First-order differential equation

Solve the following differential equation:

$$y' = t^2 - 3t^2 y$$

Solution

Rearranging:

$$\frac{dy}{dt} = t^2 - 3t^2 y$$

$$dy = (t^2 - 3t^2 y)dt$$

$$dy = (1 - 3y)t^2 dt$$

Assuming $1 - 3y \neq 0$:

$$\frac{dy}{(1 - 3y)} = t^2 dt$$

Integrating both sides:

$$\frac{-\ln(|3y - 1|)}{3} = \frac{t^3}{3} + C$$

Using the initial condition:

$$\frac{-\ln(|3 - 1|)}{3} = C$$

$$C = -0.231$$

$$\frac{-\ln(|3y - 1|)}{3} = \frac{t^3}{3} - 0.231$$

Solving for y (optional):

$$\ln(|3y - 1|) = -t^3 + 3 \cdot 0.231$$

$$y = \frac{e^{-t^3 + 0.693} + 1}{3}$$