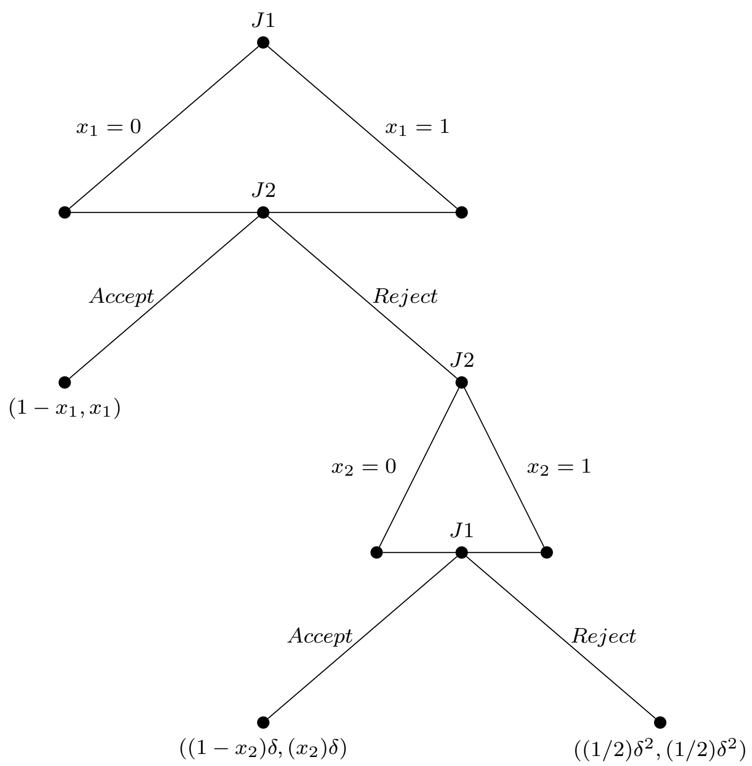
Dynamic negotiation game with infinite strategies

J1 and J2 find \$1 on the floor and must decide how to split it. The way they will split it is through a sequence of offers. First, J1 will offer to keep an amount of $1 - x_1$, with x_1 corresponding to J2. After seeing the offer, J2 must accept or reject it. If he accepts, the game ends. If he rejects, in the next period he will offer to split the weight such that he keeps x_2 and J1: $1 - x_2$. If J1 accepts, the game ends; if he rejects, they split the weight equally in a third period. Both players consider that, in present terms, the subjective valuation of \$1 today is different from \$1 tomorrow. The discount factor for J1 and J2 is $\delta \in (0; 1)$ (the same for both).

1. Determine the perfect outcome in subgames.
2. Suppose that in the third period, J1 would keep \$1. Determine the perfect outcome in subgames.
3. Now assume that, since J2 realizes that J1 would finally keep \$1 if they do not reach an agreement, he tells his mother, who establishes that if they do not manage to agree in the first two periods with the established mechanism, in the third period she will keep the money. What will have motivated J2 to tell his mother if he would ultimately not give him the money?

Solutions

1. The game represented in extensive form:



Starting from the final node we know that the player must accept or reject, $J1$ accepts if:

$$(1 - x_2)\delta \geq (1/2)\delta^2$$

(we assume he accepts when indifferent). Therefore:

$$1 - x_2 \geq (1/2)\delta$$

$$1 - (1/2)\delta \geq x_2$$

Since $J2$'s objective before making the offer is to get the highest possible x_2, this leads him to choose $x_2 = 1 - (1/2)\delta$, as in this way $J1$ accepts the offer. Therefore, so far we know that in the second period $J2$ offers $x_2 = 1 - (1/2)\delta$ and $J1$ accepts. Previously, in period 1, $J1$ has to make the offer and as he is rational, he knows that if his offer is rejected the result will be that $J2$ gets to play and he makes the offer $x_2 = 1 - (1/2)\delta$ and finally this offer will be accepted and the payments would be:

