Aggregate demand and elasticity

Consider a market with two consumers whose demand functions are given by:

$$X_1(p) = \begin{cases} 0, & \text{if } p > 4, \\ 16 - 4p, & \text{if } p \leq 4 \end{cases}$$

$$X_2(p) = \begin{cases} 0, & \text{if } p > 10, \\ 20 - 2p, & \text{if } p \leq 10 \end{cases}$$

a) What is the market aggregate demand function?

b) When $p = 2$, what is the price elasticity for each individual and for the market?

Solution

(a) Market Aggregate Demand

The market demand is obtained by adding the individual demands:

$$Q(p) = X_1(p) + X_2(p).$$

We analyze the intervals according to the price p:

- **$p > 10$**:

  $$X_1(p) = 0 \quad (\text{since } p > 4), \quad X_2(p) = 0 \quad (\text{since } p > 10).$$

  Therefore,

  $$Q(p) = 0 \quad \text{for } p > 10.$$

- **$4 < p \leq 10$**:

  $$X_1(p) = 0 \quad (\text{since } p > 4), \quad X_2(p) = 20 - 2p \quad (\text{since } p \leq 10).$$

  Therefore,

  $$Q(p) = 0 + (20 - 2p) = 20 - 2p \quad \text{for } 4 < p \leq 10.$$

- **$p \leq 4$**:

  $$X_1(p) = 16 - 4p, \quad X_2(p) = 20 - 2p.$$

  Therefore,

  $$Q(p) = (16 - 4p) + (20 - 2p) = 36 - 6p \quad \text{for } p \leq 4.$$

  Hence, the aggregate demand is:

$$Q(p) = \begin{cases} 0, & \text{if } p > 10, \\ 20 - 2p, & \text{if } 4 < p \leq 10, \\ 36 - 6p, & \text{if } p \leq 4. \end{cases}$$

(b) Price Elasticity of Demand when $p = 2$

The *point price elasticity* of a demand function $X(p)$ is defined as:

$$E = \frac{p}{X(p)} \frac{dX(p)}{dp}.$$

1. Elasticity of X_1 at $p = 2$

For $p \leq 4$, we have $X_1(p) = 16 - 4p$.

$$\frac{dX_1}{dp} = -4, \quad X_1(2) = 16 - 4 \cdot 2 = 8.$$

Hence,

$$E_1 = \frac{p}{X_1(p)} \cdot \frac{dX_1}{dp} = \frac{2}{8} \cdot (-4) = -1.$$

This means that when the price increases by 1%, the demand falls by 1%, given that previously we were at the point $Q = 8, p = 2$.

2. Elasticity of X_2 at $p = 2$

For $p \leq 10$, we have $X_2(p) = 20 - 2p$.

$$\frac{dX_2}{dp} = -2, \quad X_2(2) = 20 - 2 \cdot 2 = 16.$$

Hence,

$$E_2 = \frac{p}{X_2(p)} \cdot \frac{dX_2}{dp} = \frac{2}{16} \cdot (-2) = -\frac{1}{4}.$$

This means that when the price increases by 1%, the demand falls by 0.25%, given that previously we were at the point $Q = 16, p = 2$.

3. Elasticity of the market demand $Q(p)$ at $p = 2$

For $p \leq 4$, the market demand is $Q(p) = 36 - 6p$.

$$\frac{dQ}{dp} = -6, \quad Q(2) = 36 - 6 \cdot 2 = 24.$$

Hence,

$$E_Q = \frac{p}{Q(p)} \cdot \frac{dQ}{dp} = \frac{2}{24} \cdot (-6) = -\frac{1}{2}.$$

This means that when the price increases by 1%, the market demand falls by 0.5%, given that previously we were at the point $Q = 24, p = 2$.