Elasticity and revenue maximization

Consider the bus travel market, where the demand function is given by

$$P = 100 - \frac{Q}{10},$$

where Q is the number of trips per day and P is the price.

a) Compute the total expenditure if $P = 50$, and the price elasticity of demand.

b) In the previous situation, the bus company wants to increase its revenue; *how could it do so?*

c) Solve the above points for $P = 75$.

(a) Total Expenditure and Demand Elasticity for $P = 50$

- **Find the quantity demanded.** From the demand function:

$$P = 100 - \frac{Q}{10} \quad \Longrightarrow \quad \frac{Q}{10} = 100 - P \quad \Longrightarrow \quad Q = 10(100 - P).$$

For $P = 50$:

$$Q = 10\,(100 - 50) = 10 \times 50 = 500.$$

- **Total expenditure (or the firm's revenue).** Let total expenditure be $\text{TE} = P \times Q$. For $P = 50$:

$$\text{TE} = 50 \times 500 = \mathbf{25{,}000}.$$

- **Price elasticity of demand.** From $Q(P) = 1000 - 10P$, we differentiate with respect to P:

$$\frac{dQ}{dP} = -10.$$

The elasticity of demand at the point (Q, P) is defined as:

$$E = \frac{dQ}{dP}\,\frac{P}{Q}.$$

For $P = 50, Q = 500$:

$$E = (-10)\,\frac{50}{500} = -10 \times \frac{1}{10} = \mathbf{-1}.$$

In absolute value, demand is *unit elastic* at this point.

(b) How Could the Firm Increase Its Revenue?

At $P = 50$, the elasticity is -1, which means the demand is at the *unit elasticity* level. For a linear demand, this point typically maximizes total revenue. Therefore:

$$\text{TE} = P \times Q$$

reaches its maximum at $P = 50$.

(c) Results for $P = 75$

- **Quantity demanded.** For $P = 75$:

$$Q = 10(100 - 75) = 10 \times 25 = 250.$$

- **Total expenditure.**

$$\text{TE} = 75 \times 250 = \mathbf{18{,}750}.$$

- **Price elasticity of demand.** Again, with $\frac{dQ}{dP} = -10$:

$$E = \frac{dQ}{dP}\,\frac{P}{Q} = (-10)\,\frac{75}{250} = -10 \times \frac{3}{10} = \mathbf{-3}.$$

In absolute value, demand is *elastic* (greater than 1).

If the firm wants to *increase* revenue starting from $P = 75$, because demand is elastic ($|E| = 3 > 1$), lowering the price would proportionally increase the quantity demanded, thus raising total expenditure (or revenue).