Marshallian Demands for an n-Good max{} Utility

Consider a consumer whose utility function is

$$u(x_1, x_2, \ldots, x_n) \;=\; \max\{\, x_1,\; x_2,\; \ldots,\; x_n\},$$

with prices $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and income m. We want to find the Marshallian (ordinary) demands $x_i(p, m)$ and the indirect utility function $v(p, m)$.

Solution

1. **Utility maximization problem:**

$$\max_{x_1,x_2,\ldots,x_n} \max\{x_1, x_2, \ldots, x_n\} \quad \text{subject to} \quad \sum_{i=1}^{n} p_i x_i = m, \quad x_i \geq 0.$$

2. **Maximizing $\max\{x_1,\ldots,x_n\}$ under a budget:**

   Because the utility is the maximum of all x_i, the consumer wants to make one of the x_i's as large as possible. Intuitively, the best option is to spend all income on the *cheapest* good.

   - Let $p_{i^*} = \min\{p_1,\ldots,p_n\}$ be the lowest price among all goods.
   - Then using all income on good i^* yields $x_{i^*} = \dfrac{m}{p_{i^*}}$ and $x_j = 0$ for $j \neq i^*$.
   - This choice maximizes $\max\{x_1,\ldots,x_n\}$ because buying any more expensive good would reduce the quantity the consumer can afford and thus reduce the overall maximum.

3. **Marshallian demands:**

   If there is a *unique* lowest price p_{i^*}, the corner solution is:

$$x_{i^*}(p,m) = \frac{m}{p_{i^*}}, \quad x_j(p,m) = 0 \text{ for all } j \neq i^*.$$

   If there is a *tie* (multiple goods share the same lowest price), the consumer is indifferent among any combination of those lowest-priced goods. Any such combination gives the same utility $\frac{m}{\min_i p_i}$.

4. **Indirect utility function:**

   Spending all income on the good with the lowest price p_{i^*} yields a maximum quantity of $\dfrac{m}{p_{i^*}}$. Hence, the indirect utility is

$$v(p,m) = \max_i \frac{m}{p_i} = \frac{m}{\min_i p_i}.$$

   This reflects that utility is the largest x_i the consumer can buy, achieved by dedicating all income to the cheapest good.