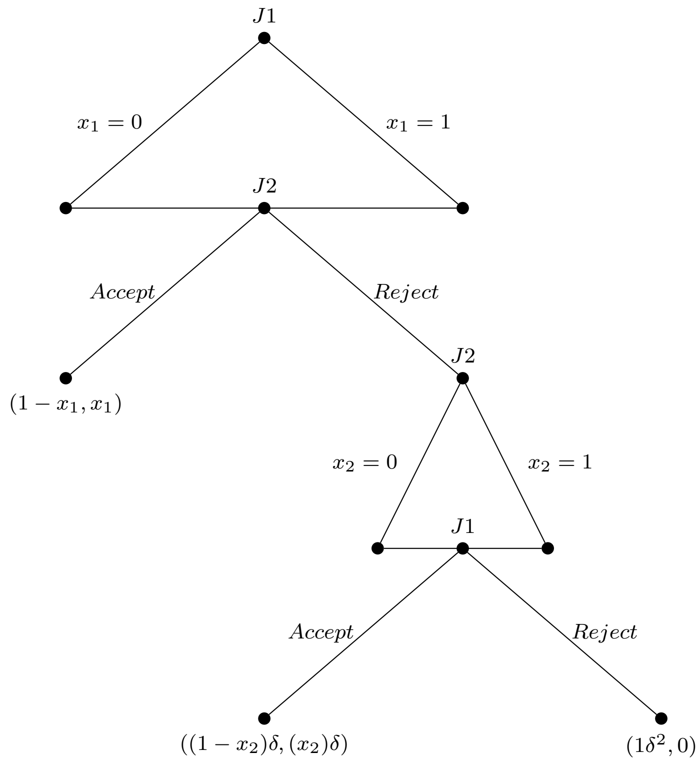
$$\left(\overbrace{(1 - (1 - (1/2)\delta))\delta}^{(1-x_2)\delta}; \overbrace{(1 - (1/2)\delta)\delta}^{(x_2)\delta}\right)$$

Since this is the end of the game $J1$ in the first period has to make an offer such that:

$$x_1 \geq (1 - (1/2)\delta)\delta$$

For $J2$ to accept, and as $J1$ wants to maximize his payment which is $1 - x_1$, he offers the minimum possible which is: $x_1 = (1 - (1/2)\delta)\delta$. The perfect outcome in subgames is then that in the first period $J1$ offers $x_1 = (1 - (1/2)\delta)\delta$ and $J2$ accepts.

2. Now the payments of the third period change, so the game written in extensive form is:



This affects the bargaining power in the last period, now $J2$ has to make an offer such that:

$$(1 - x_2)\delta \geq 1\delta^2$$
$$(1 - x_2) \geq \delta$$
$$1 - \delta \geq x_2$$

As $J2$ wants to maximize his payment (by increasing x_2), he will offer: $x_2 = 1 - \delta$. $J1$, one period earlier, has to make an offer again knowing that in case $J2$ rejects it, the outcome of the game will be:

$$\left(\overbrace{1 - (1 - \delta)\delta}^{(1-x_2)\delta}; \overbrace{(1 - \delta)\delta}^{(x_2)\delta}\right)$$

Therefore, he has to fulfill the following condition so that $J2$ accepts in the first period:

$$x_1 \geq (1 - \delta)\delta$$

And as $J1$ wants to minimize the value of x_1 to get the highest possible payment, he offers $(1-\delta)\delta = x_1$ and the game would end in the first period again.

3. Now the extensive form of the game would be as follows: [Image of extensive form game not translatable]

   Following previous reasonings, the first condition that must be met: $(1 - x_2)\delta \geq 0$, then:

$$x_2 = 1$$

   Therefore, the payments would be:

$$\left(\overbrace{(1 - 1)\delta}^{(1-x_2)\delta}; \overbrace{1\delta}^{x_2\delta}\right)$$

   So the condition in the first period:

$$\delta = x_1$$

   The perfect result in subgames would then be that $J1$ offers $x_1 = \delta$ and $J2$ accepts. What is happening here is that $J1$ loses bargaining power in the final stage of the game, this allows $J2$ to make a lower offer and for this to be accepted. Comparing the perfect results in subgames we have the following:

- First case: $x_1 = (1 - (1/2)\delta)\delta$
- Second case: $x_1 = (1 - \delta)\delta$
- Third case: $x_1 = \delta$

Dividing the 3 cases by δ as this appears in all 3 situations:

- First case: $x_1 = 1 - (1/2)\delta$
- Second case: $x_1 = 1 - \delta$
- Third case: $x_1 = 1$

Here we can see that the situation that is most advantageous for $J2$ is the third one, and for $J1$ his best situation is the second one.