

Parabolic Limit

Solve the following limit using the parabolic limit method.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6} \quad (1)$$

Solution

Approaching the function through a parabola

$$y = m(x - x_0)^2 + y_0 \quad (2)$$

In this case

$$x_0 = 0, \quad y_0 = 0 \quad (3)$$

Therefore

$$y = m(x)^2 \quad (4)$$

Replacing in our function

$$\lim_{x \rightarrow 0} \frac{x(mx)^3}{x^2 + (mx)^6} \quad (5)$$

Operating, we obtain

$$\lim_{x \rightarrow 0} \frac{x^4 m^3}{x^2(1 + x^4 m^6)} = 0 \quad (6)$$

Then, after simplifying

$$\lim_{x \rightarrow 0} \frac{m^3 x^2}{1 + x^4 m^6} = 0 \quad (7)$$

Since the numerator tends to 0 and the denominator tends to 1