$$z = f(2, -1) + \frac{\partial z}{\partial x}(2, -1) \cdot (x - 2) + \frac{\partial z}{\partial y}(2, -1) \cdot (y + 1)$$

$$z = 5 + 4(x - 2) - 2(y + 1)$$